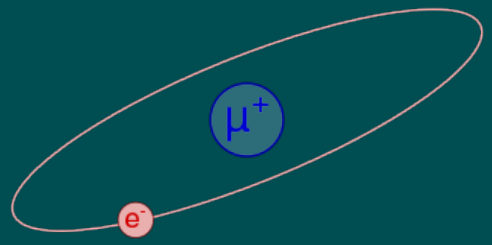

Physics of muonium and muonium-antimuonium oscillations

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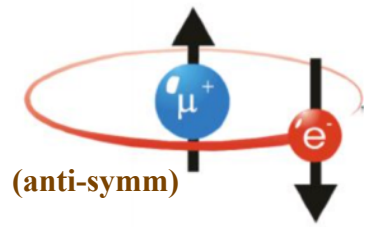
R. Conlin and AAP
arXiv: 2005.10276 [hep-ph]

LOI: https://www.snowmass21.org/docs/files/summaries/RF/SNOWMASS21-RF5_RF0-TF0_TF6_Alexey_Petrov-088.pdf

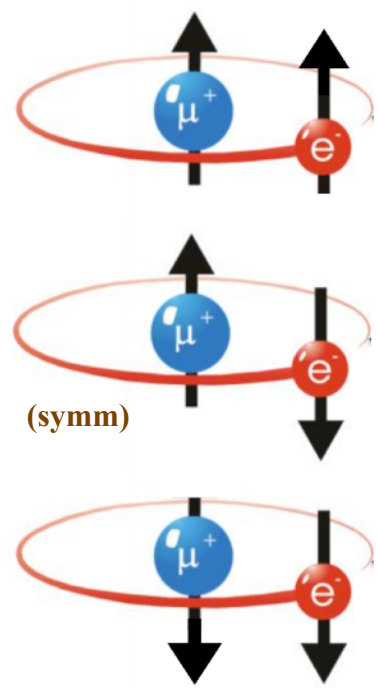
Muonium: just like hydrogen, but simpler!

- Muonium: a bound state of μ^+ and e^-
  - $(\mu^+\mu^-)$ bound state is a *true muonium*



Spin-0 (singlet) paramuonium

- Muonium lifetime $\tau_{M_\mu} = 2.2\ \mu s$
  - main decay mode: $M_\mu \to e^+e^-\bar{\nu}_\mu\nu_e$
  - annihilation: $M_\mu \to \bar{\nu}_\mu\nu_e$



Spin-1 (triplet) orthomuonium

- Muonium's been around since 1960's
  - used in chemistry
  - QED bound state physics, etc.
  - New Physics searches (oscillations)

Hughes (1960)

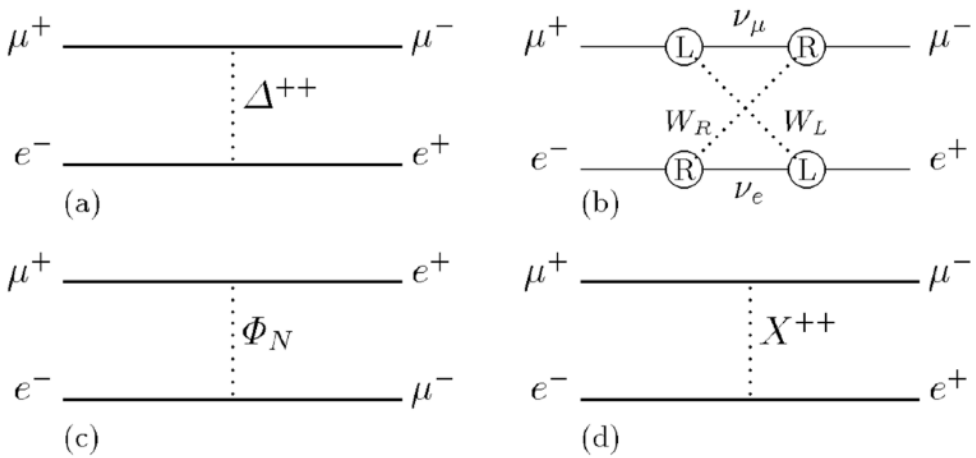
The masses of singlet and triplet are almost the same!

Muonium oscillations: just like $B^0\bar{B}^0$ mixing, but simpler!

★ Lepton-flavor violating interactions can change $M_\mu \to \overline{M}_\mu$

Pontecorvo (1957)
Feinberg, Weinberg (1961)

- Such transition amplitudes are tiny in the Standard Model
  - … but there are plenty of New Physics models where it can happen



$$\sim \left(\bar{\mu}\Gamma e\right)\left(\bar{\mu}\Gamma e\right)$$

effective operator

  - theory: compute transition amplitudes for ALL New Physics models!
  - experiment: produce M_μ but see for decay products of $\overline{M}_\mu$

Combined evolution = flavor oscillations

- If there is an interaction that couples M_μ and $\overline{M}_\mu$ (both SM or NP)
  - combined time evolution: non-diagonal Hamiltonian!

$$i\frac{d}{dt}\begin{pmatrix}|M(t)\rangle \\ |\overline{M}(t)\rangle\end{pmatrix} = \left(m - i\frac{\Gamma}{2}\right)\begin{pmatrix}|M(t)\rangle \\ |\overline{M}(t)\rangle\end{pmatrix}$$

  - diagonalization: new mass eigenstates:

$$|M_{\mu_{1,2}}\rangle = \frac{1}{\sqrt{2}}\left[|M_\mu\rangle \mp |\overline{M}_\mu\rangle\right]$$

  - new mass eigenstates: mass and lifetime differences

$$\left.\begin{aligned}\Delta m &\equiv M_1 - M_2, \\ \Delta\Gamma &\equiv \Gamma_2 - \Gamma_1.\end{aligned}\right\} \quad x = \frac{\Delta m}{\Gamma}, \qquad y = \frac{\Delta\Gamma}{2\Gamma} \quad \text{(small)}$$

These mass and width difference are observable quantities

Combined evolution = flavor oscillations

- Study oscillations via decays: amplitudes for $M_\mu \to f$ and $\overline{M}_\mu \to \bar{f}$
  - possibility of flavor oscillations ($M_\mu \to \overline{M}_\mu \to \bar{f}$)

$$
\begin{aligned}
|M(t)\rangle &= g_+(t)\,|M_\mu\rangle + g_-(t)\,|\overline{M}_\mu\rangle\,, \\
|\overline{M}(t)\rangle &= g_-(t)\,|M_\mu\rangle + g_+(t)\,|\overline{M}_\mu\rangle\,,
\end{aligned}
$$

with

$$
\begin{aligned}
g_+(t) &= e^{-\Gamma_1 t/2} e^{-i m_1 t}\left[1 + \frac{1}{8}(y - ix)^2 (\Gamma t)^2\right], \\
g_-(t) &= \frac{1}{2} e^{-\Gamma_1 t/2} e^{-i m_1 t}\,(y - ix)\,(\Gamma t)\,.
\end{aligned}
$$

  - time-dependent width: $\Gamma(M_\mu \to \bar{f})(t) = \frac{1}{2} N_f\,|A_f|^2\, e^{-\Gamma t}\,(\Gamma t)^2\, R_M(x,y)$
  - oscillation probability: $P(M_\mu \to \overline{M}_\mu) = \frac{\Gamma(M_\mu \to \bar{f})}{\Gamma(M_\mu \to f)} = R_M(x,y) = \frac{1}{2}\left(x^2 + y^2\right)$

Oscillation parameters: introduction

- Mixing parameters are related to off-diagonal matrix elements
  - heavy and light intermediate degrees of freedom

$$\left(m - \frac{i}{2}\Gamma\right)_{12} = \frac{1}{2M_M}\langle \overline{M}_\mu | \mathcal{H}_{\rm eff} | M_\mu \rangle + \frac{1}{2M_M}\sum_n \frac{\langle \overline{M}_\mu | \mathcal{H}_{\rm eff} | n \rangle \langle n | \mathcal{H}_{\rm eff} | M_\mu \rangle}{M_M - E_n + i\epsilon}$$

Local at scale $\mu = M_\mu$: only Δm
lepton number change $\Delta L_\mu = 2$

Bi-local at scale $\mu = M_\mu$: both Δm and $\Delta\Gamma$
lepton number changes: $(\Delta L_\mu = 1)^2$
or $(\Delta L_\mu = 0)(\Delta L_\mu = 2)$

  - each term has contributions from different effective Lagrangians

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm eff}^{\Delta L_\mu = 0} + \mathcal{L}_{\rm eff}^{\Delta L_\mu = 1} + \mathcal{L}_{\rm eff}^{\Delta L_\mu = 2}$$

  - … all of which have a form $\mathcal{L}_{\rm eff} = -\frac{1}{\Lambda^2}\sum_i c_i(\mu) Q_i,$ with $\Lambda \sim \mathcal{O}(TeV)$

Mass difference = real (dispersive) part; width difference: imaginary (absorptive) part

Mass difference

- Mass difference comes from the dispersive part

$$x = \frac{1}{2M_M\Gamma}\mathrm{Re}\left[2\langle\overline{M}_\mu\left|\mathcal{H}_{\rm eff}\right|M_\mu\rangle + \langle\overline{M}_\mu\left|i\int d^4x\ \mathrm{T}\left[\mathcal{H}_{\rm eff}(x)\mathcal{H}_{\rm eff}(0)\right]\right|M_\mu\rangle\right]$$

  - consider only $\Delta L_\mu = 2$ Lagrangian contributions (largest?)

$$\mathcal{L}_{\rm eff}^{\Delta L_\mu=2} = -\frac{1}{\Lambda^2}\sum_i C_i^{\Delta L=2}(\mu)Q_i(\mu)$$

  - leading order: all heavy New Physics models are encoded in (the Wilson coefficients of) the five dimension-6 operators

$$\begin{aligned}
Q_1 &= (\overline{\mu}_L\gamma_\alpha e_L)(\overline{\mu}_L\gamma^\alpha e_L)\,, \quad Q_2 = (\overline{\mu}_R\gamma_\alpha e_R)(\overline{\mu}_R\gamma^\alpha e_R)\,,\\
Q_3 &= (\overline{\mu}_L\gamma_\alpha e_L)(\overline{\mu}_R\gamma^\alpha e_R)\,, \quad Q_4 = (\overline{\mu}_L e_R)(\overline{\mu}_L e_R)\,,\\
Q_5 &= (\overline{\mu}_R e_L)(\overline{\mu}_R e_L)\,.
\end{aligned}$$

  - need to compute matrix elements for both singlet and triplet states

Mass difference: matrix elements

- QED bound state: know leading order wave function!
  - spacial part is the same as in Hydrogen atom

$$\varphi(r) = \frac{1}{\sqrt{\pi a_{M_\mu}^3}} e^{-\frac{r}{a_{M_\mu}}}$$

  - can unambiguously compute decay constants and mixing MEs (QED)

$$\langle 0| \overline{\mu}\gamma^\alpha\gamma^5 e \left| M_\mu^P \right\rangle = i f_P p^\alpha, \quad \langle 0| \overline{\mu}\gamma^\alpha e \left| M_\mu^V \right\rangle = f_V M_M \epsilon^\alpha(p),$$

$$\langle 0| \overline{\mu}\sigma^{\alpha\beta} e \left| M_\mu^V \right\rangle = i f_T \left(\epsilon^\alpha p^\beta - \epsilon^\beta p^\alpha\right),$$

  - in the non-relativistic limit all decay constants $f_P = f_V = f_T = f_M$

$$f_M^2 = 4\frac{|\varphi(0)|^2}{M_M}$$ (QED version of Van Royen-Weisskopf)

  - NR matrix elements: “vacuum insertion” = direct computation

Mass difference: results

- Spin-singlet muonium state:
  - matrix elements:

$$\langle \bar{M}_\mu^P | Q_1 | M_\mu^P \rangle = f_M^2 M_M^2, \quad \langle \bar{M}_\mu^P | Q_2 | M_\mu^P \rangle = f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^P | Q_3 | M_\mu^P \rangle = -\frac{3}{2} f_M^2 M_M^2, \quad \langle \bar{M}_\mu^P | Q_4 | M_\mu^P \rangle = -\frac{1}{4} f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^P | Q_5 | M_\mu^P \rangle = -\frac{1}{4} f_M^2 M_M^2.$$

$$x_P = \frac{4(m_{red}\alpha)^3}{\pi \Lambda^2 \Gamma} \left[C_1^{\Delta L=2} + C_2^{\Delta L=2} - \frac{3}{2} C_3^{\Delta L=2} - \frac{1}{4} \left(C_4^{\Delta L=2} + C_5^{\Delta L=2} \right) \right]$$

- Spin-triplet muonium state:
  - matrix elements

$$\langle \bar{M}_\mu^V | Q_1 | M_\mu^V \rangle = -3 f_M^2 M_M^2, \quad \langle \bar{M}_\mu^V | Q_2 | M_\mu^V \rangle = -3 f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^V | Q_3 | M_\mu^V \rangle = -\frac{3}{2} f_M^2 M_M^2, \quad \langle \bar{M}_\mu^V | Q_4 | M_\mu^V \rangle = -\frac{3}{4} f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^V | Q_5 | M_\mu^V \rangle = -\frac{3}{4} f_M^2 M_M^2.$$

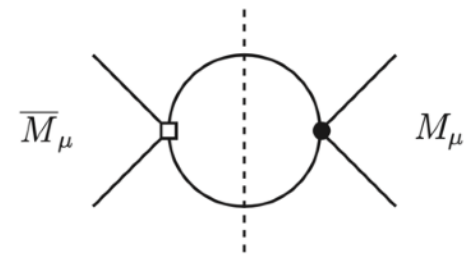
$$x_V = -\frac{12(m_{red}\alpha)^3}{\pi \Lambda^2 \Gamma} \left[C_1^{\Delta L=2} + C_2^{\Delta L=2} + \frac{1}{2} C_3^{\Delta L=2} + \frac{1}{4} \left(C_4^{\Delta L=2} + C_5^{\Delta L=2} \right) \right]$$

Experimental constraints on x result on experimental constraints on Wilson coefficients $C_k^{\Delta L=2}$ that encode all information about possible New Physics contributions

R. Conlin and AAP, arXiv: 2005.10276

Width difference

- Width difference comes from the absorptive part
  - light SM intermediate states ($e^+e^-, \gamma\gamma, \bar{\nu}\nu, etc.$)
  - $\bar{\nu}\nu$ state gives parametrically largest contribution



$$y = \frac{1}{2M_M\Gamma}\mathrm{Im}\left[\langle\overline{M}_\mu\left|i\int d^4x\ \mathrm{T}\left[\mathcal{H}_{\rm eff}(x)\mathcal{H}_{\rm eff}(0)\right]\right|M_\mu\rangle\right]$$
$$= \frac{1}{M_M\Gamma}\mathrm{Im}\left[\langle\overline{M}_\mu\left|i\int d^4x\ \mathrm{T}\left[\mathcal{H}_{\rm eff}^{\Delta L_\mu=2}(x)\mathcal{H}_{\rm eff}^{\Delta L_\mu=0}(0)\right]\right|M_\mu\rangle\right]$$

New Physics $\Delta L_\mu = 2$ contribution

$$\mathcal{L}_{\rm eff}^{\Delta L_\mu=2} = -\frac{1}{\Lambda^2}\sum_i C_i^{\Delta L=2}(\mu)Q_i(\mu)$$
$$Q_6 = (\overline{\mu}_L\gamma_\alpha e_L)\left(\overline{\nu_{\mu L}}\gamma^\alpha\nu_{eL}\right),$$
$$Q_7 = (\overline{\mu}_R\gamma_\alpha e_R)\left(\overline{\nu_{\mu L}}\gamma^\alpha\nu_{eL}\right)$$

Standard Model $\Delta L_\mu = 0$ contribution

$$\mathcal{L}_{\rm eff}^{\Delta L_\mu=0} = -\frac{4G_F}{\sqrt{2}}\left(\overline{\mu}_L\gamma_\alpha e_L\right)\left(\overline{\nu_{eL}}\gamma^\alpha\nu_{\mu L}\right)$$

Width difference: results

- Spin-singlet muonium state:

$$y_P = \frac{G_F}{\sqrt{2}\Lambda^2}\frac{M_M^2}{\pi^2\Gamma}(m_{red}\alpha)^3\left(C_6^{\Delta L=2} - C_7^{\Delta L=2}\right)$$

- Spin-triplet muonium state:

$$y_V = -\frac{G_F}{\sqrt{2}\Lambda^2}\frac{M_M^2}{\pi^2\Gamma}(m_{red}\alpha)^3\left(5C_6^{\Delta L=2} + C_7^{\Delta L=2}\right)$$

- Note: y has the same $1/\Lambda^2$ suppression as the mass difference!

R. Conlin and AAP, arXiv: 2005.10276

Experimental results from 1999

- MACS (1999): observed $5.7 \times 10^{10}\, M_\mu$ atoms after 4 months of running
  - magnetic field is taken into account (suppression factor)

Interaction type	2.8 μT	0.1 T	100 T
SS	0.75	0.50	0.50
PP	1.0	0.9	0.50
$(V \pm A) \times (V \pm A)$ or $(S \pm P) \times (S \pm P)$	0.75	0.35	0.0
$(V \pm A) \times (V \mp A)$ or $(S \pm P) \times (S \mp P)$	0.95	0.78	0.67

L. Willmann, et al. PRL 82 (1999) 49

  - no oscillations have been observed

Experimental constraints

- We can now put constraints on the Wilson coefficients of effective operators from experimental data (assume single operator dominance)
  - presence of the magnetic field

$$P(M_\mu \to \overline{M}_\mu) \leq 8.3 \times 10^{-11}/S_B(B_0)$$

  - no separation of spin states: average

$$P(M_\mu \to \overline{M}_\mu)_{\rm exp} = \sum_{i=P,V} \frac{1}{2S_i+1} P({M_\mu}^i \to {\overline{M}_\mu}^i)$$

  - set Wilson coefficients to one, set constraints on the scale probed

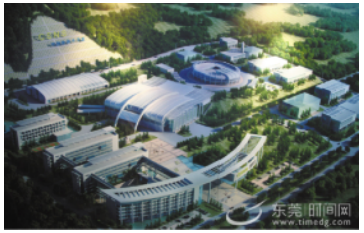
Operator	Interaction type	$S_B(B_0)$ (from [9])	Constraints on the scale Λ, TeV
Q_1	$(V-A)\times(V-A)$	0.75	5.4
Q_2	$(V+A)\times(V+A)$	0.75	5.4
Q_3	$(V-A)\times(V+A)$	0.95	5.4
Q_4	$(S+P)\times(S+P)$	0.75	2.7
Q_5	$(S-P)\times(S-P)$	0.75	2.7
Q_6	$(V-A)\times(V-A)$	0.75	0.58×10^{-3}
Q_7	$(V+A)\times(V-A)$	0.95	0.38×10^{-3}

R. Conlin and AAP, arXiv: 2005.10276

What do we need from the Snowmass process?

- Collaboration with experimentalists:
  - decays: can $M_\mu^P \to \gamma\gamma, M_\mu^V \to e^+e^-, M_\mu^V \to \gamma\gamma\gamma$ be measured?
    - » can $M_\mu \to invisible$ (SM: $M_\mu \to \nu_e \bar{\nu}_\mu$) be measured directly?

      Gninenko, Krasnikov, Matveev. Phys.Rev. D87 (2013) 015016
  - oscillations: new experiment(s) to improve bounds?



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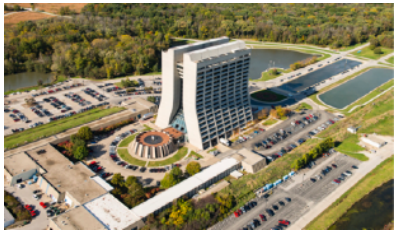


FNAL

ORNL?

    - » Are time-dependent oscillations studies possible?
- Collaboration with theorists:
  - matching NP models to EFT operators & complementarity with $\mu \to e\gamma$, $\mu \to 3e$, etc. and other collider measurements

    Crivellin et al Phys. Rev. D 99, 035004 (2019)
  - what models of NP can be <u>better</u> probed by muonium oscillations?

Conclusions and things to take home

- Muonium is the simplest atom (a bound state of μ^+ and e^-)
  - a heavy-light state that can exhibit flavor oscillations (like K, B, and D mesons)
  - oscillations probe New Physics without complications of QCD

- We discussed modern approach to phenomenology of muonium mixing
  - mass differences Δm (heavy NP intermediate states)
  - lifetime differences $\Delta\Gamma$ (SM intermediate state, NP in ΔL_μ operators)

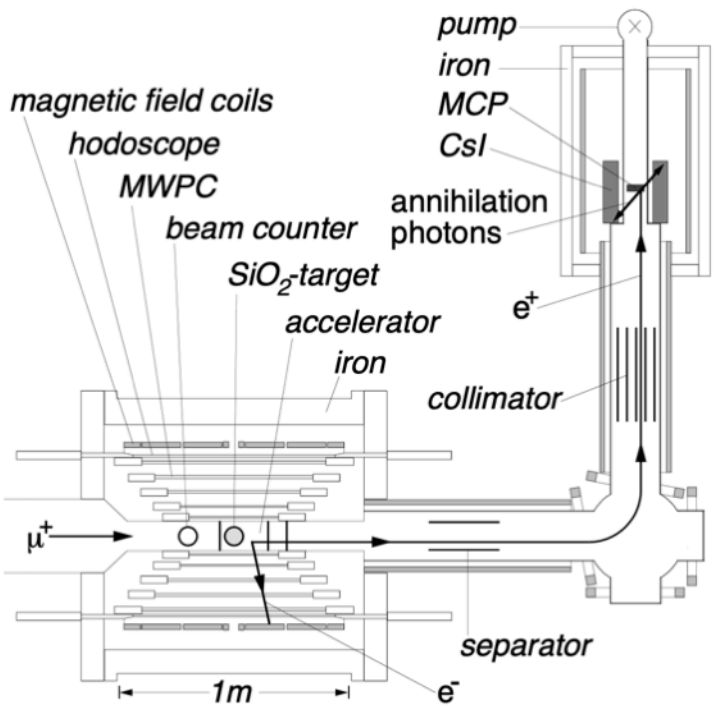
- We used EFT to compute oscillation parameters
  - results can be matched to particular models of New Physics
  - found that both Δm and $\Delta\Gamma$ parametrically scale as $\mathcal{O}(\Lambda^{-2})$

- Last experimental data is from 1999! Need new data!
  - we already probe LHC energy domain!


The Evening Herald
EXTRA!
WEATHER
FROM THE WVLRLD

1999: experimental setup and constraints

- Similar experimental set ups for different experiments
  - example: MACS at PSI
  - idea: form M_μ by scattering muon (μ^+) beam on SiO_2 target
- A couple of "little inconveniences":
  - ➡ how to tell f apart from $\bar{f}$?
    - $M_\mu \to f$ decay: $M_\mu \to e^+e^-\bar{\nu}_\mu\nu_e$
    - $\overline{M}_\mu \to \bar{f}$ decay: $\overline{M}_\mu \to e^+e^-\bar{\nu}_e\nu_\mu$
    - $\bar{f}$: fast e^- (~53 MeV), slow e^+ (13.5 eV)
  - ➡ oscillations happen in magnetic field
    - … which selects M_μ vs. $\overline{M}_\mu$



Muonium-Antimuonium Conversion Spectrometer (MACS)

L. Willmann, et al. PRL 82 (1999) 49

The most recent experimental data comes from 1999! Time is ripe for an update!

Effective Lagrangians and lifetime difference

- Effective Lagrangians for $\Delta L_\mu = 0$, $\Delta L_\mu = 1$, and $\Delta L_\mu = 2$

$$\mathcal{L}_{\rm eff}^{\Delta L_\mu=0} = -\frac{4G_F}{\sqrt{2}} \left(\overline{\mu}_L \gamma_\alpha e_L\right) \left(\overline{\nu_{eL}} \gamma^\alpha \nu_{\mu L}\right)$$

$$\begin{aligned}
\mathcal{L}_{\rm eff}^{\Delta L_\mu=1} = & -\frac{1}{\Lambda^2} \sum_f \Big[\left(C_{VR}^f \, \overline{\mu}_R \gamma^\alpha e_R + C_{VL}^f \, \overline{\mu}_L \gamma^\alpha e_L \right) \, \overline{f} \gamma_\alpha f \\
& + \left(C_{AR}^f \, \overline{\mu}_R \gamma^\alpha e_R + C_{AL}^q \, \overline{\mu}_L \gamma^\alpha e_L \right) \, \overline{f} \gamma_\alpha \gamma_5 f \\
& + m_e m_f G_F \left(C_{SR}^f \, \overline{\mu}_R e_L + C_{SL}^f \, \overline{\mu}_L e_R \right) \, \overline{f} f \\
& + m_e m_f G_F \left(C_{PR}^f \, \overline{\mu}_R e_L + C_{PL}^f \, \overline{\mu}_L e_R \right) \, \overline{f} \gamma_5 f \\
& + m_e m_f G_F \left(C_{TR}^f \, \overline{\mu}_R \sigma^{\alpha\beta} e_L + C_{TL}^f \, \overline{\mu}_L \sigma^{\alpha\beta} e_R \right) \, \overline{f} \sigma_{\alpha\beta} f + h.c. \Big],
\end{aligned}$$

$$\mathcal{L}_{\rm eff}^{\Delta L_\mu=2} = -\frac{1}{\Lambda^2} \sum_i C_i^{\Delta L=2}(\mu) Q_i(\mu)$$

$$Q_6 = \left(\overline{\mu}_L \gamma_\alpha e_L\right) \left(\overline{\nu_{\mu L}} \gamma^\alpha \nu_{eL}\right), \quad Q_7 = \left(\overline{\mu}_R \gamma_\alpha e_R\right) \left(\overline{\nu_{\mu L}} \gamma^\alpha \nu_{eL}\right)$$

- $\Delta\Gamma$: naively $\mathcal{O}(\Lambda^{-4})$ from double $\Delta L_\mu = 1$ insertion! But not always…

Effective Lagrangians and particular models

- Effective Lagrangian approach encompasses all models
  - lets look at an example of a model with a doubly charged Higgs Δ^{--}
  - this is common for the left-right models, etc.

$$\mathcal{L}_R = g_{\ell\ell}\overline{\ell}_R \ell^c \Delta + H.c.,$$

  - integrate out Δ^{--} to get

$$\mathcal{H}_\Delta = \frac{g_{ee} g_{\mu\mu}}{2M_\Delta^2} \left(\overline{\mu}_R \gamma_\alpha e_R\right) \left(\overline{\mu}_R \gamma^\alpha e_R\right) + H.c.,$$

  - match to $\mathscr{L}_{\rm eff}^{\Delta L=2}$ to see that $M_\Delta = \Lambda$ and

$$C_2^{\Delta L=2} = g_{ee} g_{\mu\mu}/2.$$