

Radiative Muon Capture in the Mu2e-II Era

Mu2e-II Workshop

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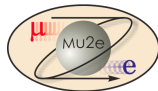
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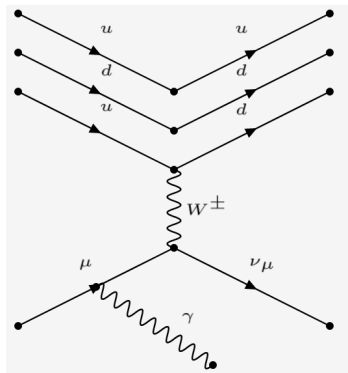
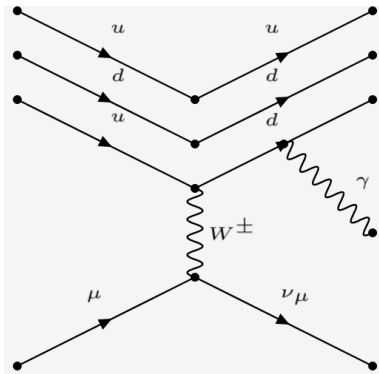
September 23, 2020



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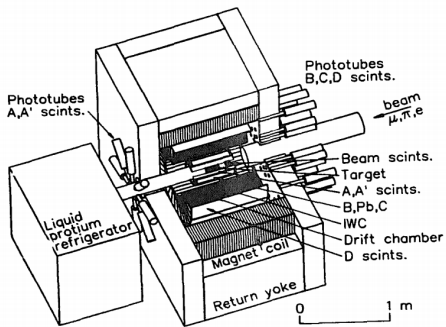


Introduction

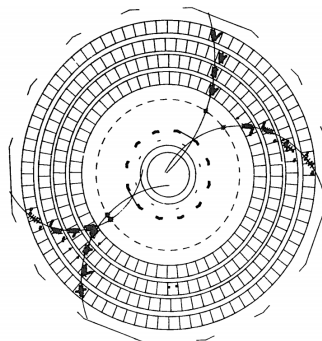


- Radiative muon capture (RMC) is an intrinsic background at Mu2e and Mu2e-II, where photon conversions (both in materials and virtual conversions) can mimic signals
- The maximum photon energy kinematically allowed is dependent on the nuclear target
- As the $\mu^- \rightarrow e^-$ conversion energy is greater than the $\mu^- \rightarrow e^+$ conversion energy, this is typically a more significant background for the latter search
- This talk summarizes our current understanding of RMC and its potential impacts for Mu2e-II

TRIUMF RMC spectrometer



(a) TRIUMF detector

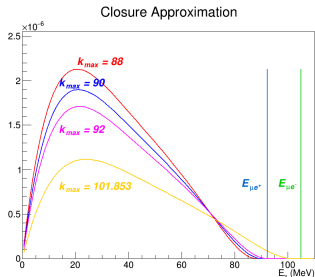


(b) TRIUMF tracking

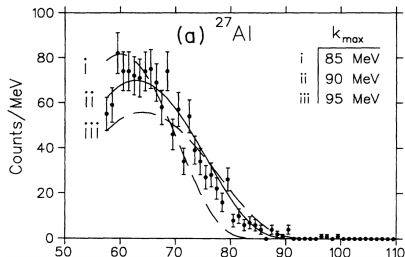
- In the 1990's, the TRIUMF collaboration measured the RMC spectra of 13 nuclear targets
- They used a tracking spectrometer to measure the photon energies, using a thin lead foil to convert the photons into e^\pm pairs that were then reconstructed
- By requiring the e^\pm tracks to be consistent with a conversion occurring in the lead converter, they were only reconstructing the real photon spectra for RMC
- They were not sensitive to the virtual photon conversions that would occur in the stopping target
- The TRIUMF data is the largest RMC statistics available

(a,b) Wright et al. (1992), docdb-21926

TRIUMF RMC measurement on Al



(a) RMC spectra



(b) 1992 TRIUMF Al data

- The RMC photon energy spectrum is modeled by the closure approximation:

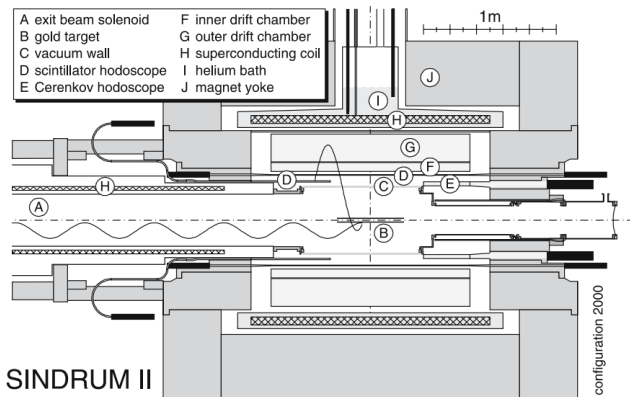
$$\frac{dN}{dx} = \frac{e^2}{\pi} \frac{k_{\max}^2}{m_\mu^2} (1 - \alpha) \cdot (1 - 2x + 2x^2) \cdot x \cdot (1 - x)^2$$

where $x = E_\gamma/k_{\max}$ and $\alpha = (N - Z)/A$ (Christillin (1979), [docdb-1224](#))

- For example, TRIUMF measured on aluminum:
 - ▶ $k_{\max} = 90.1 \pm 1.8$ MeV (though we found $\sigma_{k_{\max}} \sim 0.4\text{-}0.5$ MeV)
 - ▶ $\text{Br}(\text{RMC} > 57 \text{ MeV}) = 1.43 \cdot 10^{-5} \pm 0.13$
- They found the endpoint on all of the targets was several MeV below the kinematic limit
- The closure approximation is a simple model though, used to predict the total rates and the bulk of the spectrum shape, not to model the high momentum region accurately

(b) *Armstrong et al. (1992)*, [docdb-1192](#)

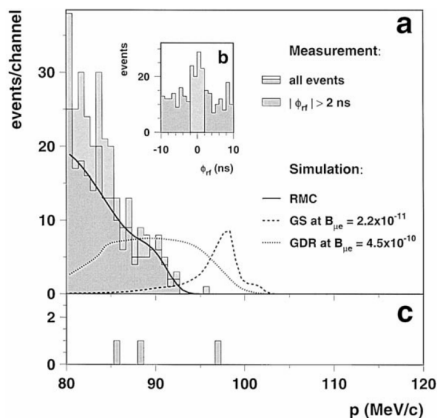
SINDRUM-II experiment



- SINDRUM-II was the previous muon conversion experiment where the current upper limits come from
- RMC was a background for them, where they had higher statistics than TRIUMF near the endpoint of the spectrum
- SINDRUM-II only saw the high momentum e^{\pm} from RMC conversions, but was therefore sensitive to both the real and virtual photon contributions to RMC

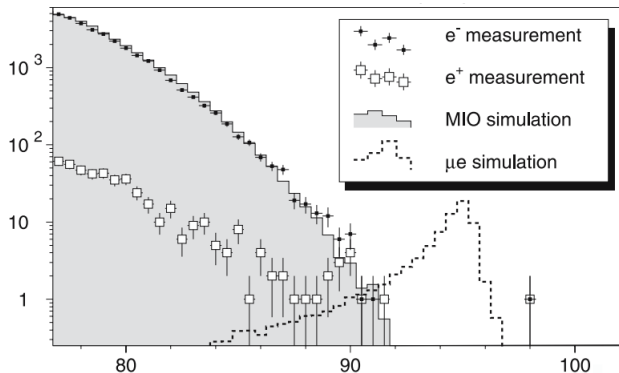
SINDRUM-II (2006), [docdb-688](#)

SINDRUM-II Ti data



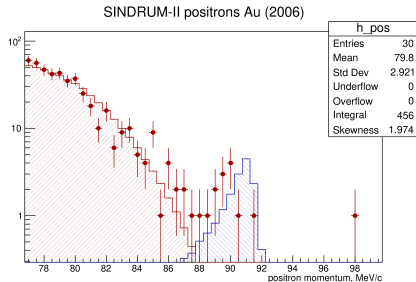
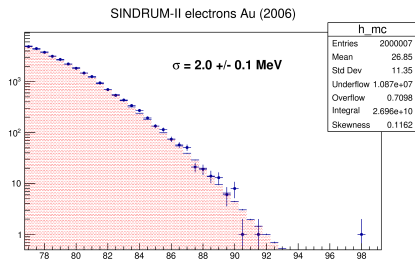
- The current $\mu^- \rightarrow e^+$ limits in the field of a nucleus come from SINDRUM-II's Ti data:
 - ▶ $Br(\mu^- Ti \rightarrow e^+ Ca) < 1.7 \cdot 10^{-12}$ (GS)
- In analyzing their RMC data, they found it wasn't well described by the closure approximation alone
- They added a contribution corresponding to the $^{48}Ti(\mu^-, \nu_\mu \gamma)^{48}Sc(0^+, 6.68 \text{ MeV})$ reaction, not considered by their closure approximation
- The transition added had a branching fraction smaller than would be seen by TRIUMF

SINDRUM-II Au data



- The most stringent $\mu^- \rightarrow e^-$ limits in the field of a nucleus come from SINDRUM-II:
 - ▶ $Br(\mu^- Au \rightarrow e^- Au) < 7 \cdot 10^{-13}$
- There was about $2\times$ more muon stops in the Au data than the 1997 Ti data but no $\mu^- \rightarrow e^+$ limit was published, though they did publish the positron data
- Pasha and I worked on understanding the positron spectrum, using the electron spectrum to create a parameterized detector response model

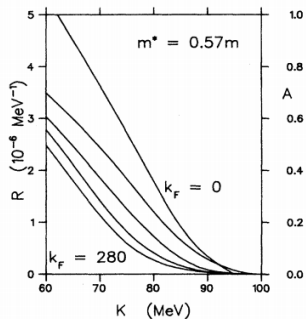
RMC at SINDRUM-II



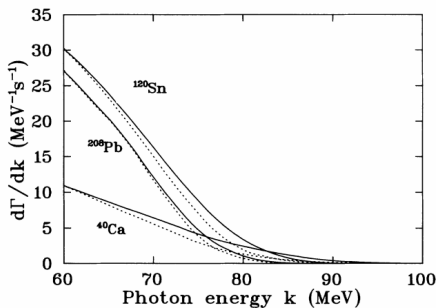
(a) Detector response parameterization (b) RMC fit and the $\mu^- \rightarrow e^+$ signal

- We analyzed their data using the DIO data to tune a mock detector response model, which was able to describe their data well (plot (a))
- There is an excess in the tail of the positron spectrum, not addressed by the authors
- **This excess is not explained by the closure approximation**
- The excess is not described well by $\mu^- \rightarrow e^+$ though ($p = 0.004$ for fit a in [89,92] MeV)
- **A possible explanation is exclusive transitions not considered by the closure approximation**
- The branching ratio of this transition would be too small for TRIUMF to see, similar to what SINDRUM-II proposed to describe the Ti positron data
- A background like this may not be removed by data driven side-band background modeling

(a,b) Details in [docdb-31019](#) and [arXiv:2009.00214](#)



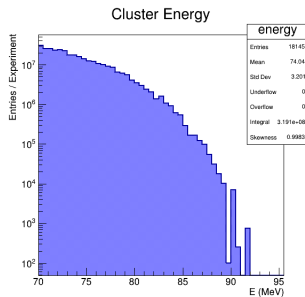
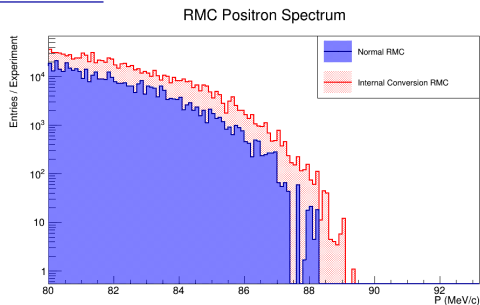
(a) RMC spectra vs k_{Fermi}



(b) Example RMC spectra

- Fearing et al. used a Fermi gas nuclear model to study the RMC spectra rather than the closure approximation
- Using this, there is no hard cutoff put in for the photon energy as previously used
- The initial plots studied appear to show a long tail in the high energy region
- Richard Hill (Associate Professor), Ryan Plestid (Post-doc), and Kaushik Borah (Ph.D. student) from the University of Kentucky are working on a relativistic Fermi gas model to understand the RMC spectra

(a) Fearing et al. (1989), [docdb-35219](#) (b) Fearing et al. (1992), [docdb-35222](#)



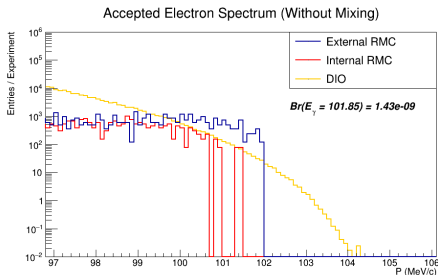
(a) Positron spectrum (AI)

(b) Photon clusters (AI)

- Internal (virtual) conversions will also contribute to the background, where **the branching ratio and spectrum for internal RMC conversions is unmeasured for all nuclei**
- The only measurement available is of internal radiative pion capture (RPC) on hydrogen, where $\text{Br}(\text{internal RPC})/\text{Br}(\text{RPC}) = 0.00694 \pm 0.00031$ (Samios, 1961)
- Assuming this fraction and spectrum, there would be $\approx 2 \times$ more positrons from internal RMC conversions than from external (real) RMC conversions in Mu2e
- The shape of the virtual photon energy and mass distributions are also unknown, though Richard, Ryan, and Kaushik are studying this as well
- (b) shows the RMC photon cluster distribution without triggering (which has been studied by Castiglia [docdb-31354](#), Harrington [docdb-28540](#), Miscetti et al. [docdb-22780](#), etc.) which can be used to measure the real photon spectrum

Implications for $\mu^- \rightarrow e^-$ on Al

- RMC assuming the measured closure approximation spectrum can be a significant background for the $\mu^- \rightarrow e^+$ search, but should not significantly impact the $\mu^- \rightarrow e^-$ search on Al
- The true RMC spectrum may not be so kind though, as SINDRUM-II has seen RMC contributions not explained by the closure approximation



- The kinematic photon energy limit on aluminum is 101.85 MeV (Compton electrons can have up to $E \approx k + \frac{m_e}{2}$ and conversion electrons/positrons can have up to $E \approx k - m_e$)
- TRIUMF RMC spectrometer only had sensitivity to, for example, exclusive photon energy transitions above the measured endpoint with $Br(\text{transition})/Br(\text{RMC} > 57 \text{ MeV}) > 10^{-3} - 10^{-4}$
- For a branching ratio of 10^{-4} at $E_\gamma = 101.85 \text{ MeV}$, this would be a **significant background to the DIO measurement**

Target dependence

- The RMC background is highly sensitive to the choice of the nuclear target
- Different nuclear targets will have different energy differences between the RMC kinematic endpoint and the $\mu^- \rightarrow e^\pm$ conversion energies
- For targets like Al, the kinematic endpoint is far enough below the conversion energy to not be a significant background for the $\mu^- \rightarrow e^-$ (but may affect the DIO measurement)
- Trade off in this case is the $\mu^- \rightarrow e^+$ conversion energy is far below the $\mu^- \rightarrow e^-$ conversion energy and the RMC kinematic endpoint, making this measurement much more difficult
- For other targets, the three endpoint energies can be much closer, where RMC can be a significant background to both searches
- For $\mu^- \rightarrow e^\pm$, a target with $M(Z,A)$ and $M(Z-2,A) < M(Z-1,A)$ would be the best candidate, as RMC would be at a lower energy for both searches (e.g. $M(^{40}\text{Ca}) < M(^{40}\text{K})$ and $M(^{40}\text{Ar}) < M(^{40}\text{K})$)
- Another consideration for the target choice is that due to Mu2e and COMET, aluminum will be the only target with high enough RMC statistics to understand what to expect from the high energy RMC tail

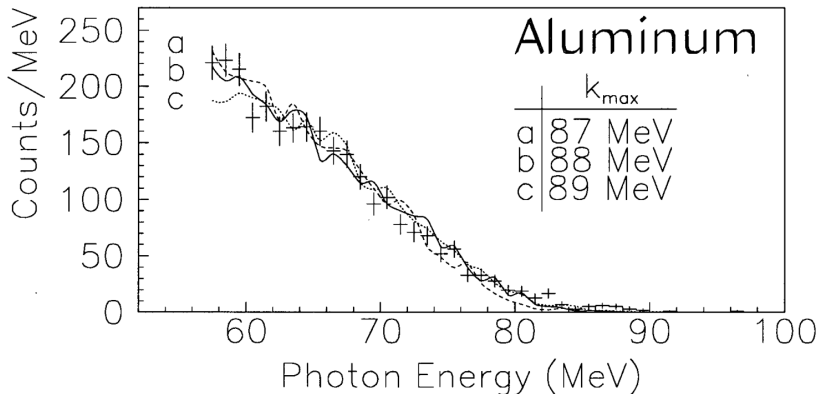
Summary

- The RMC spectrum on several nuclei was measured by TRIUMF, with endpoint values several MeV below the kinematic endpoints
- The results published by TRIUMF have several inconsistencies, such as systematically over estimated uncertainties
- SINDRUM-II found that the Ti RMC spectrum was not well described by the closure approximation alone, and added an exclusive transition on top of it to describe their data
- The positron spectrum for a gold target from SINDRUM-II's 2006 paper has an excess in the high momentum tail which is not addressed by the authors
- **This excess is inconsistent with the closure approximation**, and is further evidence there can be exclusive transitions near the end point
- **Fearing et al. predictions appear to have high momentum tails, beyond what the closure approximation would suggest**
- We're currently using the internal conversion spectrum from RPC on hydrogen

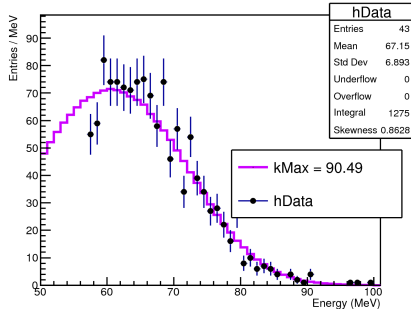
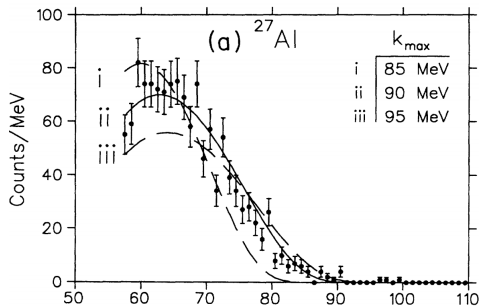
Implications for Mu2e-II

- RMC is arguably the most uncertain background for both Mu2e and Mu2e-II
- The high energy tail of the RMC real and virtual spectra are unknown, and could significantly vary between nuclear targets
- Considering an exclusive transition on aluminum at the endpoint with a rate allowed by the current data leads to the DIO background spectrum being significantly altered
- Richard, Ryan, and Kaushik are working on understanding the real and virtual spectra for RMC on nuclei
- RMC has many unknowns, but Mu2e will offer the chance to understand it better due to the much higher statistics
- The Mu2e-II target should be chosen with RMC in mind, and we should utilize the insight gained from Mu2e when making the target choice (Ti for example does a decent job, where for the kinematic limit $\Delta E(e^-, RMC) \sim 4$ MeV and $\Delta E(e^+, RMC) \sim -0.3$ MeV)

Backup slides



- Bergbusch published the Al RMC spectrum in his M.S. thesis ([docdb-21294](#))

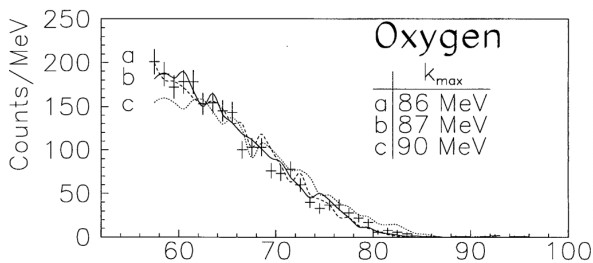


(a) TRIUMF Al data and folded spectra (b) Digitized data and our folded spectrum

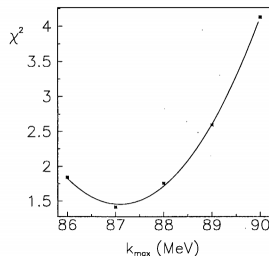
- TRIUMF published their detector response. Unfortunately, they had several inconsistencies making it difficult to understand
- Pasha, Eleonora, and I spent a significant amount of time understanding these, attempting to reproduce their results
- We had some success in reproducing their measured endpoint values for the 13 nuclear targets, but not in reproducing their uncertainties ([docdb-22262](#))

(a) *Armstrong et al. (1992), [docdb-1192](#)*

RMC at TRIUMF



(a) 1995 TRIUMF ^{16}O data



(b) ^{16}O χ^2 fit

- The published RMC endpoint on aluminum from TRIUMF ($k_{\max} = 90.1 \pm 1.8$ MeV) has a large uncertainty, which translates into a large background uncertainty for the $\mu^- \rightarrow e^+$ search
- We found the uncertainties from TRIUMF were **systematically overestimated by defining the uncertainty by $\Delta\chi^2/\text{DoF} = 1$ rather than $\Delta\chi^2 = 1$** (overestimates by $\sim \sqrt{\text{DoF}}$)
- The uncertainty on the endpoint value is more likely 0.4 - 0.5 MeV

(a,b) Bergbusch (1995), [docdb-21294](#)