High-precision determination of V_{us} and V_{ud} from lattice QCD

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(RBC and UKQCD collaborations)

Rare Processes and Precision Frontier Townhall Meeting

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Outline

leptonic Kaon/Pion decay including radiative corrections

semi-leptonic Kaon decay including radiative corrections

semi-leptonic Baryon Decays

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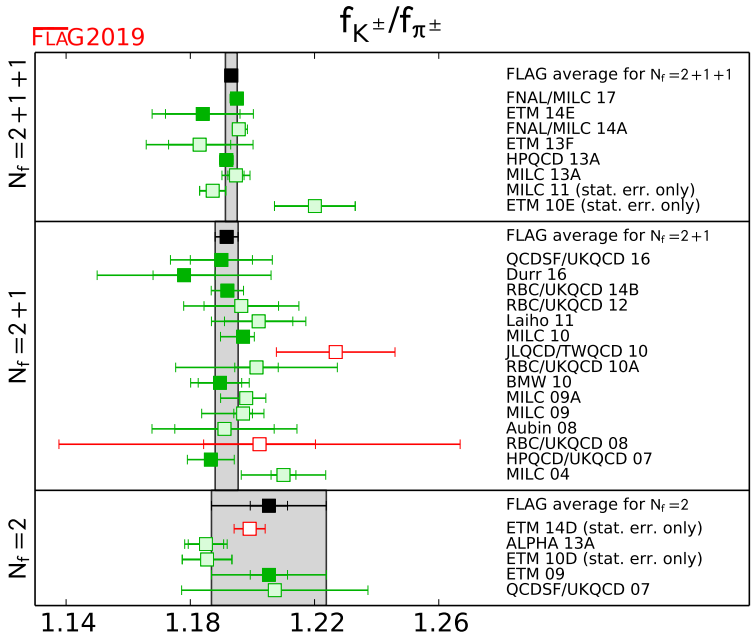
leptonic Meson Decays: Motivation

- leptonic meson decays: $K^{\pm}/\pi^{\pm} \to \ell^{\pm}\nu_{\ell}$
- tree-level decay rate (e.g. kaon decay)

$$\Gamma^0(K^+ \to \ell^+\nu_\ell) = \frac{G_F^2\,|V_{us}|^2\,f_K^2}{8\pi}\,M_K\,m_\ell^2\left(1-\frac{m_\ell^2}{M_K^2}\right)^2$$

- decay constant f_K (calculated on the lattice)



[FLAG, Eur.Phys.J.C 80 (2020), http://flag.unibe.ch/2019/]

- $\lesssim$ **1%** precision for f_K and f_π

$\to$ isospin breaking corrections become important

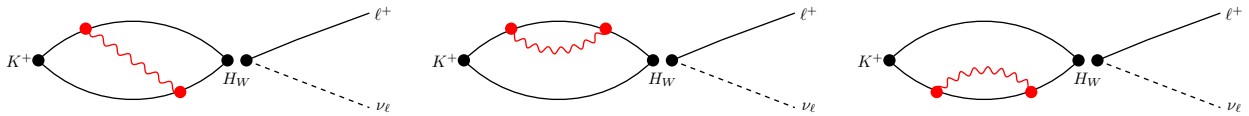
- different masses for up- and down quark (of $\mathcal{O}((m_d - m_u)/\Lambda_{\mathrm{QCD}})$)
- Quarks have electrical charge $\mathcal{O}(\alpha)$
- full QCD+QED decay rate

$$\Gamma = \Gamma^0 + \delta\Gamma = \Gamma^0(1+\delta R)$$

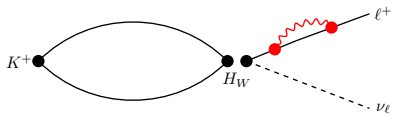
leptonic Meson Decays: QED corrections

- perturbative treatment of QED on lattice [RM123 Collaboration, Phys.Rev. **D87**, 114505 (2013)]
- formalism for $K_{\ell 2}$ [N. Carrasco *et al*, Phys.Rev. **D91**, 074506 (2015)], [V. Lubicz *et al*, Phys. Rev. **D95**, 034504 (2017)]

- quark QED corrections $\mathcal{O}(e_q^2)$

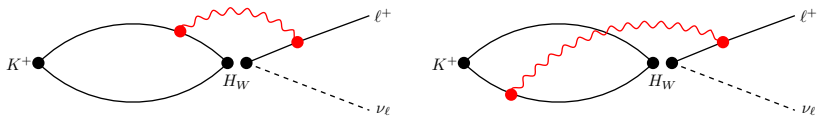


- lepton QED corrections $\mathcal{O}(e_\ell^2)$

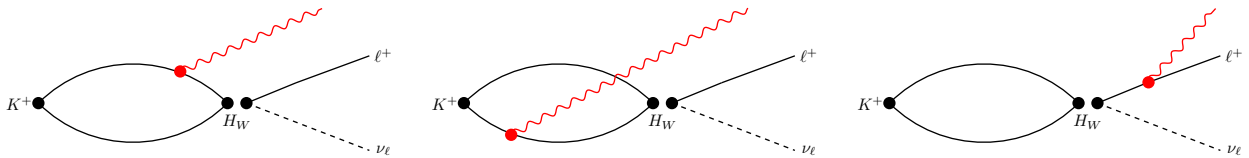


$\longrightarrow$ absorbed in renormalisation of lepton

- quark-lepton QED correction $\mathcal{O}(e_\ell e_q)$



- final state photon radiation (to cancel IR divergences)



leptonic Meson Decays: Status and Plans

- first lattice results [M. Di Carlo *et al*, Phys. Rev. D 100, 034514 (2019)], [D. Giusti *et al*, Phys. Rev. Lett. **120**, 072001 (2018)], [A. Desiderio *et al*, arXiv:2006.05358]

- RBC/UKQCD Status:
  - Work in progress: Calculation directly at the physical point
  - Full calculation of all quantities in 2-3 years including
    - continuum extrapolation
    - non-perturbative renormalisation of the weak Hamiltonian in QCD+QED
    - isospin breaking effects for sea quarks
    - lattice calculation of real photon emission
  - Exploring to calculate QED corrections in infinite volume analytically to eliminate power-law finite volume errors [X. Feng and L. Jin, Phys. Rev. **D100**, 094509 (2019)]

- inclusion of isospin breaking corrections in the calculation allows for a more precise determination of V_{us} and V_{ud}
- aim for precision of all required lattice calculations to $\lesssim 1‰$ within 10 years

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semi-leptonic Baryon Decays

Semi-leptonic Kaon Decay: Motivation

- $K_{\ell 3}$-Decay: $K \to \pi \ell \nu_\ell$
- hadronic matrix element (w/o QED corrections)

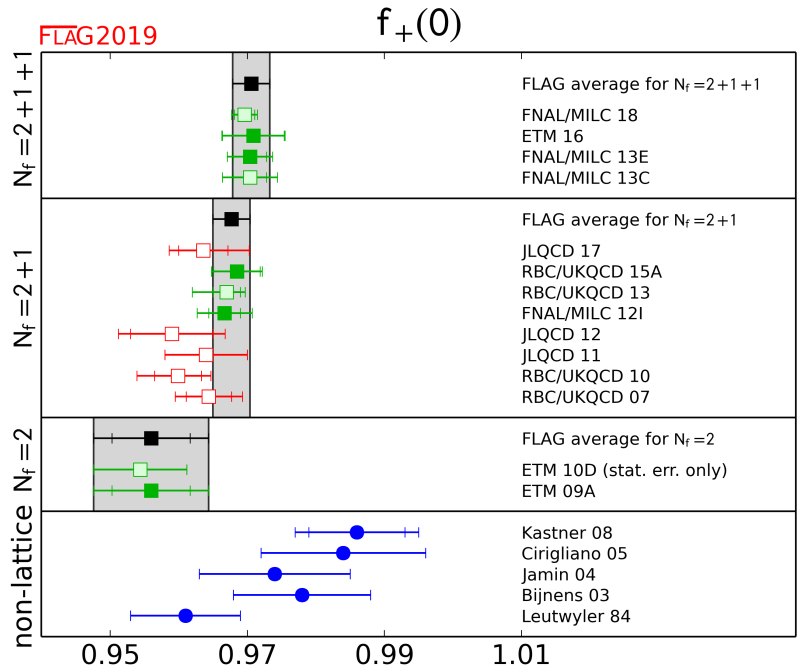
$$\langle \pi(p_\pi) | \bar{u} \gamma_\mu s | K(p_K) \rangle = f_+(q^2)(p_K + p_\pi)_\mu + f_-(q^2)(p_K - p_\pi)_\mu$$

- tree-level decay rate

$$\Gamma^0_{K_{\ell 3}} = \frac{G_F^2 |V_{us}|^2 M_K^5 C_K^2}{128\pi^3} |f_+(0)|^2 I^0_{K\ell}(\lambda_i)$$

- $I^0_{K\ell}(\lambda_i)$: phase space integration
- $f_+(0)$ from lattice QCD

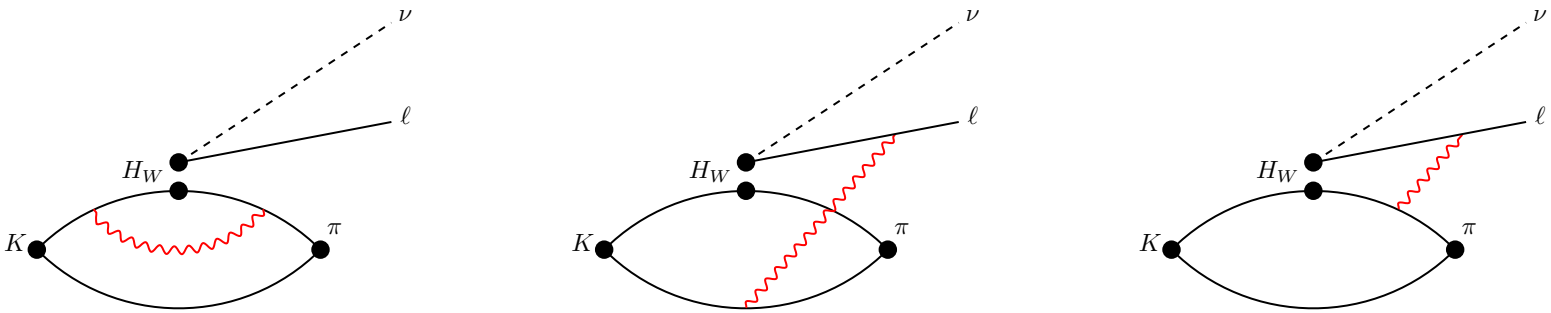
- Isospin Breaking corrections from χPT [V. Cirigliano *et al*, JHEP 11 , 006 (2008)]
- plan: *ab initio* determination from lattice QCD+QED



[FLAG, Eur.Phys.J.C 80 (2020), http://flag.unibe.ch/2019/]

Semi-leptonic Kaon Decays: Isospin Breaking corrections

- $K^{\pm}_{\ell 3}$: $K^{\pm} \to \pi^0 \ell^{\pm} \nu_\ell$ $\qquad$ $K^0_{\ell 3}$: $K^0 \to \pi^{\pm} \ell^{\mp} \nu_\ell$
- examples of diagrams



- final state photon radiation to cancel IR divergences (detailed discussion in [C. Sachrajda *et al*, PoS LATTICE2019 (2019)])
- lattice calculation to scan the whole 2D Daliz plot of allowed kinematics

- Within 10 years: complete calculation at physical point at ‰-level precision

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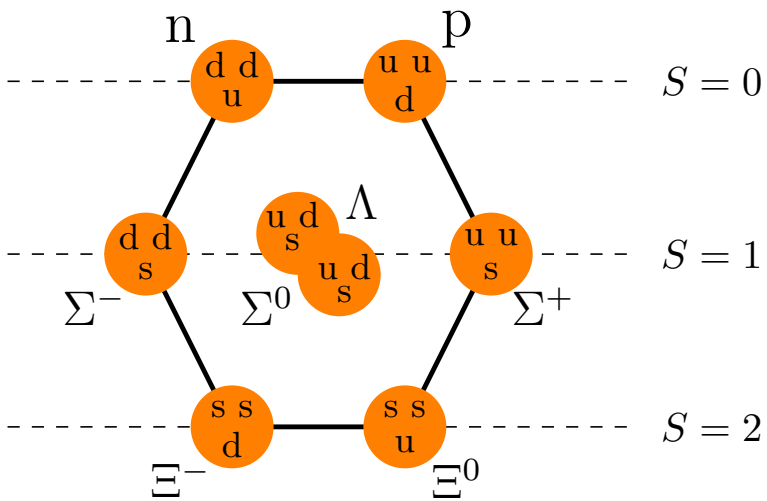
semi-leptonic Baryon Deacys

- Decays $B_i \to B_f \ell^- \overline{\nu}_\ell$ with $s \to u$ within the Baryon octet
- determination of V_{us}
- Branching fractions [PDG]



decay	$\mathcal{B}(B_f \to B_i \ell \nu_\ell)$	experiment
$\Sigma^- \to n e^- \overline{\nu}_e$	$(1.017 \pm 0.034) \times 10^{-3}$	SPS 1983
$\Sigma^- \to n \mu^- \overline{\nu}_\mu$	$(4.5 \pm 0.4) \times 10^{-4}$	BNL 1971
$\Xi^0 \to \Sigma^+ e^- \overline{\nu}_e$	$(2.52 \pm 0.08) \times 10^{-4}$	NA48 2007
$\Xi^0 \to \Sigma^+ \mu^- \overline{\nu}_\mu$	$(2.33 \pm 0.35) \times 10^{-6}$	NA48 2013
$\Xi^- \to \Sigma^0 e^- \overline{\nu}_e$	$(8.7 \pm 1.7) \times 10^{-5}$	SPS 1983
$\Xi^- \to \Sigma^0 \mu^- \overline{\nu}_\mu$	$< 8 \times 10^{-4}$	BNL 1974
$\Xi^- \to \Lambda e^- \overline{\nu}_e$	$(5.63 \pm 0.31) \times 10^{-4}$	SPS 1983
$\Xi^- \to \Lambda \mu^- \overline{\nu}_\mu$	$(3.5^{+3.5}_{-2.2}) \times 10^{-4}$	BNL 1974
$\Lambda \to p e^- \overline{\nu}_e$	$(8.32 \pm 0.14) \times 10^{-4}$	SPS 1983
$\Lambda \to p \mu^- \overline{\nu}_\mu$	$(1.57 \pm 0.35) \times 10^{-4}$	CERN 1972

semi-leptonic Baryon Deacys

- lattice calculation of the required hadronic matrix element

$$\begin{aligned}&\left\langle B_i \middle| \overline{u}\,\gamma_\mu(1-\gamma_5)\,s \middle| B_f \right\rangle \\ =&\overline{u}_{B_i}(p_{B_i})\Big\{\Big[\gamma_\mu f_1(q^2) - i\frac{\sigma^{\mu\nu}q_\nu}{M_{B_f}+M_{B_i}} f_2(q^2) + \frac{q_\mu}{M_{B_f}+M_{B_i}} f_3(q^2)\Big] \\ &+\Big[\gamma_\mu g_1(q^2) - i\frac{\sigma^{\mu\nu}q_\nu}{M_{B_f}+M_{B_i}} g_2(q^2) + \frac{q_\mu}{M_{B_f}+M_{B_i}} g_3(q^2)\Big]\gamma_5\Big\}u_{B_f}(p_{B_f})\end{aligned}$$

  six hadronic form factors

- Lattice calculations including Baryons suffer from exponentially growing noise-to-signal ratio

- short term ($\approx$ 2-3 years): Lattice QCD calculation at **1%** precision

- long term: going beyond **1%** precision? $\longleftrightarrow$ future experimental effort?

Thank you