Which hadrons are "easy", "difficult" or "too challenging" to study reigorously with lattice QCD (currently)

very briefly ...

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topical Snowmass workshop

Heavy-Quark Exotic Hadrons (online)

hadron spectroscopy with lattice QCD

only strong strong decays (no electro-weak decays)

only strong decay threshods matter

resonances:

- not QCD eigenstates
- inferred from decay products (like in exp)

$$H_1 H_2 \rightarrow R \rightarrow H_1 H_2$$

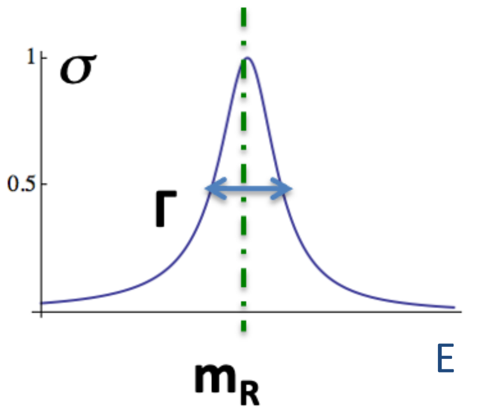
scattering amplitudes T

$\downarrow$

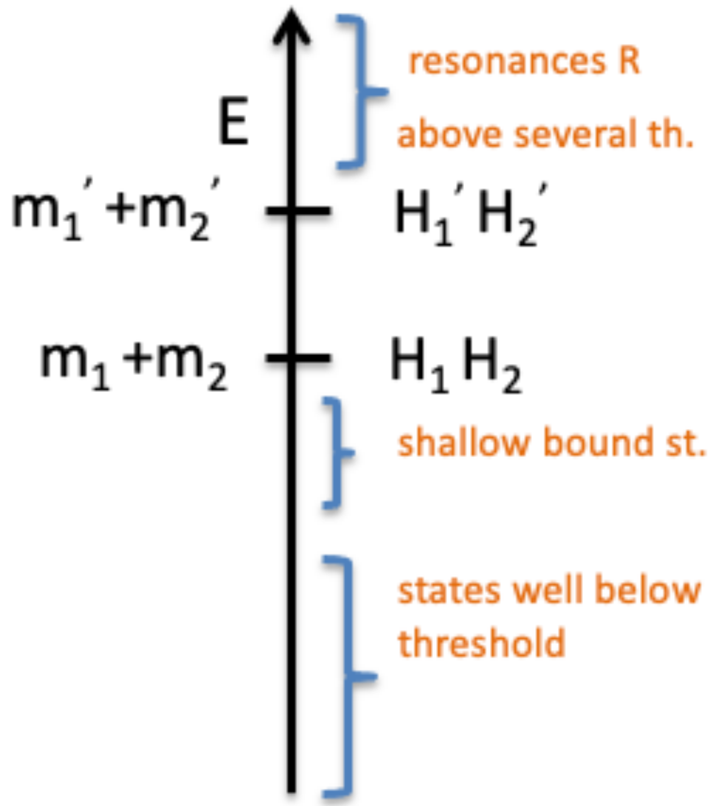
cross sections, poles of T

$\downarrow$

masses and widths of R



$$R \rightarrow H_1 H_2 \,,\, H_1' H_2'$$



hadron spectroscopy with lattice QCD

$R \rightarrow H_1 H_2 , H_1' H_2'$

"easy": below or near the lowest threshold

(still below the next threshold)

"difficult": - high above one threshold

- above two or three thresholds:

T for coupled-channel scattering

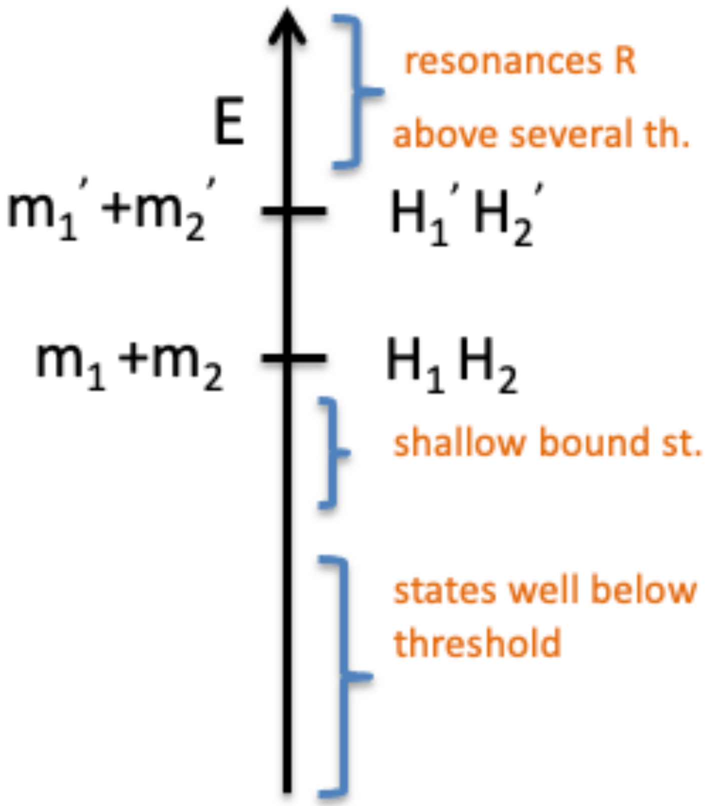
"too chalenging" (for now):

- above more than three thresholds

(if none of them can be "neglected")

- if $H_1 H_2 H_3$ decays are important

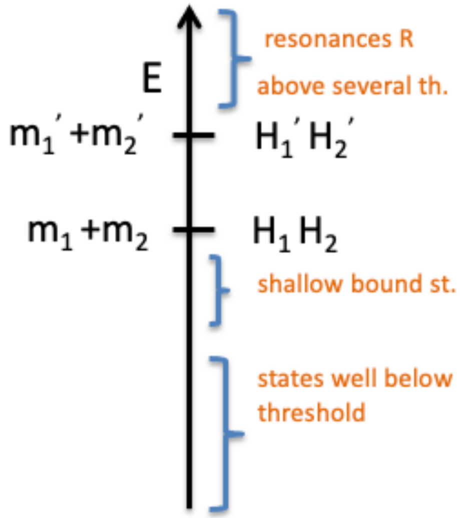
in addition to two-hadron decays



in general:

- the higher the hadron lies with respect to threshold: the more difficult

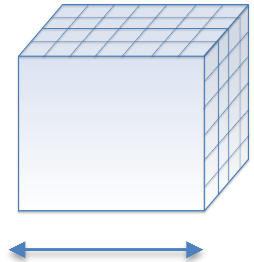
- states with two heavy quarks are more challenging if they lie high above threshold (compared to states composed only of u,d,s)



Some arguments

lattice QCD extracts energies E_n of eigenstates

$R \to H_1 H_2$

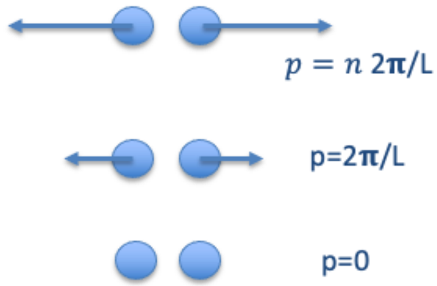


L , periodic b.c.

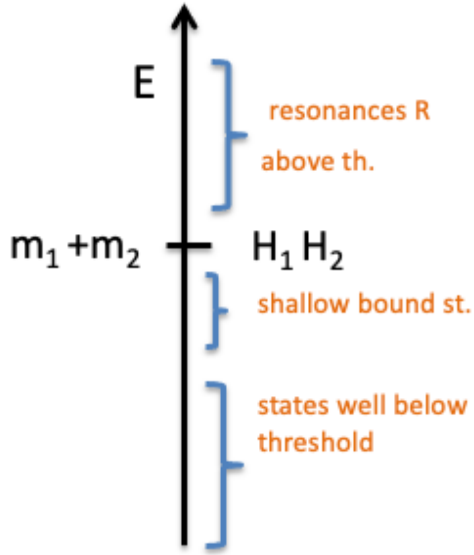
$E^{n.i.}$ (in non-interaction limit)

$H_1 H_2$ (cm frame)

$$E^{n.i.} = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}$$



$E=m_1+m_2$

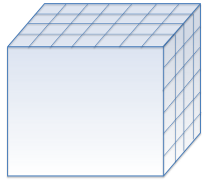


one has to extract ALL eigen-energies below the energy of interest

(can not focus on just particular higher-lying energy window like in exp)

exp (L=inf): continous spectrum above th., lat (L=finite): discrete spectrum plot of non-interacting energies

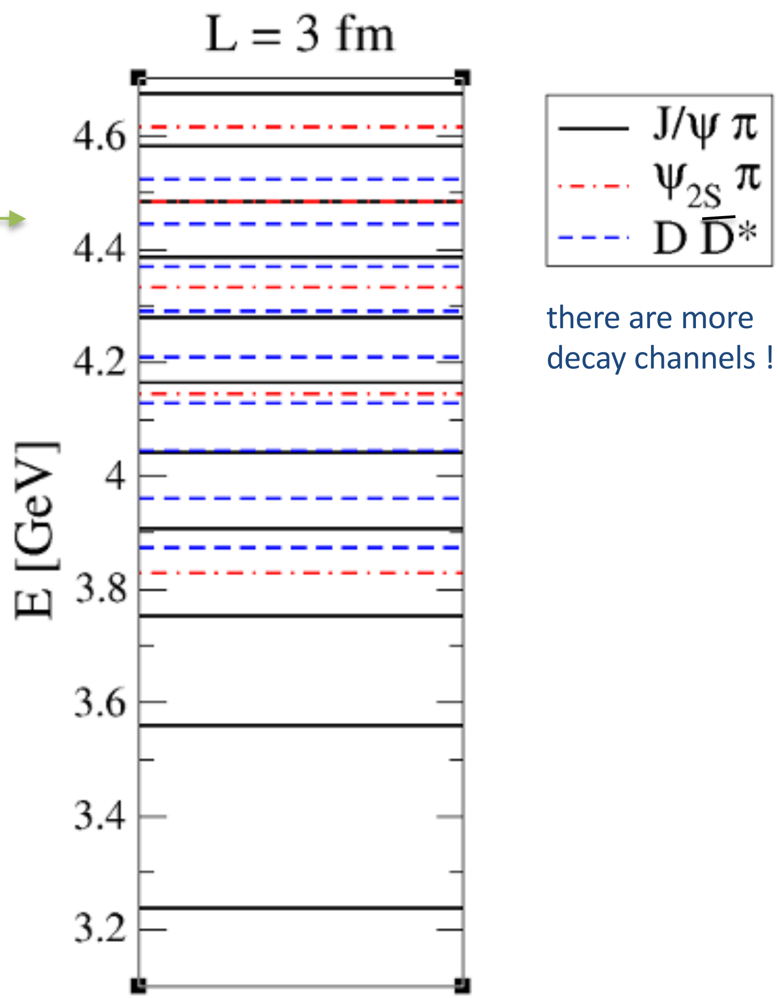
Example: channel Z_c , $J^P=1^+$



$$E^{n.i.} = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2} \quad , \; p = n\, 2\boldsymbol{\pi}/L$$

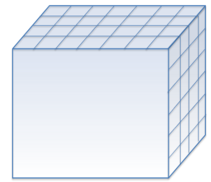
$Z_c^+(4430)$

$Z_c^+(3900)$



there are more decay channels !

Example: channel Z_c , $J^P=1^+$



plot of non-interacting energies

$$E^{n.i.} = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2} \quad , \; p = n\, 2\pi/L$$

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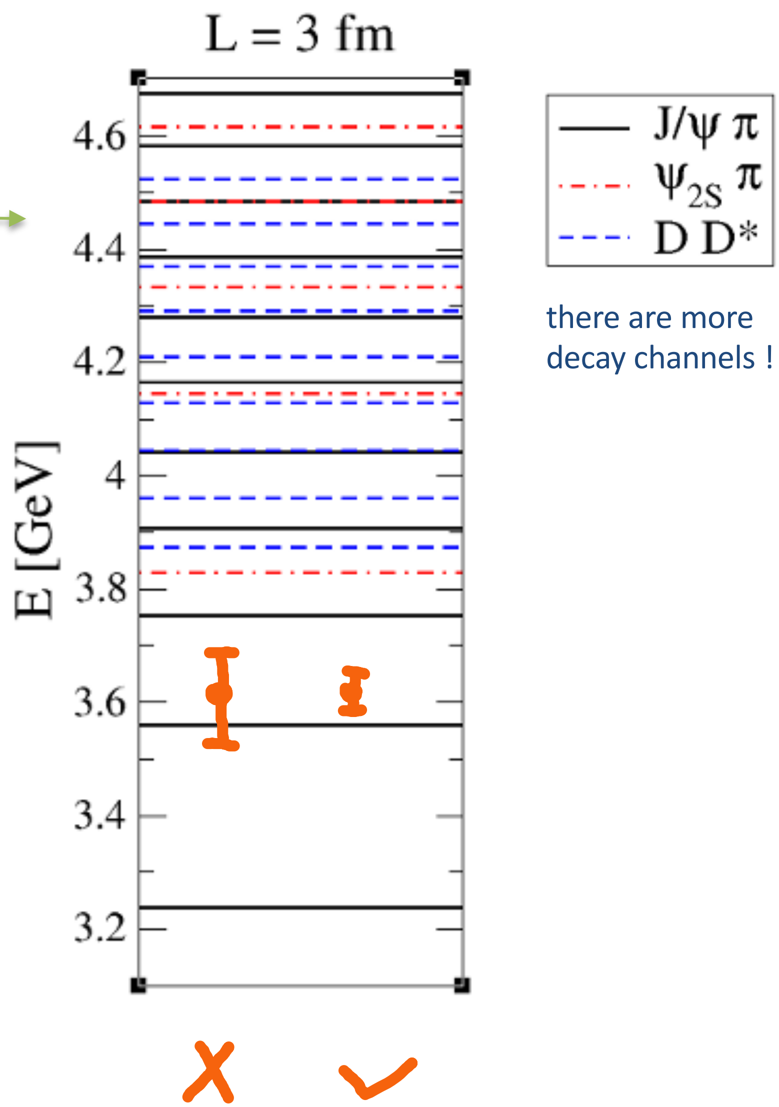


there are more decay channels !

challenge:

very accurate determination of E is needed

higher E have larger statistical noise

One needs to resolve

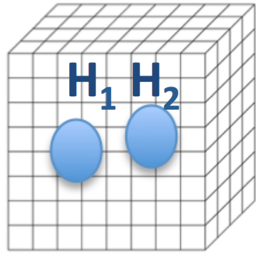
$$\Delta E = \mathrm{E} - E^{n.i.}$$

Scattering matrix T can be determined only when ΔE are resolved

Extracting scattering matrix from lattice (generalized Luscher's method)

Resonance above one threshold

$$R \to H_1 H_2 \qquad T(E) \underset{\text{Luscher's method}}{\leftarrow} E_n$$



Lattice simulation of one-channel scattering via Luscher's method: doable

Resonance above two or more thresholds

most of exotic hadrons are above more than one threshold:
for example Zc(4430), X(6900), Zb

$$R \to H_1 H_2,\ H_1' H_2'$$

channel $a:\quad H_1 H_2$

channel $b:\quad H_1' H_2'$

$$T(E) = \begin{array}{cc} \text{a -> a} & \text{a -> b} \\ \multicolumn{2}{c}{\begin{bmatrix} T_{aa}(E) & T_{ab}(E) \\ T_{ab}(E) & T_{bb}(E) \end{bmatrix}} \\ \text{b->a} & \text{b -> b} \end{array} \quad \underset{\text{Luscher's method}}{\leftarrow} E_n$$

det[T(E) – f(E)] =0 : at given E: one equation, three unknowns

known f

Conclusion

many interesting exotic hadrons from experiment: (too) challenging for rigorous lattice QCD

some exotic hadrons from lattice (for example bb$\underline{ud}$ tetraquark below BB*): (too) difficult for exp

Identify some hadronic states that can be reliably studied with both lattice and exp !