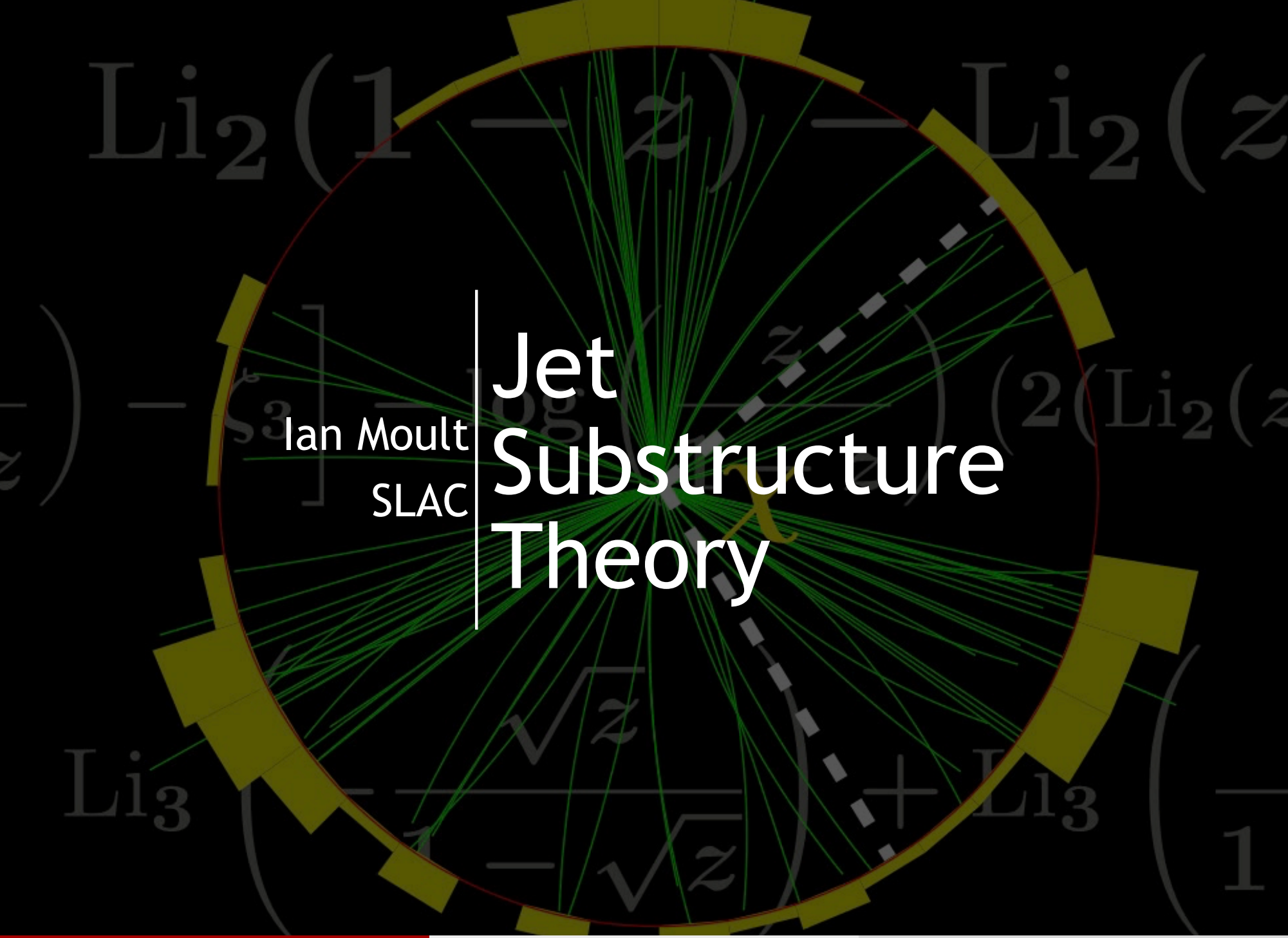
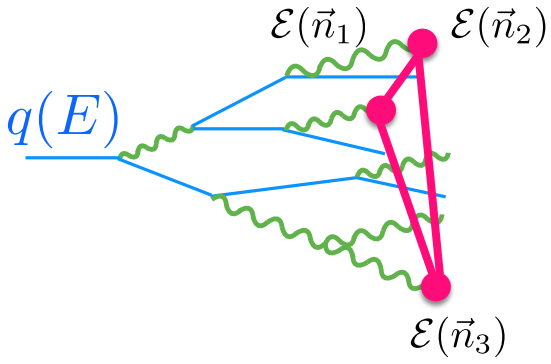


Jet Substructure Theory

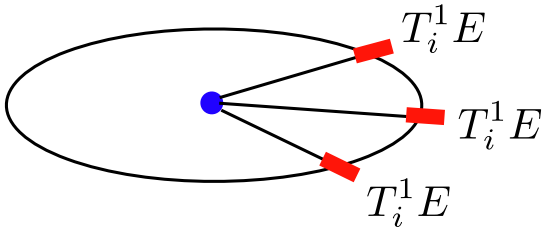
Ian Moult
SLAC

Outline

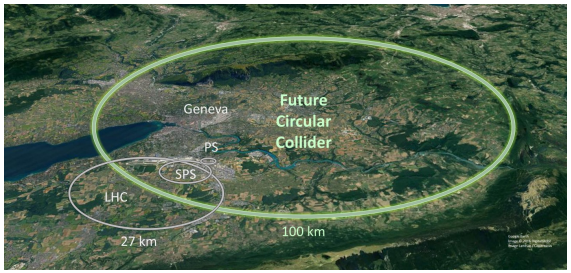
- Overview of Observables and Calculations



- Interplay of Experimental and Theoretical Issues



- Opportunities at Future Colliders (extremely brief)

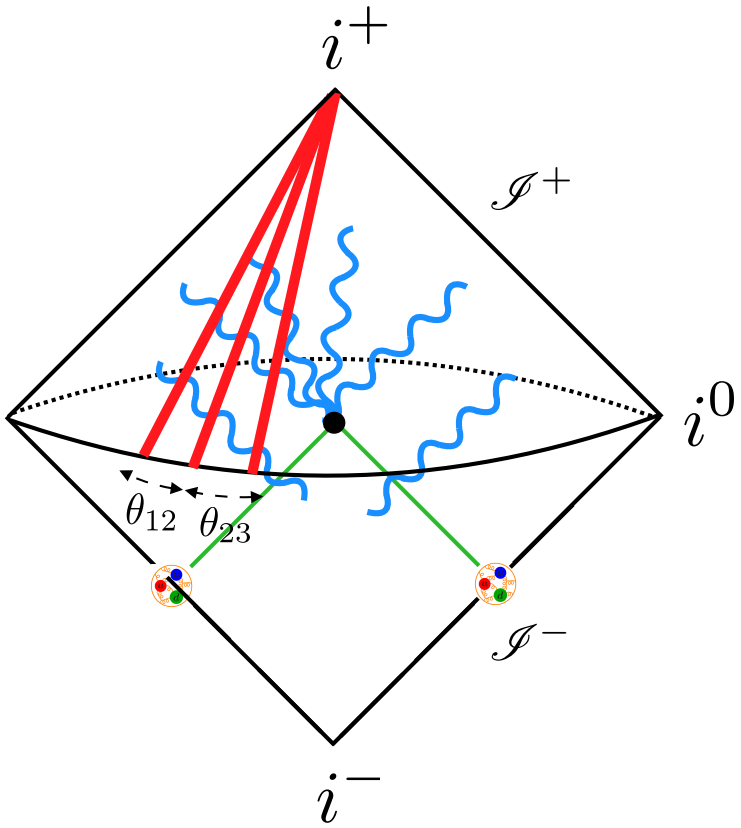


Disclaimer

- Jet substructure is by now a huge topic, to which I can not do justice in 20min.

- Topics chosen represent my personal views. Others may (do) have different views.

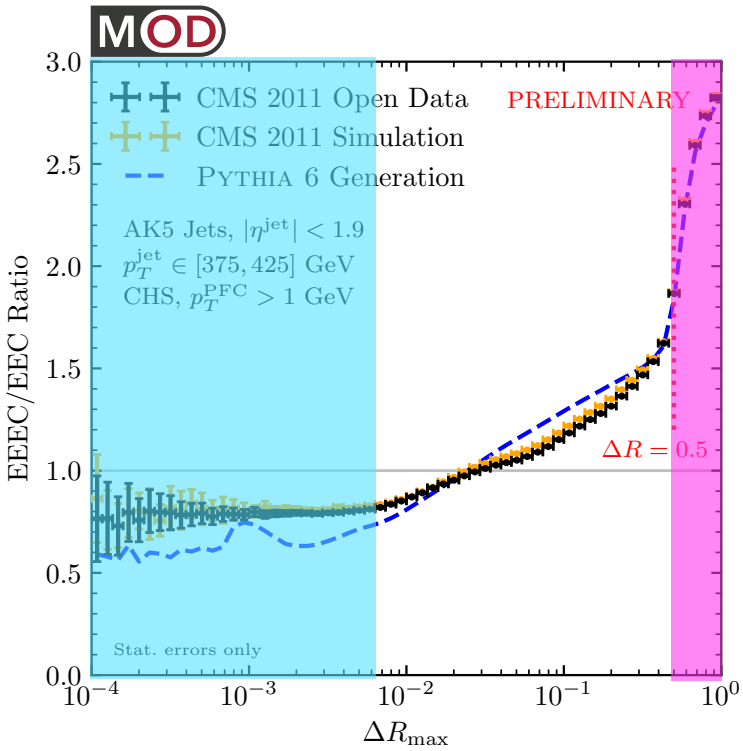
Overview of Observables and Calculations



General Classification

- Huge variety of observables exist, many of which probe similar physics.
- Can loosely classify based on the physics that is probed:
  - "Scaling" observables: probe scaling with "size". Sensitive to α_s, certain anomalous dimensions in the theory. e.g. jet mass, two-point energy correlator.
  - Multi-point correlators/multidifferential: probe kinematic structure, $1 \to n$ splitting, spin correlations, etc. e.g. Lund Plane, three-point energy correlator.
  - Jets from Massive Particles: probe properties of the massive particle. e.g. mass of top quark jets.

Scaling Observables



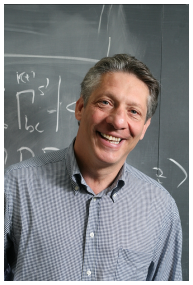
Rich Variety of Physics

- Scaling observables are those whose dominant behavior is determined by $1 \to 2$ splitting functions (alternatively twist-2 spin j operators).

Two Regimes with Very Different Physics

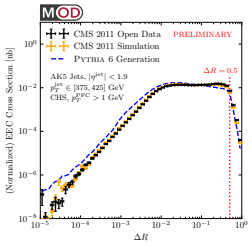
Ellis et al. 1977

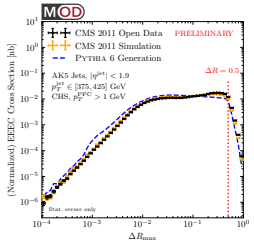
Farhi 1977

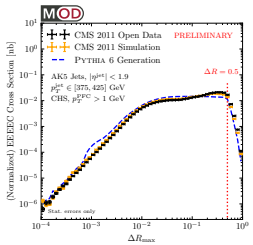


Interesting Physics at all j!

$j = 3$ $\quad$ $j = 4$ $\quad$ $j = 5$ $\quad$ $j \to \infty$







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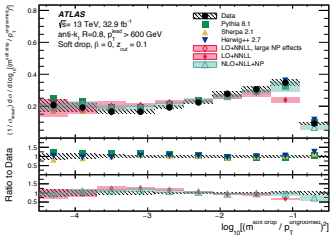


Finite spin DGLAP, q/g flavor changing splitting and evolution

Jet Calculus (Konishi, Ukawa, Veneziano, 1979)

Wilson Lines, Cusp, Collinear Anom Dims

Getting Rid of Soft Radiation

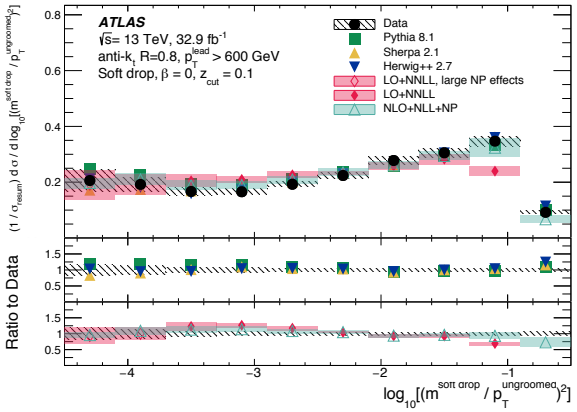
- Due to the complex environment of the LHC, it is convenient to remove sensitivity to soft radiation. The two physics regimes (finite j vs. $j \to \infty$) correspond to two different approaches to doing this:

Wilson Lines, Cusp, Collinear Anom Dims

Groom a Sudakov Observable

$$e_2^{(\beta)} = \frac{1}{p_{TJ}^2} \sum_{i<j\in J} p_{Ti} p_{Tj} (R_{ij})^\beta$$

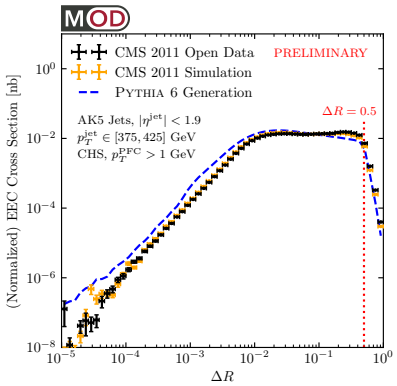
e.g. mass, energy correlation functions



Start with a Collinear Observable

$$\frac{d\sigma}{dz} = \sum_{i,j} \int d\sigma \, \frac{E_i E_j}{Q^2} \delta\left(z - \frac{1-\cos\chi_{ij}}{2}\right)$$

e.g. energy energy correlates

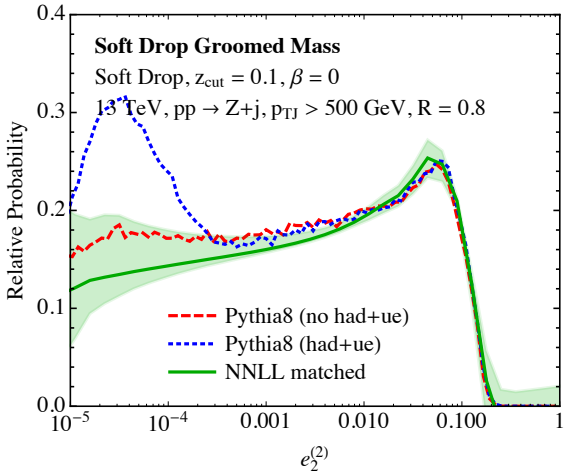


Finite spin DGLAP, q/g flavor changing

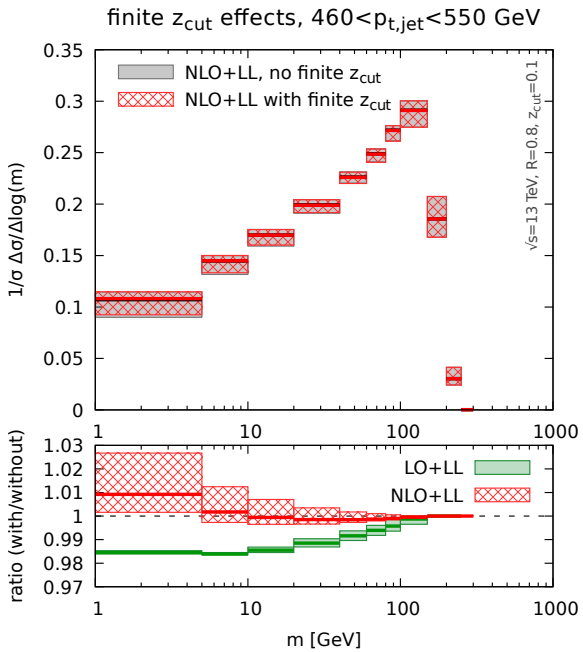
Groomed Jet Mass

- Groomed jet mass was the first precision calculation of a jet substructure observable at the LHC!
- Compute to NLL (single log).

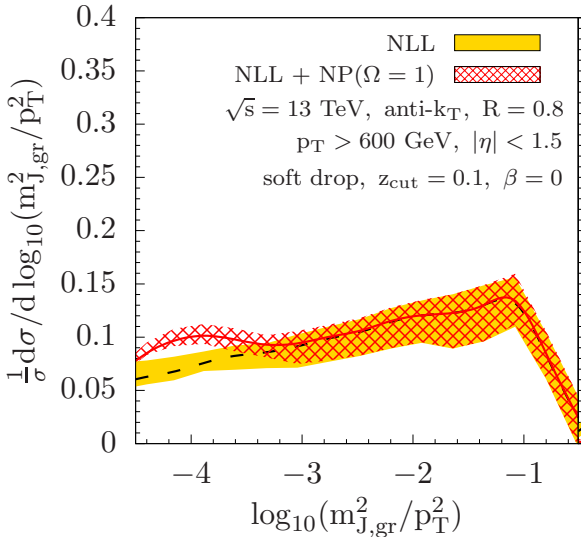
Soft Drop Mass $pp \to Z + j$



[Frye, Larkoski, Schwartz, Yan]



[Marzani, Soyez, Schunk]



[Kang, Lee, Liu, Ringer]

- Ingredients for extension to NNLL recently extracted by Larkoski et al.

Projected Energy Correlators

- Finite j DGLAP can be probed using "Projected Energy Correlators"
- We can reduce higher point correlators by integrating out shape information, keeping only the longest side x_L. This is a proxy for its size.

Projected Energy Correlator:

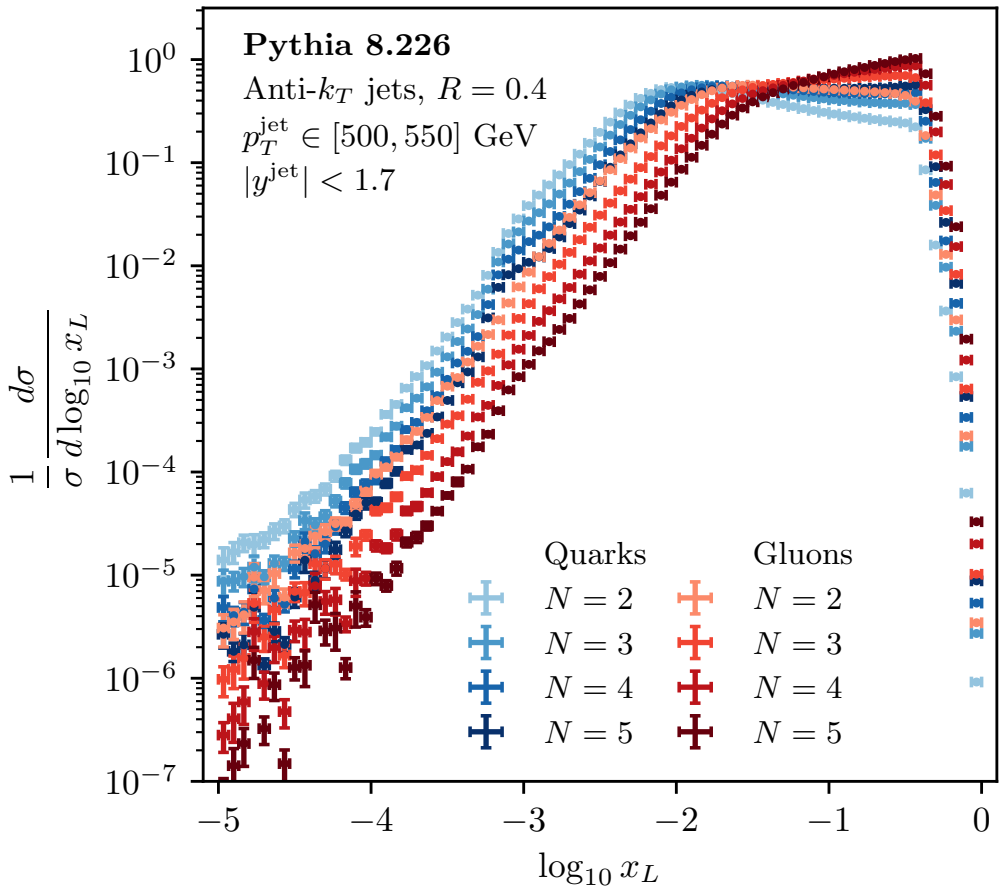
$$\frac{d\sigma^{[N]}}{dx_L} = \sum_n \sum_{1\leq i_1,\ldots,i_N\leq n} \int d\sigma_{e^+e^-\to X_n} \frac{\prod_{a=1}^N E_{i_a}}{Q^N} \cdot \delta(x_L - \max\{R_{i_1 i_2}, R_{i_1 i_3}, \ldots, R_{i_{N-1} i_N}\})$$

- Directly generalizes the two point correlator. Exhibits scaling with twist-2 spin-j anomalous dimension:

$$\boxed{\frac{d\sigma^{[\nu]}}{dx_L} = C^{[\nu]}(\alpha_s)\gamma^{\mathcal{N}=4}_{J^{[\nu]}}(\alpha_s)\,\frac{x_L^{\gamma^{\mathcal{N}=4}_{J^{[\nu]}}(\alpha_s)}}{x_L}}$$

Behavior of Projected Correlators

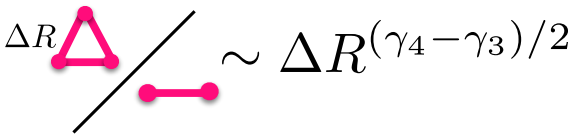
- Generalizes the two point correlator to an infinite family of single logarithmic (groomed mass like) observables.



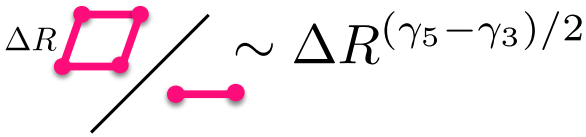
Ratios

- Multiple observables of same family $\implies$ can take ratios!
- Ratios of correlators offer a particularly robust observable.

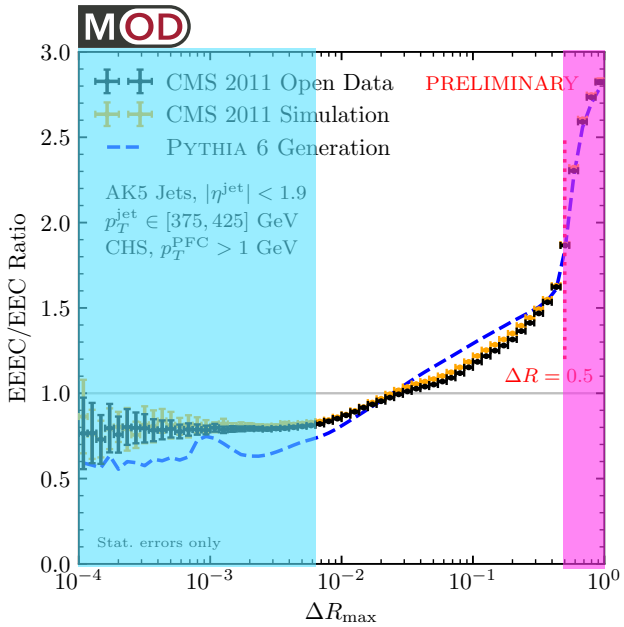
Scaling Behavior ✓

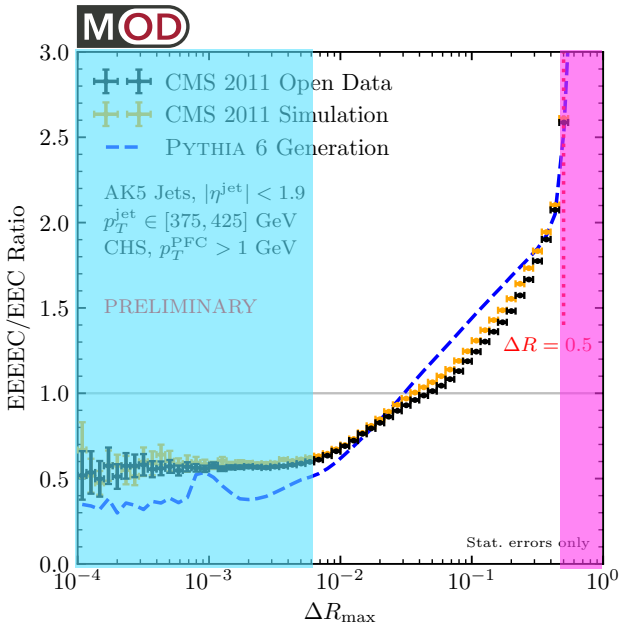


$\sim \Delta R^{(\gamma_4-\gamma_3)/2}$



$\sim \Delta R^{(\gamma_5-\gamma_3)/2}$

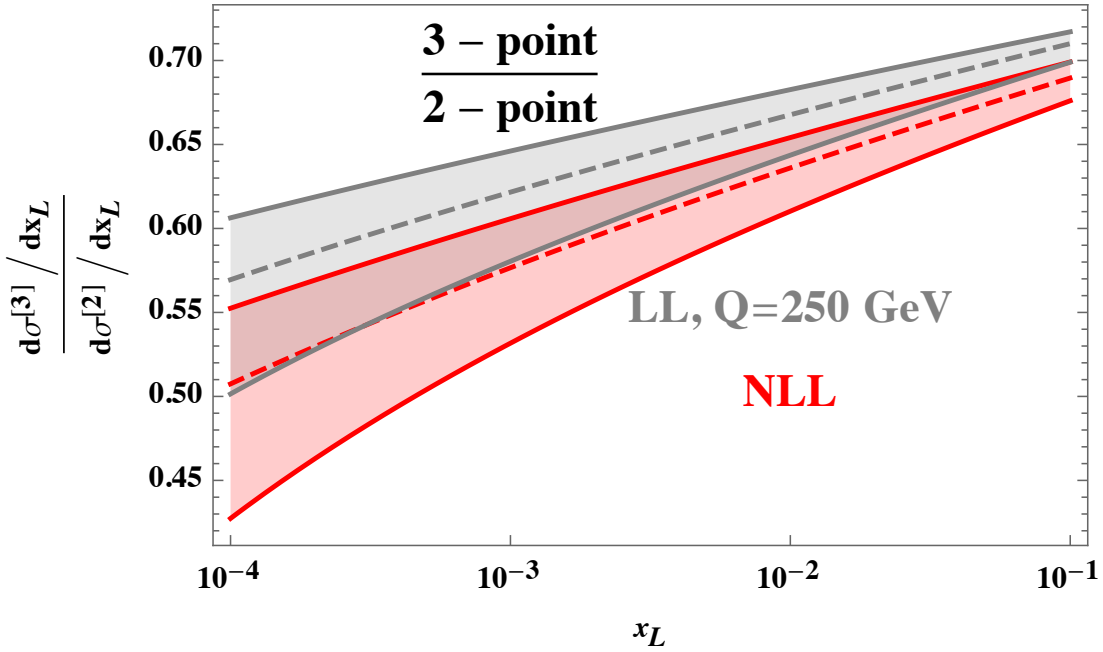




- Slope is directly proportional to α_s.

$3/2$ Ratio at NLL

- Starting to look at phenomenology: $3/2$ point ratio for quark jets.



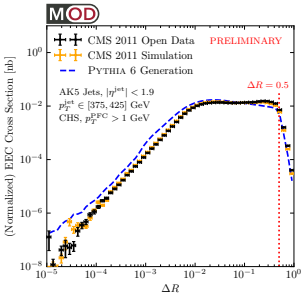
(scale variation is by a factor of 5 instead of the standard 2)

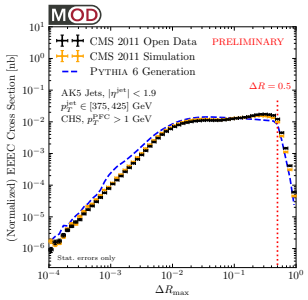
- Hope to extend to NNLL (single log) very shortly. We are missing one number, preliminary tests show significant further reduction in scale variation.

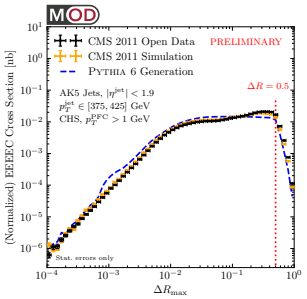
Towards Phenomenology for All j

- NNLL for all j realistic goal in near future. Should be the goal for "scaling" observables.

$j = 3$ $\quad$ $j = 4$ $\quad$ $j = 5$ $\quad$ $j \to \infty$







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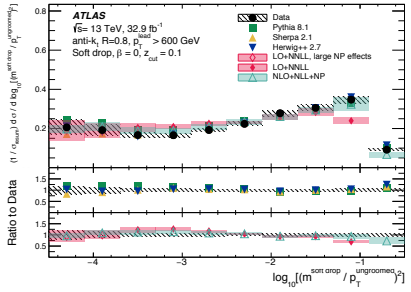
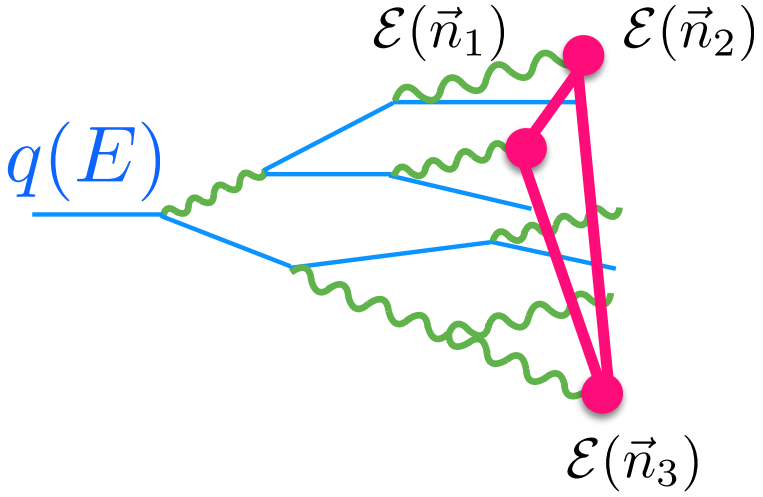


Finite spin DGLAP, q/g flavor changing splitting and evolution

Wilson Lines, Cusp, Collinear Anom Dims

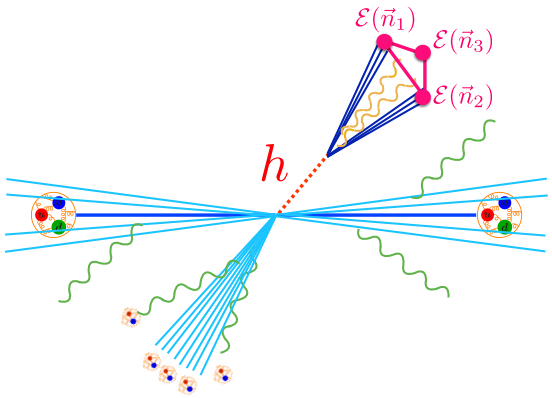
- Would greatly extend physics probed in precision jet substructure!

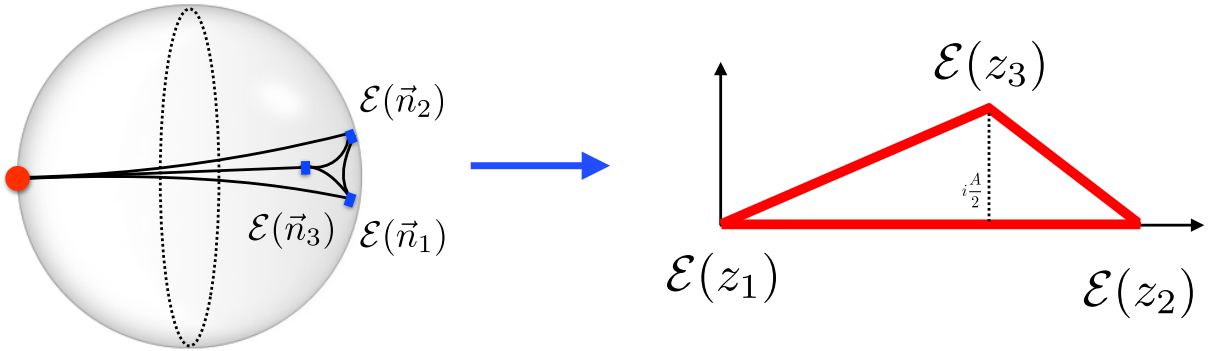
Beyond Scaling: Multi-Differential/Multi-Point



Multi-Point Energy Correlators

- Full shape dependence of higher point correlators probes detailed aspects of the underlying theory.
- Multipoint correlators are used in many jet substructure searches.
- Stereographically project to plane to deal with complex analysis instead of vectors.

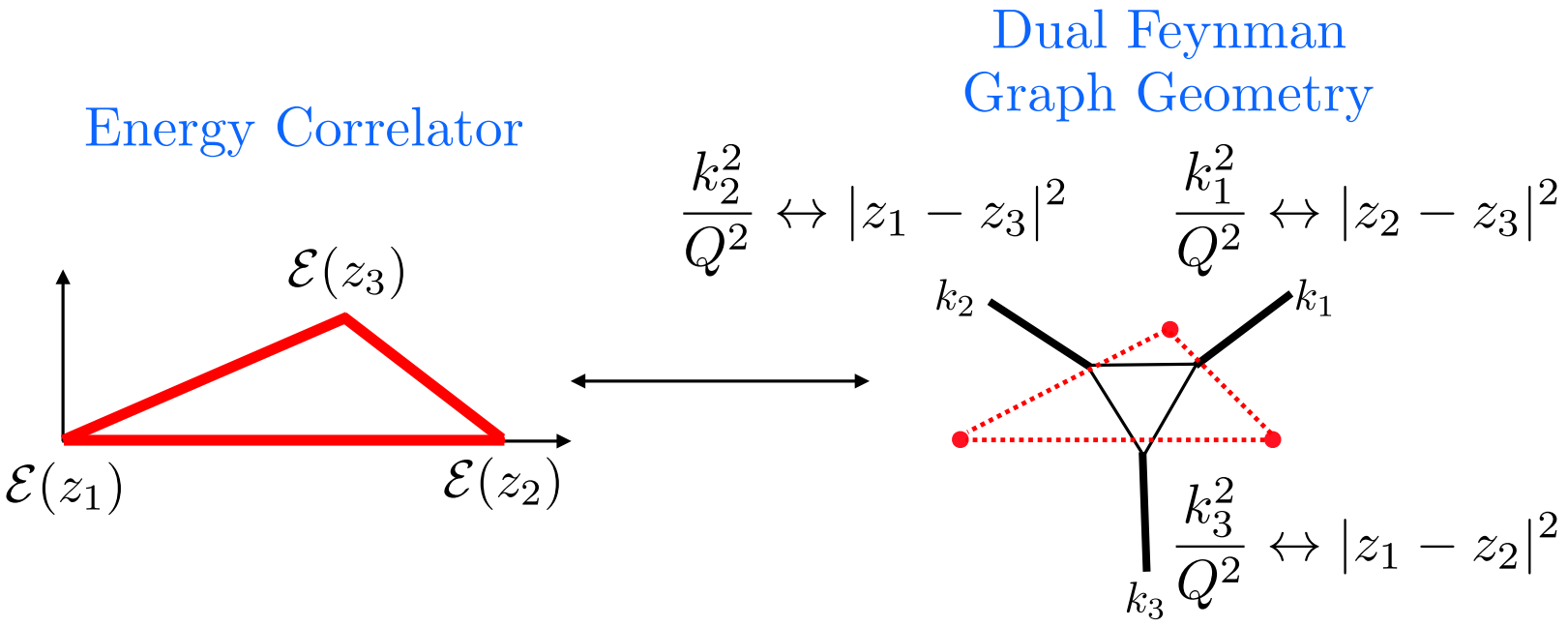




Recent Progress in Understanding Multipoint Correlators

- First calculations of multipoint correlators achieved thorugh representations as a dual Feynman loop integral, where the $|z_{ij}|^2$ are the dual coordinates:

$$x_i^\mu - x_{i+1}^\mu = p_i^\mu \,, x_{ij}^2 = (x_i - x_j)^2 = (p_i + \cdots p_{j-1})^2 \,,$$
$$x_{ij}^2 \leftrightarrow |z_{ij}|^2$$



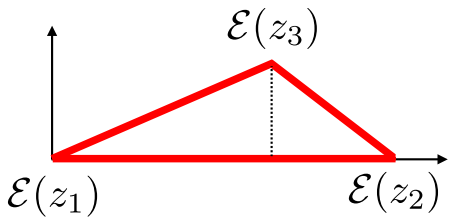
- This allows us to move to a world that we understand much better.

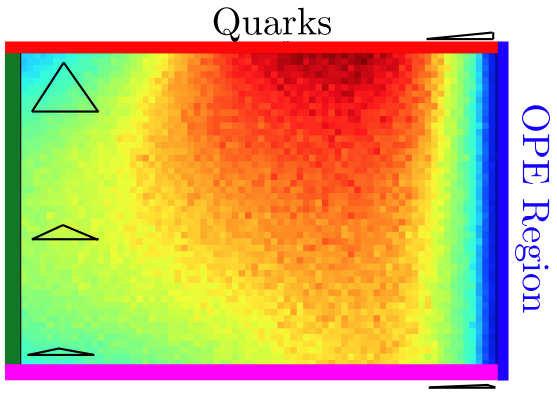
Shape Dependence: Celestial Correlators

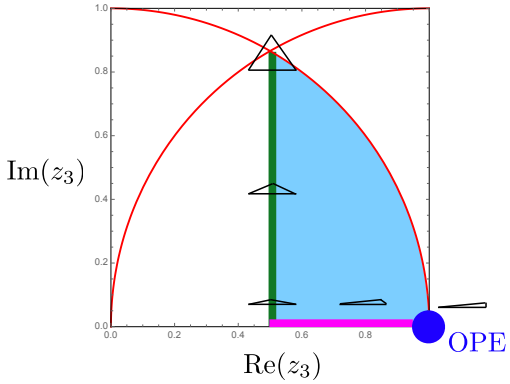
- A remarkably detailed probe of QCD in jets! (dimension associated with overall scale suppressed)

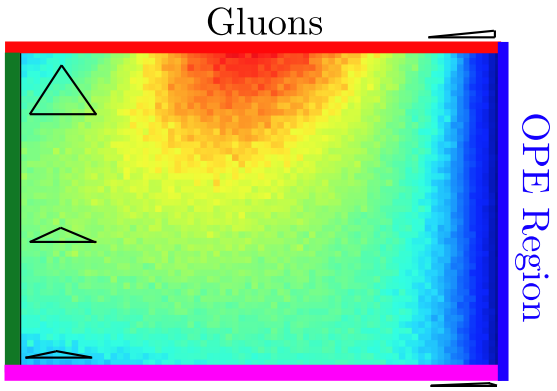
Shape ✓

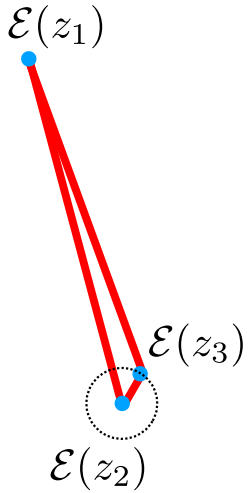
$\langle q(E)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)\rangle$









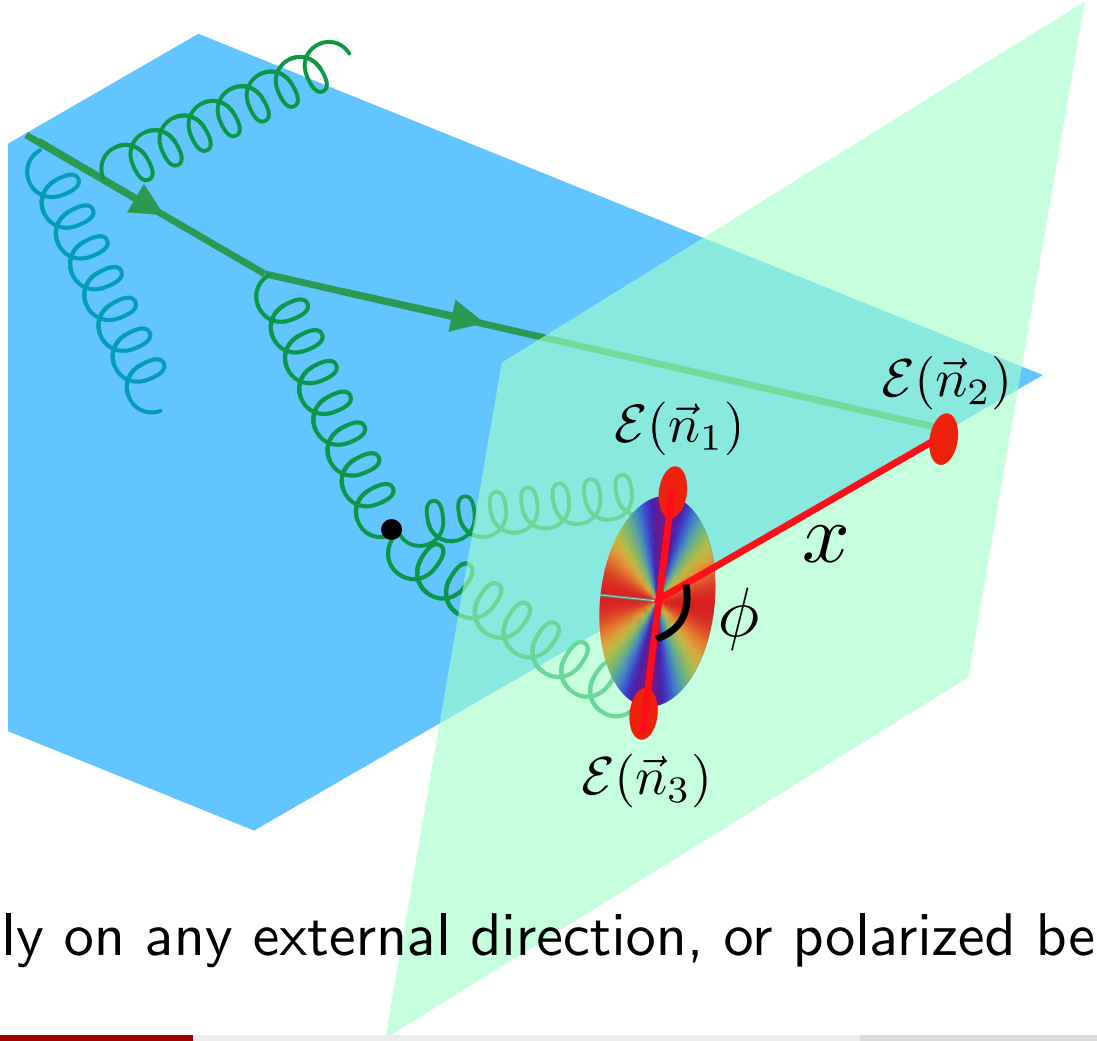


$\langle g(E)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)\rangle$

- Direct probe of full structure of $1 \to 3$ splitting.

Seeing Quantum Interference with Spinning Gluons

- Multi-point correlators allow for new opportunities: rotate the squeezed pair to reveal a $\cos(2\phi)$ interference pattern in the detector!

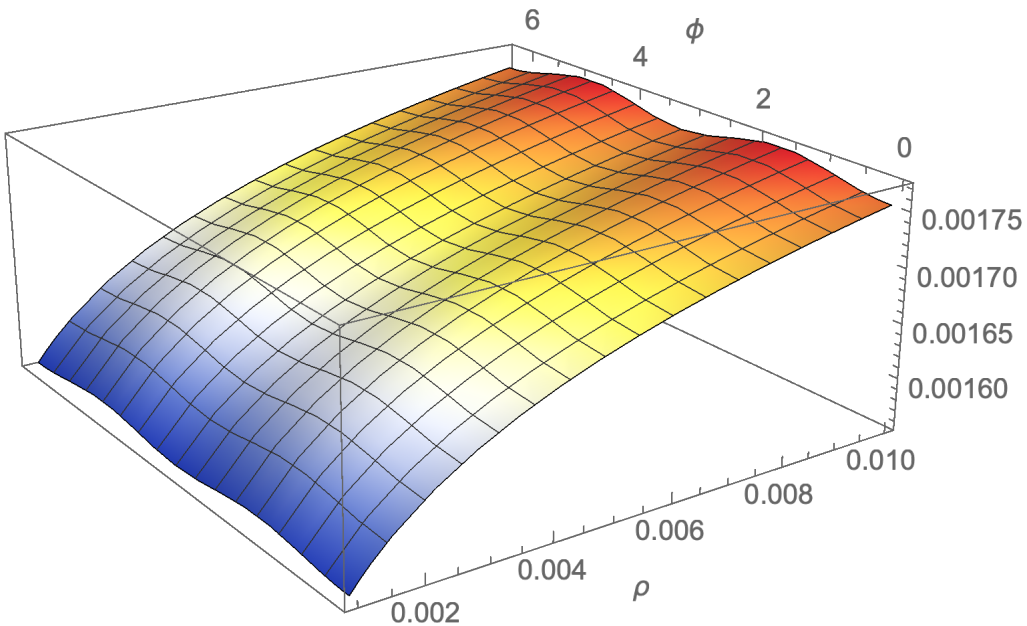


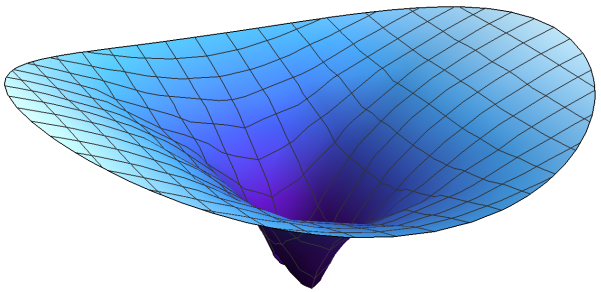
- Does not rely on any external direction, or polarized beams.

Resumming Interference

- Resummation is controlled by gluonic operators with transverse spin 2
  $\Longrightarrow$ probe of gluon spin structure!

$$\mathrm{Tr} F^{-(i_\perp} (iD^-)^{m-2} F^{-j_\perp)}$$





- Would be remarkable to probe experimentally!

The Lund Plane

- Characterize a single emission with two variables:

Past Boosts

2018: (Primary) Lund Plane density

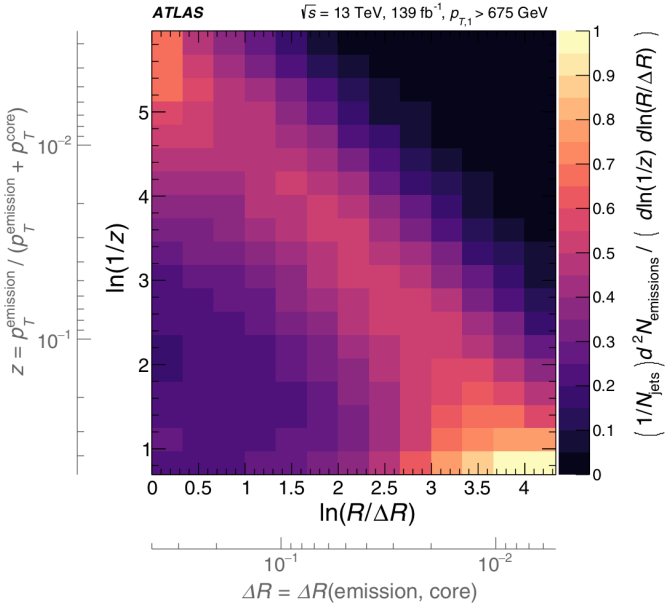
- Recluster jet constituents with Cambridge/Aachen
- Iteratively decluster $j \to j_1, j_2$
- At each step, measure

$$\Delta = \sqrt{y_{12}^2 + \phi_{12}^2},$$
$$k_t = \min(p_{t1}, p_{t2})\Delta,$$
$$z = \frac{\min(p_{t1}, p_{t2})}{p_{t1} + p_{t2}}$$

Idea: characterise radiation in jets

2019: measured by ATLAS

$$\tilde{\rho}(\Delta, kz) = \frac{1}{N_{\text{jets}}} \frac{dn_{\text{emissions}}}{d\ln 1/\Delta \, d\ln z}$$



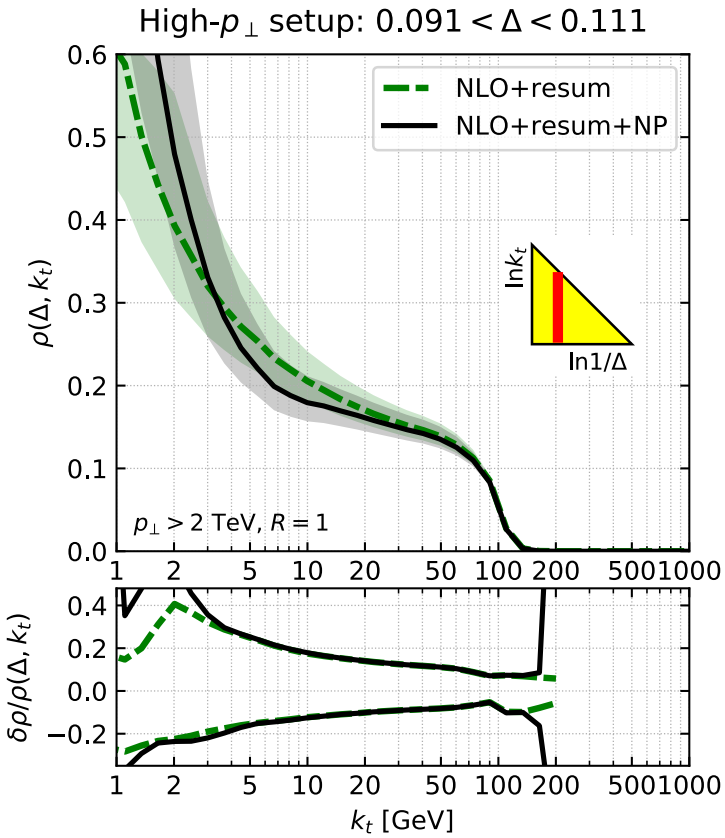
arXiv:2004.03540; CERN-EP-2020-030

Gregory

- Slices through the Lund Plane have been analytically calculated:

Boost 2020: calculating the Lund Jet Plane in QCD

Step-by-step inspection:



- Resummation of single logs
- Exact $\mathcal{O}(\alpha_s^2)$ from NLOJet++
- Match NLO+resum
- Non-perturbative corrections from Monte-Carlo

Pythia8(3 tunes), Herwig7 & Sherpa2

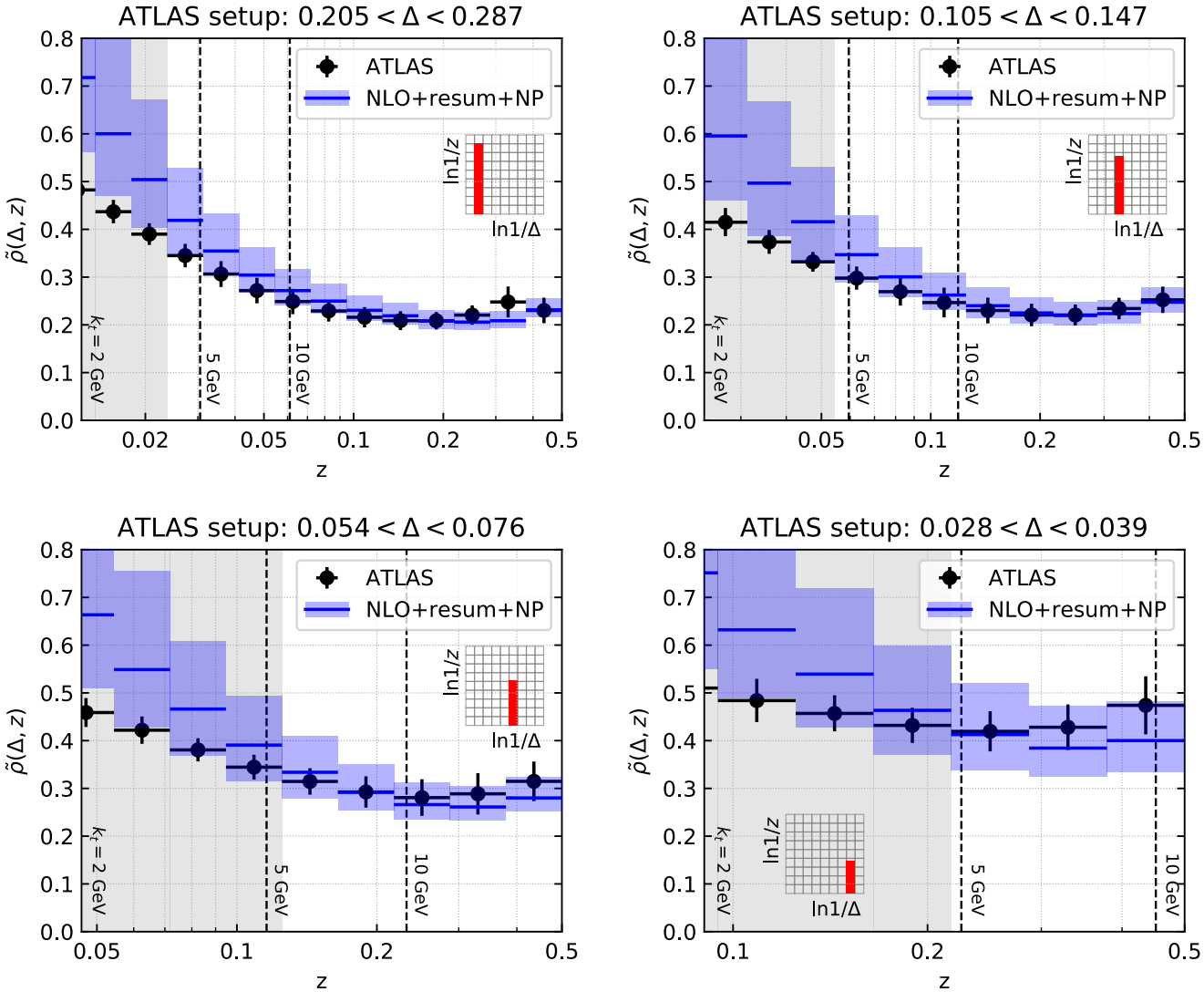
- Relevant below $k_t \sim 20-30$ GeV
- significant uncertainty below $k_t \sim 3-5$ GeV

Overal uncertainty:

5−7% at large k_t,

$\sim 20\%$ at $k_t \sim 5$ GeV

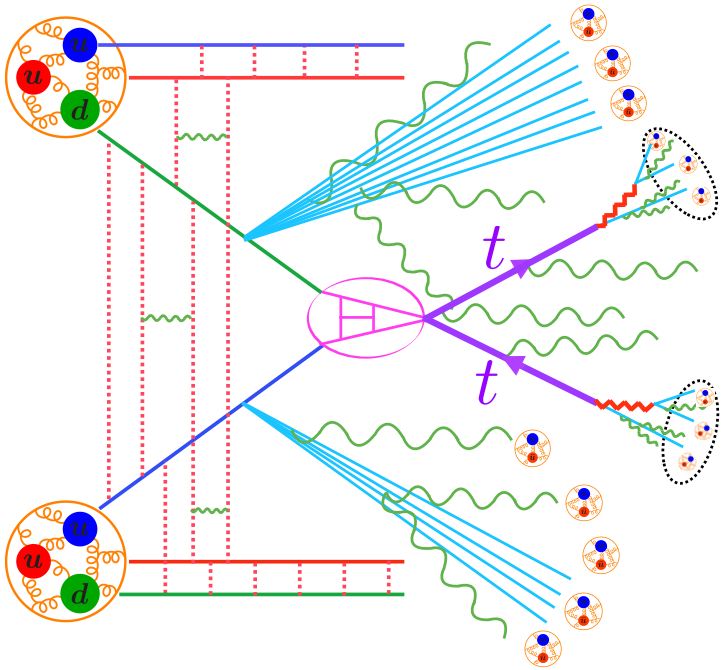
QCD predictions v. ATLAS



good agreement for $k_t \gtrsim 5$ GeV

- extra non-pert correction due to charged tracks
- gray indicates large non-pert uncert.

Jets from Massive Particles



Top Quark Mass

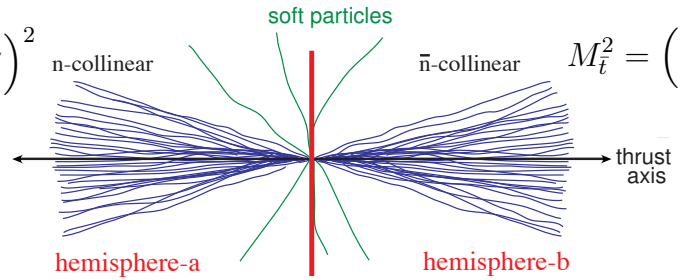
- For sufficiently inclusive event (jet) shape observables, top mass can be given a precise meaning through factorization formulas:

$$e^+e^- \to t\bar{t} \qquad \frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}$$

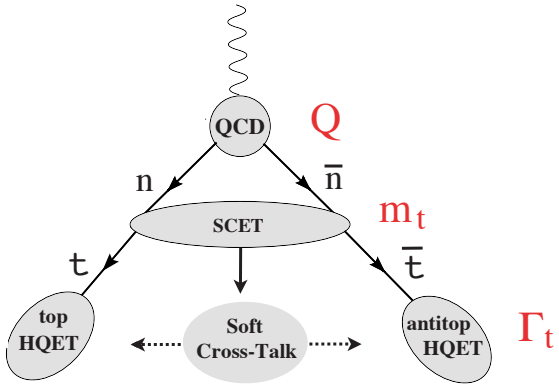
Peak Region:

$$M_{t,\bar{t}}^2 - m^2 \sim m\Gamma \ll m^2$$

$$M_t^2 = \Big(\sum_{i\in a} p_i^\mu\Big)^2$$



$$M_{\bar{t}}^2 = \Big(\sum_{i\in b} p_i^\mu\Big)^2$$



Factorization Theorem:

$$\Big(\frac{\mathrm{d}\sigma}{\mathrm{d}M_t^2 \mathrm{d}M_{\bar{t}}^2}\Big)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m)\, H_m\Big(m_J, \frac{Q}{m_J}, \mu_m, \mu\Big)$$

$$\times \int \mathrm{d}l^+ \mathrm{d}l^-\, J_B\Big(\hat{s}_t - \frac{Ql^+}{m_J}, \Gamma_t, \delta m, \mu\Big)\, J_B\Big(\hat{s}_{\bar{t}} - \frac{Ql^-}{m_J}, \Gamma_t, \delta m, \mu\Big)$$

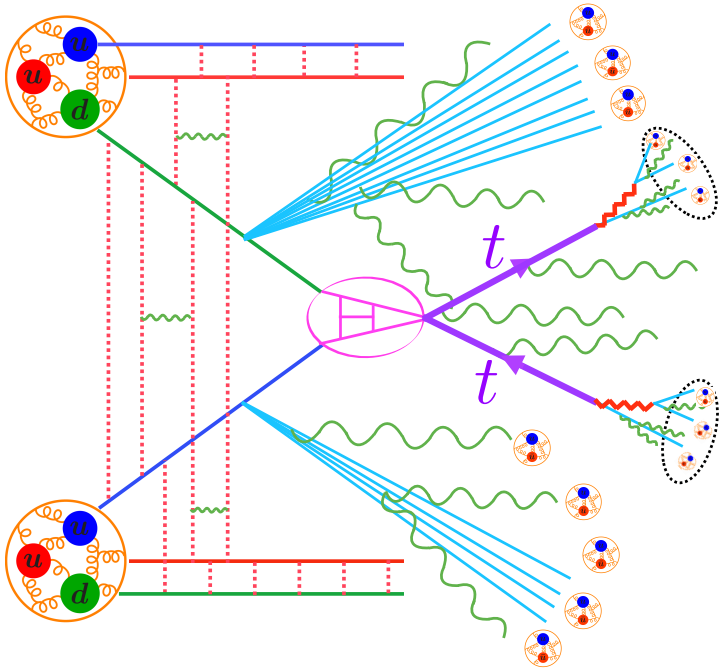
$$\times\, S_{\text{hemi}}(l^+ - k, l^- - k', \mu)\, F(k,k')$$

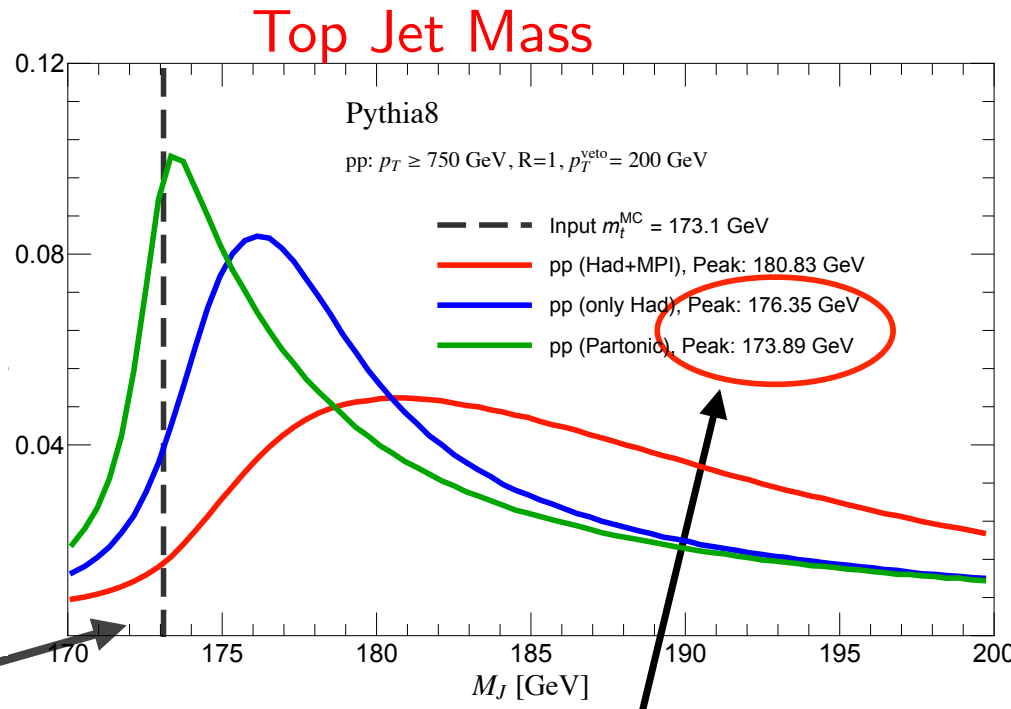
(boosted HQET) Jet Functions — Control Over Mass Scheme — Soft Function — Hadronization

Fleming, Hoang, Mantry, Stewart 2007

Top Jets in pp

- The complication of such an inclusive measurement at a hadron collider is the large contamination.





Input mass in Pythia
m_t = 173.1 GeV

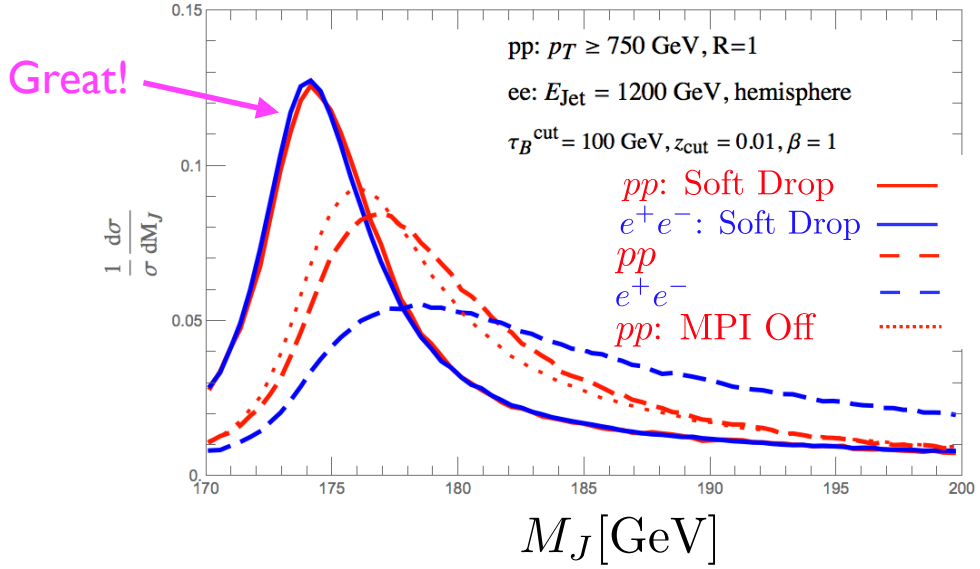
Significant contamination

[Hoang, Mantry, Pathak, Stewart]

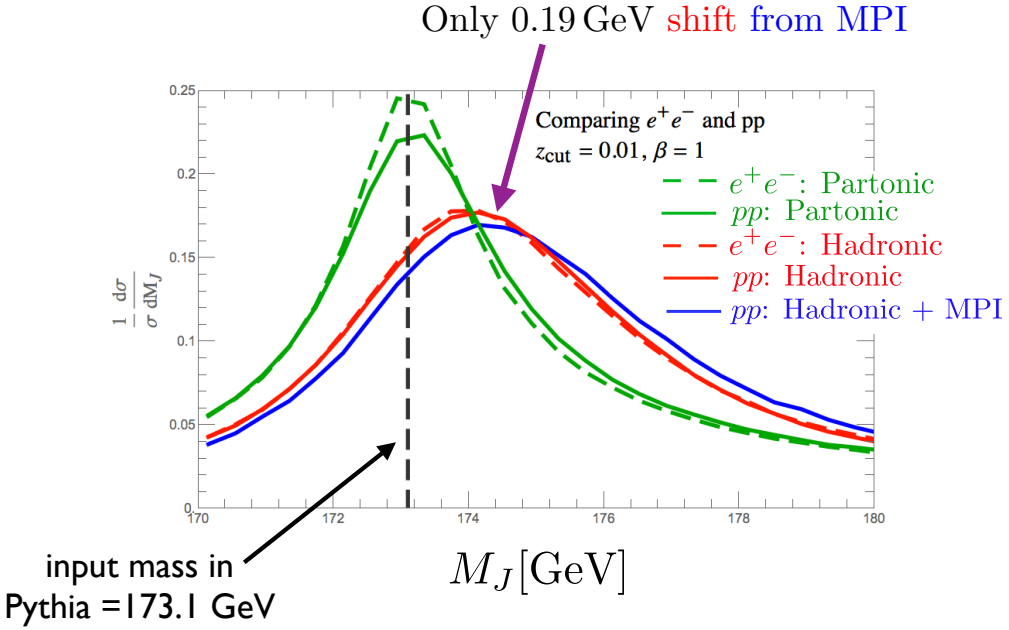
Soft Drop Top Mass

- Grooming techniques can be used to achieve a precision top mass measurement from jet substructure.
  - Hadronization uncertainty reduced ✓
  - Poorly understood contribution from MPI minimal ✓

Groomed vs. Ungroomed



Soft Drop Top Mass



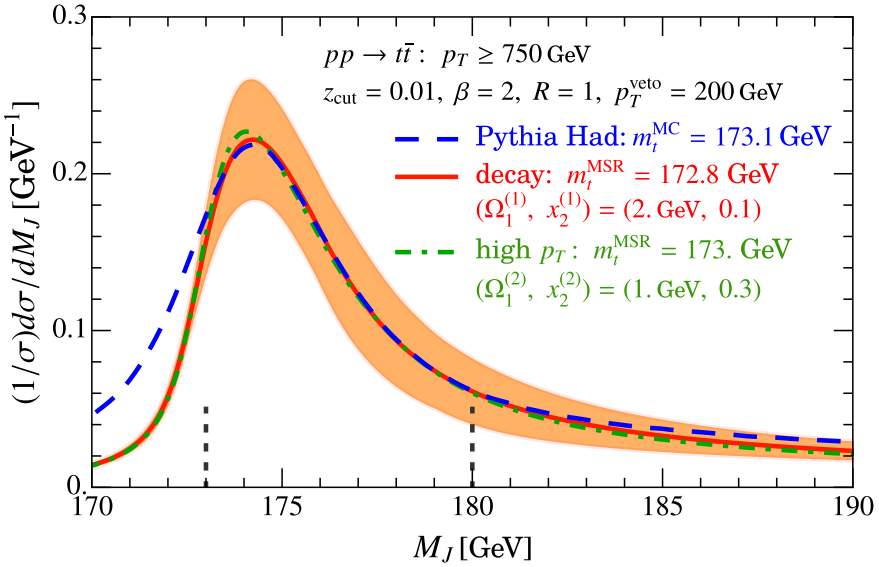
[Hoang, Mantry, Pathak, Stewart]

Soft Drop Top Mass

- All orders factorization theorem for Soft Drop Top Mass in pp:

$$\frac{d^2\sigma}{dM_J^2 d\mathcal{T}^{\rm cut}} = {\rm tr}\big[\hat{H}_{Qm}\hat{S}(\mathcal{T}^{\rm cut}, Qz_{\rm cut}, \beta, \ldots)\otimes F\big]\otimes J_B \otimes \mathcal{I}\mathcal{I}\otimes ff$$
$$\times\left\{\int d\ell dk\, J_B\Big(\hat{s}_t - \frac{Q\ell}{m}, \Gamma_t, \delta m\Big) S_C\left[\ell - \Big(\frac{k^{2+\beta}}{2^\beta Q z_{\rm cut}}\Big)^{\frac{1}{1+\beta}}, Qz_{\rm cut}, \beta\right] F_C(k)\right\}$$

Soft Drop Top Mass NLL

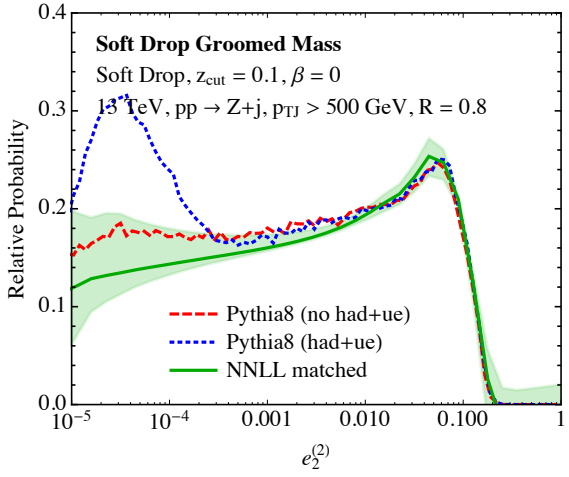


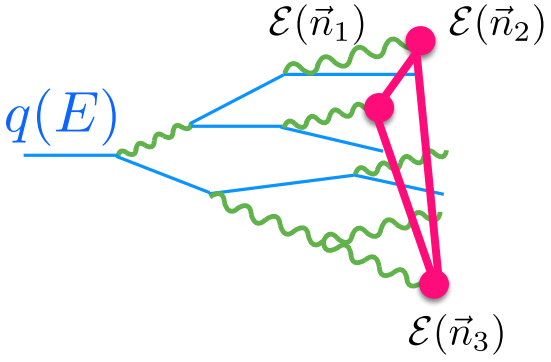
- Calculation can be extended to NNLL for a precision top mass extraction.

[Hoang, Mantry, Pathak, Stewart]

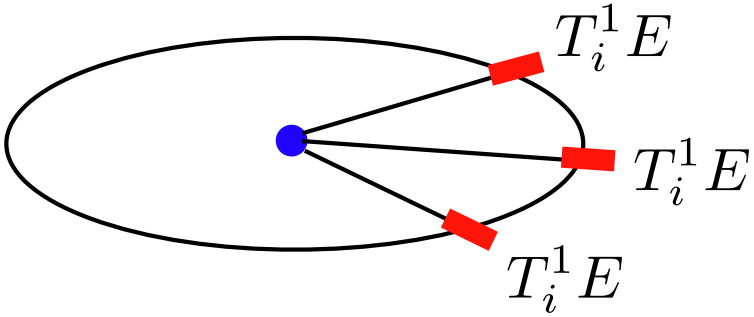
Summary

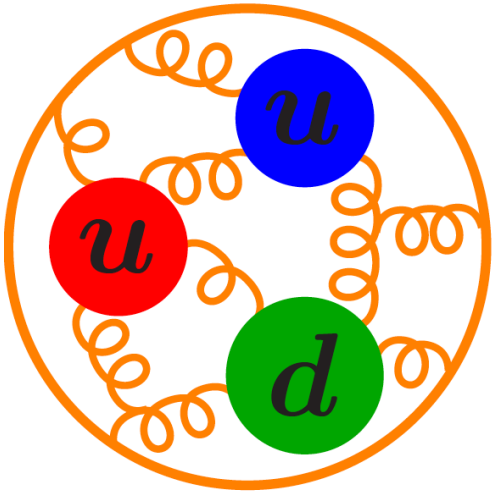
- Precise predictions for "scaling" observables: can now probe complete set of twist 2 anomalous dimensions associated with splitting functions.
- Push towards multi-point correlations in jets due to improved analytic understanding.
- Probing desired physics with theoretically simpler observables.
- Better understanding of jets from massive particles.



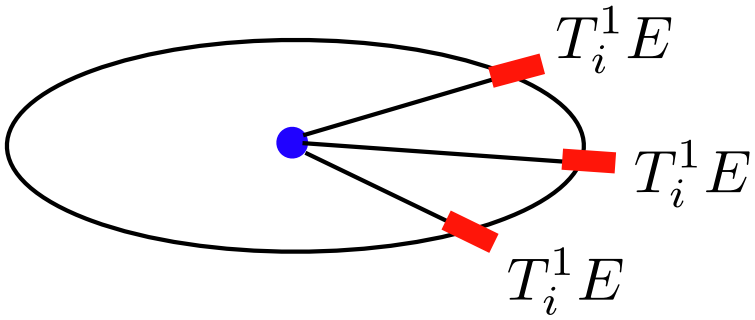


Relate Experimental, Phenomenological, and Theoretical Issues Associated with Making Measurements



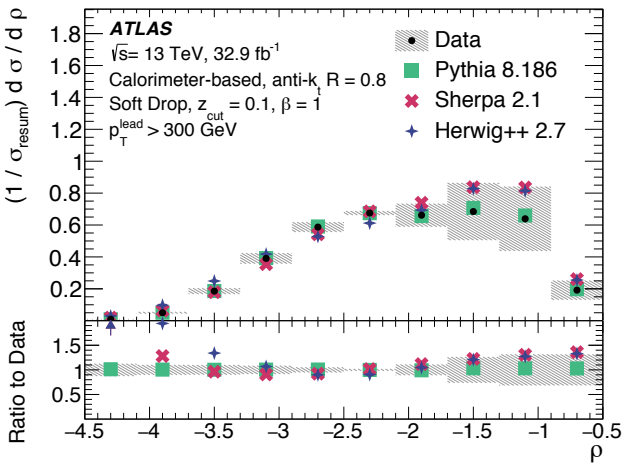


Tracking Information

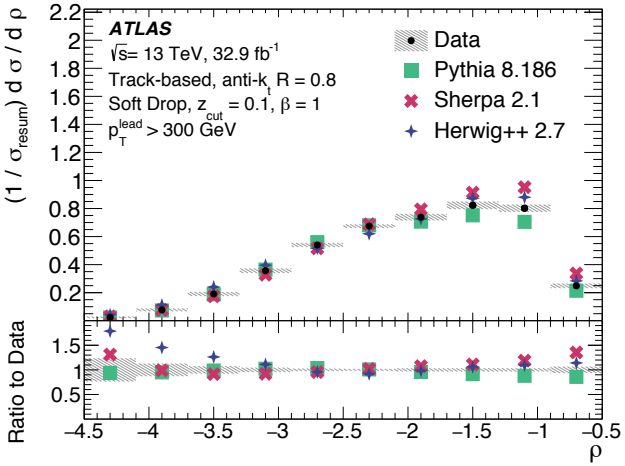


Tracks

- Tracks offer many experimental advantages: better precision, better angular resolution, etc.
- This is already true for "simple" observables like mass:



(c) $\beta = 1$, calorimeter-based



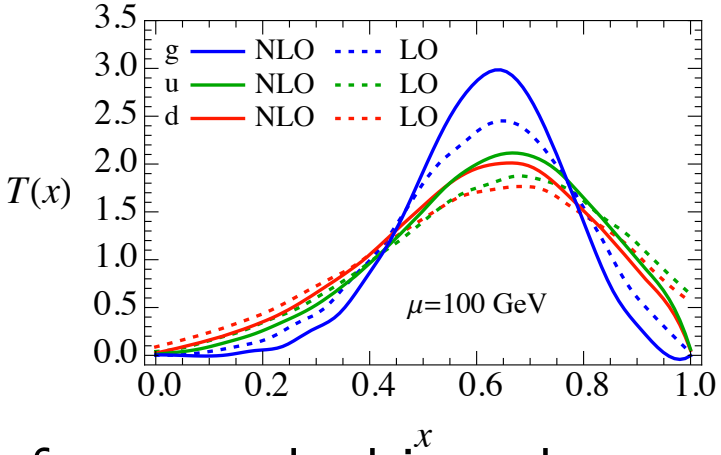
(d) $\beta = 1$, track-based

- Essential to move to multidifferential/ higher point correlations.
- Also becomes more important with increased pile up (e.g. HL-LHC).
- Incorporating tracking in higher order pertubative calculations is absolutely crucial to advance sophistication of jet substructure.

Track Functions

- Calculations on tracks are not IRC safe.
- There is an elegant formalism for incorporating tracks (Chang, Procura, Waalewijn, Thaler 2013) using Track Functions, $T_i(x)$.
- Track functions are a non-perturbative function describing energy fraction of a parton going into tracks, $\bar{p}_i^\mu = x p_i^\mu + \mathcal{O}(\Lambda_{\text{QCD}})$. (Analogous to a fragmentation function).

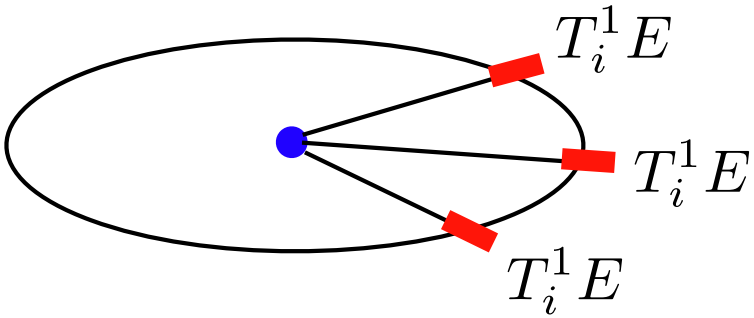
$$\int_0^1 dx\ T_i(x,\mu) = 1$$



- Evolution is perturbative.
- Unfortunately, phase space constraints from standard jet substructure observables (e.g. mass) do not interface well with tracks $\implies$ has not been used.

Tracks and Energy Correlators

- Energy correlators are weighted by energy flow through detector cells as a function of angle.
- How to go from full calorimeter to tracks? simply multiply by "average energy deposited into tracks".



$$\boxed{\mathsf{EEC}^{\mathsf{tr}}(z) = (T_q^{(1)})^2 I_1(z) + 2 T_q^{(1)} T_g^{(1)} I_2(z)}$$

$$E_i \to \int dx_i \, x_i T_i(x_i) E_i = T_i^{(1)} E_i$$

- Upshot: Any perturbative calculation of energy correlators that can be done, can also be done on tracks just by weighting pieces of calculation by $T_i^{(1)}$! (higher moments only appear as contact terms)

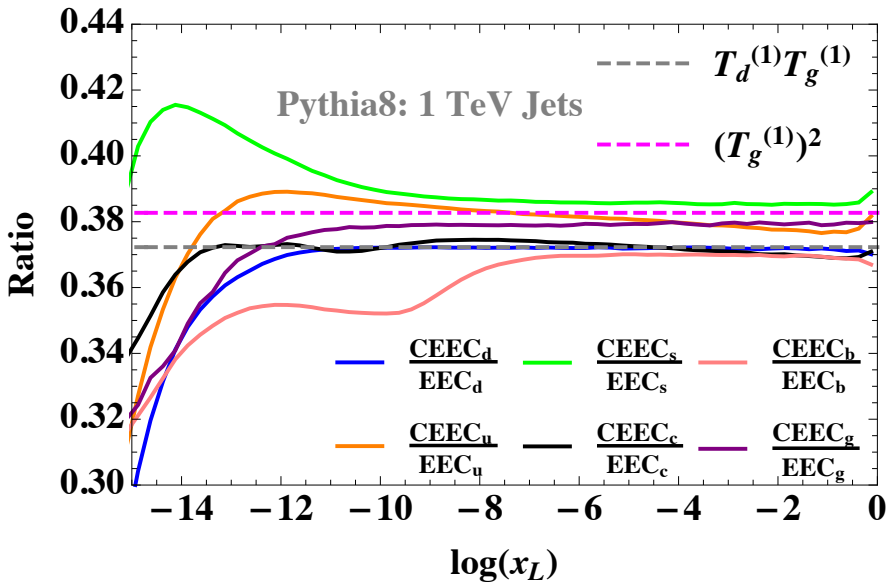
Tracks and Resummation

- Interfaces nicely with resummation. e.g.Two point correlator at LL for pure gluons:

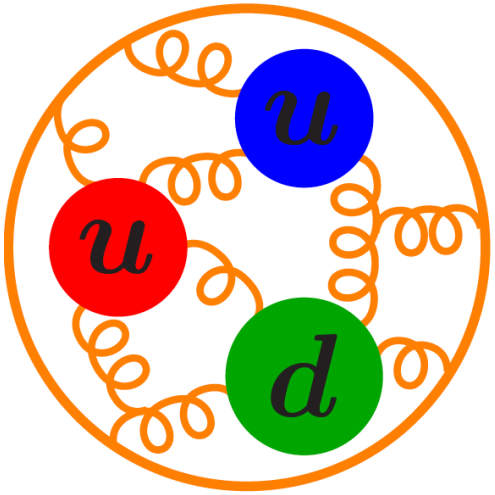
$$\Sigma^{[2]}(x_L) = \frac{1}{2}\left(\frac{\alpha_s(\sqrt{x_L}Q)}{\alpha_s(Q)}\right)^{-\frac{\gamma^{(0)}(3)}{\beta_0}}$$

$$\Sigma^{[2]}_{\text{tr}}(x_L) = \frac{1}{2}[T_g^{(1)}(Q)]^2\left(\frac{\alpha_s(\sqrt{x_L}Q)}{\alpha_s(Q)}\right)^{-\frac{\gamma^{(0)}(3)}{\beta_0}}$$

- With both quarks and gluons there is a matrix, but still straightforward...



Hadronization

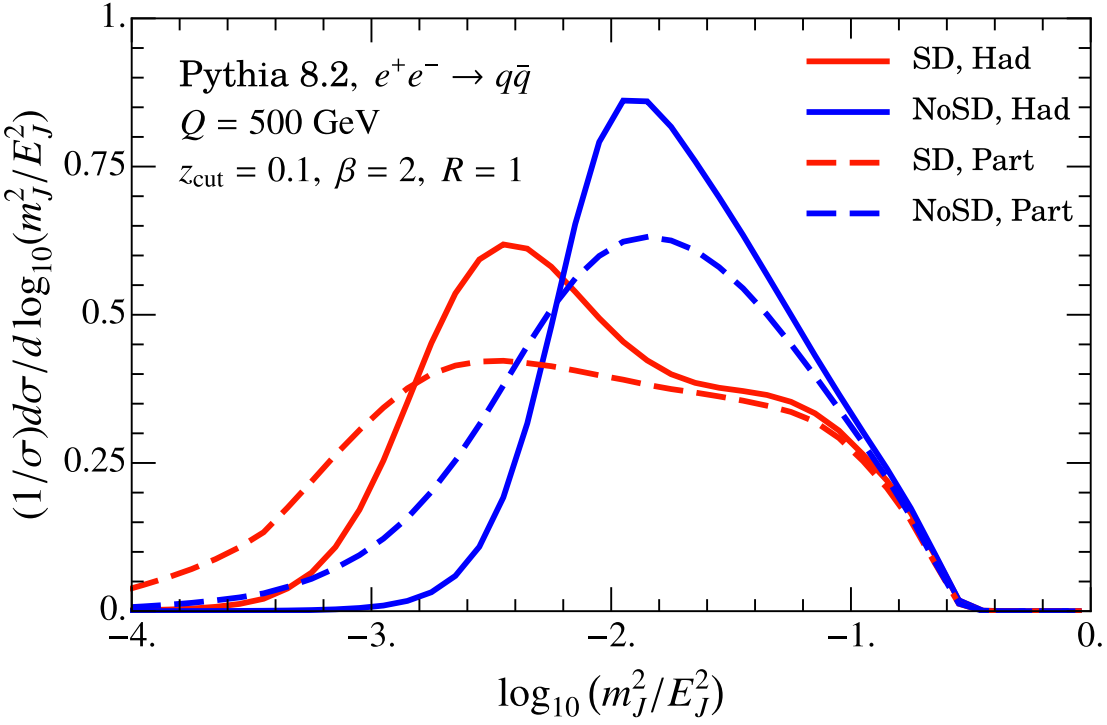


Hadronization

- Experimentalists make measurements in real world with hadrons, not quarks and gluons $\implies$ non-perturbative effects can be large, particularly for jet substructure observables that probe small scales within high energy jets.
- At this point, there are loosely three approaches:
  - Reduction via Grooming Algorithm
  - Reduction via Choice of Observable
  - First Principles Understanding

Reduction via Grooming Algorithm

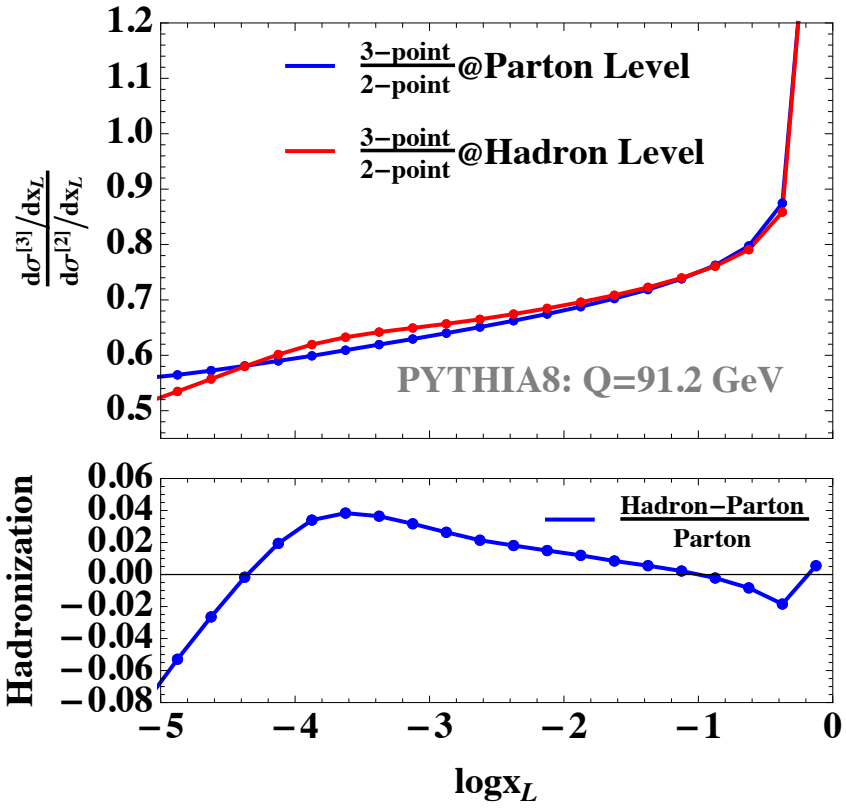
- Hadronization effects can be significantly reduced by using a grooming algorithm. This is a general approach for any scaling observable.

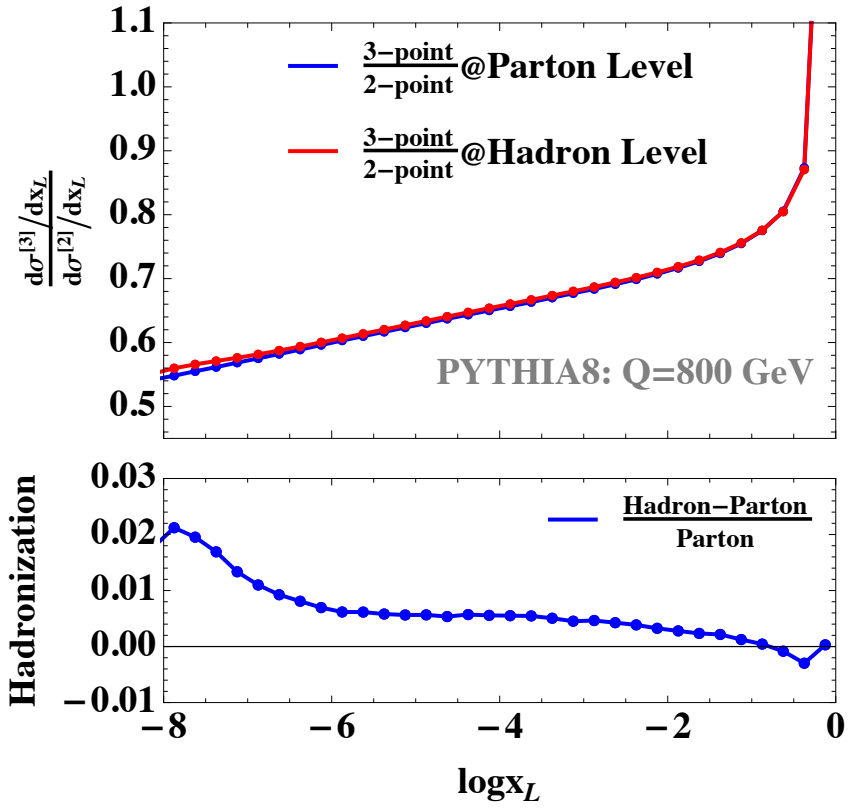


- Recent work towards improved understanding NP effects in Groomed Mass (Stewart, Pathak, Hoang).

Reduction via Choice of Observable

- Ratios of projected correlators have $\leq$ percent level NP corrections:

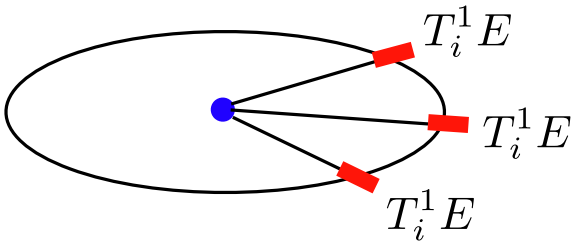




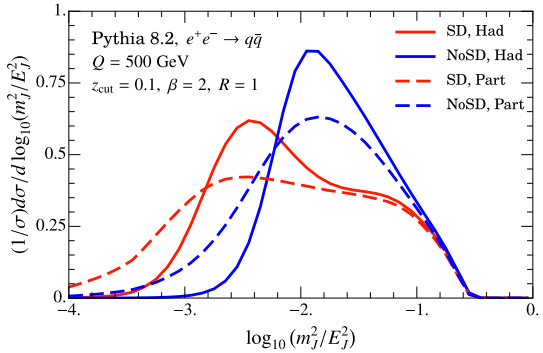
- Observable specific approach, but useful for e.g. precision α_s.

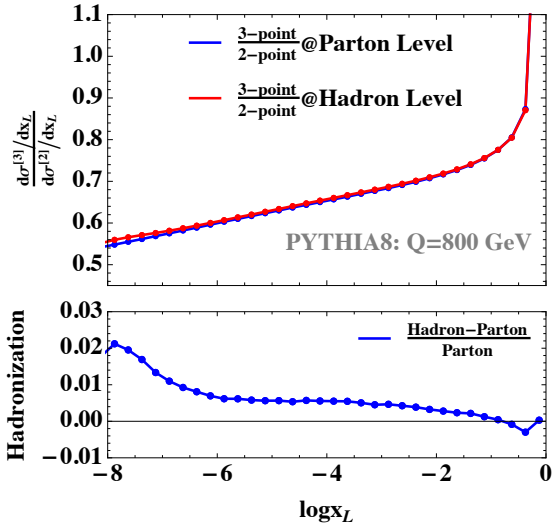
Summary

- (multi-point) Energy Correlator based observables can be computed on tracks.

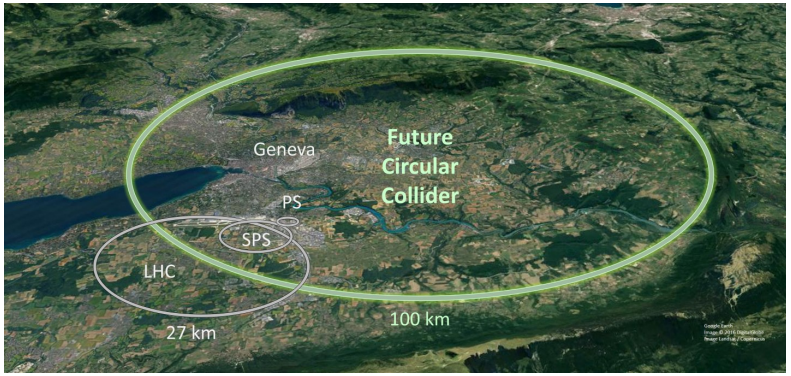


- Multiple approaches to reducing sensitivity to sensitivity to NP effects for precision QCD measurements:
  - Grooming Algorithms
  - Choice of Observable





Opportunities at Future Colliders



Future Colliders

- The study of jet substructure, originally motivated by improving our ability to search for new physics at the LHC, has revolutionized our theoretical and experimental understanding of jets. The study of QCD at any future collider will be strongly influenced by this.

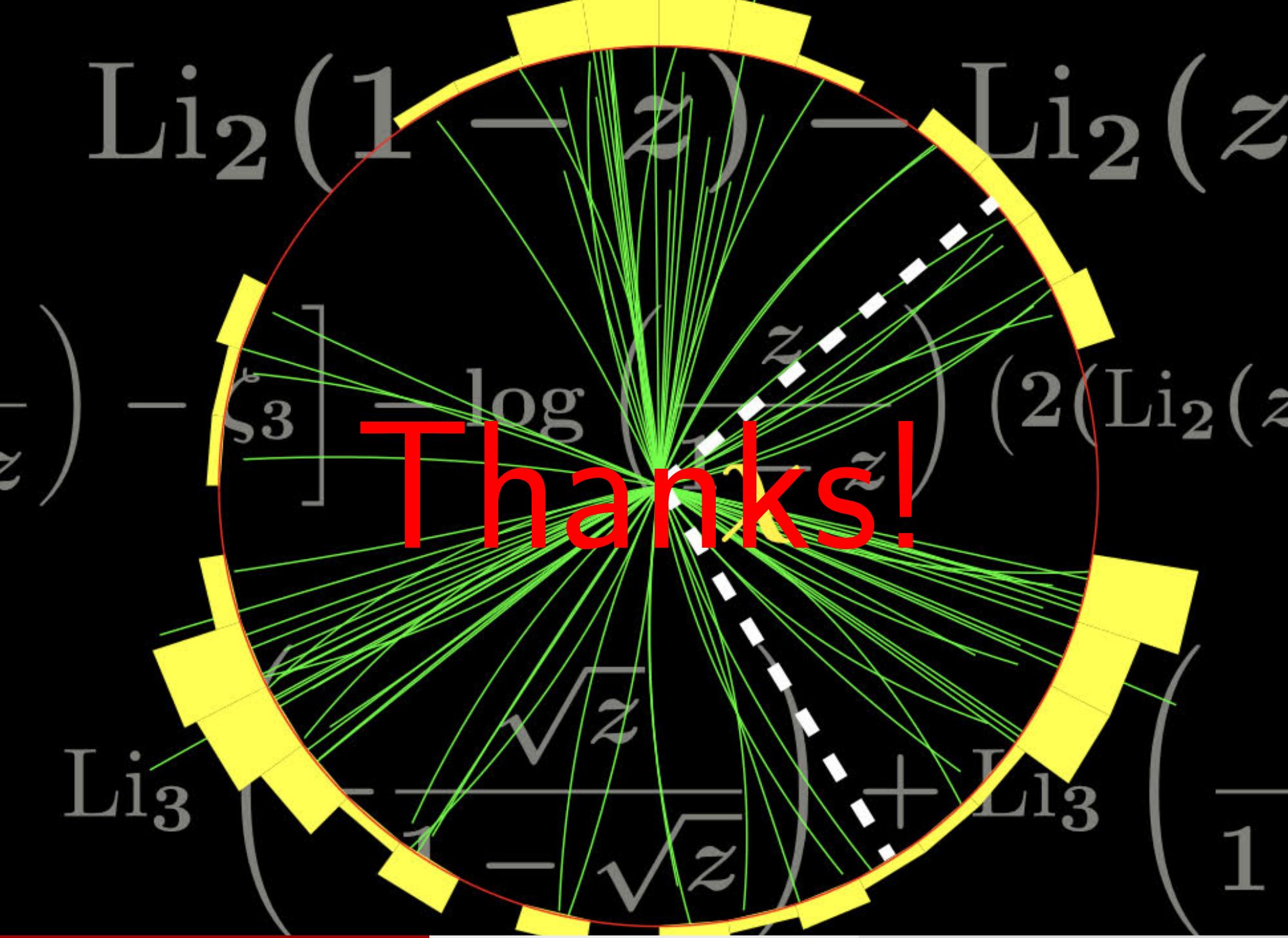
- The main lesson learned from jet substructure is how to construct observables that are sensitive to specific physics effects, and the theory/ experiment techniques to realize these. This is a paradigm shift that is more important than any particular case study.

Future Colliders

- Jet substructure for studying QCD is in its infancy. Only most basic observables have been studied. Only most basic scaling observables have been studied $\implies$ Huge room for improvement even just with standard LHC/ HL LHC.
- Higher energy colliders provide huge improvement:
  - More perturbative substructure.
  - More boosted top quarks.
  - Smaller NP effects.
- $e^+e^-/\mu^+\mu^-$ colliders would enable all these techniques to be applied in a clean context.
- Study of event shapes on samples of Higgs decays enable one to probe light quark Yukawas.
- Electron-Hadron/ electron-ion colliders can exploit jet substructure to probe nuclear structure (much recent work related to EIC)
- $\ldots$

Conclusion

- In addition to its utility as a search strategy, jet substructure provides a window into the dynamics of quantum field theory on the lightcone.
- Provides new ways to measure SM parameters, such as α_s and m_t
- Very Exciting Progress in Understanding the Field Theory Underlying Jet Substructure in the Past Year, and Hopefully Much More to Come!


Thanks!