Phenomenology of light exotic hadrons with functional methods

Gernot Eichmann
LIP Lisboa

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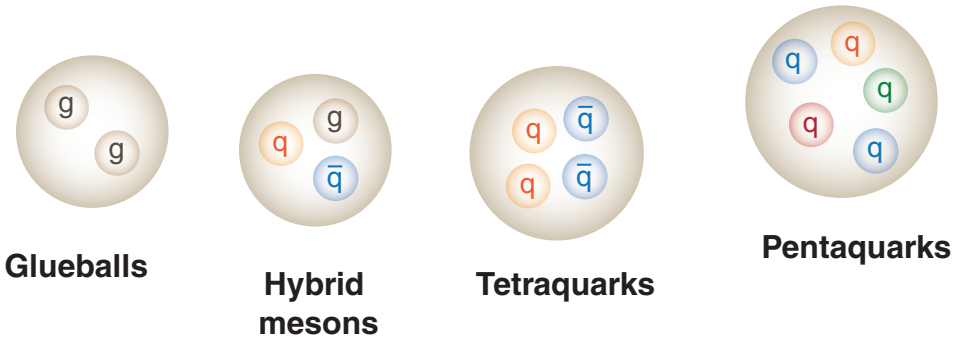
Light exotic hadrons



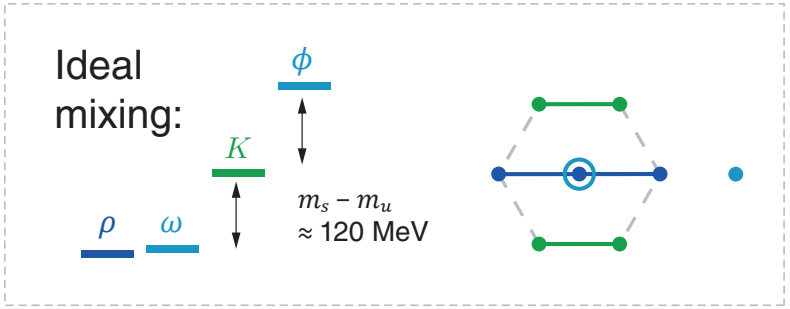
What does "exotic" mean for light hadrons?

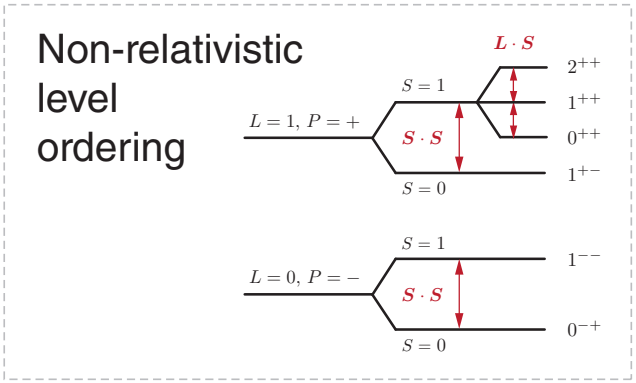
- Exotic quantum numbers: $1^{-+}, \ldots$
  → **Hybrids?** $\pi_1(1400)$, $\pi_1(1600)$
  Meyer, Swanson, Prog. Part. Nucl. Phys. 82 (2015), 1502.07276 [hep-ph]

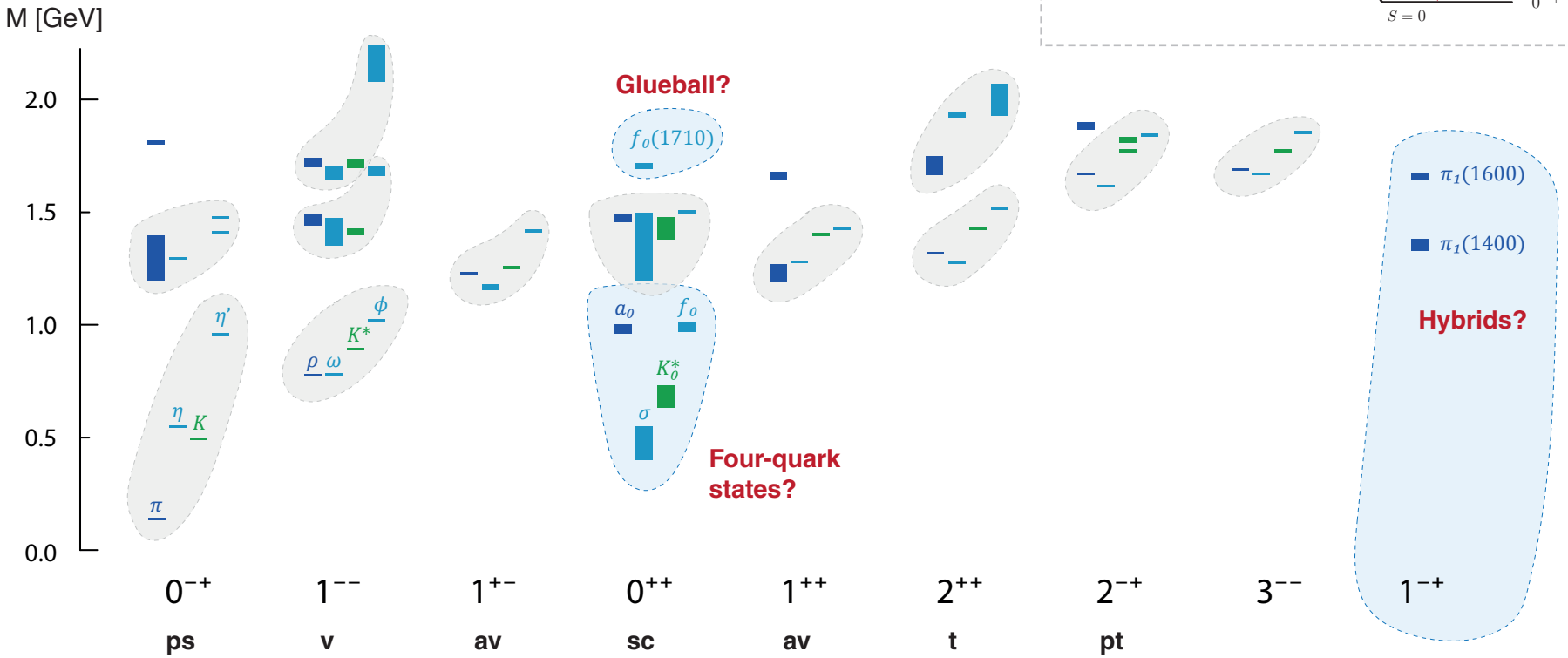
- Overpopulation of multiplets, unconventional properties
  → **Glueballs?** $f_0(1??0)$
  Ochs, J. Phys. G 40 (2013), 1301.5183 [hep-ph]
  → **Four-quark states?** σ, κ, a_0, f_0, ...
  Pelaez, Phys. Rept. 658 (2016), 1510.00653 [hep-ph]

Light exotic mesons

Light meson spectrum
(PDG 2020)

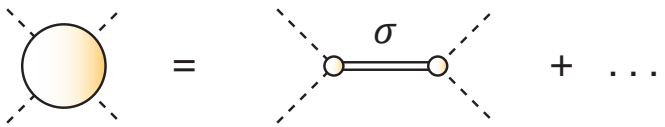


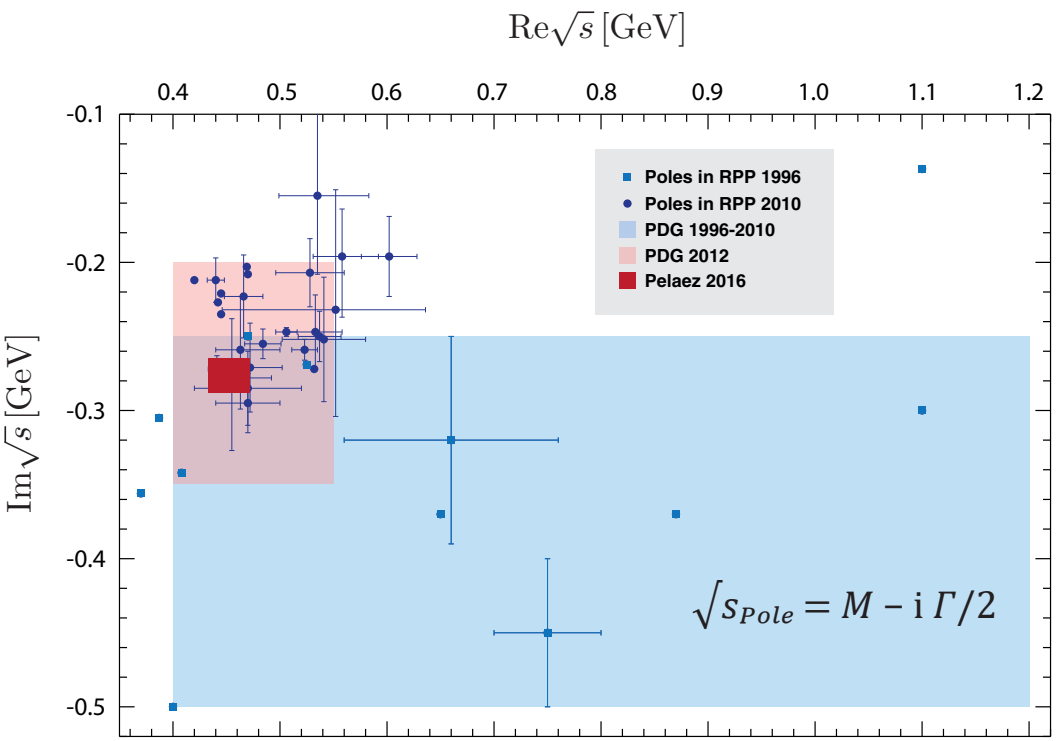




Light scalar mesons

$\sigma/f_0(500)$ is a **resonance** in $\pi\pi$ scattering:





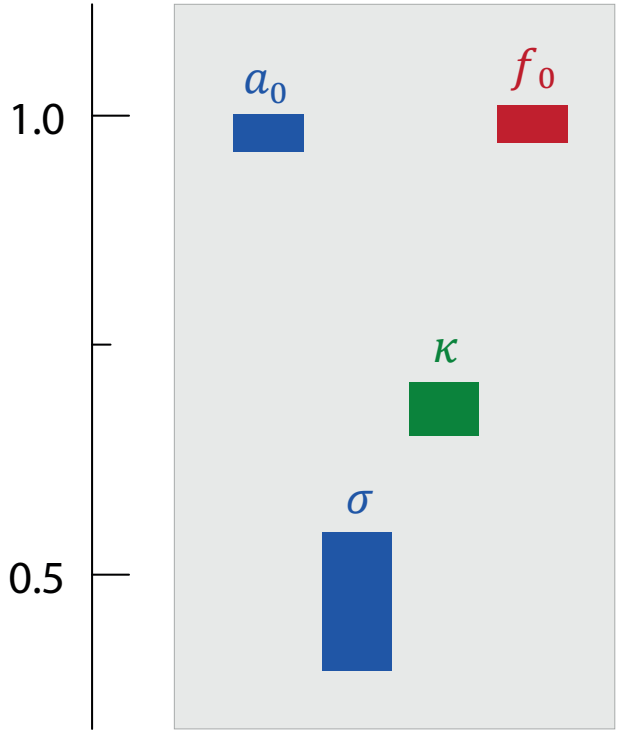
Pelaez, Phys. Rept. 658 (2016) 1

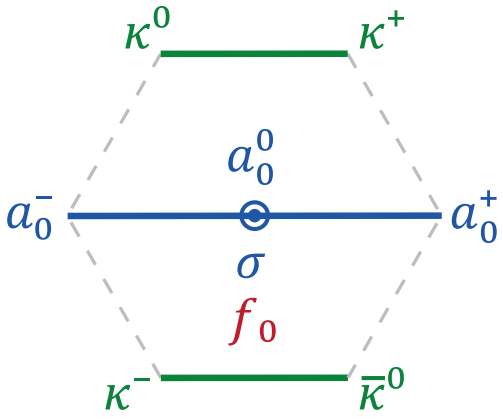
- **PDG 2010:** "$f_0(600)$"
  $\sqrt{s} \sim$ (400...1200) − i (250...500) MeV
- **Dispersive analyses:**
  $\sqrt{s} \sim$ 450(20) − i 275(10) MeV

  Caprini, Colangelo, Leutwyler 2006
  Garcia-Martin, Kaminski, Pelaez, Ruiz de Elvira 2011
  Moussallam 2011
  Masjuan, Ruiz de Elvira, Sanz-Cillero 2014
  Pelaez 2016
- **PDG 2012:** "$f_0(500)$"
  $\sqrt{s} \sim$ (400...550) − i (200...350) MeV
- Pole locations from **lattice QCD**
  Briceno, Dudek, Edwards, Wilson, PRL 118 (2017), PRD 97 (2018)

Light scalar mesons

Light scalar (0^{++}) **mesons** don't fit into the conventional meson spectrum:





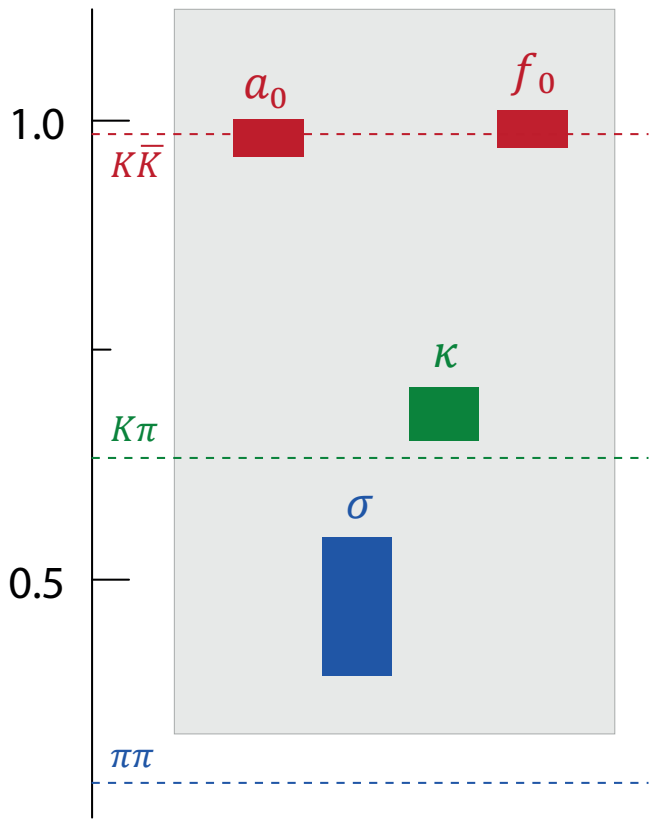
f_0	(980 MeV)	$s\bar{s}$
κ	(680 MeV)	$u\bar{s}, d\bar{s}$
a_0	(980 MeV)	$u\bar{u}, d\bar{d}, u\bar{d}$
σ	(500 MeV)	

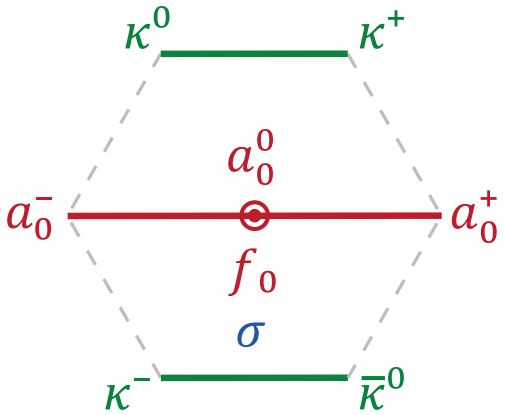
- Why are a_0, f_0 mass-degenerate?
- Why are their **decay widths** so different?

  $\Gamma(\sigma, \kappa) \approx 550$ MeV
  $\Gamma(a_0, f_0) \approx 50$–$100$ MeV
- Why are they so **light**?
  Scalar mesons ~ **p-waves,** should have
  masses similar to axialvector & tensor mesons ~ 1.3 GeV

Light scalar mesons

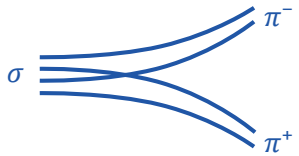
What if they were **tetraquarks** (diquark-antidiquark)? Jaffe 1977, Close, Tornqvist 2002, Maiani, Polosa, Riquer 2004





f_0 (980 MeV) } $u s \bar{u} \bar{s}$, ...
a_0 (980 MeV) }
κ (800 MeV) $u s \bar{u} \bar{d}$, ...
σ (500 MeV) $u d \bar{u} \bar{d}$

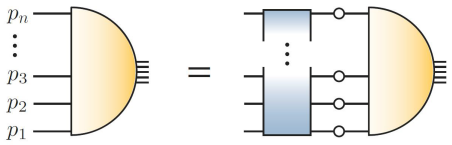
- Explains **mass ordering** & **decay widths**: f_0 and a_0 couple to K$\bar{\text{K}}$, large widths for σ, κ
- Alternative: **meson molecules?** Weinstein, Isgur 1982, 1990; Close, Isgur, Kumano 1993



- **Non-q$\bar{\text{q}}$ nature** of σ supported by dispersive analyses, unitarized ChPT, large Nc, extended linear σ model, quark models Pelaez, Phys. Rept. 658 (2016)

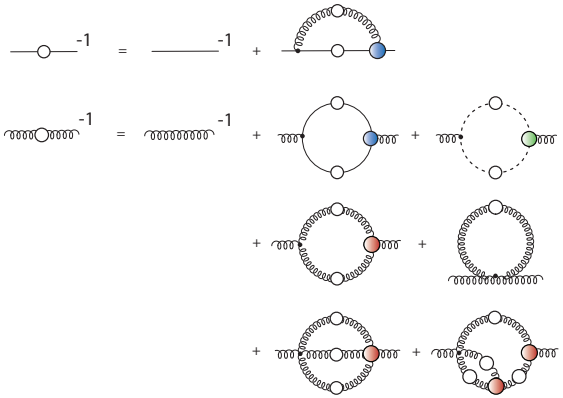
Functional methods

- Hadronic **bound-state equations** (BSEs, Faddeev eqs, ...)

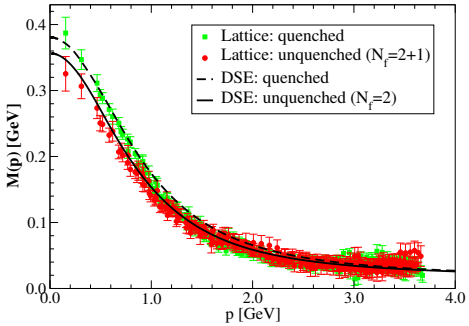


"QFT analogue of Schrödinger eq."
→ hadron masses & "wave functions"
→ **spectroscopy calculations**

- Ingredients: **QCD's n-point functions,** Satisfy Dyson-Schwinger equations (**DSEs**): QCD's quantum eqs. of motion

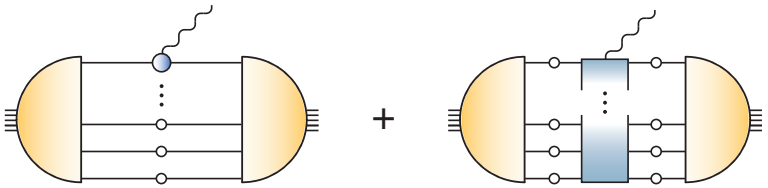


...



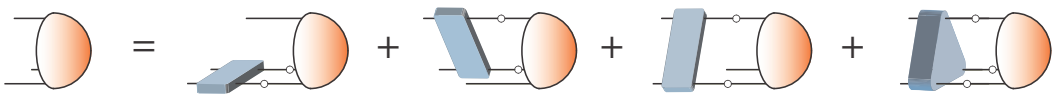
→ running **quark mass**

- Structure calculations: form factors, PDFs, GPDs, two-photon processes, ...

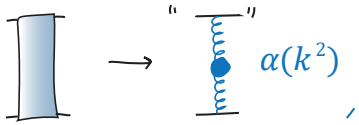


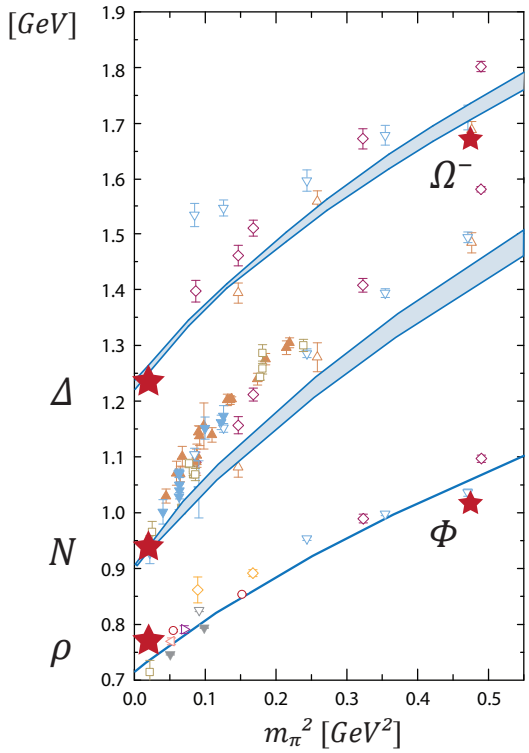
Baryons

- Covariant **3-quark Faddeev equation** for baryons
  GE, Alkofer, Nicmorus, Krassnigg, PRL 104 (2010)



so far mostly rainbow-ladder

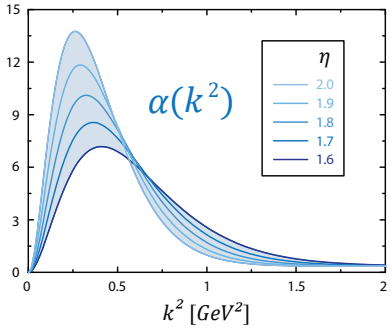




- Octet and decuplet spectra
  Sanchis-Alepuz, Fischer, PRD 90 (2014)
- Elastic and transition form factors
  GE, PRD 84 (2011), Sanchis-Alepuz, Fischer, EPJA 52 (2016)
- Heavy baryons
  Qin, Roberts, Schmidt, PRD 97 (2018), FBS 60 (2019)
- Similar results for **quark-diquark,** describes excitation spectrum well
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

Diquark clustering in baryons?
Barabanov et al., 2008.07630 [hep-ph]

Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog. Part. Nucl. Phys. 91 (2016), 1606.09602

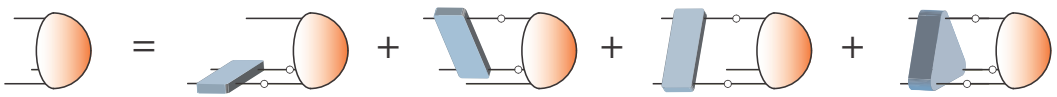


Maris, Tandy, PRC 60 (1999), Qin et al., PRC 84 (2011)

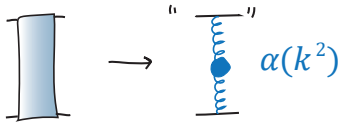
Scale set by f_π, shape parameter → bands

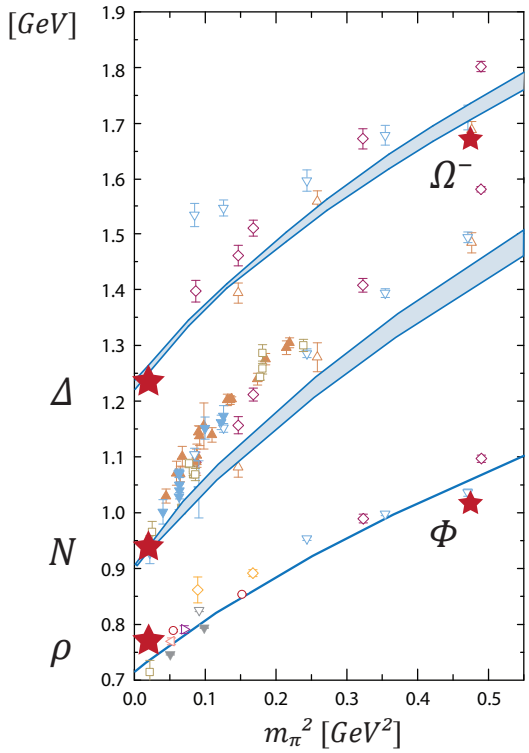
Baryons

- Covariant **3-quark Faddeev equation** for baryons
  GE, Alkofer, Nicmorus, Krassnigg, PRL 104 (2010)

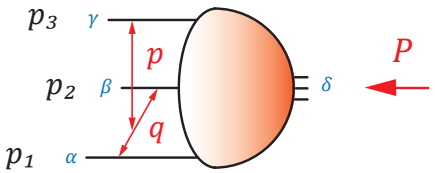


so far mostly rainbow-ladder





Keep full structure of baryon's Faddeev amplitude:



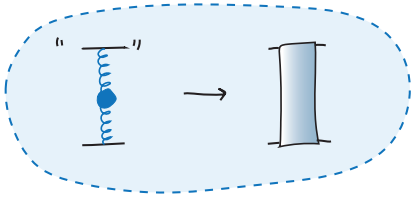
$$\Psi_{\alpha\beta\gamma\delta}(p,q,P) = \sum_i f_i(p^2, q^2, p\cdot q, p\cdot P, q\cdot P)\ \tau_i(p,q,P)_{\alpha\beta\gamma\delta}$$

Lorentz-invariant dressing functions

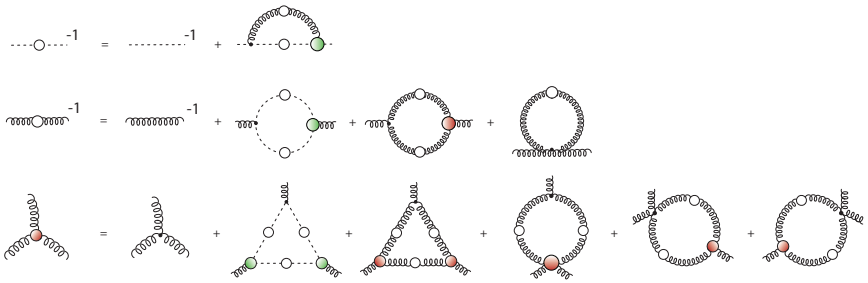
Dirac-Lorentz tensors: 64 (128) for spin 1/2 (3/2)

Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, Prog. Part. Nucl. Phys. 91 (2016), 1606.09602

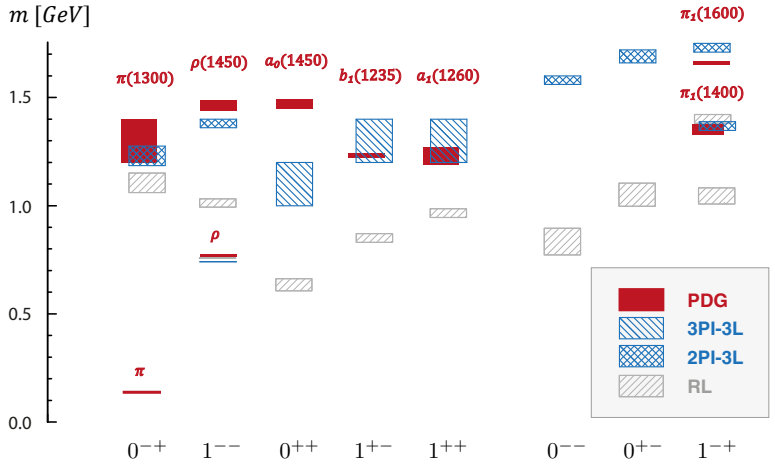
Towards ab-initio

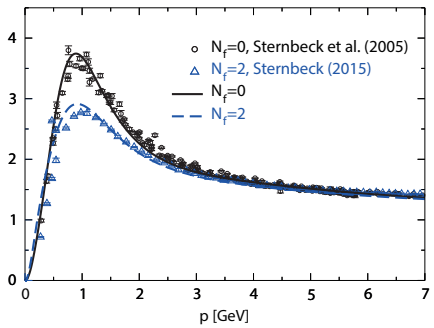


- Include higher n-point functions (solve DSEs)
  GE, Williams, Alkofer, Vujinovic, PRD 89 (2014),
  Williams, EPJA 51 (2015), Huber, PRD 101 (2020)



- Beyond rainbow-ladder calculations improve **light-meson spectrum**
  Williams, Fischer, Heupel, PRD 93 (2016)





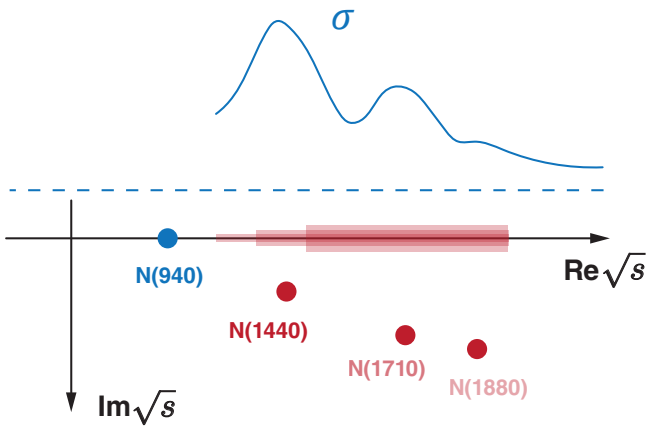
Gluon propagator:
DSE vs. lattice

Williams, Fischer, Heupel,
PRD 93 (2016)

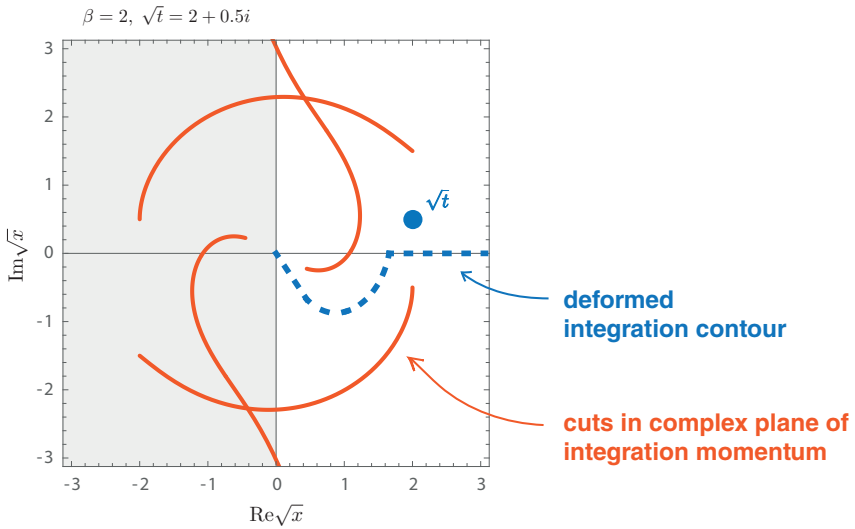
- Fruitful interplay between **DSEs, FRG, lattice**
  Cyrol, Mitter, Pawlowski, Strodthoff, PRD 97 (2018)
  Oliveira, Silva, Skullerud, Sternbeck, PRD 99 (2019)

Resonances

- Most hadrons are **resonances** and decay ⇔ poles in complex momentum plane



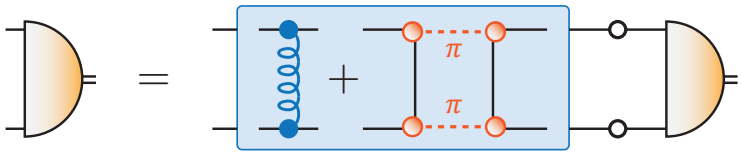
- **Contour deformations** as tool to go beyond thresholds



GE, Duarte, Peña, Stadler, PRD 100 (2019)

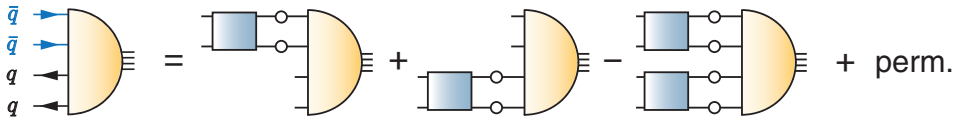
- BSE kernel must be aware of decay channels: ρ meson becomes resonance

Williams, PLB 798 (2019), Miramontes, Sanchis-Alepuz, EPJA 55 (2019)



Four-quark states

- Light scalar mesons (σ, κ, a_0, f_0) as **four-quark states:**
  GE, Fischer, Heupel, PLB 753 (2016)



$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2, q^2, k^2, \{\omega_j\}, \{\eta_j\}\right) \tau_i(p,q,k,P) \quad \otimes \quad \text{Color} \quad \otimes \quad \text{Flavor}$$

9 Lorentz invariants:

$$p^2, \quad q^2, \quad k^2, \quad P^2 = -M^2$$

$$\begin{array}{ll} \omega_1 = q\cdot k & \eta_1 = p\cdot P \\ \omega_2 = p\cdot k & \eta_2 = q\cdot P \\ \omega_3 = p\cdot q & \eta_3 = k\cdot P \end{array}$$

256 Dirac-Lorentz tensors

2 Color tensors:

$3\otimes\bar{3}, \quad 6\otimes\bar{6}$ or
$1\otimes 1, \quad 8\otimes 8$
(Fierz-equivalent)

- Group momentum variables into multiplets of **permutation group S4:** can switch off groups of variables without destroying symmetries
  GE, Fischer, Heupel, PRD 92 (2015)

$f_i\left(\mathcal{S}_0, \triangledown, \triangle, \bigcirc\right)$

Four-quark states

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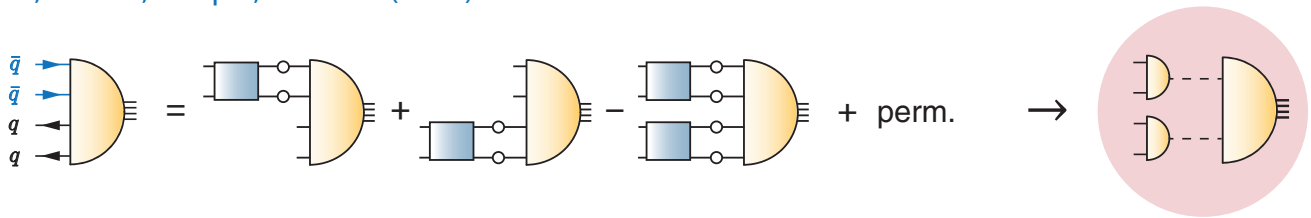
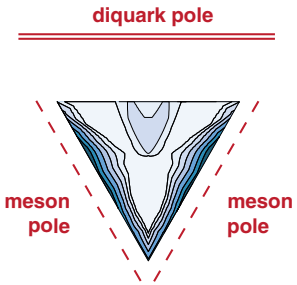
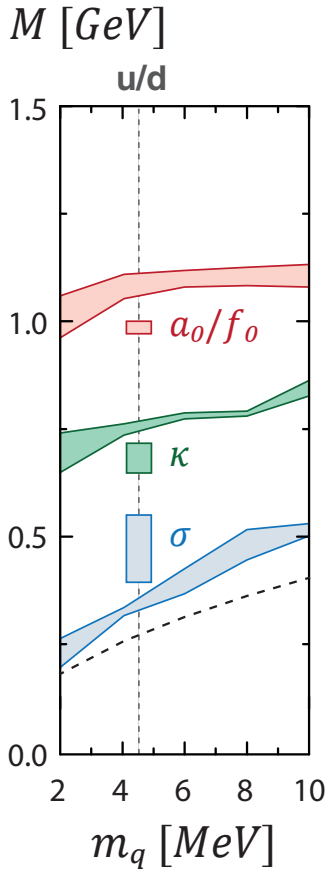
  GE, Fischer, Heupel, PLB 753 (2016)



- BSE dynamically generates **meson poles** in BS amplitude:

$f_i\,(\mathcal{S}_0, \triangledown, \triangle, \bigcirc)$ → 1500 MeV
$f_i\,(\mathcal{S}_0, \triangledown, \triangle, \bigcirc)$ → 1500 MeV
$f_i\,(\mathcal{S}_0, \triangledown, \triangle, \bigcirc)$ → 1200 MeV
$f_i\,(\mathcal{S}_0, \triangledown, \triangle, \bigcirc)$ → **350 MeV !**





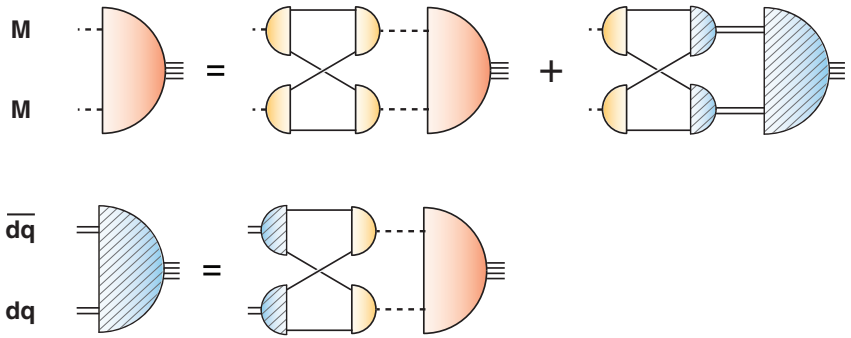
- "Light scalar mesons" look like **meson molecules,** diquark-antidiquark components almost negligible

- Lightness is inherited from pseudoscalar Goldstone bosons!

Four-quark states

Two-body formulation: **meson-meson / diquark-antidiquark,**
follows from four-quark eq. (analogue of quark-diquark for baryons)

Heupel, GE, Fischer, PLB 718 (2012)



- Interaction by **quark exchange**
- System 'wants' to be **meson-meson**-like (no diagonal dq-$\overline{dq}$ term)
- Similar results as in 4-quark approach: m_σ ~ 400 MeV, etc.

Four-quark states

Two-body formulation: **meson-meson / diquark-antidiquark,**
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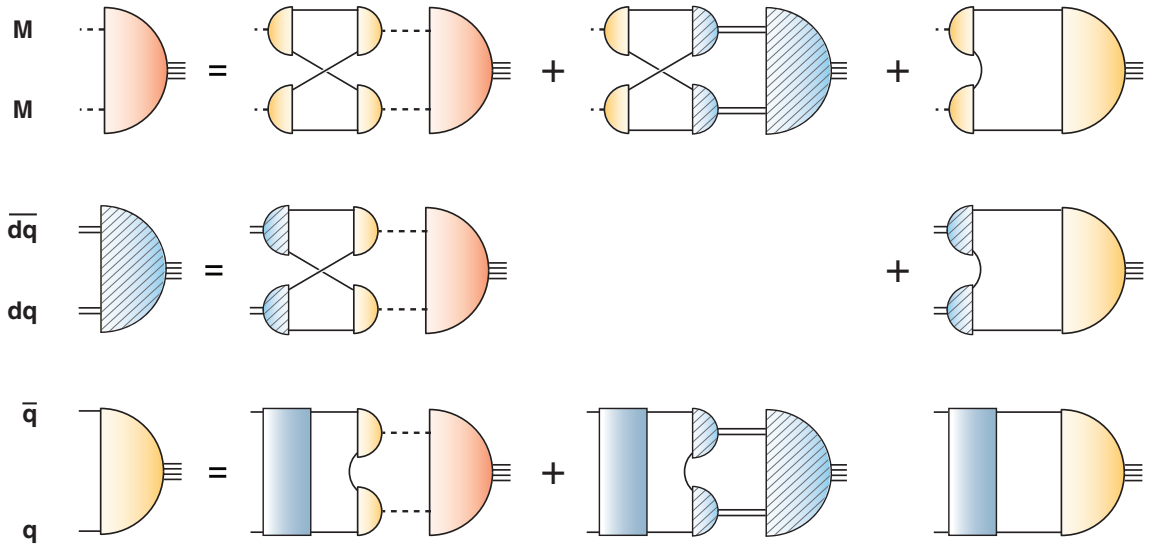
Heupel, GE, Fischer, PLB 718 (2012)



Include **mixing with $q\bar{q}$:**
$\pi\pi$ still dominant

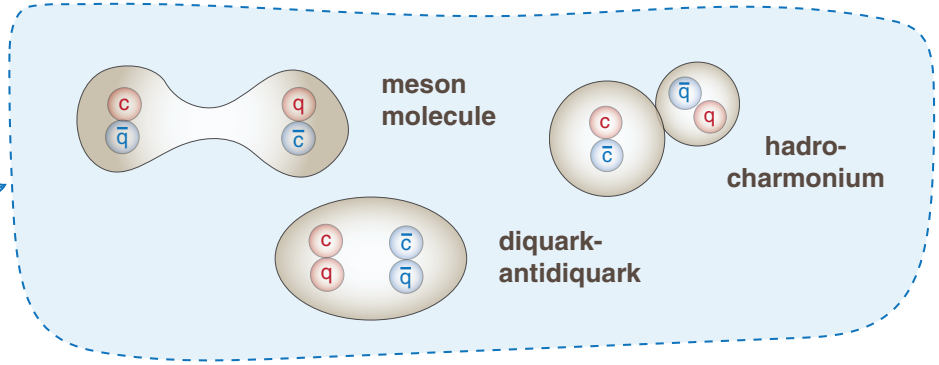
Santowsky, GE, Fischer, Wallbott, Williams, PRD 102 (2020)

[MeV]	ground state mass	first excitation
$\pi\pi$	416 ± 26	970 ± 130
$\pi\pi + 0^+0^+$	416 ± 26	970 ± 130
$q\bar{q}$	667 ± 2	1036 ± 8
$\pi\pi + q\bar{q}$	472 ± 22	1080 ± 280
$\pi\pi + 0^+0^+ + q\bar{q}$	456 ± 24	1110 ± 110

Four-quark states

- Heavy-light **four-quark states:** what is their internal decomposition?

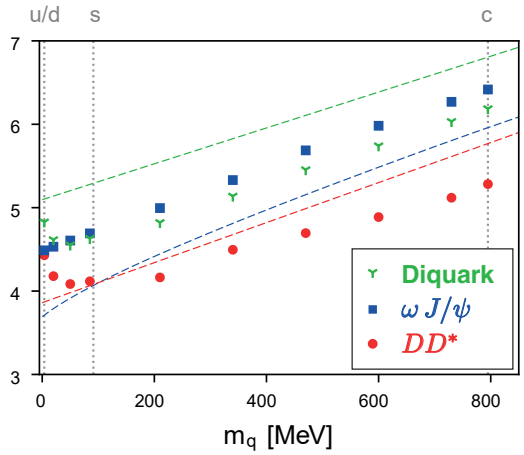
$cq\bar{q}\bar{c}$

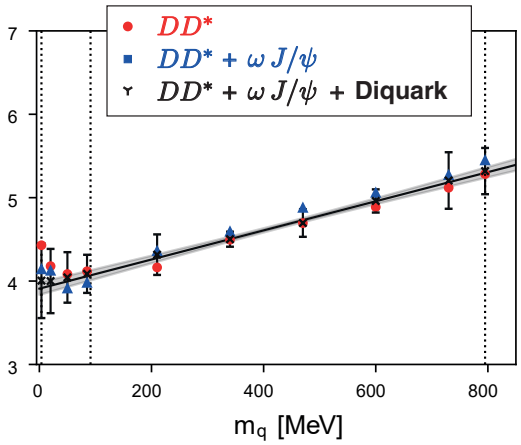


- **Four-quark BSE:** all mix together

$M_{cq\bar{q}\bar{c}}$ [GeV] in X(3872) channel

$m_S + m_A$
$m_\omega + m_{J/\psi}$
$m_D + m_{D^*}$





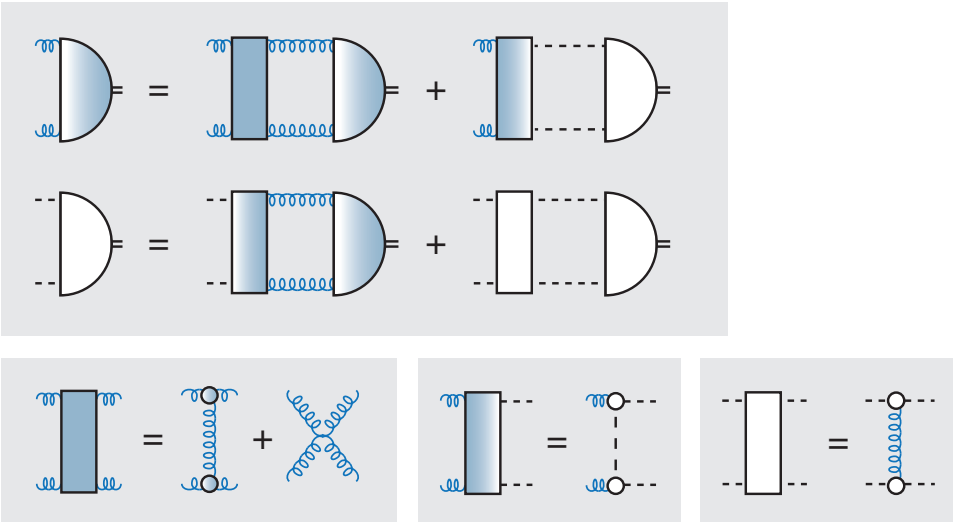
Wallbott, GE, Fischer, PRD 100 (2019), PRD 102 (2020)

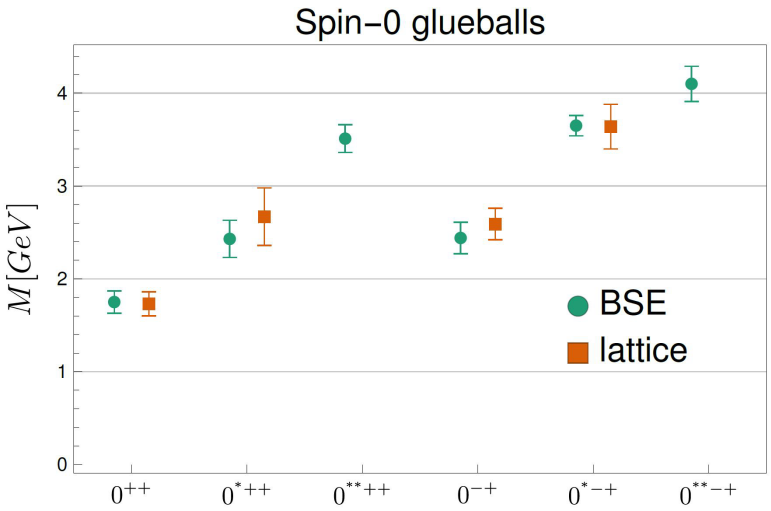
$cq\bar{q}\bar{c}$ → strong meson-meson component: DD* for X(3872), Zc(3900):

$cc\bar{q}\bar{q}$ → diquarks also play role

Glueballs

- Coupled **glueball–ghostball BSEs**
  Meyers, Swanson, PRD 87 (2013)
  Sanchis-Alepuz, Fischer, Kellermann, von Smekal, PRD 92 (2015)
  Souza, Ferreira, Aguilar, Papavassiliou, Roberts, Xu, EPJA 56 (2020)





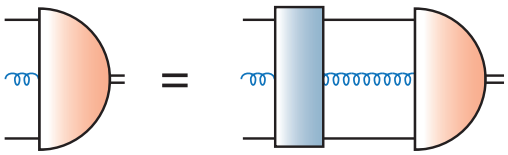
- Compute all ingredients consistently:
  no model input (only neglecting diagrams)
  Huber, Fischer, Sanchis-Alepuz, 2004.00415 [hep-ph]

Lattice:
Morningstar, Peardon, PRD 60 (1999)
New: Athenodorou, Teper, 2007.06422

- But: pure Yang-Mills theory (no quarks)

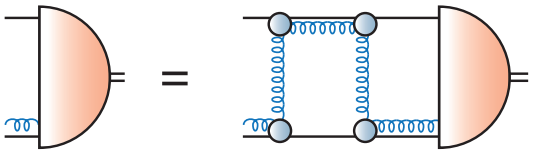
Hybrids

- **Three-body equation** (quark, antiquark, gluon)



... in progress

- **Two-body equation** [quark–gluon]–antiquark with model ansätze
  Xu, Cui, Chang, Papavassiliou, Roberts, EPJA 55 (2019)



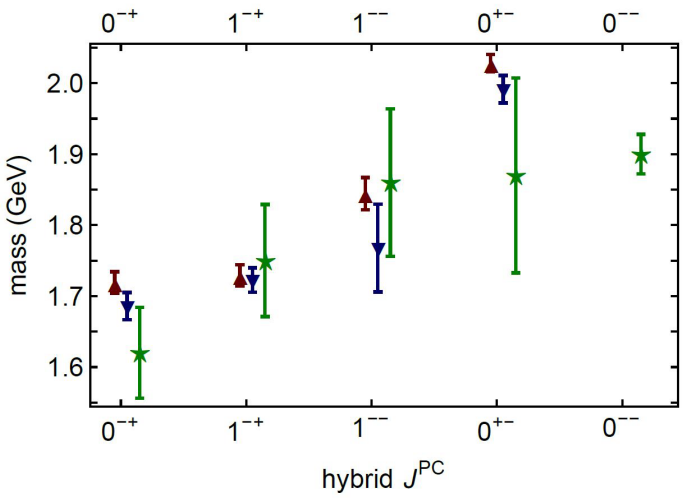


FIG. 2. Comparison between our ACM-improved spectrum (stars, green), Row 2 in Table I, and the rescaled lQCD results in Rows 3 (up-triangles, red) and 4 (down-triangles, blue).

Lattice: Dudek, Edwards, Peardon, Richards, Thomas, PRD 82 (2010)

The road ahead

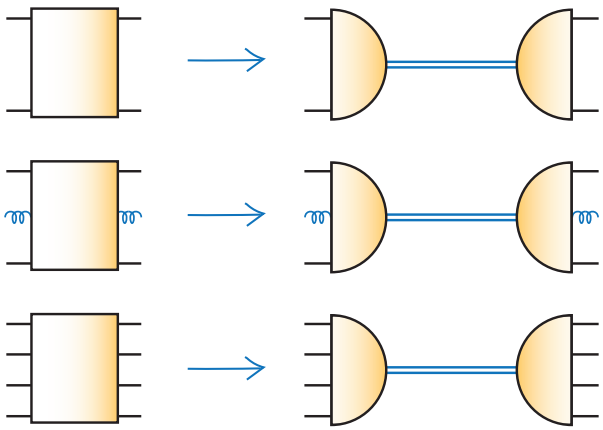
- **Four-quark states:**
  GE, Fischer, Heupel, Santowsky, Wallbott, 2008.10240
- Towards ab-initio calculations
  (include higher n-point functions)
- Resonance properties
- Hybrid mesons
- Exotic baryons ~ **pentaquarks:**
  Meson-baryon interactions, admixture of 5q (...) components?
- Exotic dibaryons ~ **hexaquarks**:
  from quarks and gluons to nuclei

Thank you!

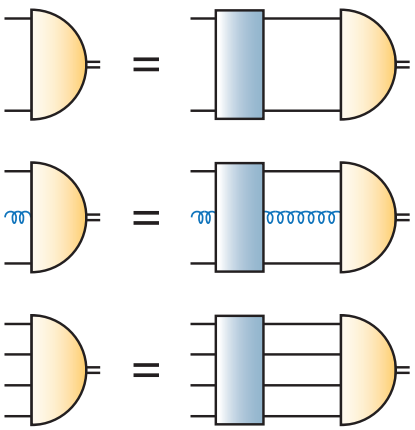
Backup slides

Some thoughts

- Same spectral representation for **all** correlation functions that produce $q\bar{q}$:



- **Without** truncations, BSEs for qq, $qq\bar{q}\bar{q}$, $q\bar{q}g$, ... should produce **same spectrum**



→ harder to include $qq\bar{q}\bar{q}$, $q\bar{q}g$

→ harder to include $q\bar{q}$ components

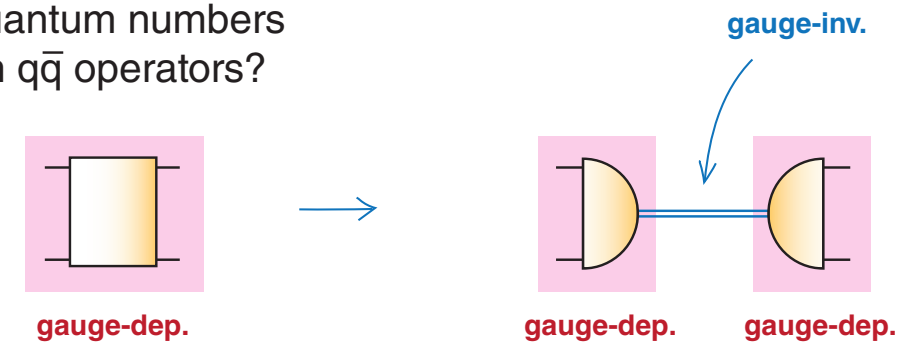
With truncations, $q\bar{q}$ ($qq\bar{q}\bar{q}$, $q\bar{q}g$, ...) BSE should give more reliable spectrum for $q\bar{q}$ ($qq\bar{q}\bar{q}$, $q\bar{q}g$, ...) dominated states

Exotic quantum numbers not excluded in principle (need gluon-rich kernel in $q\bar{q}$ BSE)

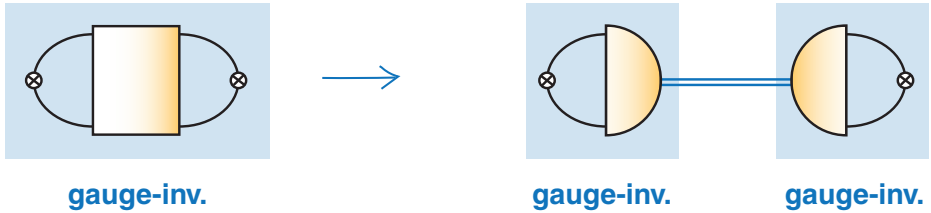
Some thoughts

- Why don't we see exotic quantum numbers in **lattice calculations** with $q\bar{q}$ operators?

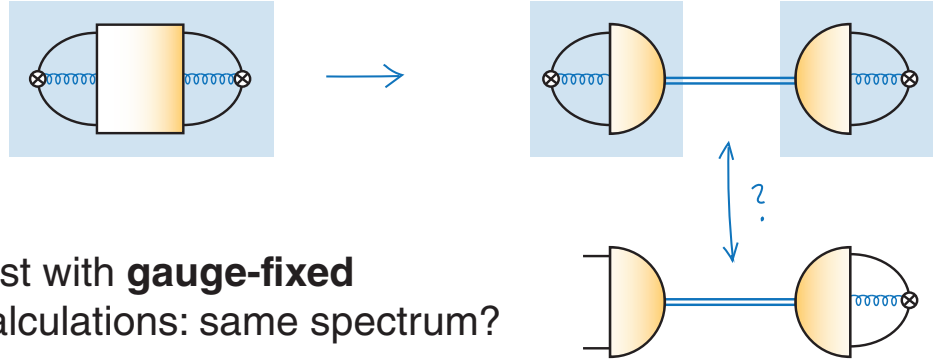
$\langle q_\alpha \, \bar{q}_\beta \, q_\gamma \, \bar{q}_\delta \rangle \;=$



$\langle (\bar{q}\,\Gamma q)(\bar{q}\,\Gamma q) \rangle \;=$



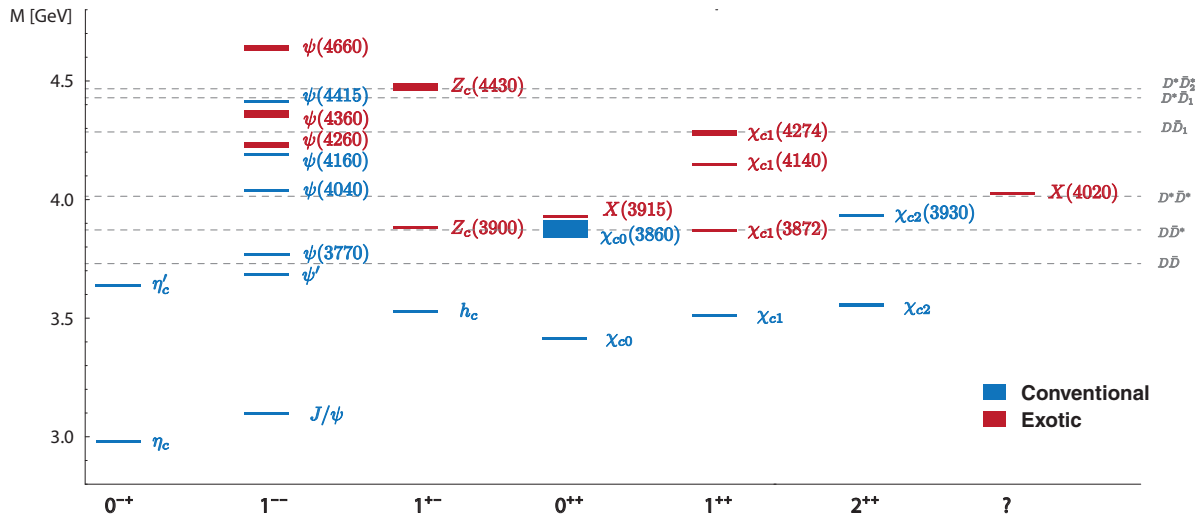
= 0 for exotic quantum nrs (?)



≠ 0 for exotic quantum nrs

Could test with **gauge-fixed** lattice calculations: same spectrum?

Exotic mesons



- Several tetraquark candidates in **charmonium spectrum**: X(3872), X(3915), Zc(3900), ...
- Z states cannot be $c\bar{c}$ since they carry charge
- Oldest tetraquark candidates: **light scalar mesons**

Reviews:

Chen, Chen, Liu, Zhu,
Phys. Rept. 639 (2016), 1601.02092

Lebed, Mitchell, Swanson
PPNP 93 (2017), 1610.04528

Esposito, Pilloni, Polosa,
Phys. Rept 668 (2017), 1611.07920

Guo, Hanhart, Meißner et al.,
Rev. Mod. Phys. 90 (2018), 1705.00141

Ali, Lange, Stone,
PPNP 97 (2017), 1706.00610

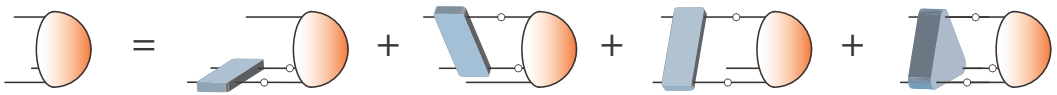
Olsen, Skwarnicki, Zieminska,
Rev. Mod. Phys. 90 (2019), 1708.04012

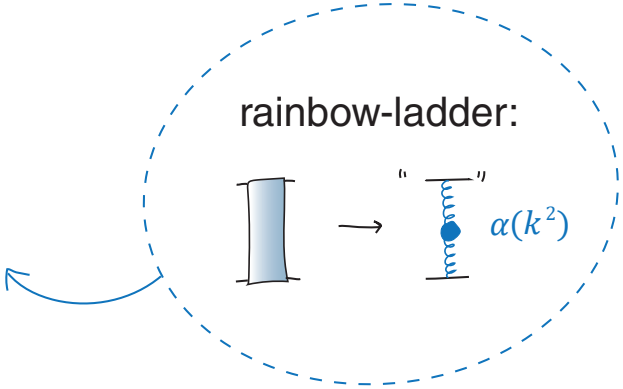
Liu, Chen, Chen, Liu, Zhu,
PPNP 107 (2019), 1903.11976

Brambilla, Eidelman, Hanhart et al.,
1907.07583

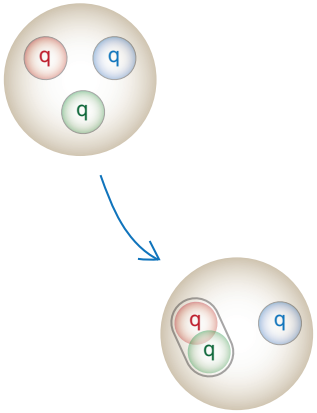
Baryon spectroscopy

- Solution of nucleon's covariant **3-body Faddeev equation**
  GE, Alkofer, Nicmorus, Krassnigg, PRL 104 (2010)





- Simpler: **quark-diquark Faddeev equation**
  Oettel et al., PRC 58 (1998), Cloet et al., FBS 46 (2009), Chen et al., PRD 97 (2018)

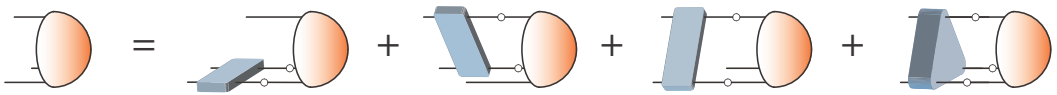


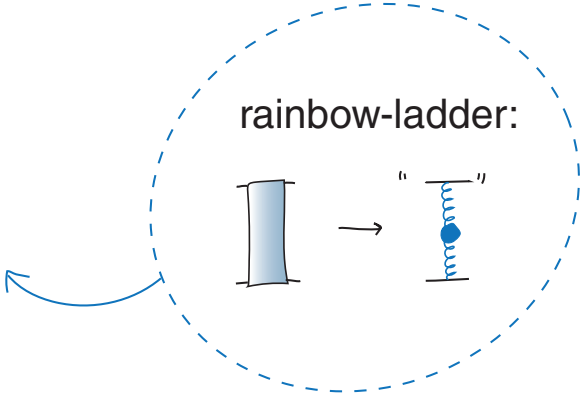
Ping-pong exchange
between quark and diquark

Calculate all ingredients in rainbow-ladder:
can compare **quark-diquark** and **3-body** directly
GE, Krassnigg, Schwinzerl, Alkofer, Ann. Phys. 323 (2008)

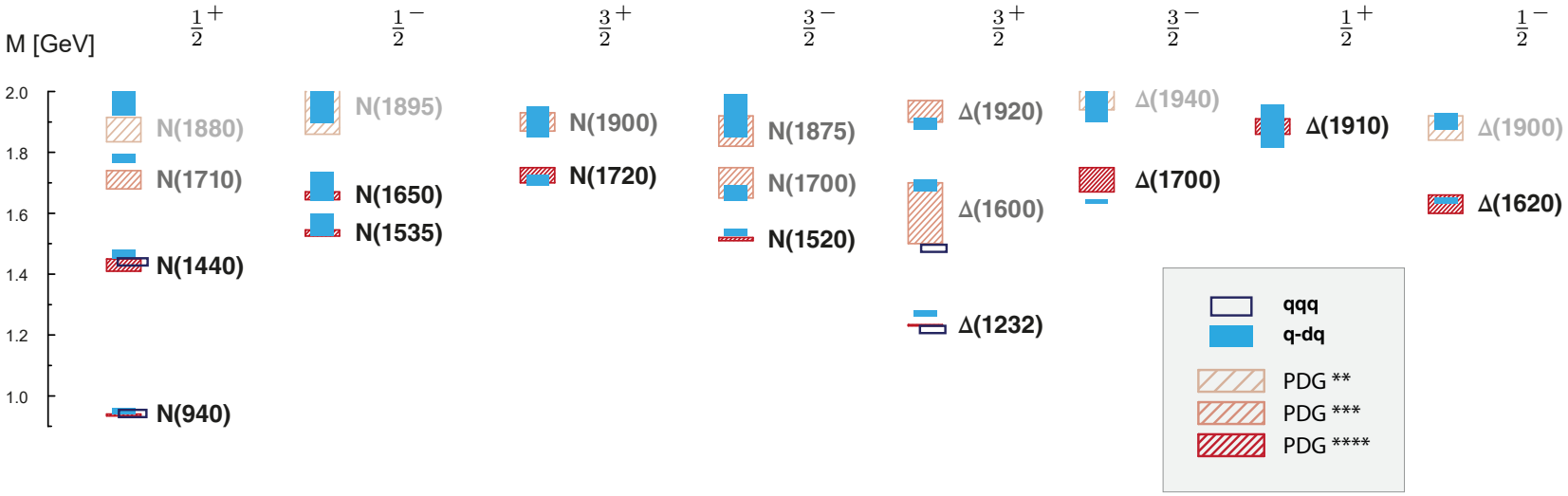
Baryon spectroscopy

- Solution of nucleon's covariant **3-body Faddeev equation**
  GE, Alkofer, Nicmorus, Krassnigg, PRL 104 (2010)



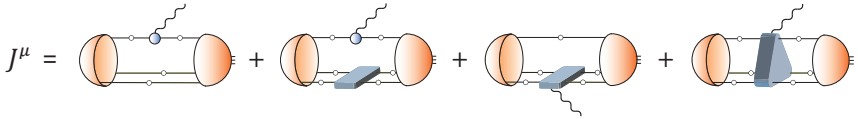


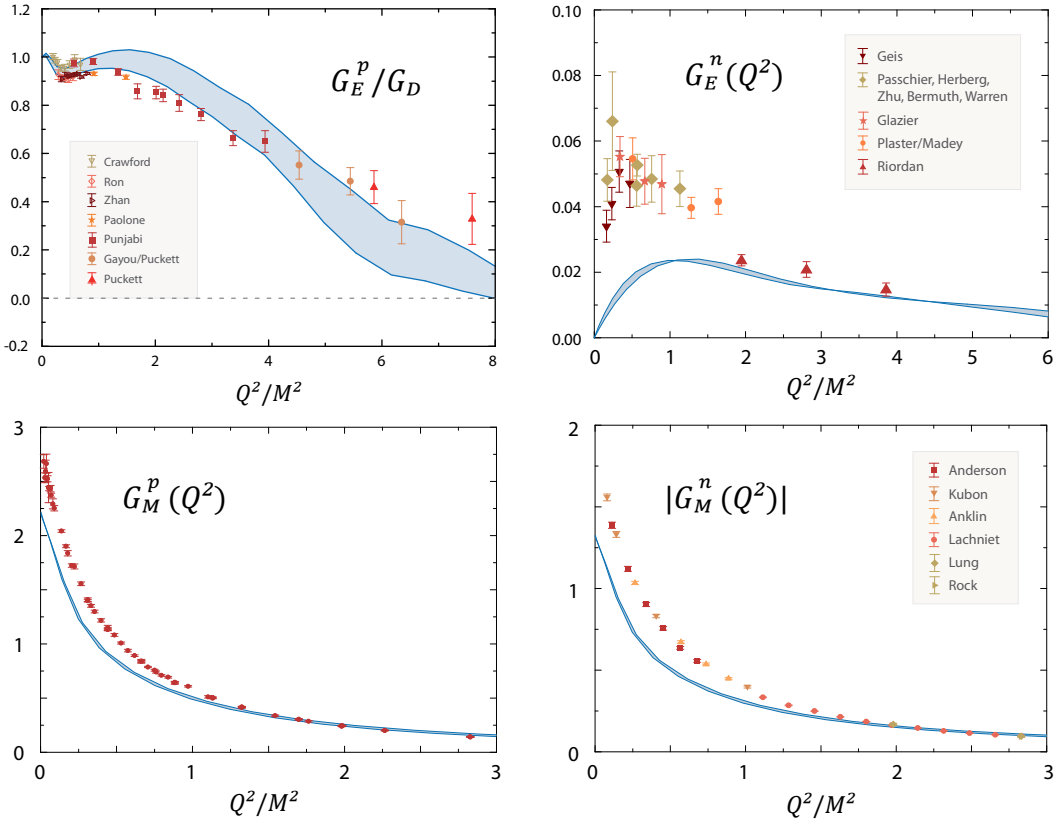
- Similar results in **quark-diquark** description
  GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)



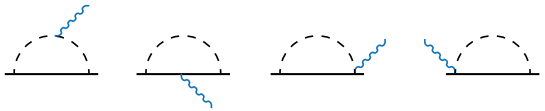
Baryon form factors

Nucleon em. form factors from three-quark equation
GE, PRD 84 (2011)





- **"Quark core without pion cloud"**



- **similar:** $N \to \Delta\gamma$ transition, axial & pseudoscalar FFs, octet & decuplet em. FFs

  Review: GE, Sanchis-Alepuz, Williams, Fischer, Alkofer, PPNP 91 (2016), 1606.09602

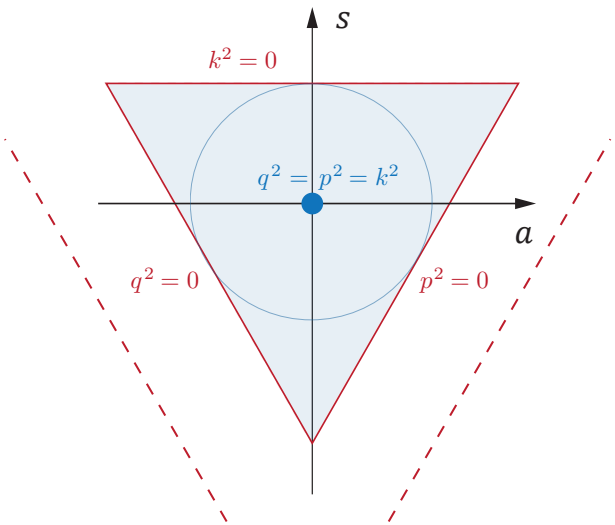
Structure of the amplitude

- **Singlet:** symmetric variable, carries overall scale:

$$\mathcal{S}_0 = \frac{1}{4}\left(p^2 + q^2 + k^2\right)$$

- **Doublet:** $\mathcal{D}_0 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix} \sqrt{3}\left(q^2 - p^2\right) \\ p^2 + q^2 - 2k^2 \end{bmatrix}$

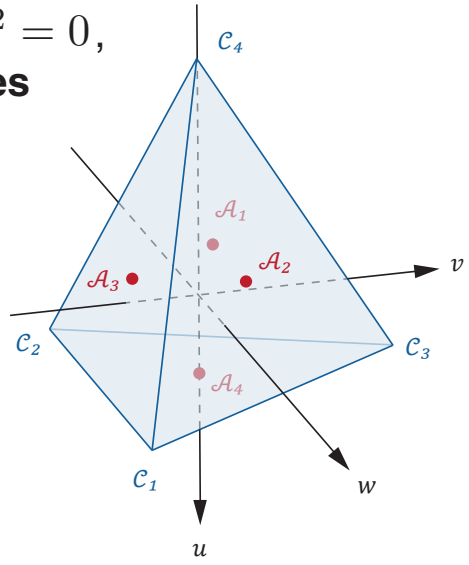
  Mandelstam triangle,
  outside: **meson and diquark poles!**



Lorentz invariants can be grouped into **multiplets of the permutation group S4:**
GE, Fischer, Heupel, PRD 92 (2015)

- **Triplet:** $\mathcal{T}_0 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix} 2\left(\omega_1 + \omega_2 + \omega_3\right) \\ \sqrt{2}\left(\omega_1 + \omega_2 - 2\omega_3\right) \\ \sqrt{6}\left(\omega_2 - \omega_1\right) \end{bmatrix}$

  tetrahedron bounded by $p_i^2 = 0$,
  outside: **quark singularities**

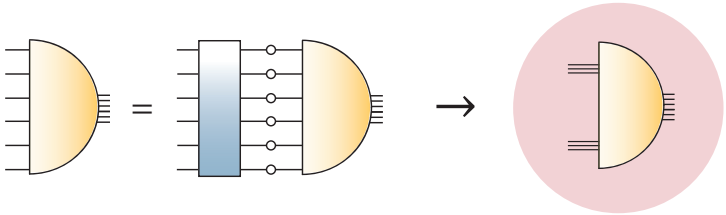


- Second **triplet:**
  3dim. sphere

$$\mathcal{T}_1 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix} 2\left(\eta_1 + \eta_2 + \eta_3\right) \\ \sqrt{2}\left(\eta_1 + \eta_2 - 2\eta_3\right) \\ \sqrt{6}\left(\eta_2 - \eta_1\right) \end{bmatrix}$$

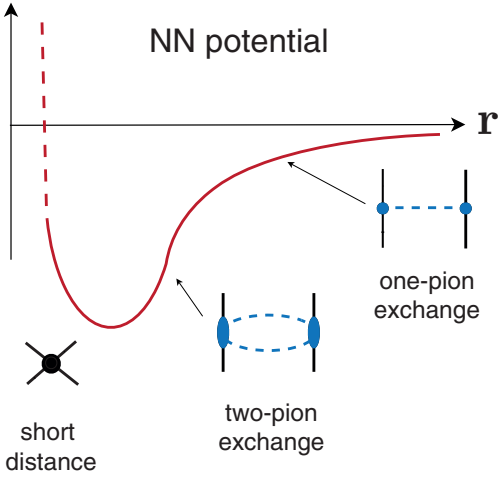
Nucleons in nuclei

Transition from quarks & gluons to **light nuclei:**

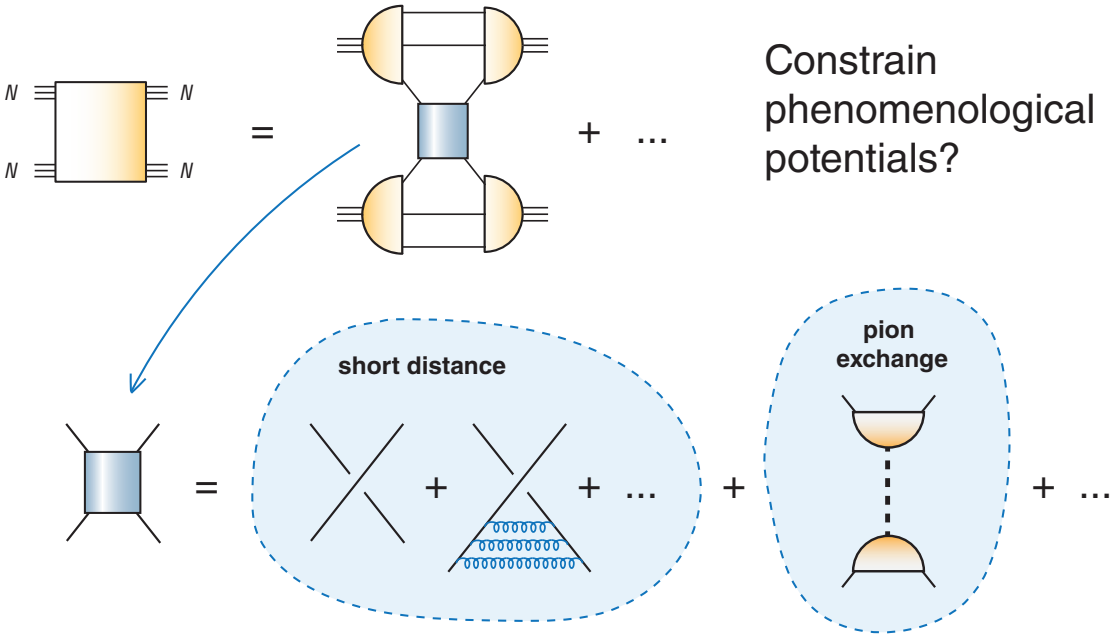


- Relativistic structure of **deuteron**
- Exotic dibaryons and hypernuclei
- **Short-range correlations**
- **EMC effect:** overlapping nucleons in nuclei?

Microscopic origins of **short-range nuclear force?**



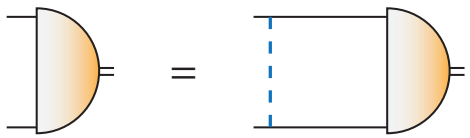
Weise, Nucl. Phys. A805 (2008)



Constrain phenomenological potentials?

Bound states & resonances

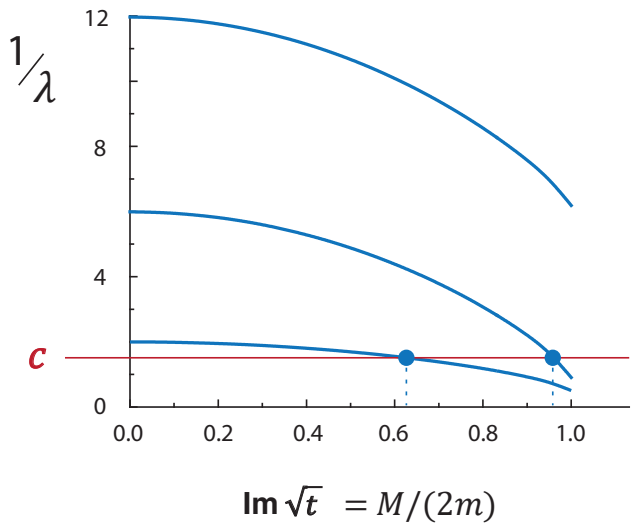
- **Homogeneous BSE:**

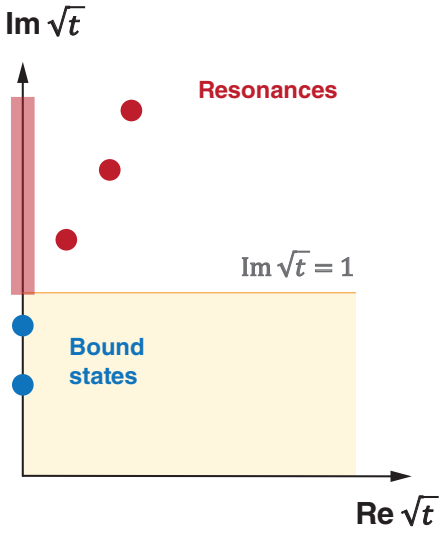


$$\psi(X,Z,t) = c \int dx \int dz \; K(X,x,Z,z,t) \; G_0(x,z,t) \; \psi(x,z,t)$$

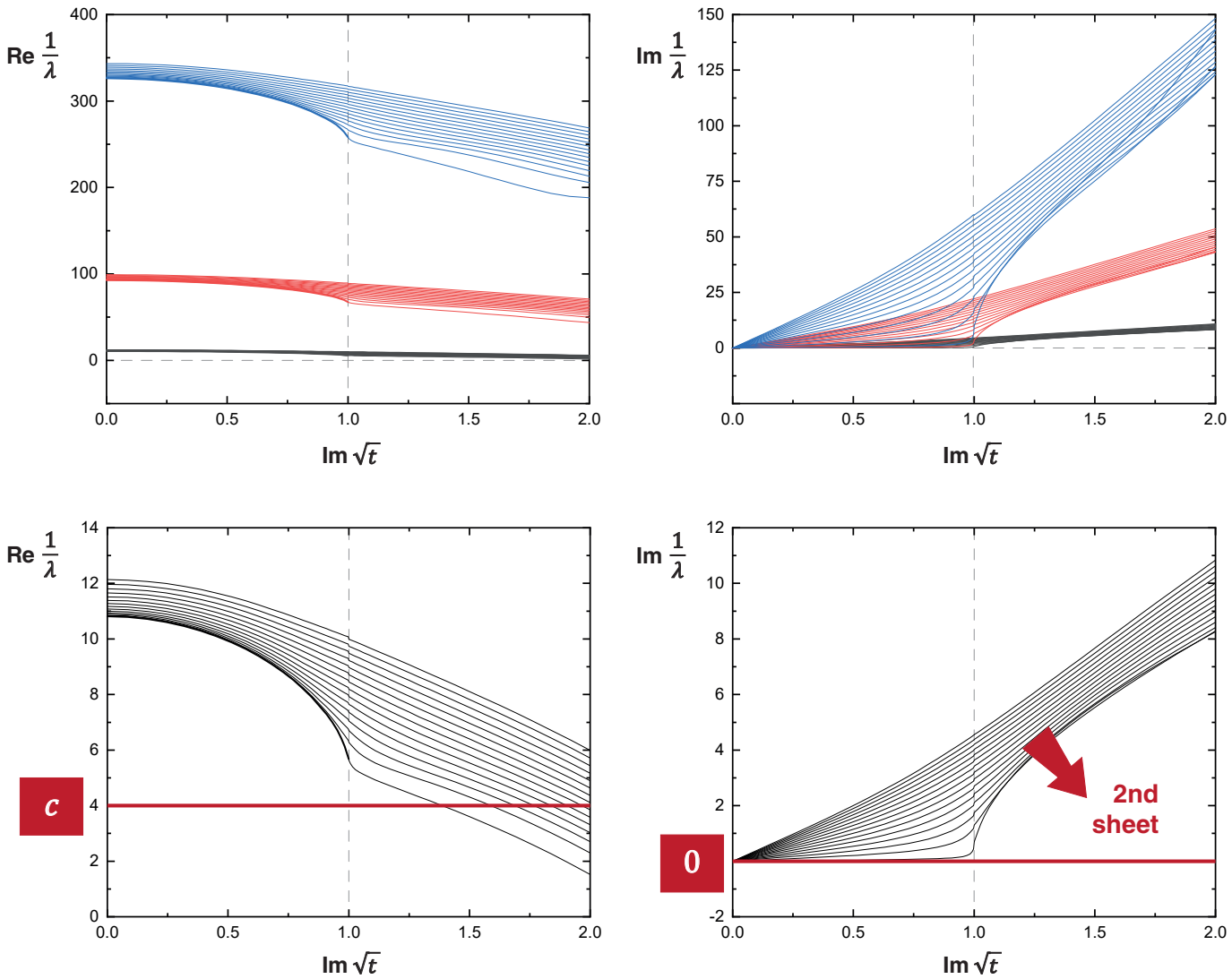
$$\Rightarrow \quad \psi(t) = c\, K G_0(t)\, \psi(t)$$

$$\Rightarrow \quad \frac{1}{\lambda(t)} \overset{!}{=} c$$





BSE Eigenvalues



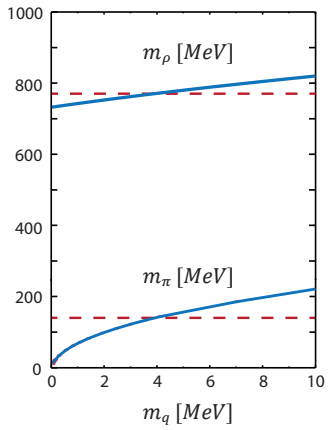
$$\frac{1}{\lambda(t)} \overset{!}{=} c + 0 \cdot i$$

still valid for **complex poles:**

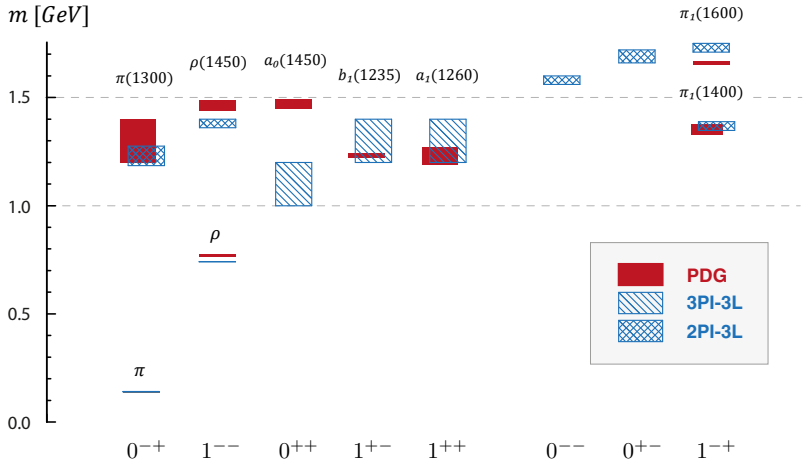
can detect resonances from **homogeneous BSE**

Mesons

- Pion is **Goldstone boson:** $m_\pi{}^2 \sim m_q$



- **Light meson spectrum** beyond rainbow-ladder

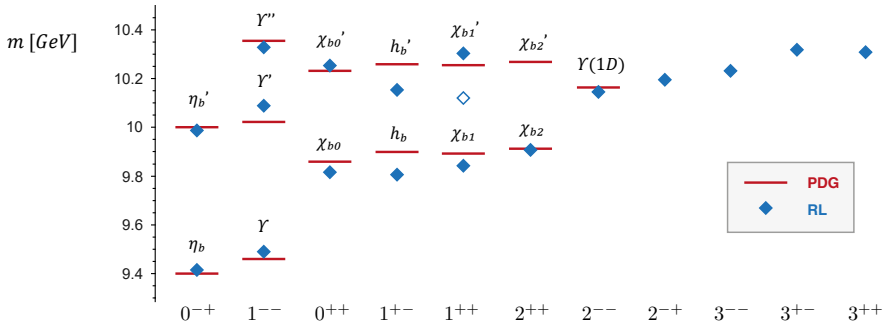


Williams, Fischer, Heupel, PRD 93 (2016)

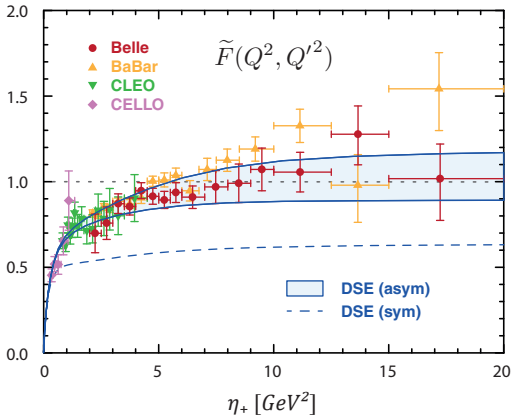
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

- **Bottomonium spectrum**

Fischer, Kubrak, Williams, EPJ A 51 (2015)



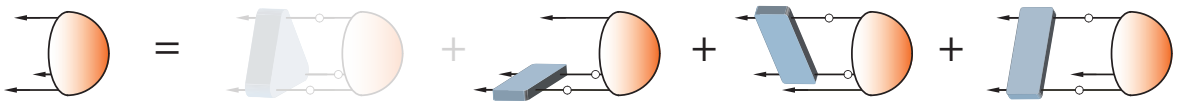
- **Pion transition form factor**



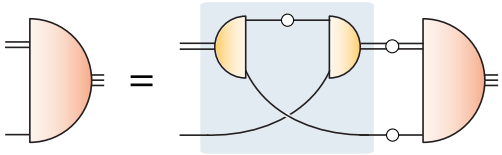
GE, Fischer, Weil, Williams, PLB 774 (2017)

Bound-state equations

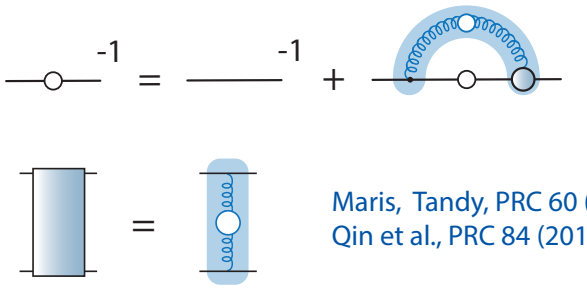
Bethe-Salpeter equation for baryons: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016), 1606.09602



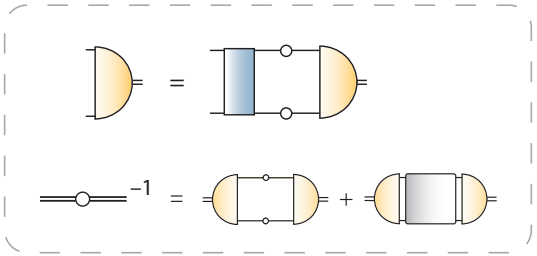
Quark-diquark approximation:



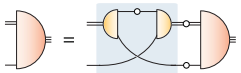
Rainbow-ladder:

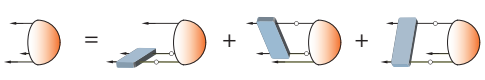


Maris, Tandy, PRC 60 (1999),
Qin et al., PRC 84 (2011)



DSE / BSE / Faddeev landscape

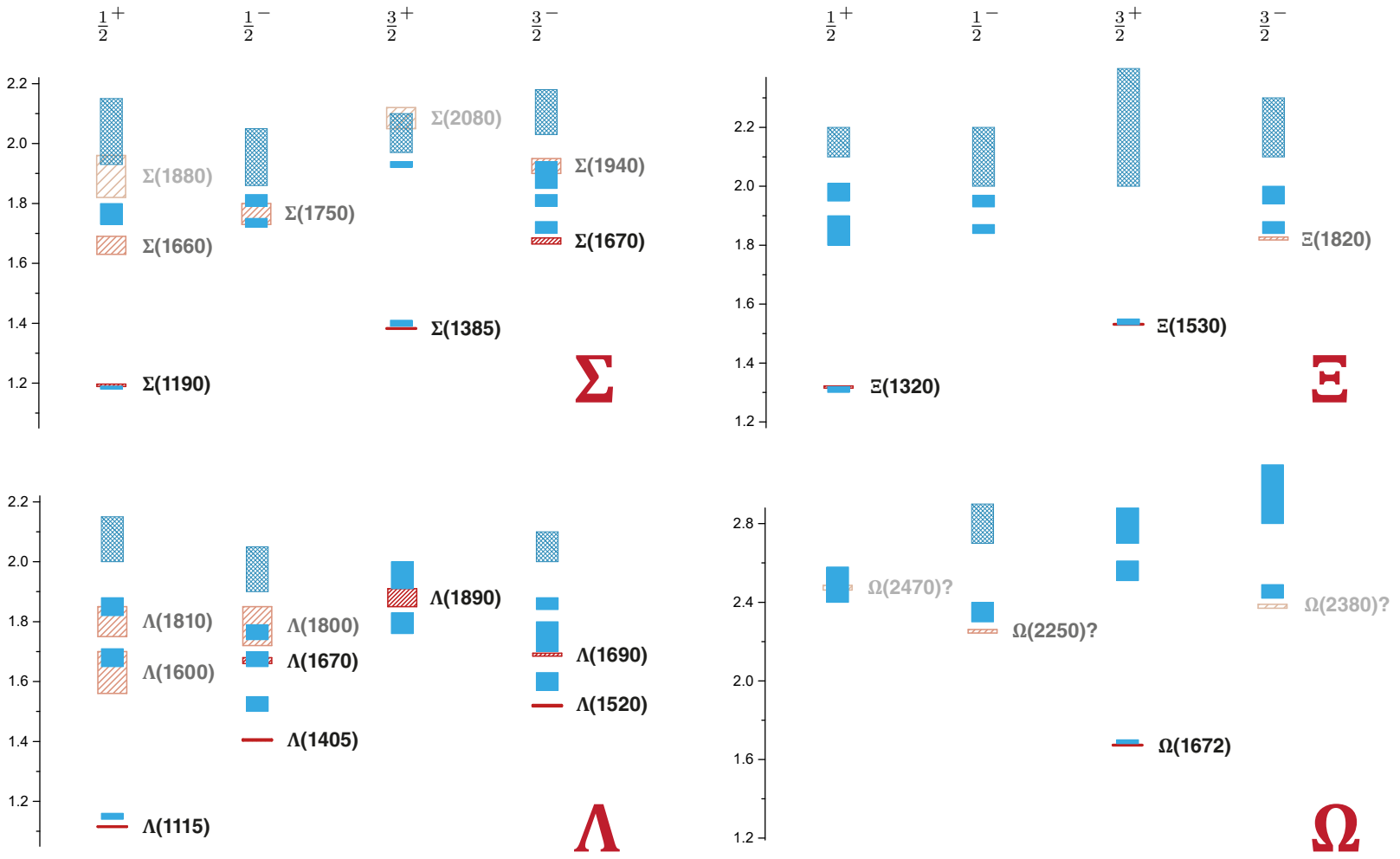




		NJL / contact	q-dq model	DSE (RL)		DSE (bRL)
u/d	N, Δ masses	√	√	√	√	√
	N, Δ em. FFs	√	√	√	√	
	$N \to \Delta\gamma$	√	√	√	√	
	N^*, Δ^* masses (+)	√	√	√	√	
	$N \to N^*\gamma$	√	√			
	N^*, Δ^* masses (-)	√	√	√	√	
	$N \to N^*\gamma$					
s	ground states	√	√	√	√	
	excited states	√	√	√	√	
	em. FFs & TFFs				√	
c, b	ground states	√	√		√	
	excited states		√		√	
		Cloet, Thomas, Roberts, Bashir, Segovia, Chen, Wilson, Lu, . . .	Oettel, Alkofer, Roberts, Cloet, Segovia, Chen, El-Bennich, . . .	GE, Alkofer, Nicmorus, Sanchis-Alepuz, Fischer	GE, Sanchis-Alepuz, Fischer, Alkofer, Qin, Roberts	Sanchis-Alepuz, Williams, Fischer

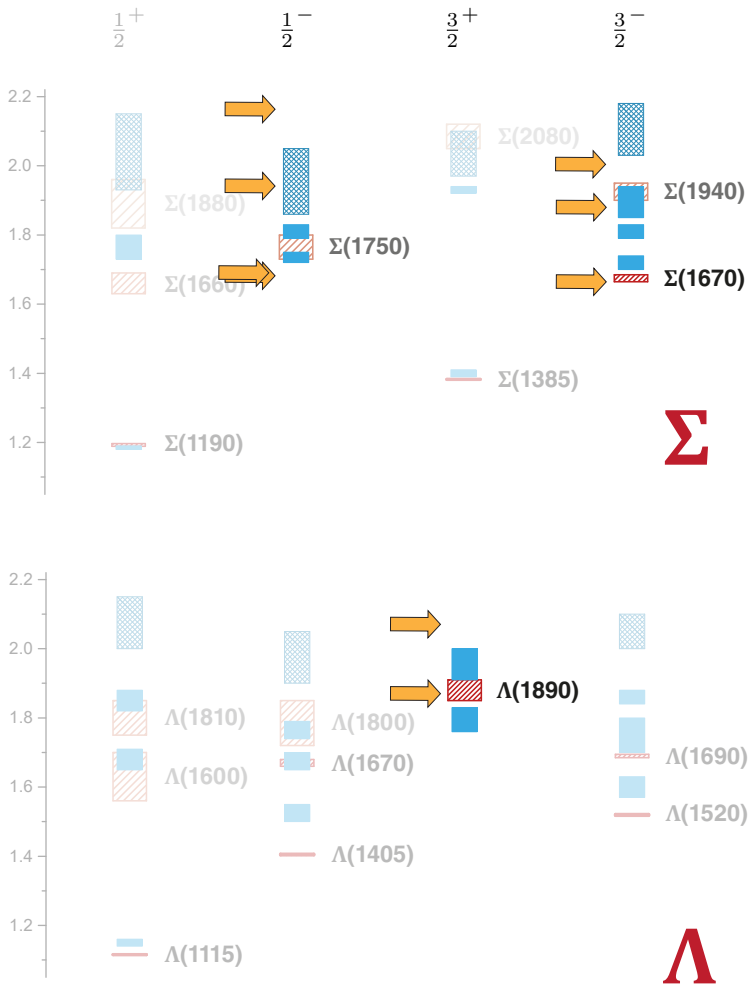
√ . . . before 2015
√ . . . after 2015

Strange baryons



GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

Strange baryons



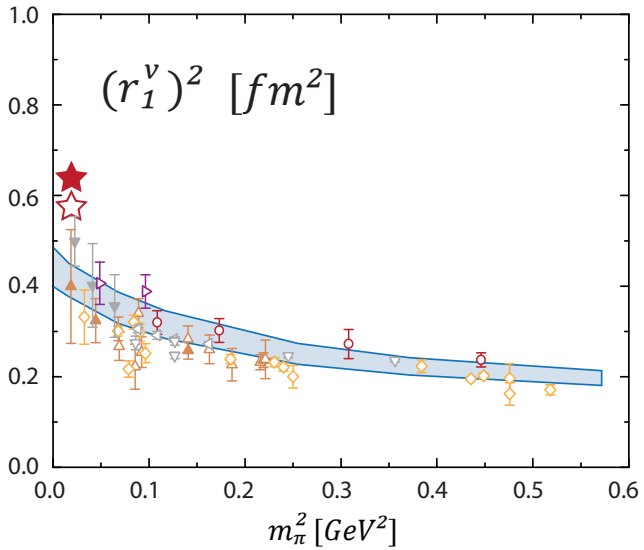
New states from Bonn-Gatchina

Sarantsev et al., 1907.13387 [nucl-ex]

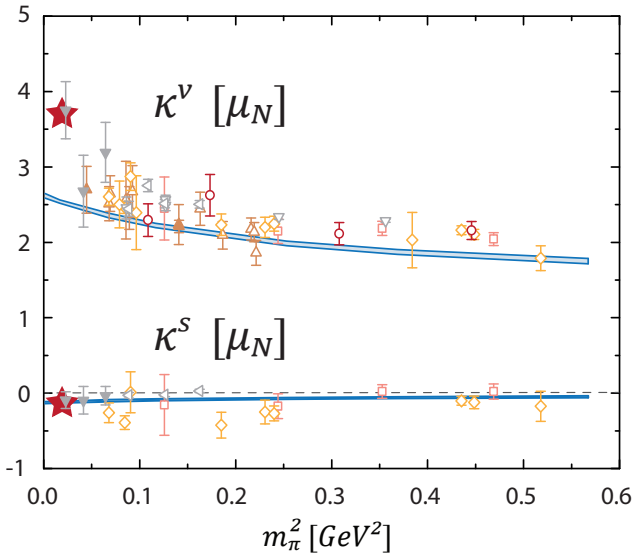
GE, Fischer, FBS 60 (2019), Fischer, GE, PoS Hadron 2017

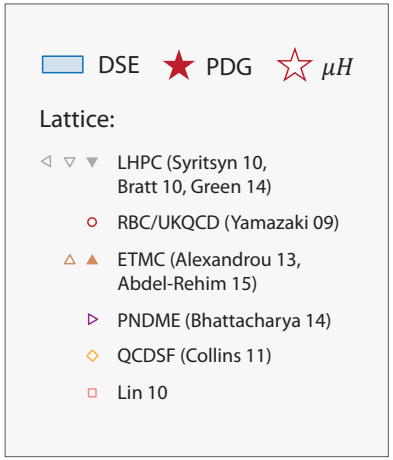
Nucleon em. form factors

Nucleon charge radii:
isovector (p-n) Dirac (F1) radius

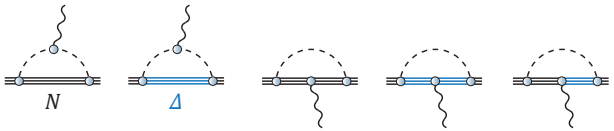


Nucleon magnetic moments:
isovector (p-n), isoscalar (p+n)





- **Pion-cloud effects** missing (⇒ divergence!), agreement with lattice at larger quark masses.



- **But:** pion-cloud **cancels** in κ^s ⇔ **quark core**

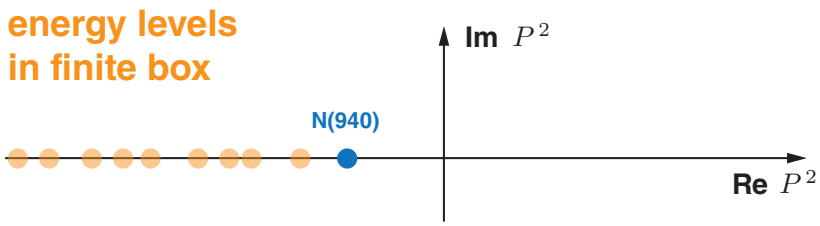
Exp: $\kappa^s = -0.12$
Calc: $\kappa^s = -0.12(1)$ **!!**

GE, PRD 84 (2011)

Resonances?

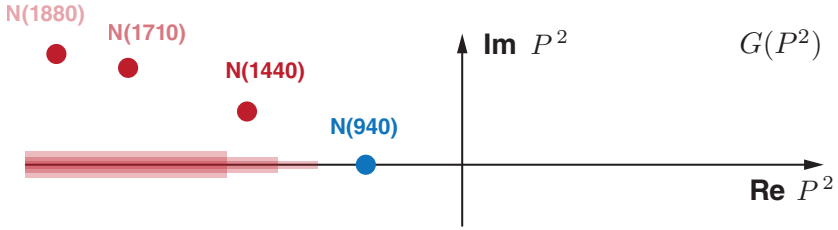
Lattice QCD:

$$\langle \dots \rangle = \int \mathcal{D}[\psi, \bar{\psi}, A]\, e^{-S[\psi, \bar{\psi}, A]} \; (\dots)$$



- **Finite volume:**
  bound states & scattering states

vary volume,
Luescher method

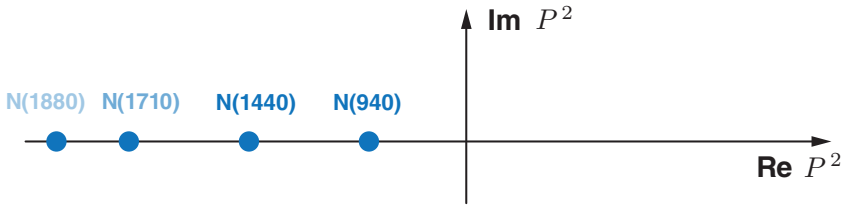


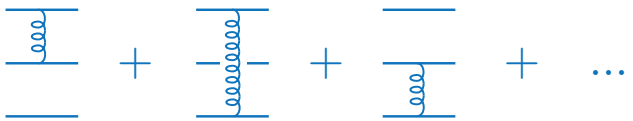
- **Infinite volume:**
  Bound states, resonances,
  branch cuts

Resonances?

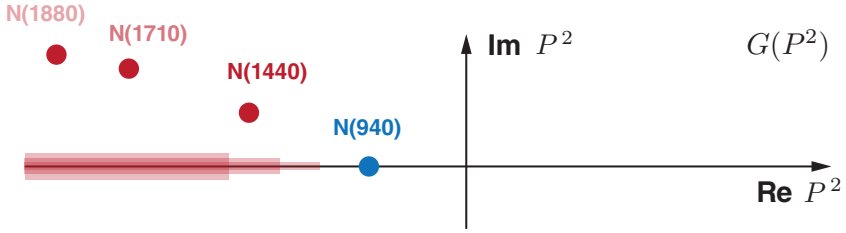
In terms of quarks and gluons?

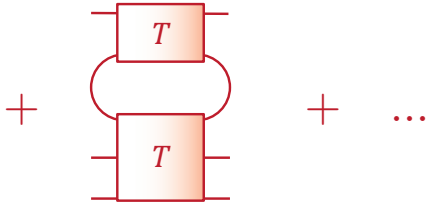
Bound states:



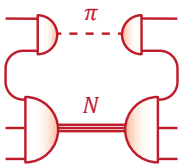


Resonances by **meson-baryon interactions:**





Both **bound states** and **resonances**
must be generated from quark-gluon structure!



Analogue for $\rho \to \pi\pi$:

Williams, 1804.11161 [hep-ph],
Miramontes, Sanchis-Alepuz,
1906.06227 [hep-ph]