Transverse-Momentum-Dependent Parton Distributions from Lattice QCD

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Yong Zhao
Brookhaven National Laboratory

Letter of Interest

Contact: yzhao@bnl.gov

Collaboration:

Markus Ebert (MIT & MPP),

Jian Liang (U. Kentucky),

Yizhuang Liu (T.-D. Lee Institute & U. Regensburg),

Phiala Shanahan (MIT),

Iain Stewart (MIT),

Michael Wagman (FNAL),

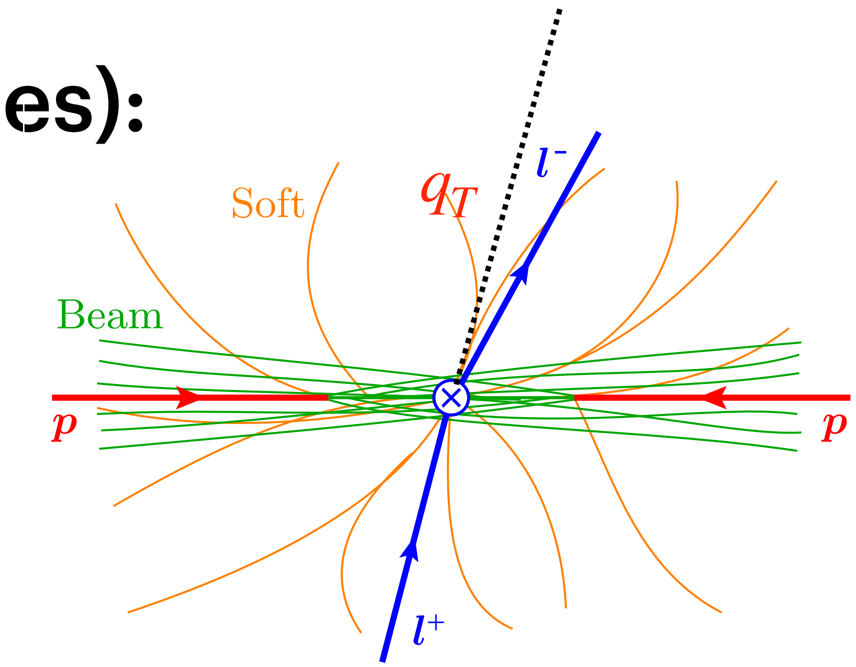
Wei Wang (SJTU),

Yong Zhao (BNL)

Nonperturbative inputs for TMD phenomenology

- **TMD factorization (for Drell-Yan processes):**

$$\frac{d\sigma}{dQdYd^2q_T} = \sum_{i,j} H_{ij}(Q,\mu) \int d^2 b_T \, e^{i\vec{b}_T\cdot\vec{q}_T} \times f_i^{\mathrm{TMD}}(x_a, \vec{b}_T, \mu, \zeta_a) f_j^{\mathrm{TMD}}(x_b, \vec{b}_T, \mu, \zeta_b)$$

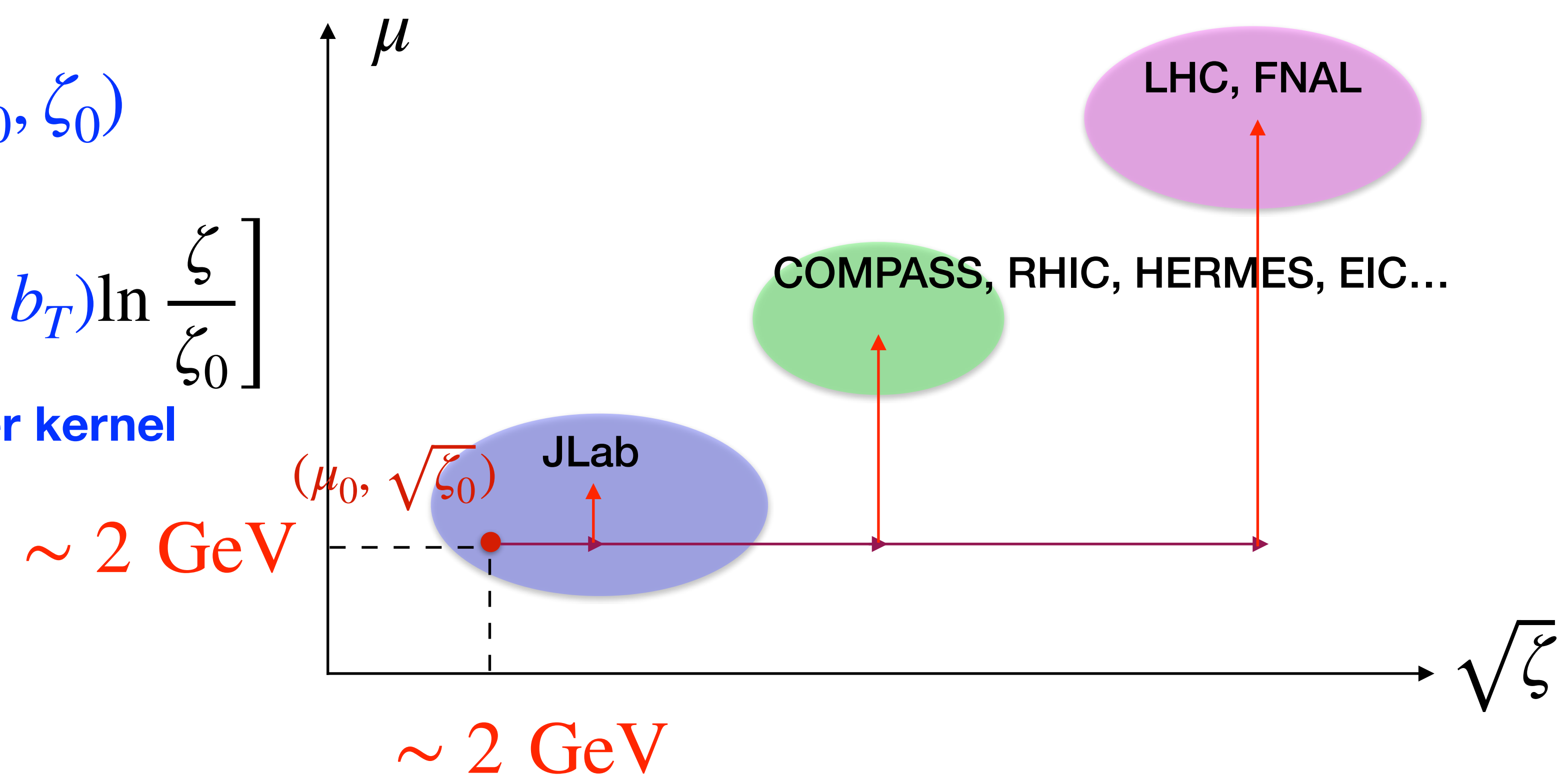


- **TMD evolution:**

$$f_i^{\mathrm{TMD}}(x, \vec{b}_T, \mu, \zeta) = f_i^{\mathrm{TMD}}(x, \vec{b}_T, \mu_0, \zeta_0) \times \exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \gamma_\mu^i(\mu', \zeta_0)\right] \exp\left[\frac{1}{2}\gamma_\zeta^i(\mu, b_T) \ln\frac{\zeta}{\zeta_0}\right]$$

Collins-Soper kernel

Nonperturbatively unknown when $b_T \sim 1/\Lambda_{\mathrm{QCD}}$



Lattice QCD Calculation of TMDPDFs

with Large-Momentum Effective Theory (LaMET)

X. Ji, PRL 110 (2013); SCPMA57 (2014).

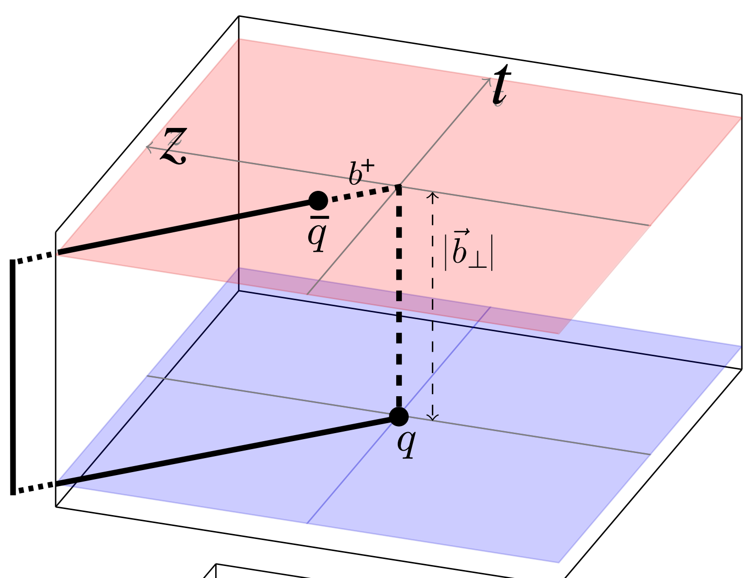
- **TMDPDF:**

$$f_i^{\rm TMD}(x,\vec{b}_T,\mu,\zeta) = \lim_{\epsilon\to0,\tau\to0} Z_{\rm UV}(\epsilon,\mu,\zeta) \times B_i(x,\vec{b}_T,\epsilon,\tau,xP^+)\Delta_S^i(b_T,\epsilon,\tau)$$

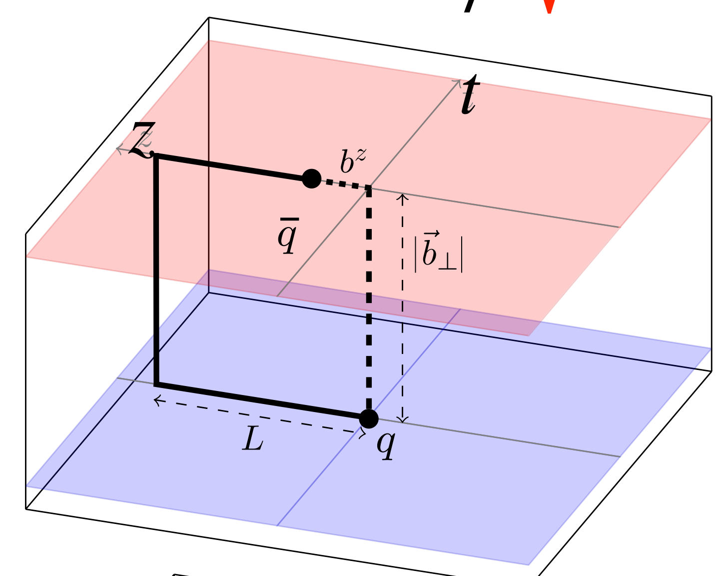
- **Quasi-TMDPDF:**

$$\tilde{f}_q^{\rm TMD}(x,\vec{b}_T,\mu,P^z) = \int \frac{db^z}{2\pi} e^{ib^z(xP^z)}\tilde{Z}_q(b^z,\mu,a) \times \tilde{B}^q(b^z,\vec{b}_T,a,L,P^z)\Big/\sqrt{\tilde{S}_q(b_T,a,L)}$$

B_q

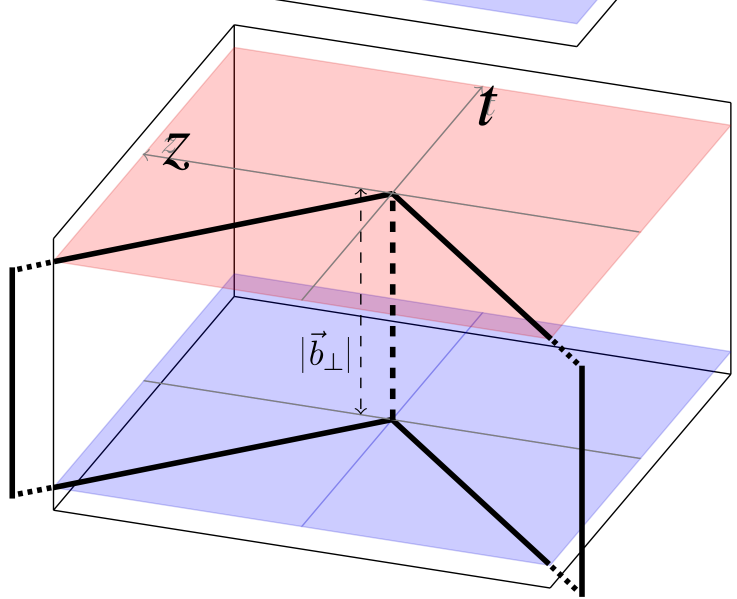


Lorentz boost and $L \to \infty$

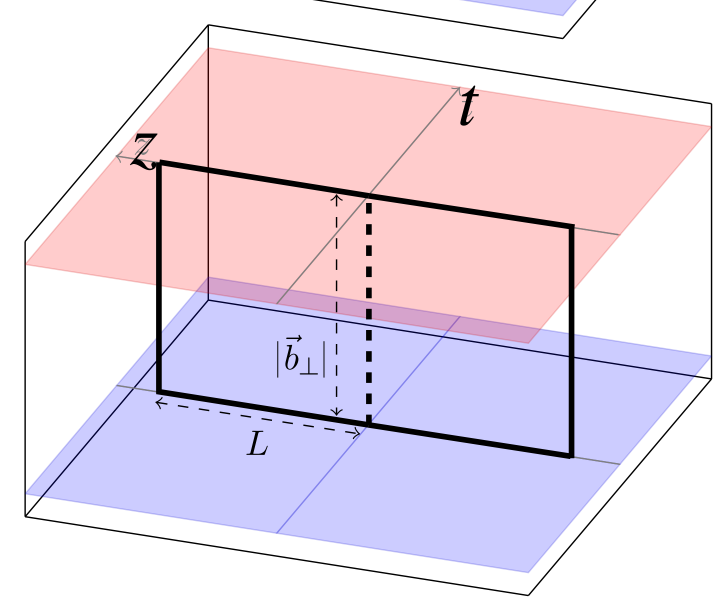


$\tilde{B}_q$

S^q



Cannot be related by Lorentz boost



$\tilde{S}^q$

Lattice QCD Calculation of TMDPDFs

with Large-Momentum Effective Theory (LaMET)

Relationship between TMDPDF and Quasi-TMDPDF:

$$\tilde{f}^{\rm TMD}_{\rm ns}(x, \overrightarrow{b}_T, \mu, P^z) = C^{\rm TMD}_{\rm ns}\left(\mu, xP^z\right) \sqrt{S^q_r(b_T, \mu)} \; \exp\left[\frac{1}{2}\gamma^q_\zeta(\mu, b_T)\ln\frac{(2xP^z)^2}{\zeta}\right]$$

$$b^z \sim \frac{1}{P^z} \ll b_T \ll L$$

$$\times f^{\rm TMD}_{\rm ns}(x, \overrightarrow{b}_T, \mu, \zeta) + \mathcal{O}\left(\frac{b_T}{L}, \frac{1}{b_T P^z}, \frac{1}{P^z L}\right)$$

- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and Y.Z., PRD99 (2019);
- Ebert, Stewart, Y.Z., PRD99 (2019), JHEP09 (2019);
- Ji, Liu and Liu, Nucl.Phys.B 955 (2020), 1911.03840;
- Schaefer and Vladimirov, Phys.Rev.D 101 (2020);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, 2004.03543.

Milestones in lattice calculations:

1. The nonperturbative Collins-Soper kernel;

2. The soft function;

3. The full TMDPDF.

The Non-perturbative Collins-Soper kernel

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z)\, \tilde{B}_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z)\, \tilde{B}_{\rm ns}^{\rm TMD}(x, \vec{b}_T, \mu, P_2^z)}$$

Ji, Sun, Xiong and Yuan, PRD91 (2015); Ebert, Stewart, Y.Z., PRD99 (2019).

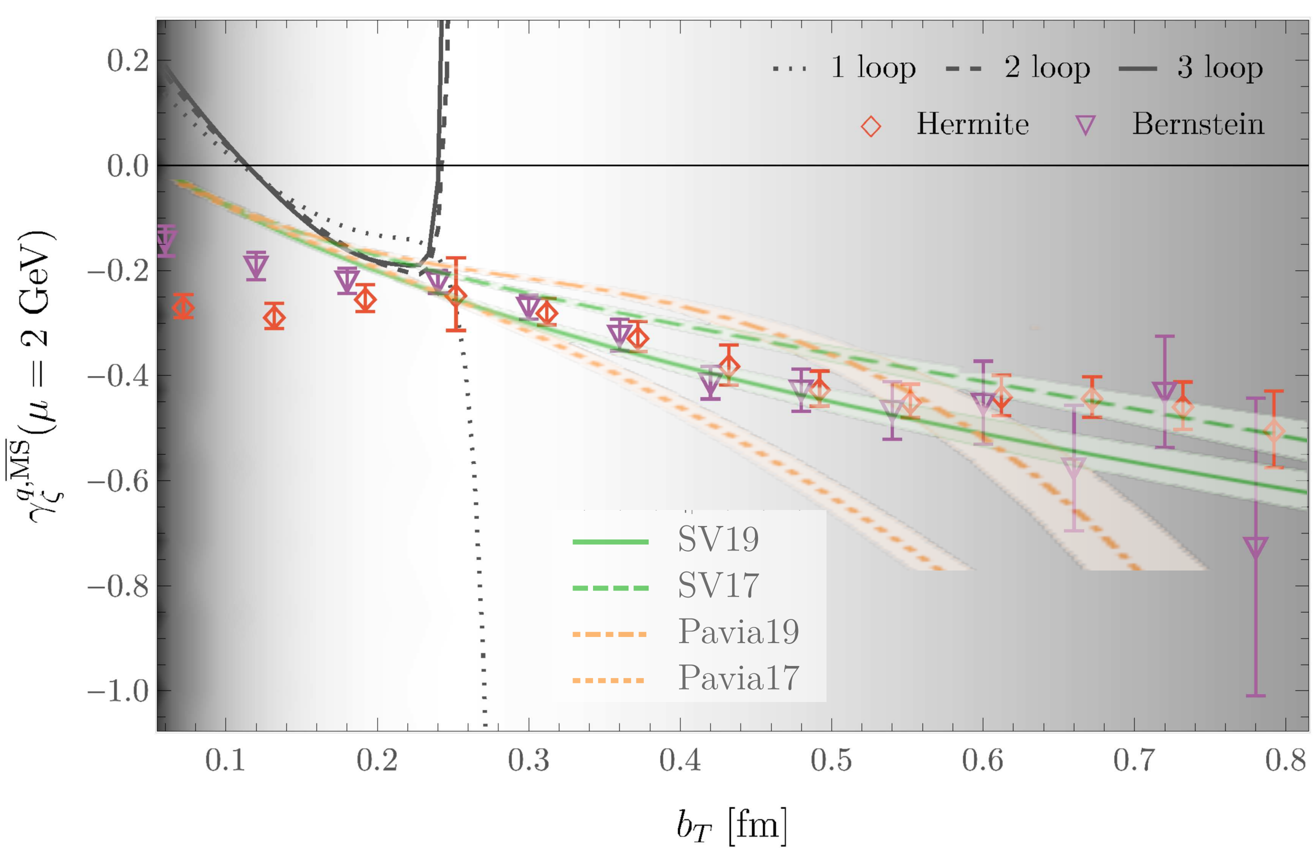
First exploratory calculation on a quenched lattice:

- Lattice renormalization and perturbative matching;

Shanahan, Wagman and Y.Z., Phys.Rev.D 101 (2020);
Constantinou, Panagopoulos and Spanoudes, PRD99 (2019);
Ebert, Stewart and Y.Z., JHEP 03 (2020).

- Extraction by direct Fourier transform (FT) or fitting to models.

FT truncation error can be improved with longer Wilson line extension or larger hadron momentum.



P. Shanahan, M. Wagman, Y.Z., Phys.Rev.D 102 (2020).

Target error:

$\lesssim 10\,\%$ for $0.2\ {\rm fm} < b_T < 1\ {\rm fm}$

Further application of forming ratios: Ratios of TMDPDFs with different spin structures.

Ebert, Schindler, Stewart, Y.Z., JHEP 09 (2020).

The TMD soft function from lattice QCD

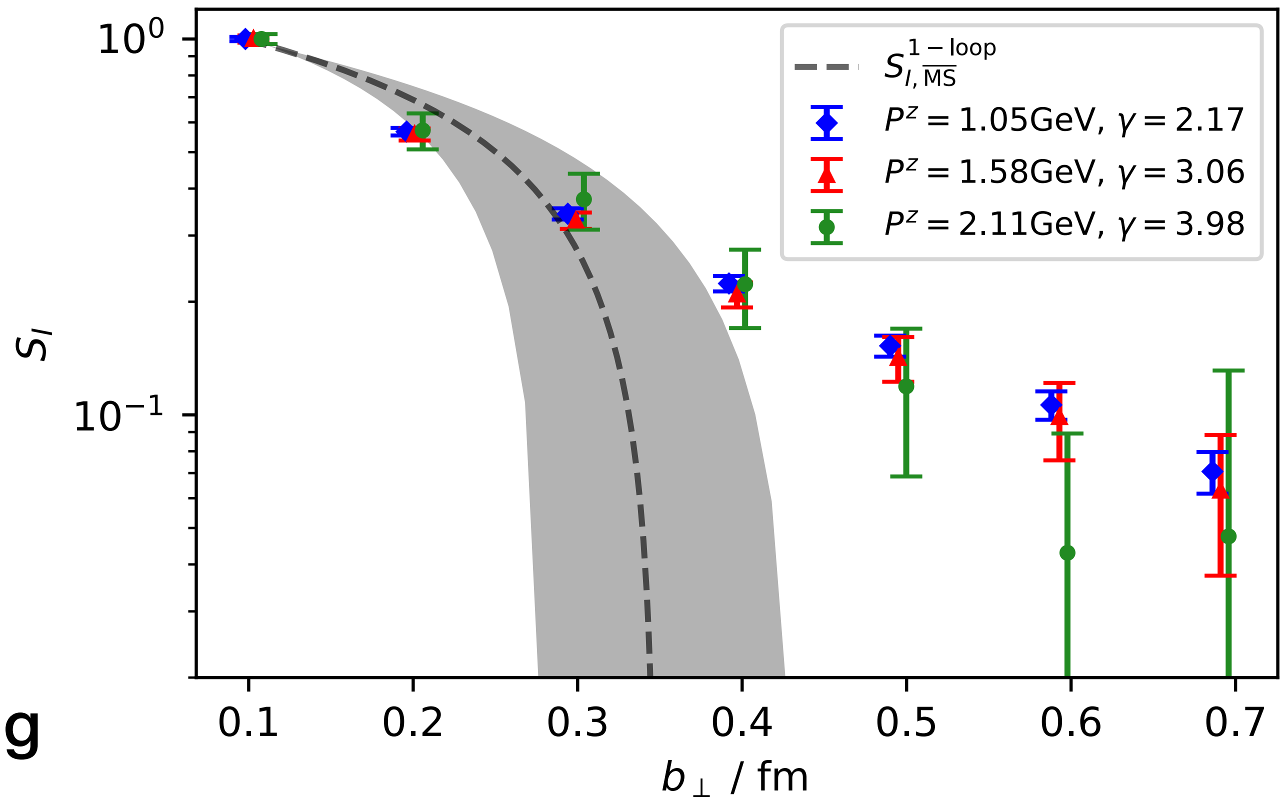
$$\left\langle \pi(-P) \middle| j_1(b_T) j_2(0) \middle| \pi(P) \right\rangle = S_q^r(b_T, \mu)\ H(x, \mu) \otimes \Phi^\dagger(x, b_T, -P^z) \otimes \Phi^\dagger(x, b_T, P^z)$$

Quasi-TMD distribution amplitude

Ji, Liu and Liu, Nucl.Phys.B 955 (2020), 1911.03840;
X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, 2004.03543.

First exploratory calculation on the lattice:

- Dynamical fermions;
- Tree-level matching and no lattice renormalization.



Q.-A. Zhang, et al. (LP Collaboration), 2005.14572.

Targets:

- Systematic control: renormalization, operator mixing, perturbative matching and power corrections.
- $\lesssim 10\,\%$ for $0.2\ \text{fm} < b_T < 1\ \text{fm}$.

Full TMDPDFs from lattice QCD

- Combination of the lattice calculations of the quasi-TMDPDF, Collins-Soper kernel and the soft function.
- Target error:
  - $\lesssim 20\,\%$ for $0.2 \text{ fm} < b_T < 1 \text{ fm}$.
- Complementing global analysis:
  - Differentiate models for TMDPDFs;
  - Comparison to results from global fits;
  - Predictions for certain TMDPDFs of which there is very little data.