Precise lattice QCD calculations of kaon and pion decay parameters and first-row CKM unitarity tests

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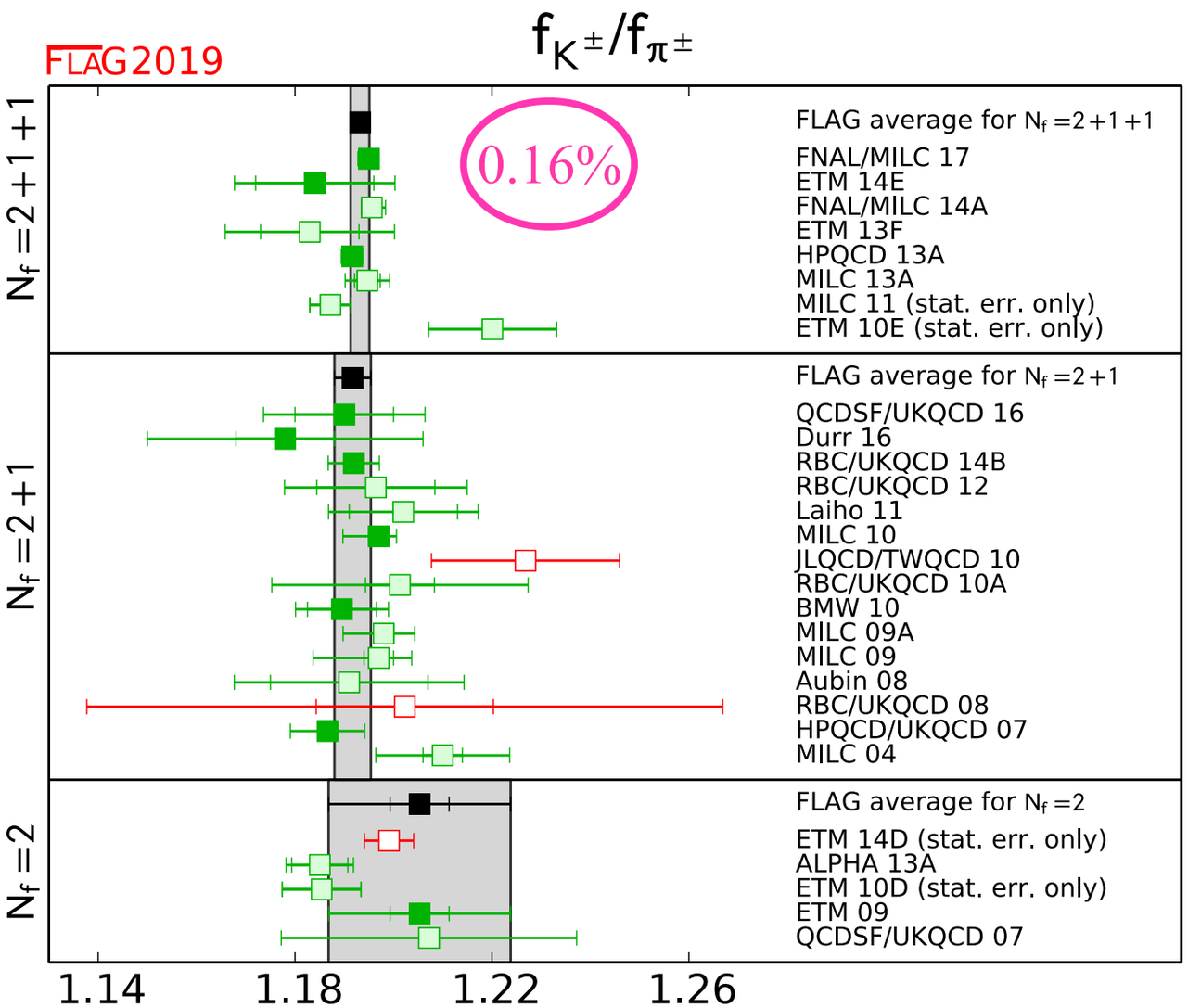
Snowmass Rare and Precision Frontier Town Hall
02 October 2020

Leptonic decay constants

S. Aoki et al [FLAG review 2019, arXiv:1902.08191]



small errors due to

- ✦ **physical light quark masses**
- ✦ improved light-quark actions
- ✦NPR or no renormalization

Lattice input for CKM determination:

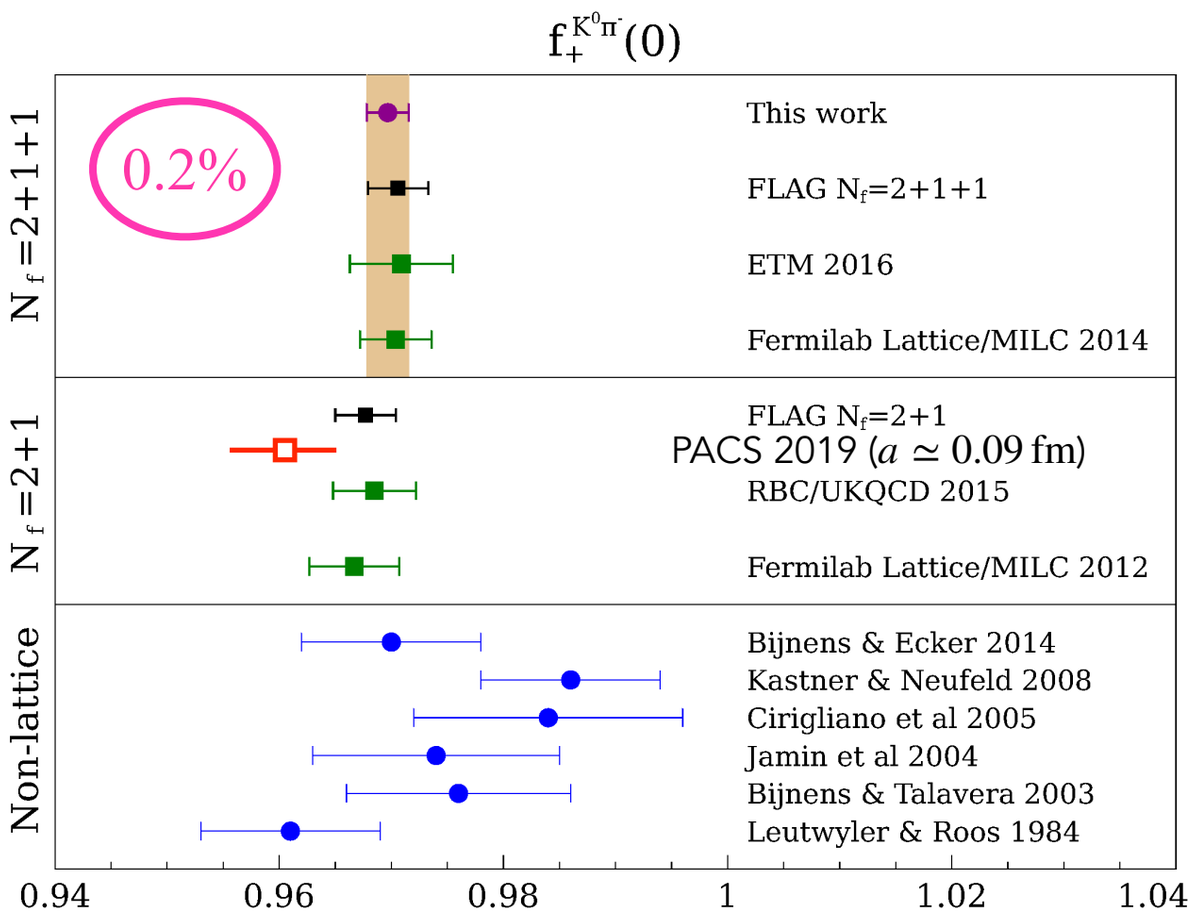
$$\frac{\Gamma(K^+ \to l^+\nu_l(\gamma))}{\Gamma(\pi^+ \to l^+\nu_l(\gamma))} \propto \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{f_{K^\pm}^2}{f_{\pi^\pm}^2}\frac{\left(1+\delta^l_{EM,K}\right)}{\left(1+\delta^l_{EM,\pi}\right)}$$

Needed to relate pure QCD decay constant to experiment. Currently estimated phenomenologically.
[Cirigliano et al, arXiv:1107.6001, RMP 2012]

Can be used with $|V_{ud}|$ from nuclear β decay to determine $|V_{us}|$.

$K_{\ell 3}$ form factor

A. Bazavov et al [FNAL/MILC, arXiv:1809.02827, 2019 PRD]



Lattice input for CKM determination:

$$\Gamma_{K_{l3(\gamma)}} \propto |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \left(1 + \delta_{\rm EM}^{Kl} + \delta_{\rm SU(2)}^{K\pi}\right)$$

Needed to relate pure QCD form factor to experiment. Mode dependent.

Needed to include charged kaon decay in the experimental average.

[Cirigliano et al, arXiv:1107.6001, RMP 2012].

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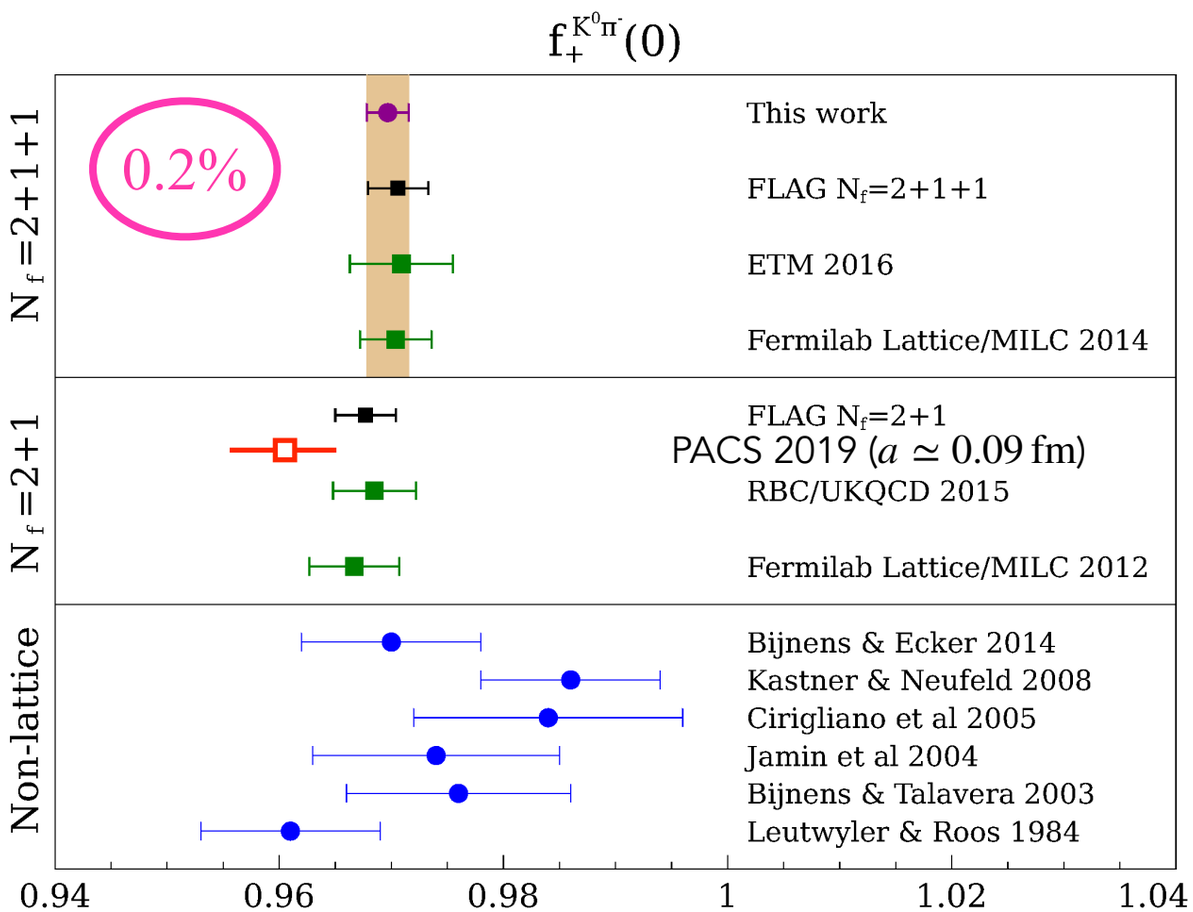

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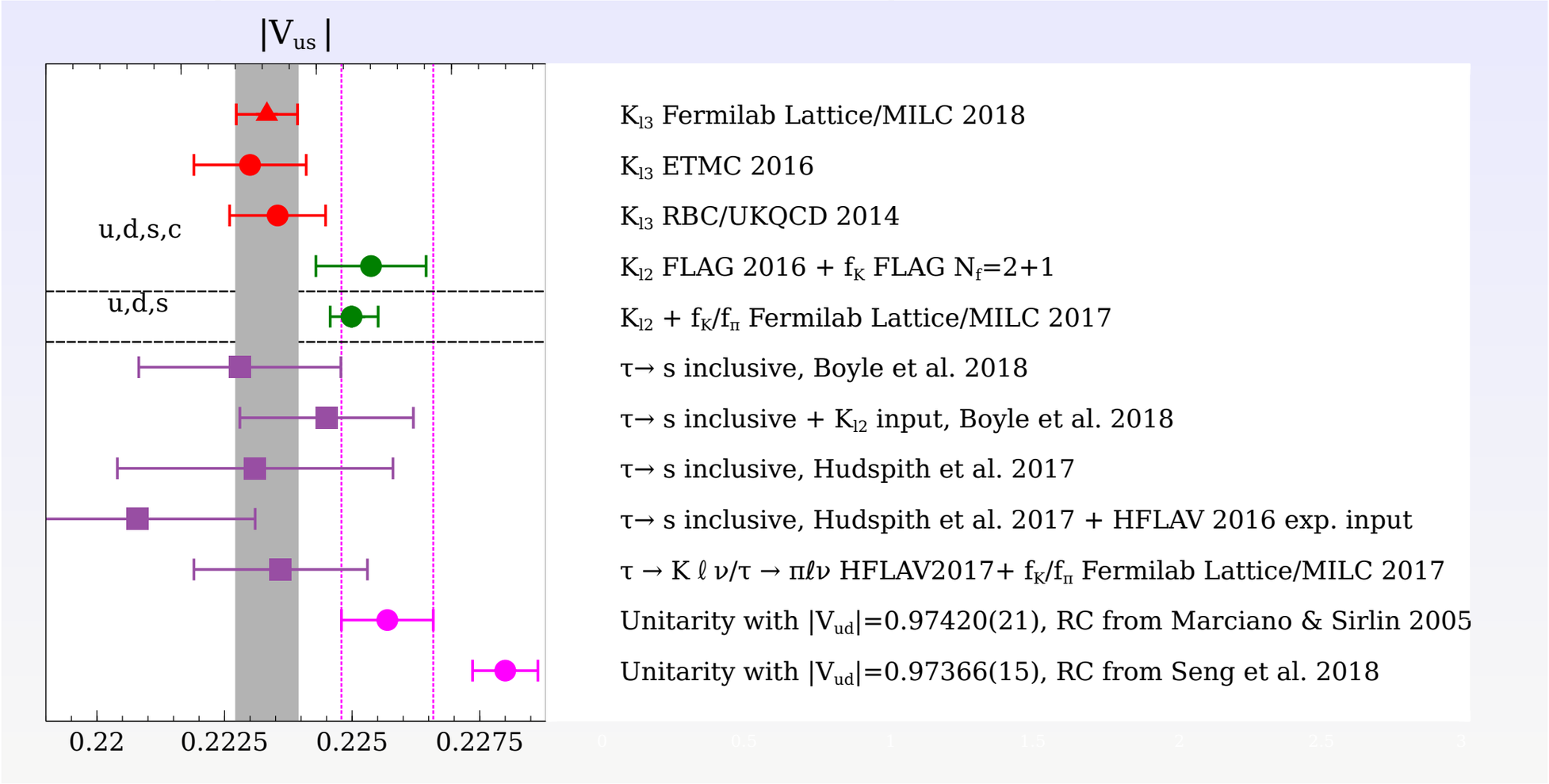
➠Lattice results have reached the ``QED wall":
- QED effects now important source of uncertainty
- further improvement of theory error requires inclusion of QED in lattice calculation

Implications for $|V_{us}|$

A. Bazavov et al [FNAL/MILC, arXiv:1809.02827, 2019 PRD]



Tensions with leptonic determinations:

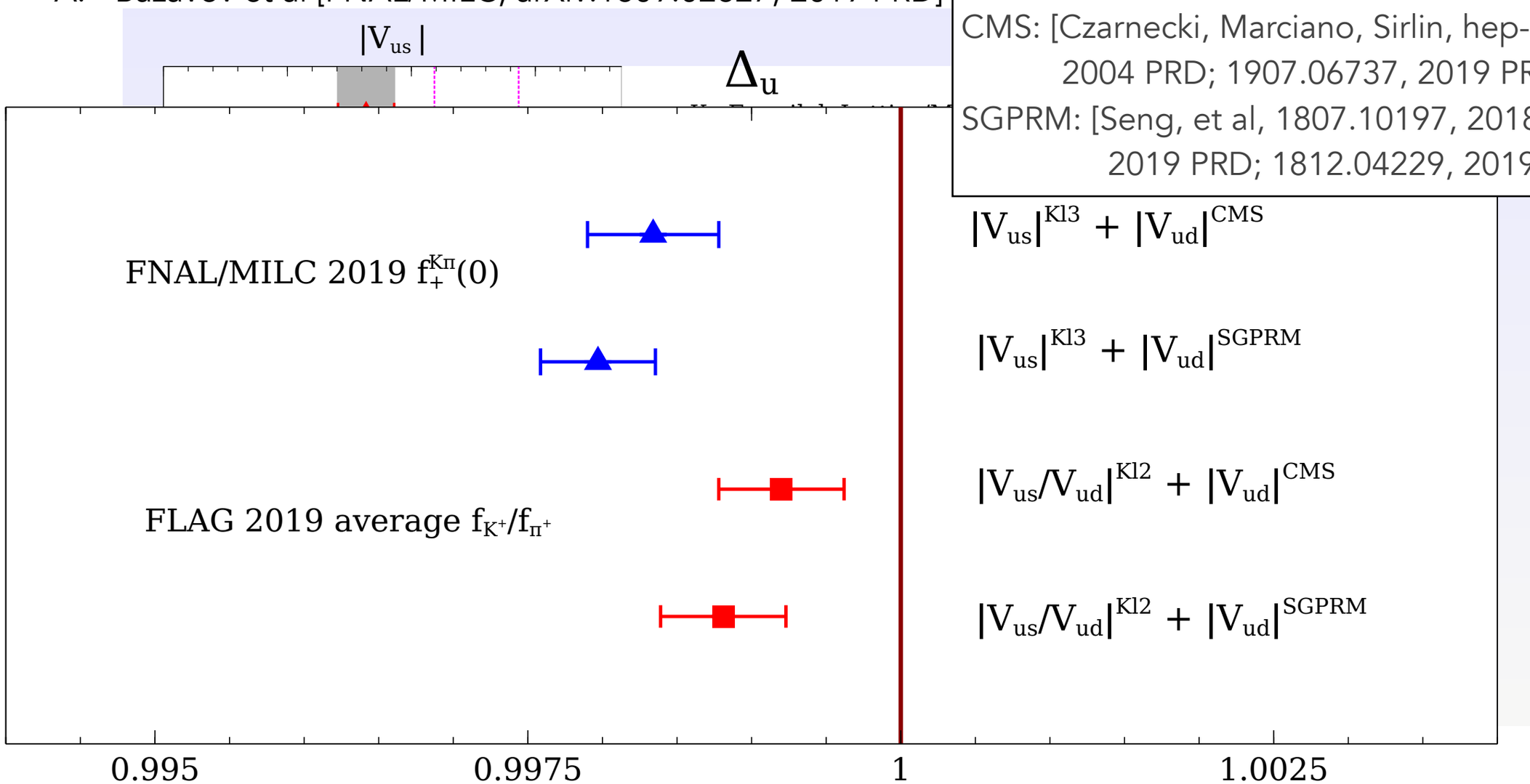
- $\Gamma^{\rm exp}_{K_{\ell 2}} + f_{K^\pm}$: 1.6σ
- $\Gamma^{\rm exp}_{K_{\ell 2}} + f_{K^\pm}/f_{\pi^\pm} + |V_{ud}|$: 2.2σ

Tension with CKM unitarity: 2-5σ

Implications for $|V_{us}|$

A. Bazavov et al [FNAL/MILC, arXiv:1809.02827, 2019 PRD]

CMS: [Czarnecki, Marciano, Sirlin, hep-ph/0406324, 2004 PRD; 1907.06737, 2019 PRD]
SGPRM: [Seng, et al, 1807.10197, 2018 PRL; 1812.03353, 2019 PRD; 1812.04229, 2019 PRL]



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On-going projects

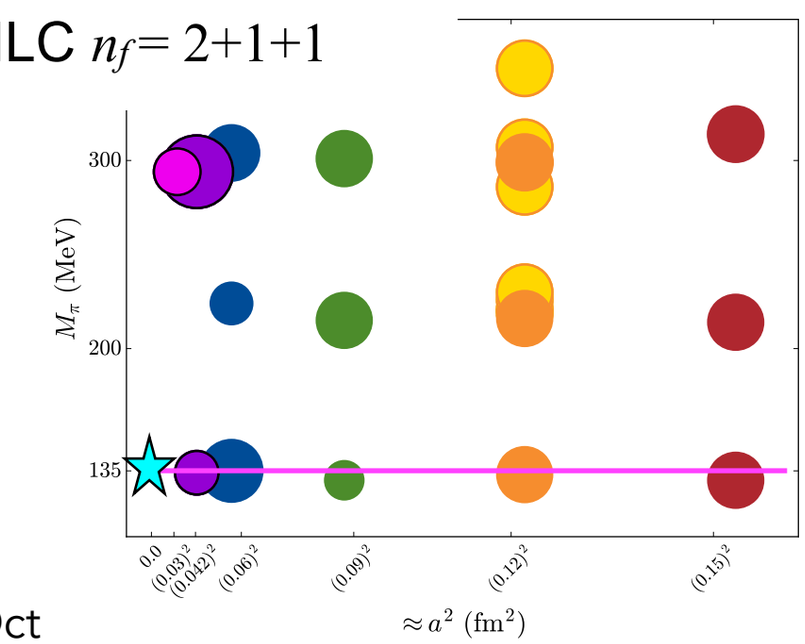
- 1st row CKM unitarity test without using $|V_{ud}|$ from nuclear decay. Currently:

$$\Delta_u = -0.0151(38)_{f_+(0)}(36)_{f_{K^\pm}/f_{\pi^\pm}}(36)_{\rm exp}(27)_{\rm EM}$$ 2.2σ tension

  - refining analysis of leptonic decay constants to combine with $K_{\ell 3}$ form factor, taking correlations into account.
  - determine lattice spacing from different quantity (not $\pi_{\ell 2}$ and f_π, which uses $|V_{ud}|$) ➠ enables $|V_{ud}|$ determination from f_π

- Continue program on MILC HISQ ensembles

MILC n_f= 2+1+1



Future possibilities

- ☆ Calculate QED corrections (including radiative corrections) directly in lattice QCD+QED.
  - Methods being developed for isospin and radiative corrections. First lattice results [DiCarlo, et al, 1904.0873, 2020 PRD; Kane at al, 1907.00279; Desiderio et al, 2006.05358]
  - builds on our program calculating QED corrections to $(g-2)_\mu$ LO HVP [S. Gottlieb, LOI presentation in RF3]
  - extend to B, D decays. [A. Kronfled, LOI presentation in RF1]
- ☆ Rare kaon processes: promising channels for New Physics searches + active exp. program (NA62, KOTO, KOTO-II, KLEVER, LHCb)
  - quantification of long-distance contributions
  - already first results by RBC/UKQCD [N. Christ et al, 1608.07585, 2016 PRD; 1910.10644, 2019 PRD]
  - plan to build on infrastructure for QED corrections and inclusive decay rates [W. Jay, LOI presentation in RF1]

Future possibilities

- ☆ Structure-dependent radiative corrections to $|V_{ud}|$ determinations from nuclear decay.
  - Help disentangle current tensions
  - First lattice calculation [Feng at al, 2003.09798, 2020 PRL]
  - would build on infrastructure for QED and radiative corrections
- ☆ $|V_{us}|$ from strange hadronic τ decay
  - alternate determination
  - Expect improved experimental measurements (Belle II, LHCb, …)
    ➠ potential for competitive result
  - builds on lattice HVP infrastructure
  - first lattice results from RBC/UKQCD

    [P. Boyle, 1803.07228, 2018 PRL]