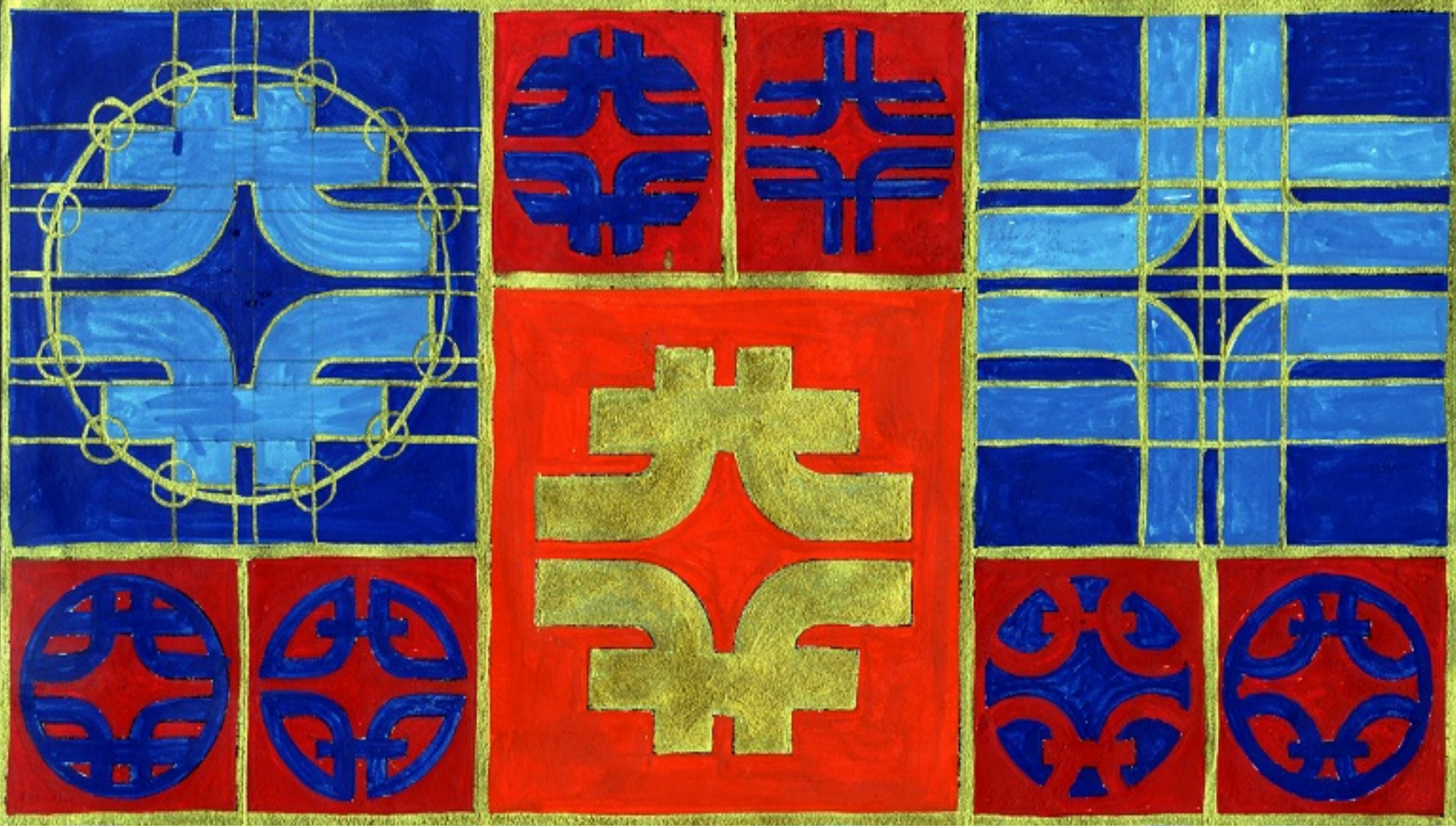

Inclusive B-Meson Decays from Lattice QCD

William I. Jay (Fermilab) and Thomas DeGrand (CU Boulder)

Rare Processes and Precision Frontier Townhall — 10/2/2020

Snowmass2021 - Letter of Interest

Lattice-QCD studies of inclusive B-meson decays

Snowmass Topical Groups: (check all that apply □/■)

■ (RF1) Weak decays of b and c quarks
■ (TF02) Effective field theory techniques
■ (TF05) Lattice gauge theory
■ (TF06) Theory techniques for precision physics
■ (TF07) Collider phenomenology
■ (CompF2) Theoretical Calculations and Simulation

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Link to LOI

Inclusive B-Meson Decays from Lattice QCD

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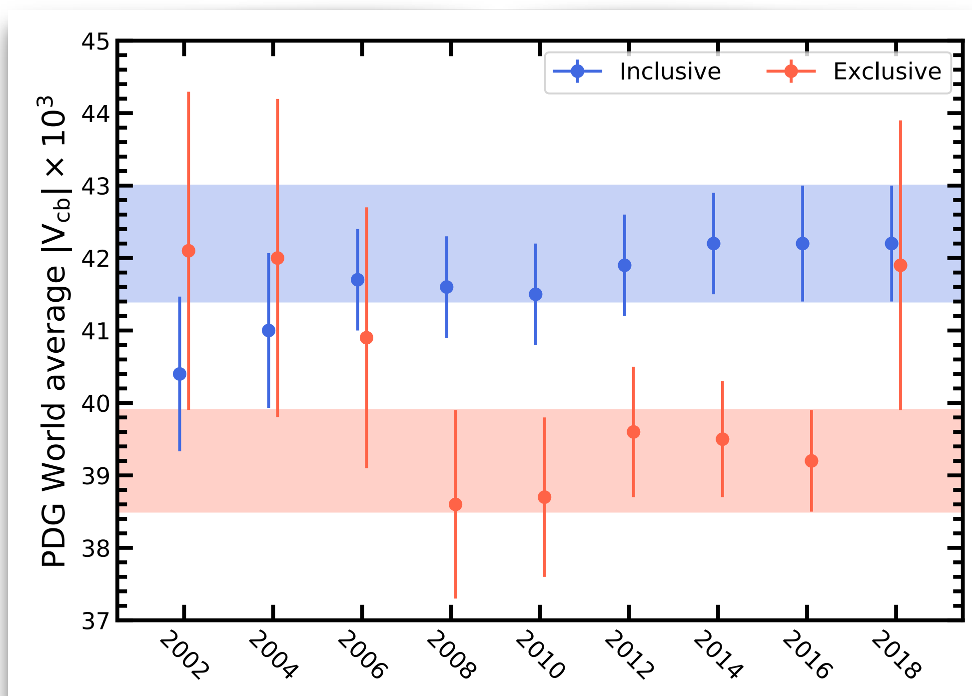
Experimental Tension

Tension in inclusive vs. exclusive determinations of

- $|V_{cb}|$ from $B \to D^* \ell \nu$ has 3.3σ tension
- $|V_{cb}|$ from $B \to D \ell \nu$ has 2.0σ tension
- $|V_{ub}|$ from $B \to \pi \ell \nu$ has 2.8σ tension



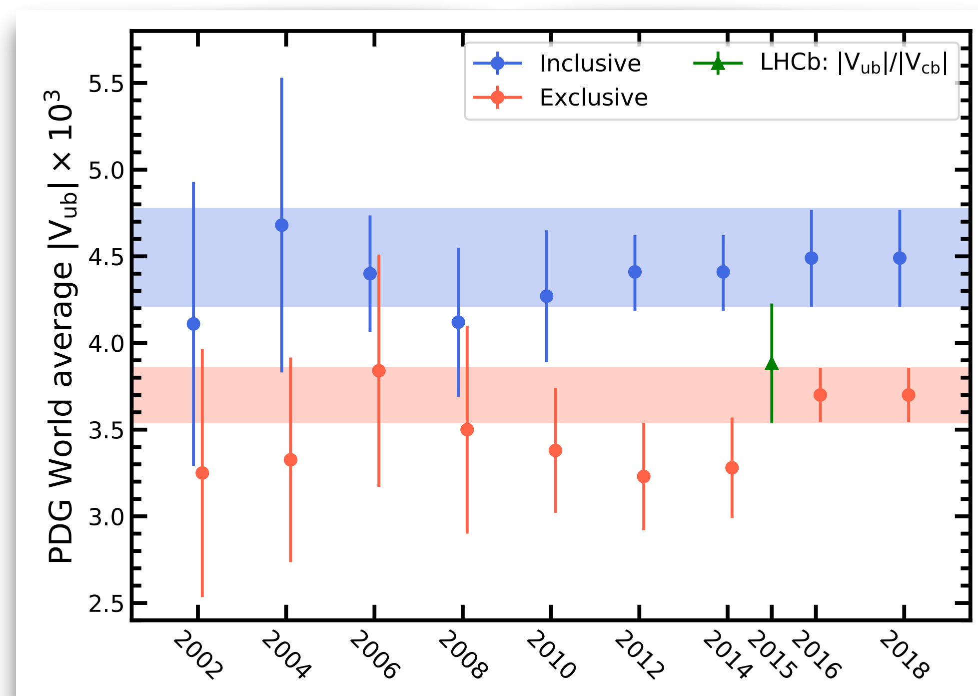


Image credit: Bouchard, Cao, Owen, arXiv:1902.09412

Experimental Tension

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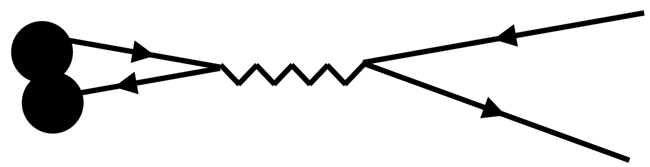
The most precise theoretical calculations employ different theoretical frameworks

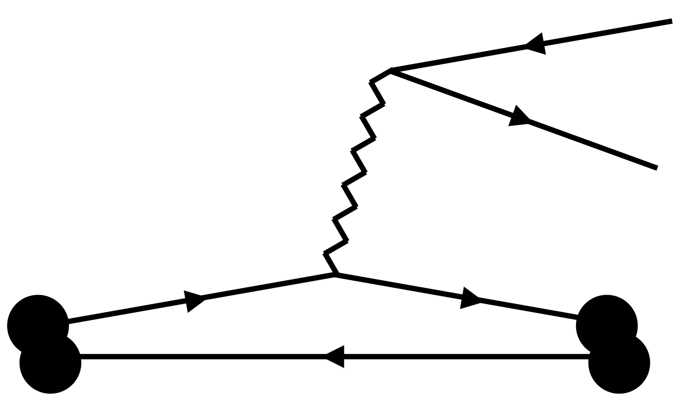
- Inclusive decays: continuum heavy quark EFT + operator product expansion
- Exclusive decays: numerical lattice gauge theory

The CKM Matrix on the Lattice

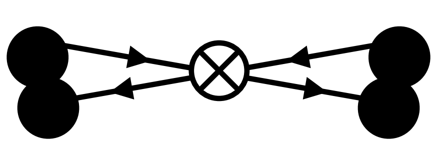
Leptonic decays



Semi-leptonic decays



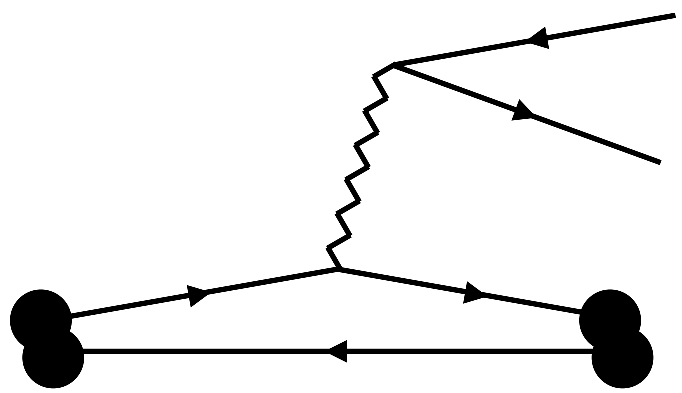
Meson Mixing



$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \ell\nu \\ & K \to \pi\ell\nu & B \to \pi\ell\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d|\bar{B}_d\rangle & \langle B_s|\bar{B}_s\rangle & \end{pmatrix}$$

Exclusive Semi-leptonic Decays

Via numerical lattice QCD



(form factors) ∝ (matrix elements)

$$f_J(p) \propto \langle \text{final} | \, J(p) \, | \text{initial} \rangle$$

- Methodology is well established
- Systematic effects are well understood
- Calculations are underway using physical quark masses: u, d, s, c, and b.
- Coming soon:

» B-meson decay form factors at the 1% level

» D-meson decay form factors at sub-percent level

Frontier calculations

The physics of Euclidean correlation functions:

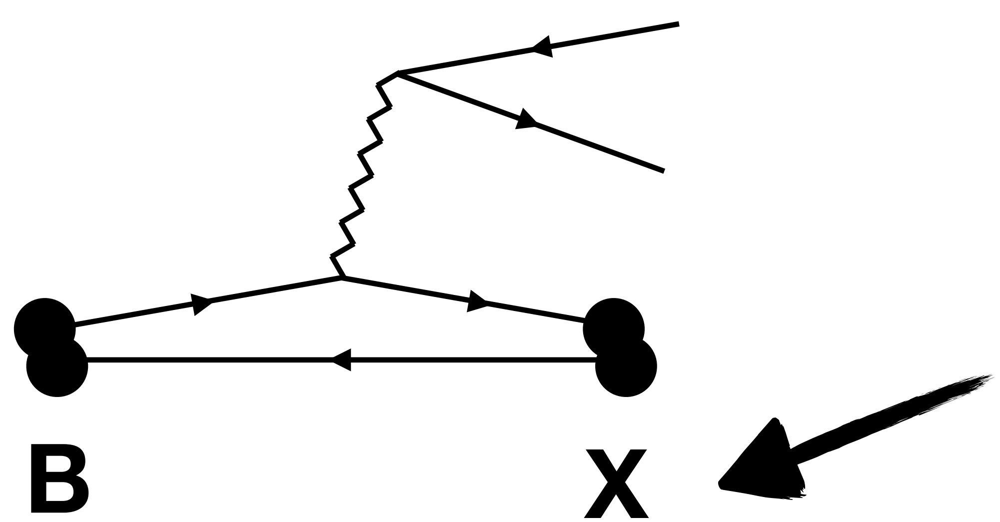
- 2-point functions: masses, decay constants ✓
- 3-point functions: form factors ✓

- 4-point functions:
  - Flavor physics: Inclusive B-meson decays
  - Kaon physics: K_L-K_S mixing, ε_K, rare kaon decays
  - Neutrino physics: $0\nu\beta\beta$-decay
  - Hadron structure: “hadronic tensor” $H_{\mu\nu}$

Frontier calculations

The physics of Euclidean correlation functions

- 2-point functions: masses, decay constants
- 3-point functions: form factors
- 4-point functions:



Sum over all hadronic final states X

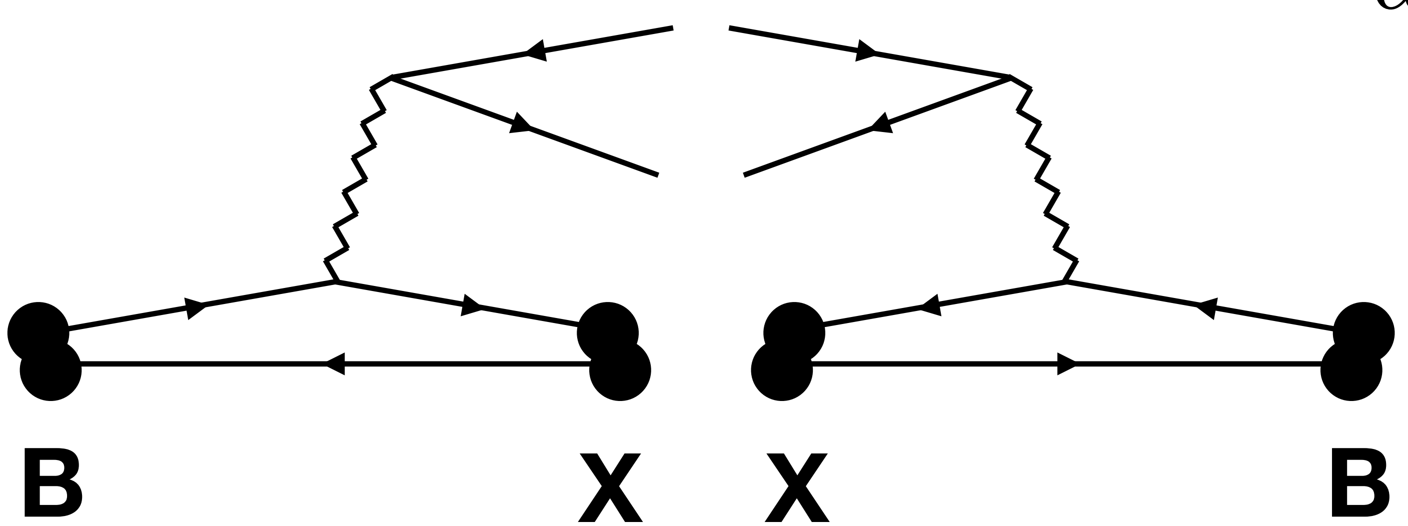
Frontier calculations

The physics of Euclidean correlation functions

- 2-point functions: masses, decay constants
- 3-point functions: form factors
- 4-point functions:

$$d\sigma \propto \mathcal{M}\mathcal{M}^\dagger$$

$$\sum_X$$



Frontier calculations

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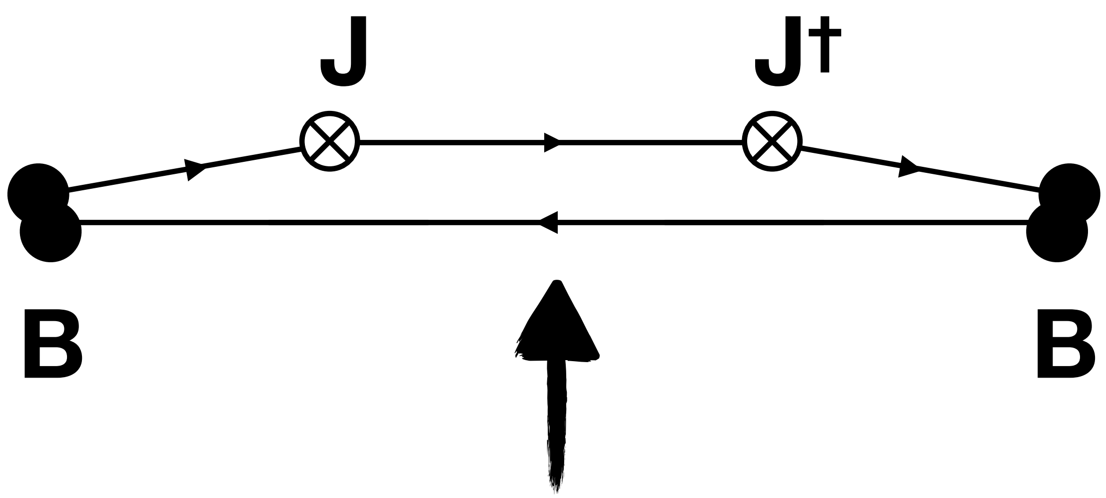

$\propto W_{\mu\nu}$

Automatic sum over intermediate states X

“Hadronic tensor” ∋ structure functions

Proposals for Inclusive B-decays

There have been several recent proposals for calculating inclusive decays using numerical lattice QCD

- Hashimoto PTEP (2017) 5, 053B03, **arXiv:1703.01881**
- Hansen, Meyer, Robaina: PRD 96 (2017) 9, 094513. **arXiv:1704.08993**
- Gambino and Hashimoto: PRL 125 (2020) 3, 032001. **arXiv:2005.13730**
- + proceedings from recent lattice conferences

Two technical challenges

- 4pt functions are numerically challenging
  - Techniques do exist already
  - Community rapidly developing new efficient techniques
- The Euclidean calculation occurs for spacelike kinematics
  - Analytic continuation is required to the physical region
  - A numerically delicate inverse transform is necessary
  - Also needed for lattice PDFs
  - A variety of options have been proposed:

    Backus-Gilbert, Bayesian reconstruction, Bayes-Gauss-Fourier transform, Maximum Entropy Methods, decomposition with Chebyshev polynomials, …

The inverse problem

- “Wick rotation” back to Minkowski space from discrete Euclidean data is an ill-posed inverse problem

Euclidean correlator

Kernel function

Physical “spectral density”

$$G(\tau_i) = \int_0^\infty d\omega K(\tau_i, \omega)\rho(\omega)$$

$$G_i = T_{ij}\rho_j$$

$$T_{ij} = \Delta\omega K_i(\omega_j)$$

Ill-posed, since T_{ij} is not square ($i \ll j$)

The inverse problem

- “Wick rotation” back to Minkowski space from discrete Euclidean data is an ill-posed inverse problem

Euclidean
cor

Kernel

al
al density”

Must augment problem with additional physics knowledge: smoothness, regularity, etc…

$$G_i = T_{ij}\rho_j$$

$$T_{ij} = \Delta\omega K_i(\omega_j)$$

Ill-posed because T_{ij} is not square ($i \ll j$)

The inverse problem

- “Wick rotation” back to Minkowski space from discrete Euclidean data is an ill-posed inverse problem
- Which methods work best? What are their limitations in principle and in practice?
- Transforming / smearing existing experimental data may allow for more direct comparison to lattice results. For example:
  - Poggio, Quinn, and Weinberg PRD 13:1958 (1976)

Summary

- Understanding the discrepancy between inclusive and exclusive determinations of CKM elements is a long-standing problem
- Frontier calculations using numerical lattice QCD will determine matrix elements for inclusive decays of B-mesons non-perturbatively
- The techniques developed will have broad applications throughout lattice QCD, e.g., in hadronic structure
- Reaching physical kinematics from Euclidean space requires solving a delicate inverse problem
- Look for important progress over the next 5-10 years.