Radiative kaon decays: long-distance effects

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Rare Processes and Precision Frontier Townhall Meeting, 2-3 October 2020

Standard Model predictions and new physics in rare Kaon Decays [SNOWMASS21-RF2-RF0-124]
L. Cappiello, O. Catà, G. D'Ambrosio

Matching long and short distances in the form factors for kaon decays [SNOWMASS21-RF2-RF0-125]
G. D'Ambrosio, D. Greynat, M. Knecht

Rare kaon decays proceed through FCNC

$\longrightarrow$ they are suppressed in the SM

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Two broad classes:

- SD dominated rare decays $\Longrightarrow K \to \pi\nu\bar{\nu}$, $K \to \pi\pi\nu\bar{\nu}$

- LD dominated rare decays $\Longrightarrow K \to \gamma^{(*)}\gamma^{(*)}, K \to \pi\gamma^{(*)}, K \to \pi\gamma\gamma^{(*)}, K \to \pi\pi\gamma^{(*)}$...

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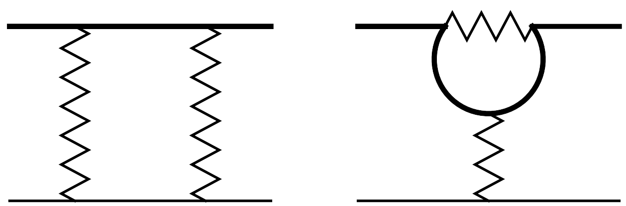

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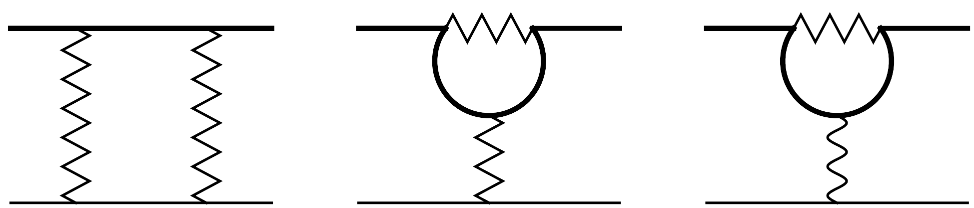

For reviews, see V. Cirigliano et al, Rev. Mod. Phys. 84, 399 (2012)
L. Littenberg, G. Valencia in PDG

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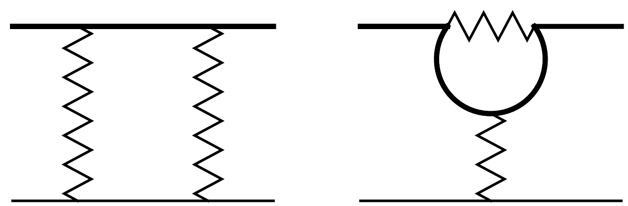

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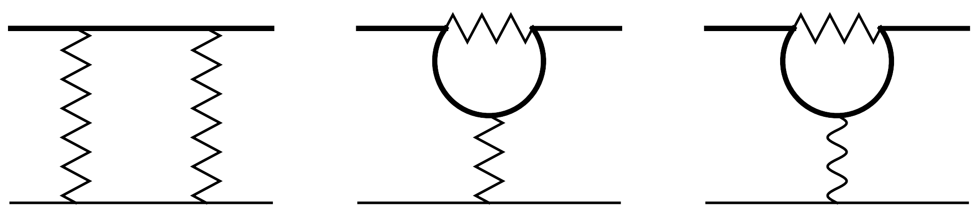

LD part difficult to assess!

Several tools have been explored in order to evaluate the corresponding matrix elements

Taking $K \to \pi \ell^+ \ell^-$ as a case study

- Chiral perturbation theory [$4m_\ell^2 \leq s \leq (M_K - M_\pi)^2$]

  - one loop

    G. Ecker et al., Nucl Phys B 291, 692 (1987)

    B. Ananthanarayan and I. S. Imsong, J. Phys. G 39, 095002 (2012)

  - beyond one loop

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- Chiral perturbation theory and large-N_c

  S. Friot et al., Phys Lett B 595, 301 (2004)

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- Lattice QCD $\longrightarrow$ talk by A. Portelli

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Extract LECs from a subclass of processes and make predictions for the remaining processes

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Extract LECs from a subclass of processes and make predictions for the remaining processes

Try to estimate the LECs $\longrightarrow$ difficult to extend simple resonance saturation that works rather well in the strong sector to the weak sector

Extracting LECs from experimental data

Effective Lagrangian for $\Delta S = 1$ transitions

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = \underbrace{G_8 F^4 \langle \lambda^6 D^\mu U^\dagger D_\mu U \rangle}_{K\to 2\pi, 3\pi, \gamma\gamma} + \underbrace{G_8 F^2 \sum_{i=1}^{37} N_i W_i}_{\text{rad. K decays}} + \cdots$$

Radiative kaon decays are generally dominated by Bremsstrahlung $\longrightarrow$ extraction of LECs from experiment difficult $\longrightarrow$ look for the interference term between Bremsstrahlung and electric/magnetic form factors

Already measured

$K^\pm \to \pi^\pm \gamma^*$	$N_{14} - N_{15}$	[NA48/2]
$K_S \to \pi^0 \gamma^*$	$2N_{14} + N_{15}$	[NA48/1]
$K^\pm \to \pi^\pm \pi^0 \gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$	[NA48/2]
$K^\pm \to \pi^\pm \gamma\gamma$	$N_{14} - N_{15} - 2N_{18}$	[NA48/2]

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Need also Dalitz plot from

$$K^\pm \to \pi^\pm \pi^0 \gamma^* \quad N_{14} + 2N_{15} - 3N_{16} + 3N_{17}$$

H. Pichl, Eur. Phys. J. C 20, 371 (2001)

L. Cappiello, O. Catà, G. D'Ambrosio, D. N. Gao, Eur. Phys. J. C 72, 1872 (2012)

L. Cappiello, O. Catà, G. D'Ambrosio, Eur. Phys. J. C 78, 265 (2018)

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First measurement by NA48/2 promising!

J. R. Batley *et al.* [NA48/2], Phys. Lett. B 788, 522 (2019)

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$\longrightarrow$ Predictions for

$K_L \to \pi^+ \pi^- \gamma^*$	$N_{14} - N_{15} - 3N_{16} + 3N_{17}$
$K_S \to \pi^+ \pi^- \gamma^*$	$N_{14} - N_{15} - 3N_{16} - 3N_{17}$
$K_S \to \pi^+ \pi^- \pi^0 \gamma$	$7N_{14} - 7N_{16} + 5N_{15} + 5N_{17}$

Extracting LECs from experimental data: perspectives for the future

More precise experimental determination of Dalitz-plot structure, more data will be collected and analysed at NA62

Improve theoretical study of the amplitude (full one-loop calculation)

Towards a phenomenological determination of (some) LECs

Focus on the CP conserving decays

$$K^\pm \to \pi^\pm \gamma^* \to \pi^\pm \ell^+ \ell^- \qquad\qquad K_S \to \pi^0 \gamma^* \to \pi^0 \ell^+ \ell^-$$

Good reasons to do so:

- already quite well studied experimentally (esp. $K^\pm$)
- more data will become available in the future (NA62, LHCb)
- analogues, in the kaon sector, of $b \to s\ell^+\ell^-$ transitions
- any LFUV effect invoked in order to explain the anomalies seen at LHCb might also manifest itself here   A. Crivellin et al., Phys. Rev. D 93, 074038 (2016)

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General structure of the amplitude [to first order in G_F and in α]

$$\begin{aligned}
\mathcal{A}^{K\to\pi\ell^+\ell^-}(s) &= -e^2 \times \bar{\mathrm{u}}(p_-)\gamma_\rho \mathrm{v}(p_+) \times \frac{1}{s} \times \left\{ i\int d^4x\, \langle \pi(p) | T\{ j^\rho(0) \mathcal{L}^{\Delta S=1}_{\text{non-lept}}(x)\} | K(k)\rangle \right.\\
&\left. - \left(\frac{G_{\mathrm{F}}}{\sqrt{2}} V_{us}V_{ud}\right) \times \frac{C_{7V}(\nu)}{4\pi\alpha} \times s \times \langle \pi(p) | (\bar{s}\gamma^\rho d)(0) | K(k)\rangle \right\}\\
&= -e^2 \times \bar{\mathrm{u}}(p_-)(\not{k} - \not{p})\mathrm{v}(p_+) \times \frac{W^{\mathrm{LD}}_{K\pi}(z;\nu) + W^{\mathrm{SD}}_{K\pi}(z;\nu)}{16\pi^2 M_K^2}
\end{aligned}$$

$\nu \equiv$ QCD renormalization scale, $s \equiv (p_+ + p_-)^2,\ z \equiv s/M_K^2$

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Low-energy structure of the amplitude beyond one-loop

$$W_{K\pi;+,S} \equiv W^{\rm LD}_{K\pi}(z;\nu) + W^{\rm SD}_{K\pi}(z;\nu) = G_F M_K^2 [a_{+,S} + b_{+,S} z] + W^{\pi-{\rm loop}}_{K\pi}(z;\alpha_{+,S},\beta_{+,S})$$

G. D'Ambrosio et al., JHEP 9808, 004 (1998)

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G. D'Ambrosio et al., JHEP 9808, 004 (1998)

- Not a complete two-loop representation, but neglected two-loop effects are small

G. D'Ambrosio, D. Greynat and M. Knecht, JHEP 1902, 049 (2019)

- α_+, β_+ $[\alpha_S, \beta_S]$ from slope and curvature of $K^+ \to \pi^+\pi^+\pi^-$ $[K_S \to \pi^0\pi^+\pi^-]$ Dalitz-plot

$$\alpha_+ = -20.84(74)\cdot 10^{-8}, \quad \beta_+ = -2.88(1.08)\cdot 10^{-8}, \quad \alpha_S = -6.81(74)\cdot 10^{-8}, \quad \beta_S = -1.5(1.1)\cdot 10^{-8}$$

J. Bijnens et al. Nucl. Phys. B 648, 317 (2003)

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- $a_+ \sim N_{14} - N_{15} + \text{chiral logs} + \mathcal{O}(p^6)$, $a_S \sim 2N_{14} + N_{15} + \text{chiral logs} + \mathcal{O}(p^6)$, $b_{+,S} \sim \mathcal{O}(p^6)$
- $G_F M_K^2 a_{+,S} = W_{K\pi;+,S}(0)$, $\quad G_F M_K^2 b_{+,S} = W'_{K\pi;+,S}(0) - \frac{1}{60}\left(\frac{M_K^2}{M_\pi^2}\right)^2 \left(\alpha_{+,S} - \beta_{+,S}\frac{s_0}{M_K^2}\right)$

Outline of a phenomenological determination of $a_{+,S}$ and $b_{+,S}$

$$W_{K\pi}(z) = W_{K\pi}^{\pi\pi}(z) + W_{K\pi}^{K\pi}(z) + W_{K\pi}^{K\bar{K}}(z) + \cdots + W_{K\pi}^{\mathrm{pQCD}}(z;\nu) + W_{K\pi}^{\mathrm{SD}}(z;\nu)$$

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$$W_{K\pi}(z) = W_{K\pi}^{\pi\pi}(z) + W_{K\pi}^{K\pi}(z) + W_{K\pi}^{K\bar{K}}(z) + W_{K\pi}^{\mathrm{res}}(z;\nu) + W_{K\pi}^{\mathrm{SD}}(z;\nu)$$

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- $W_{K\pi}^{\mathrm{res}}(z;\nu)$

- should reproduce the leading short-distance behaviour of $W_{K\pi}^{\mathrm{pQCD}}(z)$ for large euclidian z ($q^2 = s < 0$)

$$\lim_{q\to\infty} i\int d^4x\, e^{iq\cdot x} T\{j^\mu(x)\mathcal{L}_{\Delta S=1}(0)\}_{\overline{\mathrm{MS}}} =$$

$$= \left(-\frac{G_{\mathrm{F}}}{\sqrt{2}} V_{us}^* V_{ud}\right) [q^\mu q^\rho - q^2\eta^{\mu\rho}] \times \bar{s}\gamma_\rho(1-\gamma_5)d \times \frac{1}{4\pi}\sum_{I=1}^{6} C_I(\nu)\,\xi_I(\alpha_s\,;\nu^2/q^2) + \mathcal{O}(q)$$

$$\xi_I(\alpha_s\,;\nu^2/q^2) = \sum_{p\geq 0}\sum_{r=0}^{p+1} \xi_{pr}^I \alpha_s^p(\nu)\ln^r(-\nu^2/q^2)$$

such as to match the ν-dependence of the SD part $W_{K\pi}^{\mathrm{SD}}(z;\nu)$

Outline of a phenomenological determination of $a_{+,S}$ and $b_{+,S}$

$$W_{K\pi}(z) = W_{K\pi}^{\pi\pi}(z) + W_{K\pi}^{K\pi}(z) + W_{K\pi}^{K\bar{K}}(z) + \cdots + W_{K\pi}^{\mathrm{pQCD}}(z;\nu) + W_{K\pi}^{\mathrm{SD}}(z;\nu)$$

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- $W_{K\pi}^{\mathrm{res}}(z;\nu)$

- should reproduce the leading short-distance behaviour of $W_{K\pi}^{\mathrm{LD}}(z)$ for large euclidian z ($q^2 = s < 0$)

- described by an infinite sum of equally-spaced (in mass^2) zero-width resonances (Regge-type spectrum)

M. A. Shifman, hep-ph/0009131

M. Golterman and S. Peris, JHEP 0101, 028 (2001)

M. Golterman, S. Peris, B. Phily and E. De Rafael, JHEP 0201, 024 (2002)

S. Friot, D. Greynat and E. De Rafael, Phys. Lett. B 628, 73 (2005)

E. de Rafael, Pramana 78, 927 (2012)

D. Greynat, E. de Rafael and G. Vulvert, JHEP 1403, 107 (2014)

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- $W_{K\pi}^{\rm res}(z;\nu)$

- should reproduce the leading short-distance behaviour of $W_{K\pi}^{\rm LD}(z)$ for large euclidian z ($q^2 = s < 0$)

- described by an infinite sum of equally-spaced (in mass^2) zero-width resonances (Regge-type spectrum)

- such a model has been constructed, with exact matching at LO and NLO

$$W_{K\pi}^{\rm res}(z;\nu) = -16\pi^2 M_K^2 \left(-\frac{G_{\rm F}}{\sqrt{2}} V_{us} V_{ud} \right) \frac{C_{K\pi} f_+^{K\pi}(z M_K^2)}{4\pi} \times \mathcal{W}_{K\pi}^{\rm res}(z;\nu)$$

$$\mathcal{W}_{K\pi}^{\rm res}(z;\nu) = \int dx \, \frac{\rho_{K\pi}^{\rm res}(x;\nu)}{x - z M_K^2 - i0}$$

$$\rho_{K\pi}^{\rm res}(s;\nu) = \sum_{n\geq 1} M^2 \mu_n(\nu)\, \delta(s - nM^2) \qquad [M \sim 1 \text{ GeV}]$$

G. D'Ambrosio, D. Greynat and M. Knecht, JHEP 1902, 049 (2019)

G. D'Ambrosio, D. Greynat and M. Knecht, Phys. Lett. B 797, 134891 (2019)

Outline of a phenomenological determination of $a_{+,S}$ and $b_{+,S}$

$$W_{K\pi}(z) = W_{K\pi}^{\pi\pi}(z) + W_{K\pi}^{K\pi}(z) + W_{K\pi}^{K\bar{K}}(z) + \cdots + W_{K\pi}^{\mathrm{pQCD}}(z;\nu) + W_{K\pi}^{\mathrm{SD}}(z;\nu)$$

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- $W_{K\pi}^{\pi\pi}(z)$

- describe the contribution from $\pi\pi$ intermediate state by an unsubtracted DR

$$W_{+}^{\pi\pi}(z) = \frac{1}{\pi}\int_0^\infty dx\, \frac{\mathrm{Abs}\, W_{+}^{\pi\pi}(x/M_K^2)}{x - zM_K^2 - i0}$$

with

$$\frac{\mathrm{Abs}\, W_{+}^{\pi\pi}(s/M_K^2)}{16\pi^2 M_K^2} = \theta(s-4M_\pi^2) \times \frac{s-4M_\pi^2}{s}\lambda_{K\pi}^{-1/2}(s) F_V^{\pi *}(s) \times f_1^{\pi^+\pi^-\to K^+\pi^-}(s)$$

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with

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- $a^{\pi\pi}_{+,S}$ and $b^{\pi\pi}_{+,S}$ are given by spectral sum rules

$$G_F M_K^2 a_+^{\pi\pi} = W_+(0) = \int_{4M_\pi^2}^{+\infty} dx\, \frac{\mathrm{Abs}\, W^{\pi\pi}_{+}(x/M_K^2)}{x}$$

$$G_F M_K^2 b_+^{\pi\pi} = W'_+(0) = M_K^2\int_{4M_\pi^2}^{+\infty} dx\, \frac{\mathrm{Abs}\, W^{\pi\pi}_{+}(x/M_K^2)}{x^2}$$

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- simple approach: unitarization through IAM method, starting from known one-loop amplitude

T. N. Truong, Phys Rev Lett 61, 2526 (1988)
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A. Dobado, J. R. Pelaez, Phys Rev D 56, 3057 (1997)
J. Nieves et al., Phys Rev D 65, 036002 (2002)

with two parameters, α_+ (slope) and β_+ (curvature), extracted from global fits to $K \to \pi\pi\pi$ Dalitz plots

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$$a_+ = -1.59(8), \quad b_+ = -0.82(6) \quad \longrightarrow \quad \frac{a_+}{b_+}\frac{M_K^2}{M_V^2} \sim 0.8 \quad \text{too much VMD} - \text{like}$$

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Towards a phenomenological determination of $a_{+,S}$ and $b_{+,S}$: perspectives for the future

More elaborate treatment of $\pi\pi$ FSI (Khuri-Treiman equations)

Add also $K\pi$ (ISI)... and $K\bar{K}$ $\longrightarrow$ coupled-channel KT analysis, cf.

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Compare with lattice results

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More data will become available

- $K^+ \to \pi^+\mu^+\mu^-$ from NA62 (also $K^+ \to \pi^+e^+e^-$? Three-parameter fit on a_+, b_+ and β_+?)

  Ľuboš Bičian, talk at ICHEP 2020

- $K_S \to \pi^0\mu^+\mu^-$ from LHCb

Thank you for your attention