Neutrino Model Predictions: Importance of Precision

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Understanding the origin of the pattern of neutrino mixing and of neutrino mass squared differences that emerged from the neutrino oscillation data in the recent years is one of the most challenging problems in neutrino physics. It is an integral part of the more general fundamental problem in particle physics of understanding the origins of flavour in the quark and lepton sectors, i.e., of the patterns of quark masses and mixing, and of the charged lepton and neutrino masses and of neutrino mixing.

"Asked what single mystery, if he could choose, he would like to see solved in his lifetime, Weinberg doesnt have to think for long: he wants to be able to explain the observed pattern of quark and lepton masses."

From Model Physicist, CERN Courier, 13 October 2017.

Of fundamental importance are also

- the determination of the status of lepton charge conservation and the nature - Dirac or Majorana - of massive neutrinos (which is one of the most challenging and pressing problems in present day elementary particle physics) (GERDA, CUORE, KamLAND-Zen, EXO, LEGEND, nEXO,...);

- determining the status of CP symmetry in the lepton sector (T2K, NOνA; T2HK, DUNE);

- determination of the type of spectrum neutrino masses possess, or the "neutrino mass ordering" (T2K + NOνA; JUNO; PINGU, ORCA; T2HKK, DUNE);

- determination of the absolute neutrino mass scale, or $\min(m_j)$ (KATRIN, new ideas; cosmology).

The program of research extends beyond 2030.

- **BS3νRM: eV scale sterile ν's; NSI's; ChLFV processes ($\mu \to e + \gamma$, $\mu \to 3e$, $\mu^- - e^-$ conversion on (A,Z)); $\nu-$related BSM physics at the TeV scale (N_{jR}, H^{--}, H^-, etc.).**

Reference Model: 3-ν mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}} \qquad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary.
ν_j, $m_j \neq 0$: Dirac or Majorana particles.

Data: 3 νs are light: $\nu_{1,2,3}$, $m_{1,2,3} \lesssim 0.5$ eV.

3-ν mixing: 3-flavour neutrino oscillations possible.

ν_μ, E; at distance L: $P(\nu_\mu \to \nu_{\tau(e)}) \neq 0$, $P(\nu_\mu \to \nu_\mu) < 1$

$P(\nu_l \to \nu_{l'}) = P(\nu_l \to \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$

Three Neutrino Mixing

$$\nu_{l\mathrm{L}} = \sum_{j=1}^{3} U_{lj}\, \nu_{j\mathrm{L}}\ .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

	n	2	3	4
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6
CP-violating phases:				
• $\nu_j -$ Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• $\nu_j -$ Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and
2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980

PMNS Matrix: Standard Parametrization

$$U = V\,P\,, \qquad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2...$

S.M. Bilenky et al., 1980

- $\Delta m^2_{\odot} \equiv \Delta m^2_{21} \cong 7.34 \times 10^{-5}\ \text{eV}^2 > 0$, $\sin^2\theta_{12} \cong 0.305$, $\cos 2\theta_{12} \gtrsim 0.306\ (3\sigma)$,
- $|\Delta m^2_{31(32)}| \cong 2.448\ (2.502) \times 10^{-3}\ \text{eV}^2$, $\sin^2\theta_{23} \cong 0.545\ (0.551)$, NO (IO) ,
- θ_{13} - the CHOOZ angle: $\sin^2\theta_{13} = 0.0222\ (0.0223)$

F. Capozzi et al. (Bari Group), arXiv:2003.08511.

• $\text{sgn}(\Delta m^2_{\text{atm}}) = \text{sgn}(\Delta m^2_{31(32)})$ not determined

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{31} > 0, \quad \text{normal mass ordering (NO)}$$

$$\Delta m^2_{\text{atm}} \equiv \Delta m^2_{32} < 0, \quad \text{inverted mass ordering (IO)}$$

Convention: $m_1 < m_2 < m_3$ - NO, $\quad m_3 < m_1 < m_2$ - IO

$$\Delta m^2_{31}(NO) = -\,\Delta m^2_{32}(IO)$$

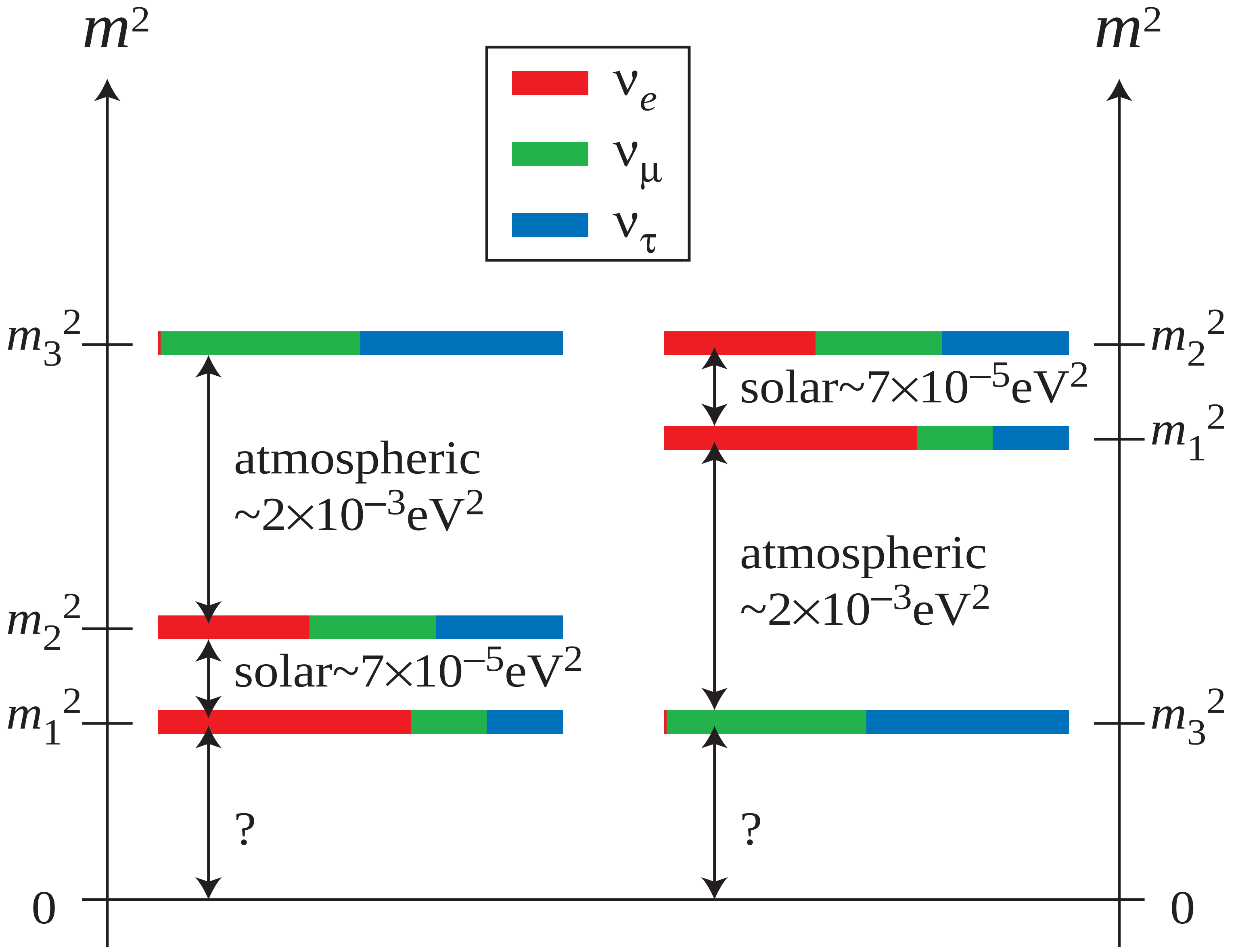
$$m_1 \ll m_2 < m_3, \qquad \text{NH},$$

$$m_3 \ll m_1 < m_2, \qquad \text{IH},$$

$$m_1 \cong m_2 \cong m_3,\ m^2_{1,2,3} >> |\Delta m^2_{31(32)}|,\ \text{QD};\ m_j \gtrsim 0.10\ \text{eV}.$$

• $m_2 = \sqrt{m_1^2 + \Delta m^2_{21}}$, $\; m_3 = \sqrt{m_1^2 + \Delta m^2_{31}}$ - NO;

• $m_1 = \sqrt{m_3^2 + \Delta m^2_{23} - \Delta m^2_{21}}$, $\; m_2 = \sqrt{m_3^2 + \Delta m^2_{23}}$ - IO;



m^2
m^2
ν_e
ν_μ
ν_τ
m_3^2
m_2^2
m_1^2
atmospheric
$\sim 2\times10^{-3}$eV^2
solar$\sim 7\times10^{-5}$eV^2
?
0
m_2^2
m_1^2
m_3^2
solar$\sim 7\times10^{-5}$eV^2
atmospheric
$\sim 2\times10^{-3}$eV^2
?
0

Table 3: Best fit values and allowed ranges at $N\sigma = 1, 2, 3$ for the 3ν oscillation parameters, in either NO or IO. The latter column shows the formal "1σ accuracy" for each parameter, defined as 1/6 of the 3σ range divided by the best-fit value (in percent).

Parameter	Ordering	Best fit	1σ range	2σ range	3σ range	"1σ" (%)
$\Delta m^2_{\odot}/10^{-5}$ eV^2	NO	7.34	7.20 – 7.51	7.05 – 7.69	6.92 – 7.91	2.2
	IO	7.34	7.20 – 7.51	7.05 – 7.69	6.92 – 7.91	2.2
$\lvert\Delta m^2_{\mathrm{A}}\rvert/10^{-3}$ eV^2	NO	2.49	2.46 – 2.53	2.43 – 2.56	2.39 – 2.59	1.4
	IO	2.48	2.44 – 2.51	2.41 – 2.54	2.38 – 2.58	1.4
$\sin^2\theta_{12}$	NO	3.04	2.91 – 3.18	2.78 – 3.32	2.65 – 3.46	4.4
	IO	3.03	2.90 – 3.17	2.77 – 3.31	2.64 – 3.45	4.4
$\sin^2\theta_{13}/10^{-2}$	NO	2.14	2.07 – 2.23	1.98 – 2.31	1.90 – 2.39	3.8
	IO	2.18	2.11 – 2.26	2.02 – 2.35	1.95 – 2.43	3.7
$\sin^2\theta_{23}/10^{-1}$	NO	5.51	4.81 – 5.70	4.48 – 5.88	4.30 – 6.02	5.2
	IO	5.57	5.33 – 5.74	4.86 – 5.89	4.44 – 6.03	4.8
δ/π	NO	1.32	1.14 – 1.55	0.98 – 1.79	0.83 – 1.99	14.6
	IO	1.52	1.37 – 1.66	1.22 – 1.79	1.07 – 1.92	9.3

$$\Delta m^2_{\odot} \equiv \Delta m^2_{21}; \quad \Delta m^2_{\mathrm{A}} \equiv \Delta m^2_{31(32)}, \text{ NO (IO)}.$$

F. Capozzi et al. (Bari Group), arXiv:1804.09678.

- Dirac phase δ: $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A^{(l,l')}_{\rm CP} \propto J_{\rm CP} \propto \sin\theta_{13}\sin\delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \text{Im}\left\{U_{e1}\,U_{\mu 2}\,U^*_{e2}\,U^*_{\mu 1}\right\} = \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta$$

Current data: $|J_{CP}| \lesssim 0.035$ (can be relatively large!); b.f.v. with $\delta = 3\pi/2$: $J_{CP} \cong -0.035$.

- Majorana phases α_{21}, α_{31}:
  - $\nu_l \leftrightarrow \nu_{l'}$, $\bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky et al., 1980;
P. Langacker et al., 1987

  - $\left|<m>\right|$ in $(\beta\beta)_{0\nu}$−decay depends on α_{21}, α_{31};
  - $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
  - BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

$$\delta \cong 3\pi/2?$$

$$\begin{aligned} J_{CP} &= \mathrm{Im}\left\{U_{e1}\, U_{\mu 2}\, U^{*}_{e2}\, U^{*}_{\mu 1}\right\} \\ &= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta \end{aligned}$$

- **Best fit value:** $\delta = 1.32\,(1.52)\pi$ **[**$1.30\,(1.54)\pi$**];**
- $\delta = 0$ **or** 2π **are disfavored at** $3.0\,(3.6)\sigma$ **[**$2.6\,(3.0)\sigma$**];**
- $\delta = \pi$ **is disfavored at** $1.8\,(3.6)\sigma$ **[**$1.7\,(3.3)\sigma$**];**
- $\delta = \pi/2$ **is strongly disfavored at** $4.4\,(5.2)\sigma$ **[**$4.3\,(5.0)\sigma$**].**
- **At** 3σ**:** δ/π **is found to lie in 0.83-1.99 (1.07-1.92) [1.07-1.97 (0.80-2.08)].**

F. Capozzi, E. Lisi *et al.*, arXiv:1804.09678 [E. Esteban *et al.*, NuFit 3.2 (Jan., 2018)]

2018 global analysis: data favors NO

IO disfavored at 3.1σ.

F. Capozzi et al., 1804.09678.

Parameter	Ordering	Best fit	1σ range	2σ range	3σ range	"1σ" (%)
$\delta m^2/10^{-5}$ eV^2	NO	7.34	7.20 – 7.51	7.05 – 7.69	6.92 – 7.90	2.2
	IO	7.34	7.20 – 7.51	7.05 – 7.69	6.92 – 7.91	2.2
$\sin^2\theta_{12}/10^{-1}$	NO	3.05	2.92 – 3.19	2.78 – 3.32	2.65 – 3.47	4.5
	IO	3.03	2.90 – 3.17	2.77 – 3.31	2.64 – 3.45	4.5
$\lvert\Delta m^2\rvert/10^{-3}$ eV^2	NO	2.485	2.453 – 2.514	2.419 – 2.547	2.389 – 2.578	1.3
	IO	2.465	2.434 – 2.495	2.404 – 2.526	2.374 – 2.556	1.2
$\sin^2\theta_{13}/10^{-2}$	NO	2.22	2.14 – 2.28	2.07 – 2.34	2.01 – 2.41	3.0
	IO	2.23	2.17 – 2.30	2.10 – 2.37	2.03 – 2.43	3.0
$\sin^2\theta_{23}/10^{-1}$	NO	5.45	4.98 – 5.65	4.54 – 5.81	4.36 – 5.95	4.9
	IO	5.51	5.17 – 5.67	4.60 – 5.82	4.39 – 5.96	4.7
δ/π	NO	1.28	1.10 – 1.66	0.95 – 1.90	0 – 0.07 $\oplus$ 0.81 – 2	16
	IO	1.52	1.37 – 1.65	1.23 – 1.78	1.09 – 1.90	9

$\delta m^2 \equiv \Delta m^2_{21}$; $\Delta m^2 \equiv \Delta m^2_{31(32)} \overset{-}{(+)} 0.5\Delta m^2_{21}$, **NO (IO).**

F. Capozzi et al. (Bari Group), arXiv:2003.08511.

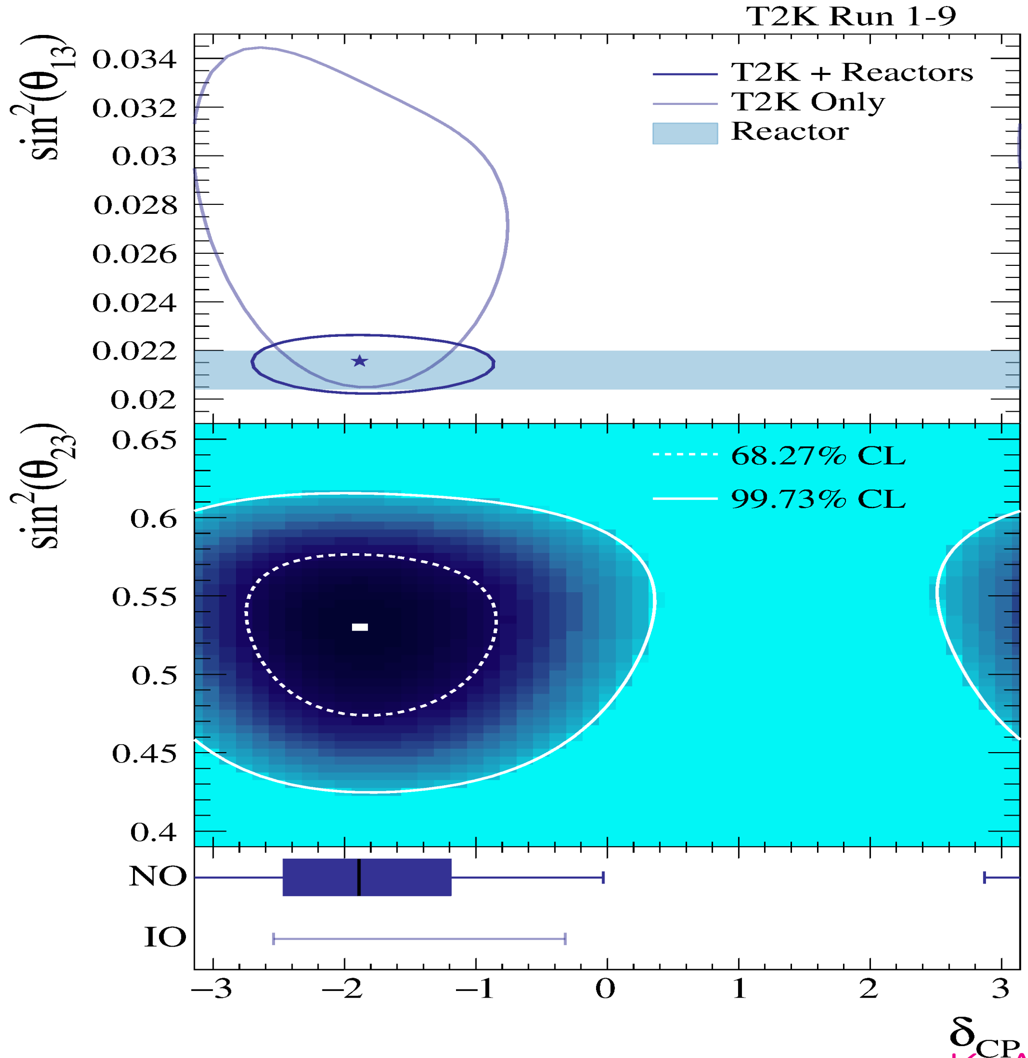
March 2020 global analysis (Bari Group):

- **Best fit value:** $\delta = 1.28\,(1.52)\pi$**;**
- $\delta = 0$ **or** 2π **are disfavored at** $2.6\,(> 5)\sigma$**;**
- $\delta = \pi$ **is allowed (disfavored) at** $1.6\,(3.2)\sigma$
- $\delta = \pi/2$ **is strongly disfavored at** $4.2\,(> 5)\sigma$
- **At** 3σ**:** δ/π **is found to lie in the intervals** $0.00 - 0.07 \oplus 0.81 - 2.00$ **(1.09-1.90).**

- **Data favors NO: IO disfavored at** 3.2σ**.**

F. Capozzi et al. (Bari Group), arXiv:2003.08511.

Latest results from T2K



K. Abe et al., 1910.03887

Best fit value: $\delta = -1.89\,(-1.38)$, NO (IO).
$\delta = 0, \pi$ ruled out at 95% CL.
At 3σ: δ is found to lie in [-3.41,-0.03] ([-2.54,-0.32]), NO (IO).

2020 global analyses after Nu2020: combine latest T2K and NOνA data.

Results on CPV due to δ and NO vs IO spectrum - inconclusive.

K.J. Kelly, P.A. Machado, S.J. Parke, Y.F. Perez Gonzalez and R. Zukanovich-Funchal,
"Back to (Mass-)Square(d) One: The Neutrino Mass Ordering in Light of Recent Data," arXiv:2007.08526 [hep-ph].

I. Esteban, M.C. Gonzalez-Garcia, M. Maltoni, T. Schwetz and A. Zhou,
"The fate of hints: updated global analysis of three-flavor neutrino oscillations," arXiv:2007.14792 [hep-ph].

Result on CPV, b.f.v.: $\delta = 197^{\circ}$, NO; $\delta = 282^{\circ}$, IO.

At 3σ: δ is found to lie in $[120^{\circ}, 369^{\circ}]$ ($[193^{\circ}, 352^{\circ}]$), NO (IO).

IO: CPV due to δ at 3σ.

IO disfavored at 1.6σ with respect to NO (2.7σ including SuperK ν_{atm} data).

Understanding the Pattern of Neutrino Mixng

The observed pattern of 3-ν mixing, is characterised by two large and one small mixing angles,

$\theta_{12} \cong 33^\circ$, $\theta_{23} \cong 45^\circ \pm 6^\circ$ **and** $\theta_{13} \cong 8.4^\circ$.

Challenge for the theory.

With the observed pattern of neutrino mixing Nature is sending us a Message. The Message is encoded in the values of the neutrino mixing angles, leptonic CP violation phases and neutrino masses. We do not know at present what is the prcise content of Nature's Message. However, on the basis of the analysis of the current ideas about the origin of the observed pattern of neutrino mixing I am led to conclude that if expressed in one word the Message most likely would read:

SYMMETRY

The Quest for Nature's Message

Towards Quantitative Understanding of U_{PMNS}, m_j

The observed pattern of 3-ν mixing, two large and one small mixing angles,
$\theta_{12} \cong 33^\circ$, $\theta_{23} \cong 45^\circ \pm 6^\circ$ **and** $\theta_{13} \cong 8.4^\circ$,
can most naturally be explained by extending the Standard Model (SM) with a flavour symmetry corresponding to a non-Abelian discrete (finite) group G_f.

$G_f = A_4$, T', S_4, A_5, D_{10}, D_{12},...

Vast literature; reviews: G. Altarelli, F. Feruglio, 1002.0211; H. Ishimori et al., 1003.3552; M. Tanimoto, AIP Conf.Proc. 1666 (2015) 120002; S. King and Ch. Luhn, 1301.1340; D. Meloni, 1709.02662; STP, 1711.10806

Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_{\odot} \cong \frac{\pi}{5.4}, \quad \theta_{23} = \theta_{\rm atm} \cong \frac{\pi}{4}(?), \quad \theta_{13} \cong \frac{\pi}{20}$

$$U_{\rm PMNS} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(?) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(?) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \sin^{-1}\frac{1}{\sqrt{3}} - 0.020$; $\theta_{12} \cong \pi/4 - 0.20$,
$\theta_{13} \cong 0 + \pi/20$, $\theta_{23} \cong \pi/4 \mp 0.10$.
- $U_{\rm PMNS}$ **due to new approximate symmetry?**

A Natural Possibility (vast literature):

$$U = U^{\dagger}_{\rm lep}(\theta^{\ell}_{ij}, \delta^{\ell})\; Q(\psi,\omega) U_{\rm TBM,BM,LC,...}\; \bar{P}(\xi_1, \xi_2)\,,$$

with

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\rm BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U^{\dagger}_{\rm lep}(\theta^{\ell}_{ij}, \delta^{\ell})$ - from diagonalization of the l^- mass matrix;
- $U_{\rm TBM,BM,LC,...}\; \bar{P}(\xi_1, \xi_2)$ - from diagonalization of the ν mass matrix;
- $Q(\psi,\omega)$, - from diagonalization of the l^- and/or ν mass matrices.

P. Frampton, STP, W. Rodejohann, 2003

$U_{\rm LC}$, $U_{\rm GRAM}$, $U_{\rm GRBM}$, $U_{\rm HGM}$:

$$U_{\rm LC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{c^{\nu}_{23}}{\sqrt{2}} & \frac{c^{\nu}_{23}}{\sqrt{2}} & s^{\nu}_{23} \\ \frac{s^{\nu}_{23}}{\sqrt{2}} & -\frac{s^{\nu}_{23}}{\sqrt{2}} & c^{\nu}_{23} \end{pmatrix}; \quad \mu - \tau \ \text{symmetry}: \quad \theta^{\nu}_{23} = \mp\pi/4\,;$$

$$U_{\rm GR} = \begin{pmatrix} c^{\nu}_{12} & s^{\nu}_{12} & 0 \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \\ -\frac{s^{\nu}_{12}}{\sqrt{2}} & \frac{c^{\nu}_{12}}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\rm HGM} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \theta^{\nu}_{12} = \pi/6\,.$$

$U_{\rm GRAM}$: $\sin^2\theta^{\nu}_{12} = (2 + r)^{-1} \cong 0.276$, $r = (1 + \sqrt{5})/2$
(GR: $r/1$; $a/b = a + b/a$, $a > b$**)**

$U_{\rm GRBM}$: $\sin^2\theta^{\nu}_{12} = (3 - r)/4 \cong 0.345$.

GRB and HG mixing: W. Rodejohann et al., 2009.

$U_{\mathrm{TBM(BM)}}$: Groups A_4, T', S_4 (S_4),... (vast literature)

(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552; S. King and Ch. Luhn, arXiv:1301.1340)

• U_{BM}: $s^2_{12} = 1/2$, $s^2_{13} = 0$, $s^2_{23} = 1/2$;
$s^2_{13} = 0$ and $s^2_{12} = 1/2$ must be corrected.

• U_{GRA}: Group A_5,...; $s^2_{13} = 0$ and possibly $s^2_{12} = 0.276$ and $s^2_{23} = 1/2$ must be corrected.

L. Everett, A. Stuart, arXiv:0812.1057;...

• U_{LC}: alternatively $U(1)$, $L' = L_e - L_\mu - L_\tau$

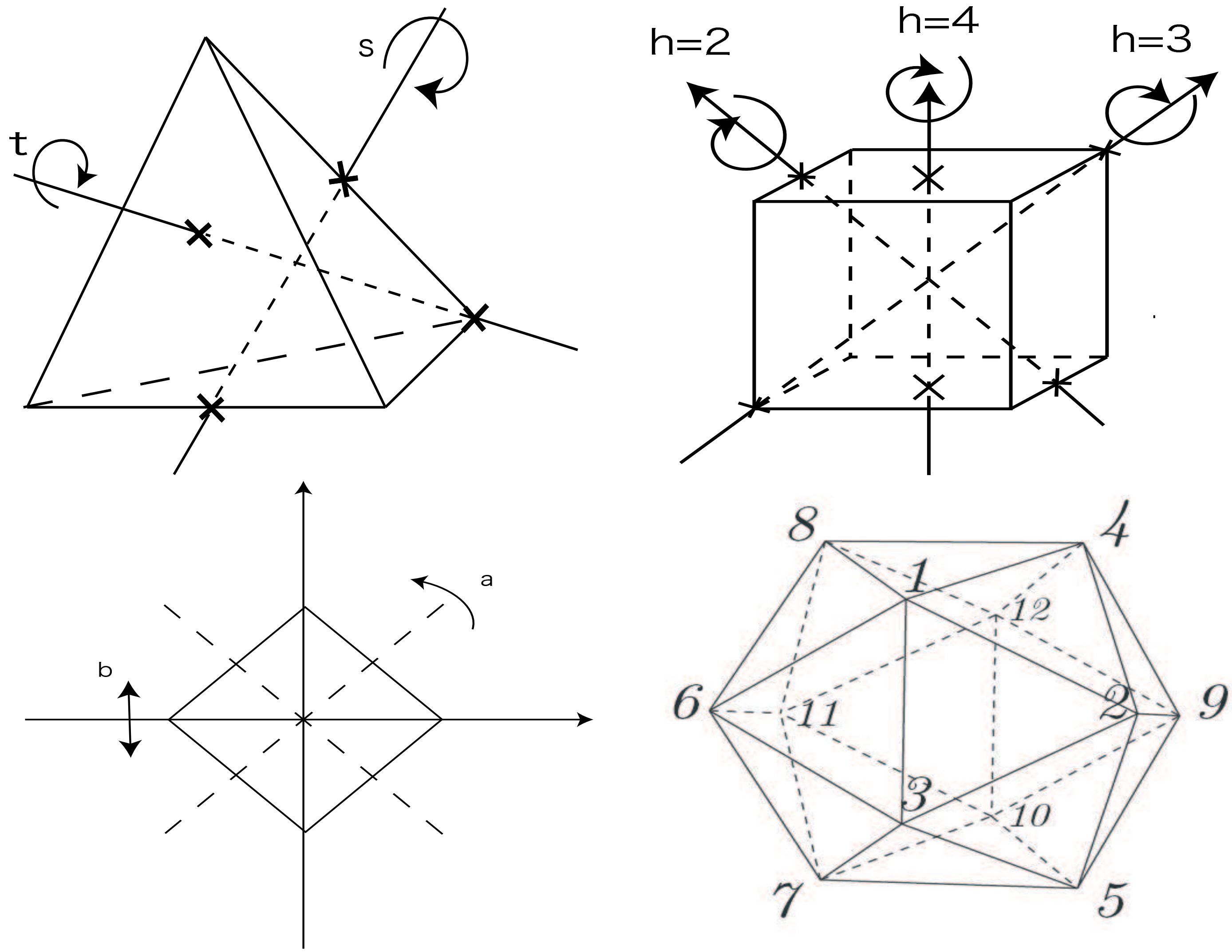
S.T.P., 1982

• U_{LC}: $s^2_{12} = 1/2$, $s^2_{13} = 0$, s^ν_{23} - free parameter;
$s^2_{13} = 0$ and $s^2_{12} = 1/2$ must be corrected.

• $U_{\rm GRB}$: Group D_{10},...; $s^2_{13} = 0$ and possibly $s^2_{12} = 0.345$ and $s^2_{23} = 1/2$ must be corrected.

• $U_{\rm HG}$: Group D_{12},...; $s^2_{13} = 0$, $s^2_{12} = 0.25$ and possibly $s^2_{23} = 1/2$ must be corrected.

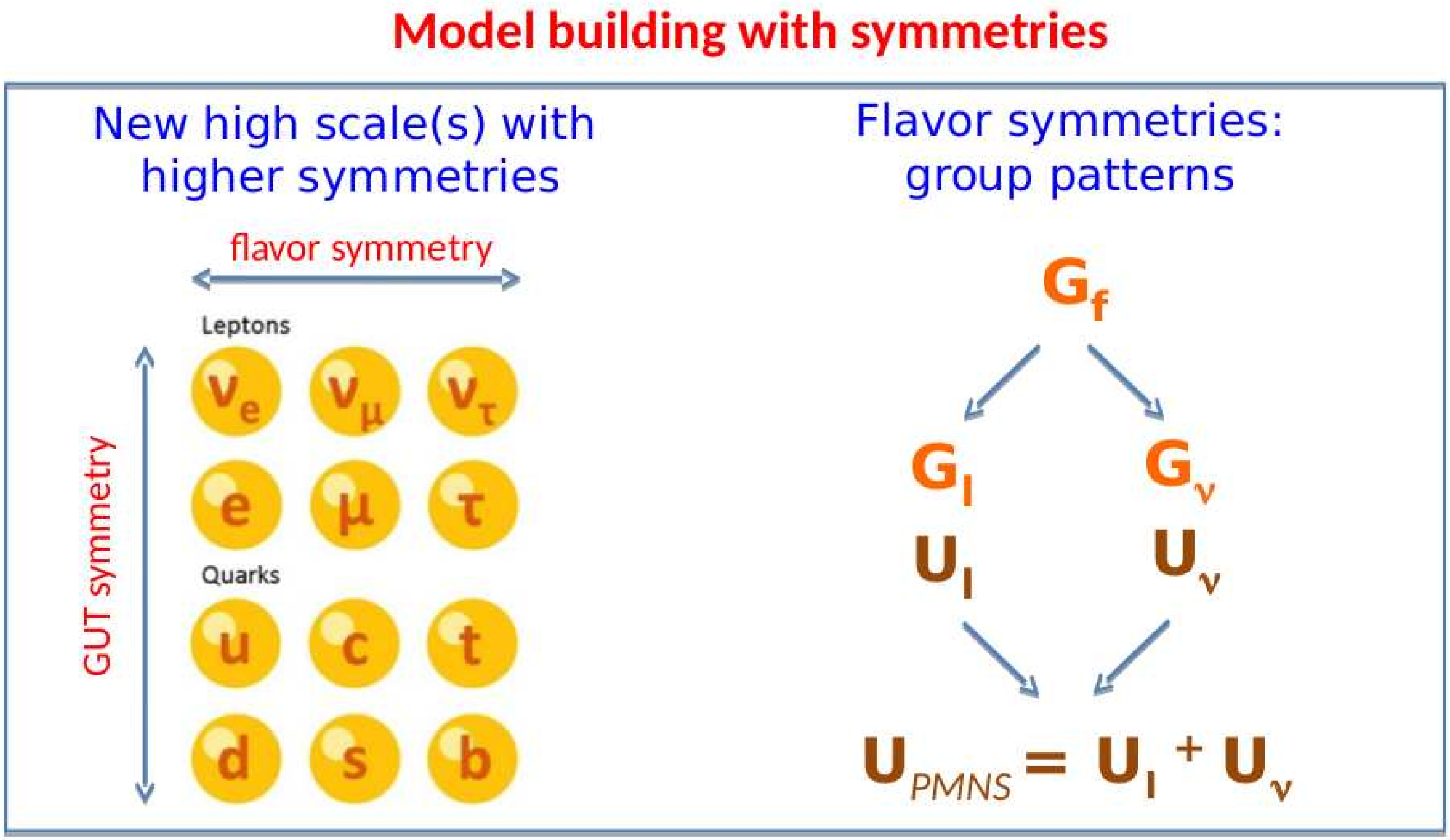
For all symmetry forms considered we have: $\theta^{\nu}_{13} = 0$, $\theta^{\nu}_{23} = \mp\pi/4$.
They differ by the value of θ^{ν}_{12}:
TBM, BM, GRA, GRB and HG forms correspond to $\sin^2\theta^{\nu}_{12} = 1/3;\ 0.5;\ 0.276;\ 0.345;\ 0.25$.



Examples of symmetries: A_4, S_4, D_4, A_5

From M. Tanimoto et al., arXiv:1003.3552

How Does it Work



E. Lisi, TAUP 2019

Examples of Predictions and Correlations

- $\sin^2\theta_{23} = \frac{1}{2}$.
- $\sin^2\theta_{23} \cong \frac{1}{2}(1 \mp \sin^2\theta_{13}) + O(\sin^4\theta_{13}) \cong \frac{1}{2}(1 \mp 0.022)$.
- $\sin^2\theta_{23} = 0.455;\ 0.463;\ 0.537;\ 0.545;\ 0.604$ **(small uncert.)**.
- $\sin^2\theta_{12} \cong \frac{1}{3}(1 + \sin^2\theta_{13}) + O(\sin^4\theta_{13}) \cong 0.340$.
- $\sin^2\theta_{12} \cong \frac{1}{3}(1 - 2\sin^2\theta_{13}) + O(\sin^4\theta_{13}) \cong 0.319$.
- **and/or** $\cos\delta = \cos\delta(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12}, ...)$,

$J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12}, ...)$,

θ^{ν}_{12},... - **known (fixed) parameters, depend on the underlying symmetry.**

Predictions and Correlations: $\delta = \delta(\theta_{ij}; \theta^{\nu}_{12})$

$U_{\nu} = U_{\mathrm{TBM,BM,GRA,GRB,HG}}\, \bar{P}(\xi_1, \xi_2)$; θ^{ν}_{12};

$U_{\ell}^{\dagger} = R_{12}(\theta^{\ell}_{12})\, Q$, $Q = \mathrm{diag}(e^{i\varphi}, 1, 1)$
(the "minimal" = simplest case ($SU(5) \times T'$,...)

$U_{\ell}^{\dagger} = R_{12}(\theta^{\ell}_{12})\, R_{23}(\theta^{\ell}_{23}) Q$, $Q = \mathrm{diag}(1, e^{-i\psi}, e^{-i\omega})$
(next-to-minimal case)

$\cos\delta = \cos\delta(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12}, ...)$,

$J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12}, ...)$,

θ^{ν}_{12},... - **known (fixed) parameters, depend on the underlying symmetry.**

For arbitrary fixed θ^{ν}_{12} and any θ_{23} ("minimal" and "next-to-minimal" cases):

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\Big[\cos 2\theta^{\nu}_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^{\nu}_{12}\right)\left(1 - \cot^2\theta_{23}\,\sin^2\theta_{13}\right)\Big]\,.$$

S.T.P., arXiv:1405.6006

This results is exact.

"Minimal" case: $\sin^2\theta_{23} = \frac{1}{2}\,\frac{1-2\sin^2\theta_{13}}{1-\sin^2\theta_{13}}$.

In all cases TBM, BM (LC), GRA, GRB, HG:

- **New sum rules relating θ_{12},θ_{13}, θ_{23} and δ;**
- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12})$.

S.T.P., arXiv:1405.6006

- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta^{\nu}_{12})$.
- TBM case: $\delta \cong 3\pi/2$ or $\pi/2$; b.f.v. of θ_{ij}:
$\delta \cong 263.5^\circ$ or 96.5°, $\cos\delta = -0.114$, $J_{CP} \cong \mp 0.034$.
- GRAM case, b.f.v. of θ_{ij}: $\delta \cong 286.8^\circ$ or 73.2°;
$\cos\delta = 0.289$, $J_{CP} \cong \mp 0.0327$.
- GRBM case, b.f.v. of θ_{ij}: $\delta \cong 258.5^\circ$ or 101.5°;
$\cos\delta = -0.200$, $J_{CP} \mp 0.0333$.
- HGM case, b.f.v. of θ_{ij}: $\delta \cong 298.4^\circ$ or 61.6°;
$\cos\delta = 0.476$, $J_{CP} \cong \mp 0.0299$.
- BM, LC cases: $\delta \cong \pi$, $\cos\delta \cong -0.978$, $J_{CP} \cong \mp 0.008$

The results shown - for NO neutrino mass spectrum; the results are practically the same for IO spectrum. (Best fit values of θ_{ij}: F. Capozzi et al., arXiv:1312.2878v1.)

S.T.P., arXiv:1405.6006

By measuring $\cos\delta$ or δ and using high precision data on θ_{12}, θ_{23} and θ_{13}, one can distiguish between different symmetry forms of $\tilde{U}_{\nu}$!

Relatively high precision measurement of δ will be performed at the future planned neutrino oscillation experiments, (DUNE, T2HK) see, e.g., R. Acciarri *et al.* [DUNE Collab.], arXiv:1512.06148, 1601.05471 and 1601.02984; K. Abe *et al.* [T2HK Proto-Collab.], arXiv:1502.05199 (PTEP 2015 (2015) 053C02).

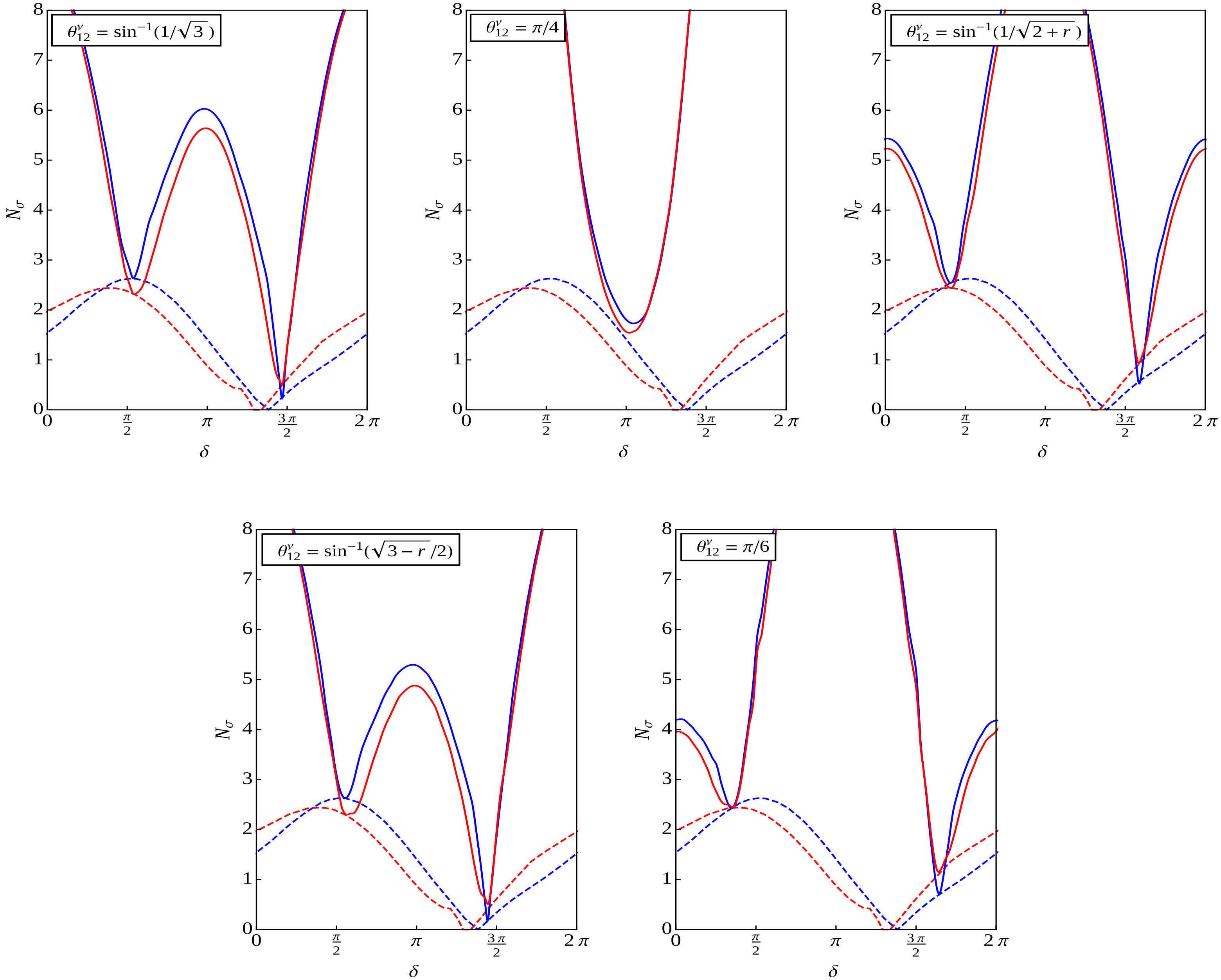
Statistical analysis, likelihood method;
input "data": $\sin^2\theta_{13}$, $\sin^2\theta_{12}$, $\sin^2\theta_{12}$, δ
from F. Capozzi et al., arXiv:1312.2878v2 (May 5, 2014).

$$L(\cos\delta) \propto \exp\left(-\frac{\chi^2(\cos\delta)}{2}\right)$$

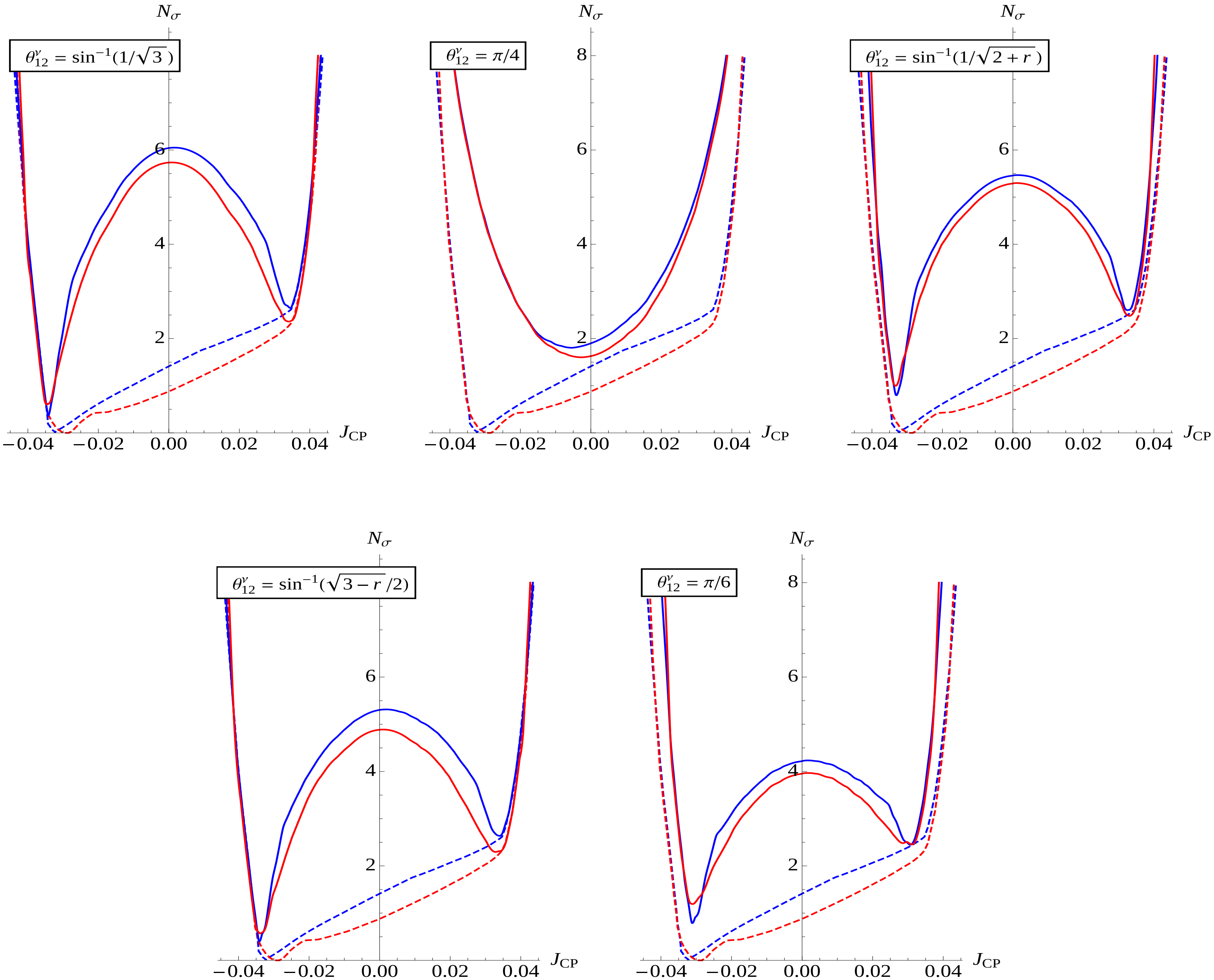
nσ confidence level interval of values of $\cos\delta$**:**

$$L(\cos\delta) \geq L(\chi^2_{\min}) \cdot L(\chi^2 = n^2)$$

I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

TBM, GRA, GRB, HG: $J_{\rm CP} = 0$ **excluded at** 5σ, 4σ, 4σ, 3σ **confidence level.**

At 3σ**:** $0.020 \leq |J_{\rm CP}| \leq 0.039$.

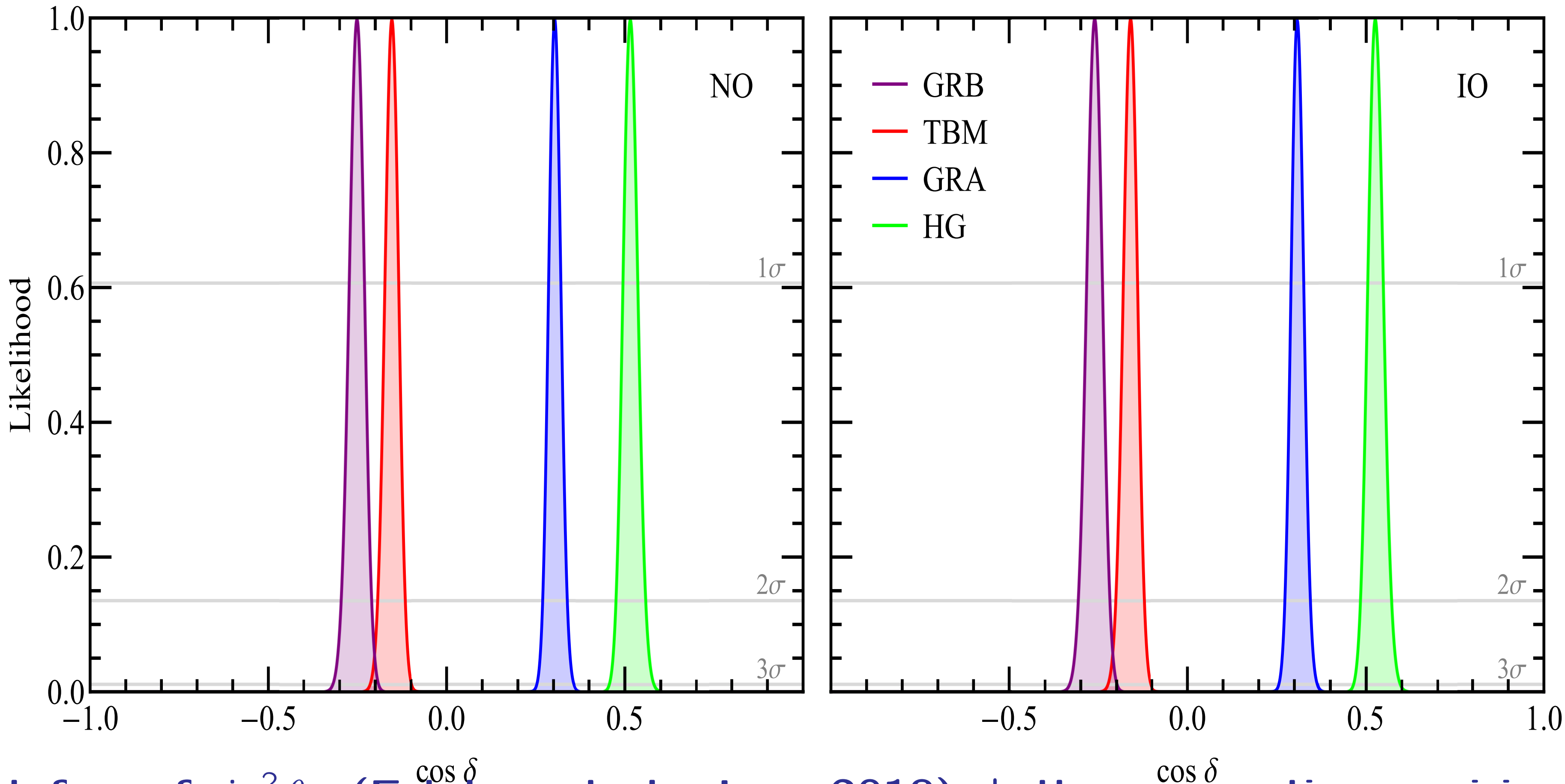
BM (LC), b.f.v.: $J_{\rm CP} = 0$;
at 3σ**:** $-0.026\ (-0.025) \leq J_{\rm CP} \leq 0.021\ (0.023)$ **for NO (IO) neutrino mass spectrum.**

Prospective precision:

$\delta(\sin^2\theta_{12}) = 0.7\%$ **(JUNO),**

$\delta(\sin^2\theta_{13}) = 3\%$ **(Daya Bay),**

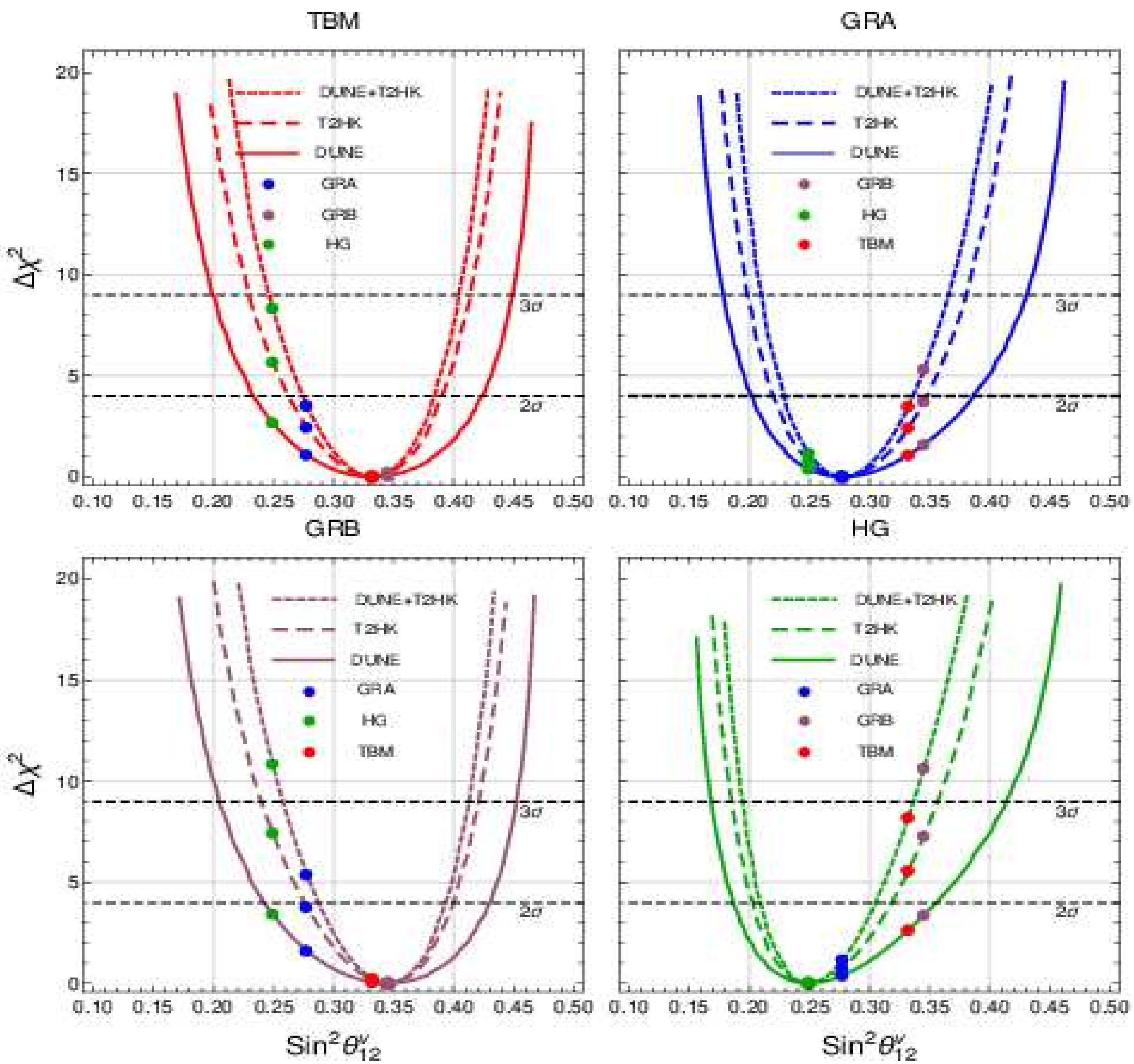
$\delta(\sin^2\theta_{23}) = 5\%$ **(T2K, NOνA combined).**



b.f.v. of $\sin^2\theta_{ij}$ (Esteban et al., Jan., 2018) + the prospective precision used.

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta^{\nu}_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^{\nu}_{12}\right)\left(1 - \cot^2\theta_{23}\sin^2\theta_{13}\right)\right] .$$

$\delta(\sin^2\theta_{23}) = 3\%$ **(T2HK, DUNE).**



Agarwalla, Chatterjee, STP, Titov, arXiv:1711.02107

GRB - HG $> 3\sigma$; GRA - GRB $\geq 2\sigma$; TMB - HG $\cong 3\sigma$; TMB - GRA $\cong 2\sigma$.
With T2HKK data - better sensitivity.

The measurement of the Dirac phase in the PMNS mixing matrix, together with an improvement of the precision on the mixing angles θ_{12}, θ_{13} and θ_{23}, can provide unique information about the possible existence of new fundamental symmetry in the lepton sector.

Prospective (useful/requested) precision:

$\delta(\sin^2\theta_{12}) = 0.7\%$ **(JUNO),**

$\delta(\sin^2\theta_{13}) = 3\%$ **(Daya Bay),**

$\delta(\sin^2\theta_{23}) = 3\%$ **(T2HK, DUNE; T2K+NOνA(?)).**

$\delta(\delta) = 10^\circ - 12^\circ$ **at** $\delta = 3\pi/2$

The Power of Data

Systematic analysis (I. Girardi *et al.*, 2016): all possible combinations of residual symmetries G_e and G_ν of the lepton flavour symmetry groups $G_f = S_4$, A_4, T' and A_5, leading to correlations between some of the three neutrino mixing angles and/or between the neutrino mixing angles and the Dirac CPV phase δ, were considered.

(A) $G_e = Z_2$ **and** $G_\nu = Z_k$, $k > 2$ **or** $Z_m \times Z_n$, $m, n \geq 2$;

(B) $G_e = Z_k$, $k > 2$ **or** $Z_m \times Z_n$, $m, n \geq 2$ **and** $G_\nu = Z_2$;

(C) $G_e = Z_2$ **and** $G_\nu = Z_2$.

In these cases $U_e^\dagger$ and/or U_ν of $U = U_e^\dagger U_\nu = (\tilde{U}_e)^\dagger \Psi \tilde{U}_\nu Q_0$, are partially (or fully) determined by residual discrete symmetries of $G_f = S_4$, A_4, T' and A_5.

More specifically:

A. $G_e = Z_2$, $G_\nu = Z_n$, $n > 2$ **or** $Z_n \times Z_m$, $n, m \geq 2$;
U_ν **fixed; A1, A2 (A3):** θ_{23}, $\cos\delta$ **(**θ_{12}, θ_{13}**) predicted.**

B. $G_e = Z_n$, $n > 2$ **or** $G_e = Z_n \times Z_m$, $n, m \geq 2$, $G_\nu = Z_2$;
U_e **fixed; B1,B2 (B3):** θ_{12}, $\cos\delta$ **(**θ_{23}, θ_{13}**) predicted.**

C. $G_e = Z_2$ **and** $G_\nu = Z_2$: θ_{12} **or** θ_{23} **or** $\cos\delta$ **predicted.**

$G_f = A_4$, S_4, T', A_5.

A_4: **3** Z_2**, 4** Z_3**, 1** $Z_2 \times Z_2$ **subgroups (total 8).**

T': **similar to** A_4.

S_4: **9** Z_2**, 4** Z_3**, 3** Z_4**, 4** $Z_2 \times Z_2$ **subgroups (total 20).**

A_5: **has 15** Z_2**, 10** Z_3**, 6** Z_5**, 5** $Z_2 \times Z_2$ **subgroups (36).**

In the case of A_4 (T') symmetry only there are 64 models (up to permutation of rows and columns).

A_4:
$(G_e, G_\nu) = (Z_2, Z_3)$, **A1 - A3;**
$(G_e, G_\nu) = (Z_2, Z_2)$, **A1 - A3;**
$(G_e, G_\nu) = (Z_3, Z_2)$, **B1 - B3;**
$(G_e, G_\nu) = (Z_2 \times Z_2, Z_2)$, **B1 - B3;**
$(G_e, G_\nu) = (Z_2, Z_2)$, **C1 - C9.**

For A_4, S_4 and A_5 the total number of models to be analysed is extremely large. However, a total of only 14 models survive the 3σ constraints on $\sin^2\theta_{ij}$ from the current data and the requirement $|\cos\delta| \leq 1$.

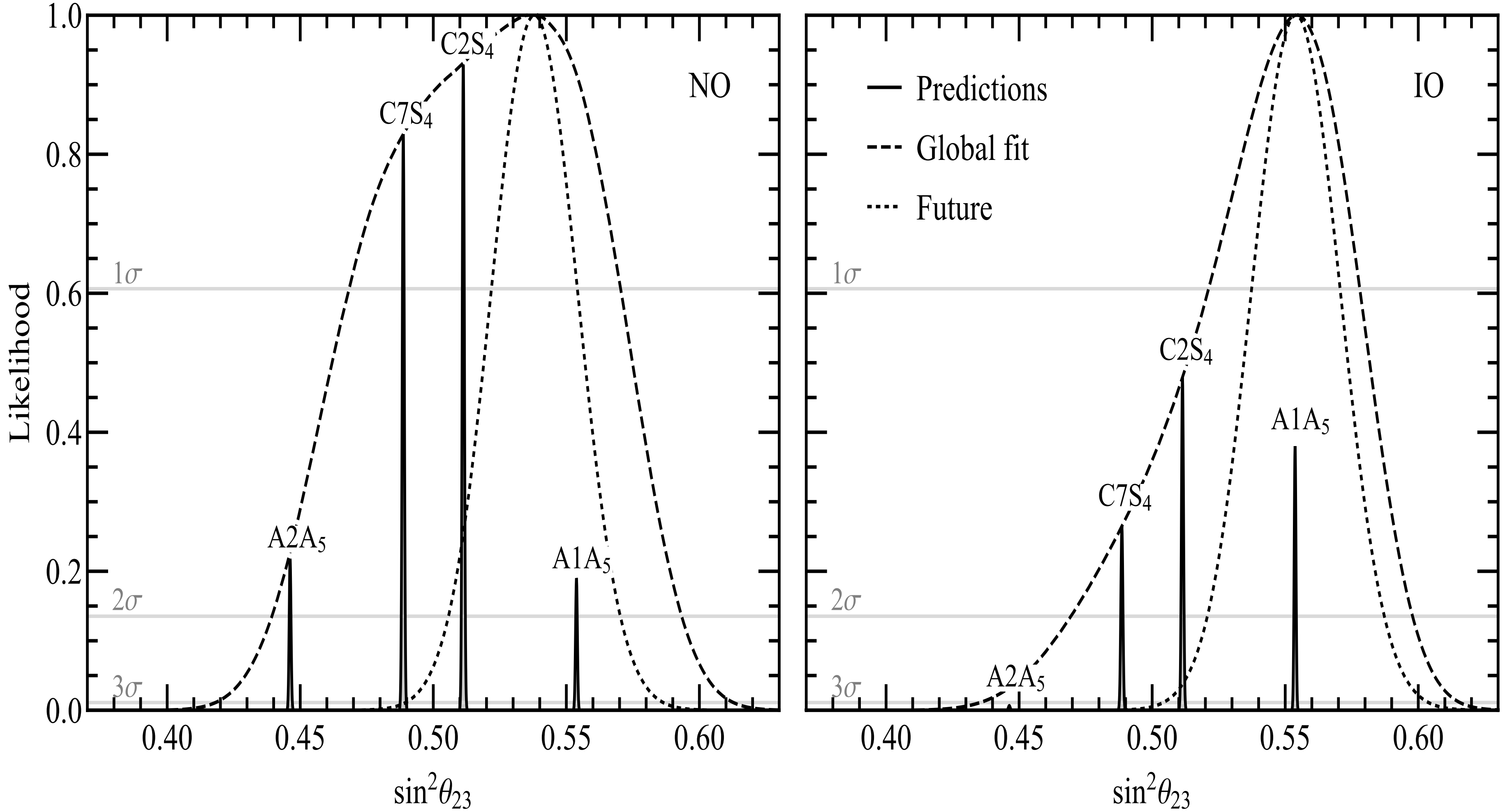
Phenomenologically Viable Predictions

A1 (A2), A_5 **(**$G_e = Z_2$**,** $G_\nu = Z_3$ **(Dirac** ν_j**)):**
$\sin^2\theta_{23} \cong 0.553\ (0.447)$; $\cos\delta \cong 0.716\ (-0.716)$.

A1, S_4: $\sin^2\theta_{23} \cong 0.5(1 - \sin^2\theta_{13}) \cong 0.489$;
$\cos\delta \cong -1$ **requires** $\sin^2\theta_{12} \cong 0.348$ **(!)**

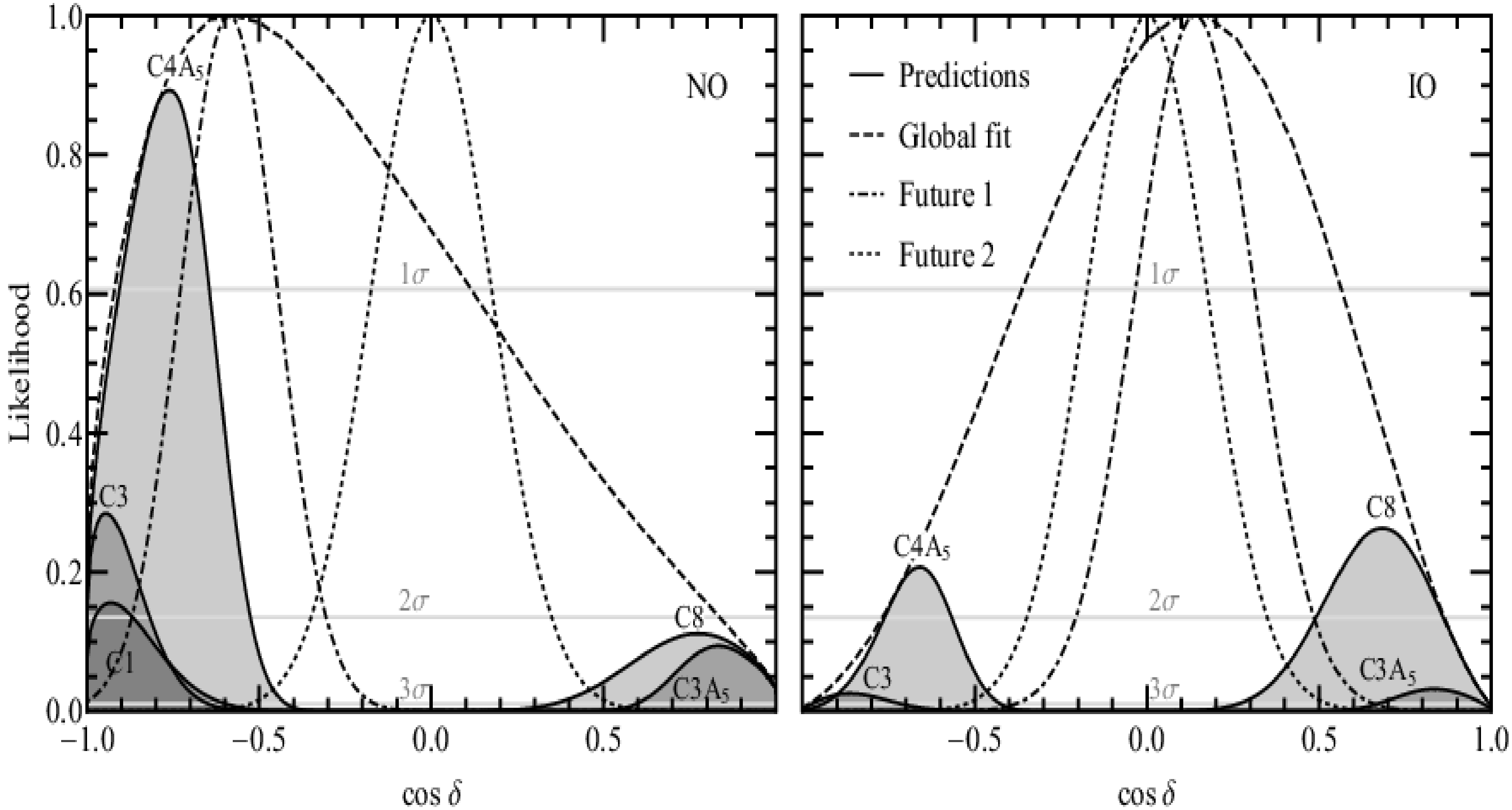
B1, A_4 **(**T'**,** S_4**,** A_5**) (**$G_e = Z_3^T$**,** $G_\nu = Z_2^S$**):**
$U_{\rm PMNS} = U_{\rm TBM} U_{13}(\theta^\nu_{13}, \delta_{13}) Q_0$;
$\sin^2\theta_{12} = 1/(3\cos^2\theta_{13}) \cong 0.340$; $\cos\delta \cong 0.570$.

B2, S_4 **(**$G_e = Z_3^T$**,** $G_\nu = Z_2^{SU}$**):**
$\sin^2\theta_{12} \cong (1 - 2\sin^2\theta_{13})/3 = 0.319$; $\cos\delta \cong -0.269$.



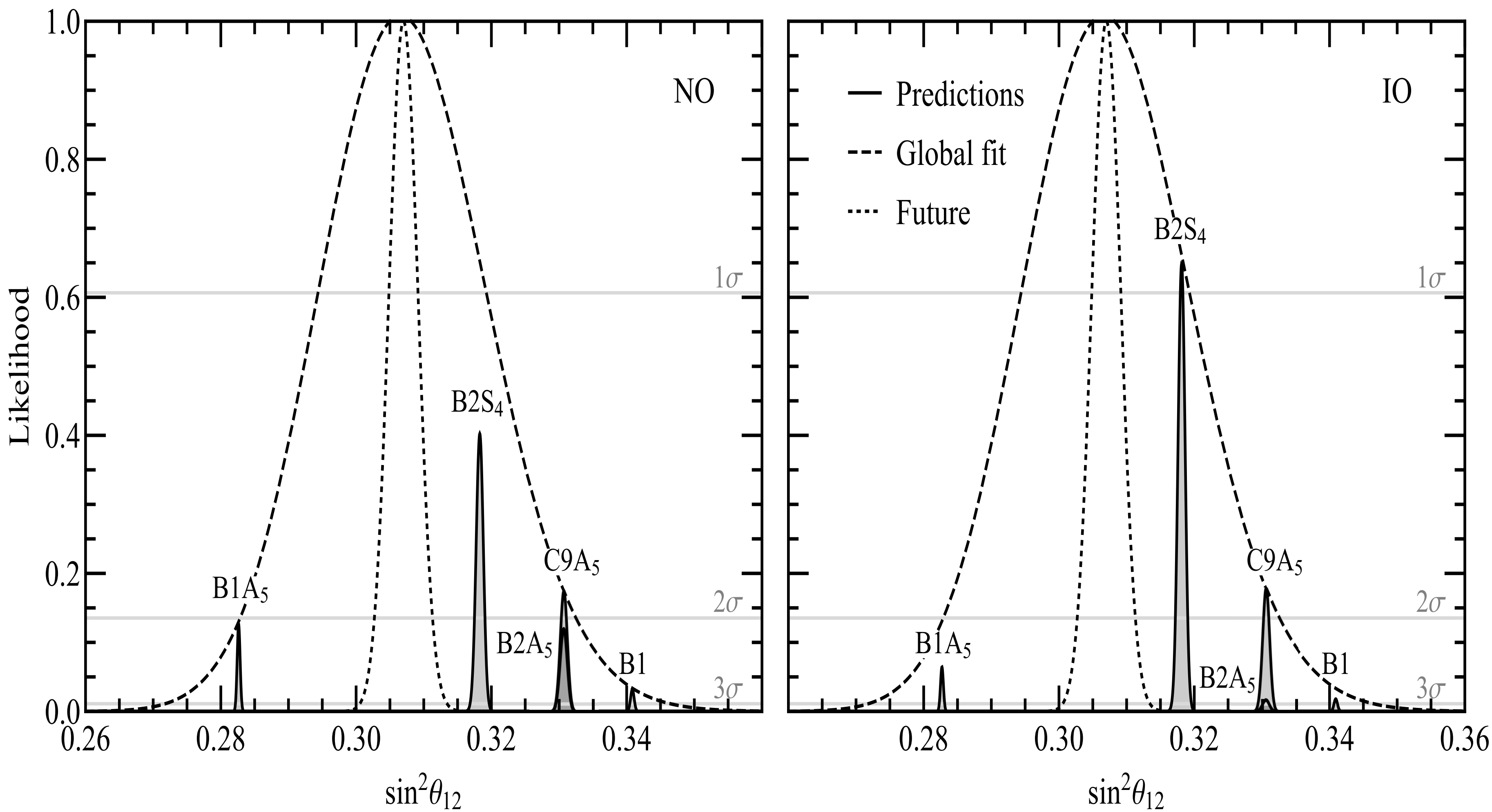
S.T.P., A. Titov, arXiv:1804.00182

Future: $\delta(\sin^2\theta_{23}) = 3\%$ **(T2HK, DUNE).**



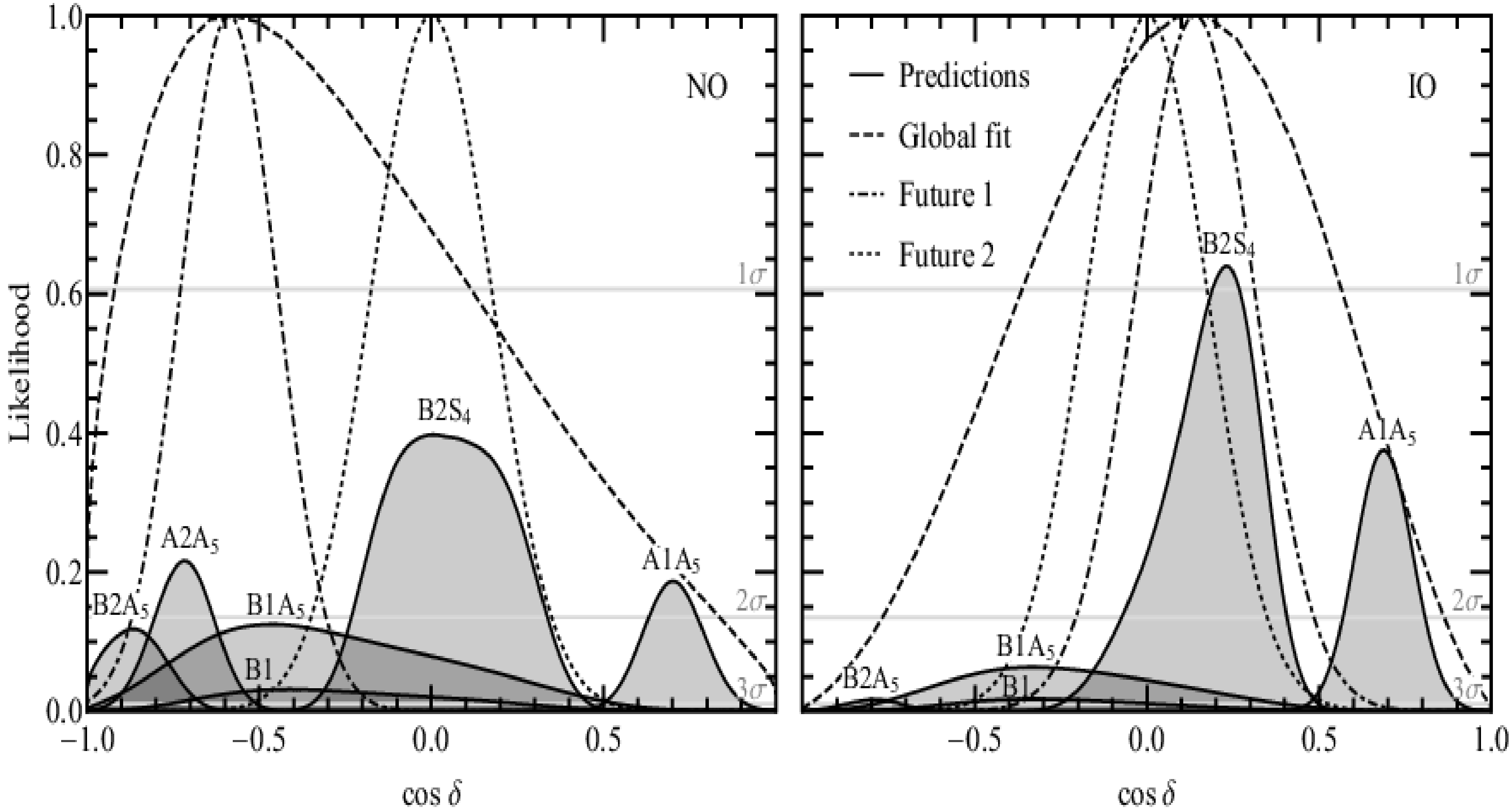
S.T.P., A. Titov, arXiv:1804.00182

Future: $\delta(\delta) = 10^\circ$**.**



S.T.P., A. Titov, arXiv:1804.00182

Future: $\delta(\sin^2\theta_{12}) = 0.7\%$ **(JUNO).**



S.T.P., A. Titov, arXiv:1804.00182

A total of 6 models would survive out of the currently viable 14 (of the extremely large number) considered if $\delta(\sin^2\theta_{23}) = 3\%$, $\delta(\sin^2\theta_{12}) = 0.7\%$ and the current b.f.v. would not change:

$A1A_5$, $C2S_4$, C3, $C3A_5$, $C4A_5$, C8.

Will be constrained further by the data on δ.

Prospective (useful/requested) precision:

$\delta(\sin^2\theta_{12}) = 0.7\%$ **(JUNO),**

$\delta(\sin^2\theta_{13}) = 3\%$ **(Daya Bay),**

$\delta(\sin^2\theta_{23}) = 3\%$ **(T2HK, DUNE; T2K+NOνA(?)).**

$\delta(\delta) = 10^\circ - 12^\circ$ **at** $\delta = 3\pi/2$

Conclusions.

- **Understanding the origin of the pattern of neutrino mixing and of neutrino mass squared differences that emerged from the neutrino oscillation data in the recent years is one of the most challenging problems in neutrino physics.**
- **The observed pattern of neutrino mixing can be due to a new basic (approximate non-Abelian discrete) symmetry of particle interactions leading to an approximate symmetry form of the PMNS matrix.**
- **The most important testable consequence of the symmetry approach to understanding the pattern of neutrino mixing is the correlation between the values of some of the neutrino mixing angles and/or the value of $\cos\delta$ and the values of the neutrino mixing angles: $\delta = \delta(\theta_{12}, \theta_{13}, \theta_{23}; \theta^{\nu}_{12})$. The second correlation depends on the underlying approximate symmetry form of the $U_{\rm PMNS}$.**

The measurement of the Dirac phase in the PMNS mixing matrix, together with an improvement of the precision on the mixing angles θ_{12}, θ_{13} and θ_{23}, can provide unique information about the possible existence of new fundamental symmetry in the lepton sector.