High-precision $\alpha_s(M_Z)$ determinations from future FCC-ee $e^+e^- \to$ *hadrons* data below the Z peak

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Proposal reminder in one slide

What:	Take data at future e^+e^- collider at multiple centre-of-mass energies below M_Z in dedicated runs.
Primary goal:	Highest precision of α_S.
Estimated costs:	$< 0.1\%$ of total project.
Estimated time:	Days to weeks.
Side benefits:	**Heavy-quark mass running**, highest precision studies of hadronization, impact on electroweak fits, improved particle identification and detector calibration.

Some “side benefits” in the QCD studies are the main topics of the EW studies and vice verse $\rightarrow$

How this talk is organized

- This talk, **for the sake of consistency** will include materials previously presented at QCD session, but with an intention to have a completely new discussion.
- Selected points from the discussion in the QCD group will be shown.
- Some EW physics cases for this proposal will be suggested for the consideration of EW physics experts.

Hope for a fruitfull discussion!

Outlook of α_s from $e^+e^- \to$ *hadrons* analyses

The classical α_s extraction analysis from the $e^+e^- \to$ *hadrons* includes the data itself, the theory predictions for the observables in the hadronic final state (event shapes or jets) and a prescription to correct these prescriptions for the non-perturbative (hadronization) effects.

- $+$ Theory precision: α_s^3+resummation
- $+$ Direct measurement
- $+$ Takes advantage of α_s running
- $-$ Model dependence: hadronization

Goal: minimize the impact of weak sides and maximise the impact of strong sides.

Data needed for the α_s extraction with the $e^+e^- \rightarrow \mathit{hadrons}$ data

- Large cross-section allows to take needed data within days for $M_{\Upsilon(nX)} < \sqrt{s} \approx M_Z$.
- Widest $\sqrt{s}$ range possible to have α_s running, but above $\Upsilon(nX)$ resonances. $M_{\Upsilon(nX)} < \sqrt{s} < +\infty$ is preferred.
- Preferably w/o $e^+e^- \rightarrow VV$ background. Avoid $\sqrt{s} > 160\,\mathrm{GeV}$ with W^+W^-, ZZ, ZH etc. background. See backups for explanation.
- Preferably w/o much $e^+e^- \rightarrow \gamma\ \mathit{hadrons}$ ISR/FSR. Avoid $\sqrt{s} = M_Z - 140\,\mathrm{GeV}$ where radiative return is large.

$M_{\Upsilon(nX)} < \sqrt{s} \approx M_Z$ **is strongly preferable.**

Theory needed for the α_s extraction with the $e^+e^- \to \mathit{hadrons}$ data

- The massless pQCD in this hadronic final state is the same for $2M_b < \sqrt{s} < 2M_t$.
- The effects of $m_b \neq 0$ decline with s, but the $R_b(s) = \frac{\sigma(e^+e^- \to b\bar{b})}{\sigma(e^+e^- \to \mathit{hadrons})}$ has non-trivial behaviour. Moreover, the α_s^2 massive b pQCD results are available since long time, and it looks that the α_s^3 massive b results are technically feasible now.

With the current theory $2M_b < \sqrt{s} < 2M_t$ seems to be the easiest for reliable calculations.

Modelling needed for the α_s extraction with the $e^+e^- \to$ *hadrons* data, I

Modelling of the hadronization is the most problematic part in the α_s extraction with this method and its largest uncertainty, e.g. see Ref. [1]. Modelling can be done

- With models in Monte Carlo event Generators (MCEGs) for arbitrary observables.
- With analytical models existing for **some** observables.

Modelling needed for the α_s extraction with the $e^+e^- \to$ *hadrons* data, II

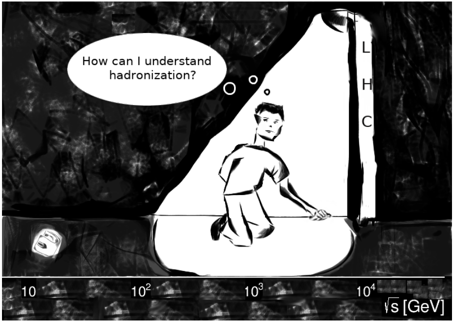
- The modern MCEGs are good at $\sqrt{s} \approx M_Z$, but not trustable for other energies[1][2].
- This is an artefact of 'chicken-or-egg' problem. The models were tuned with LEP data at $\sqrt{s} \approx M_Z$.
- The recent efforts to re-use the PETRA, TRISTAN and PEP data [3] had limited success due to huge data uncertainties.

With enough data away from Z peak, MCEG models can be re-tuned to describe the hadronization better at all scales.

Modelling needed for the extraction with the $e^+e^- \to$ *hadrons* data, II

General considerations:

- Larger hadronization corrections $\neq$ larger modelling uncertainty!
- To have smaller uncertainty on α_s one has to study hadronization and **NOT to avoid it**.



The lower energy data $M_{\Upsilon(nX)} < \sqrt{s} \approx M_Z$ is crucial.

This proposal as an extension of FCC-e^+e^- physics program

Understanding the importance of data below Z peak, the great FCC-e^+e^- CDR can be extended and include data taking in range $\sqrt{s} = 20 - 91\,\text{GeV}$[1]

$$\text{FCC-}e^+e^- = \text{Higgs factory} + \text{SuperLEP}$$
$$+ \text{SuperTRISTAN} + \text{SuperPEP} + \text{SuperPETRA}$$

Two **non-excluding** options are available:

- Dedicated: Dedicated runs with lowered beam energy.
- $e^+e^-\gamma$: γ tagging of radiative events $e^+e^- \to hadrons + \gamma$.

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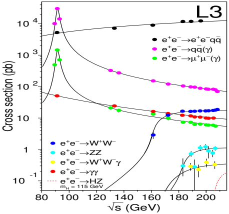
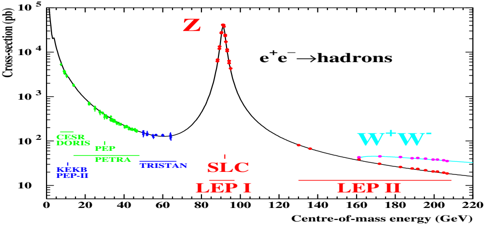
[1] The lower bound depends on the actual capabilities of the machine.

This proposal as an extension of FCC-e^+e^- physics program

- Dedicated: Perfect data, fast to collect – $10^7 - 10^9$ background free events/day (see backups) – supersedes data collected at **all** previous colliders in one day.
- $e^+e^-\gamma$: Lower data quality and numerous issues (see backups). But with and advanced FCC-*ee* detector (see backups) this option can be extremely valuable.





A perfect scenario: dedicated runs with ≈ 10 equidistant energy points in range $20-91$ GeV with $10^7 - 10^8$ events each.

Costs in terms of money, time and manpower

No detector amendments needed.	$=0$€ extra for detector construction
Running time for dedicated runs would be couple days with lower energy consumption.	≈ 0€ extra for running
The changes of beam energies would require readjustments of some magnets (but not the main ring).	Some manpower and time (some weeks?)
The data is of same type as the data at and above Z and would fit into any software/analysis for higher energy.	≈ 0€ extra for computing and physics

Costs in terms of money, time and manpower are tiny.

Selected topic from the discussions at QCD session

- Q: Such an obvious proposal should be already in CDR.
- A: No. It is not in CDR of FCC-*ee*, CEPC or any other projects.

- Q: Will be the machine performance sufficient? Lumi?
- A: Yes. Sure.

- Q: Will be the beam $\delta E/E$ good enough?
- A: Yes. Even $\pm 1\,\mathrm{GeV}$ is tolerable for QCD studies.

- Q: Has one actualy did some studies with pseudodada?
- A: Yes. Using a scenario of LEP systematics + FCC-*ee* statistics.

How the QCD-motivated LOI will benefit EW studies

The data collected at $\sqrt{s} = 20 - 91\,\mathrm{GeV}$ would be:

- Perfect data for hadronization studies and other Monte Carlo studies/tuning.
- Additional data for electroweak checks/fits.
- Unique data for the (BSM) searche.
- Particle Identification, e.g. using samples of particles with lower energies.
- **Super-bonus: b and c quark masses extraction and direct check of quark mass running using excellent b/c tagging.**

Flavour Physics super cases: m_b (or even m_c) running

With a significant amount of data it would be possible to extract not only most precise $\alpha_S(M_Z)$ but also to perform a simultaneous extration of m_b using the hadronic final state (HFS) observables like event shapes or jets and precise QCD predictions.
Two approaches can be used:

- Use inclusive HFS $e^+e^- \to \textit{hadrons}$ and observe moderate effects $\mathcal{O}(R_b(s) \times \frac{m_b^2}{s})$
- Usa b (or even c!) tagging and observe moderate effects $\mathcal{O}(\frac{m_b^2}{s})$ in $e^+e^- \to b\bar{b}\ \textit{hadrons}$ with some tagging-related systematics.

The approach dirrectly allows to check the m_b running.

EW Physics case: A_{FB}^{b} *et al.*

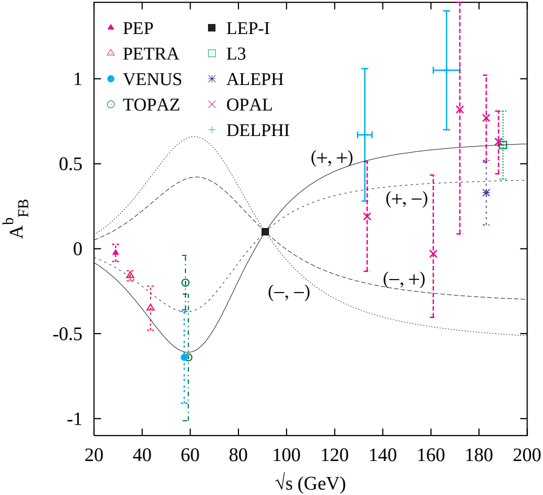


Figure: *The forward-backward asymmetry for the b-quark as a function of $\sqrt{s}$, see Ref.[4]*

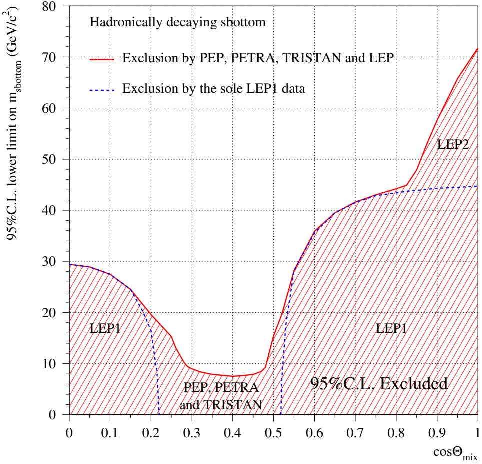
Very likely some points in the range $\sqrt{s} = 40\,\text{GeV} - M_Z$ would dramaticaly improve the precision.

Most likely the points at higher energies will be less constraining.

EW Physics case: $\gamma - Z$ interference

"*Improved experimental constraints on the hadronic interference term are obtained by including measurements of the hadronic total cross-section at centre-of-mass energies further awayfrom the Z pole than just the off-peak energies at LEP-I. Including the measurements of theTRISTAN collaborations at KEK, TOPAZ and VENUS, at* $\sqrt{s} = 58 GeV$. [5]"

BSM Physics case: BSM with σ^0_{had}



Some ranges of physics parameters can be accesed only at certain energies.

Statistics is less important. . .

See Ref. [6]

EW Physics case: uncertainties on EW observables (e.g. A_q and A_l)

Source	$\Delta R_{b\bar{b}}/R_{b\bar{b}}$	$\Delta R_{c\bar{c}}/R_{c\bar{c}}$	$\Delta A^b_{FB}/A^b_{FB}$	$\Delta A^c_{FB}/A^c_{FB}$
hadron mis-ID	4.9%	12.1%	3.8%	13.3%
muon ID	$<$0.1%	0.6%	1.5%	0.9%
p_T^{jet} cut	0.9%	0.6%	1.5%	0.2%
MDC acceptance	$<$0.1%	$<$0.1%	0.5%	0.9%
branching ratio	2.8%	5.0%	2.1%	1.2%
fragmentation parameter	1.7%	3.8%	$<$0.1%	$<$0.1%
total	6.1%	13.7%	4.9%	13.8%

Table: Summary of systematic errors from Ref. [7]

20 years later with better branching ratios and better detectors, everything what matters is **particle ID and MC** . These uncertainties will not go away w/o more work.

Conclusions

Data taking below Z peak at FCC-*ee* is a **fast**, **low-cost** extension of the FCC program with **guaranteed** huge physics returns with the **most precise** α_S measurement among them and a lot of input for EW physics.

Backups and discussion

Origin of systematics uncertainties related to $e^+e^- \to VV$

The $e^+e^- \to VV$ processes can be simulated and calculated quite precisely since a long time.

Nevertheless, the measurements of the $e^+e^{\to}Z/\gamma \to \mathit{hadrons}$ with $e^+e^- \to VV$ still have related uncertainties. This is related to the way the measurements of $e^+e^{\to}Z/\gamma \to \mathit{hadrons}$ are done:

- Measure events with hadrons in final state, e.g. event shapes.
- Apply cuts to to reduce the amount of $e^+e^- \to V_1V_2, V_1 \to q_1q_2, V_2 \to l\nu$ (semileptonic) and $e^+e^- \to V_1V_2, V_1 \to q_1q_2, V_2 \to q_2q_3$ (allhadronic) events
- Subtract from the distributions after the cuts the "MC-simulated" reminder of $e^+e^- \to V_1V_2, V_1 \to q_1q_2, V_2 \to l\nu$ and $e^+e^- \to V_1V_2, V_1 \to q_1q_2, V_2 \to q_2q_3$ events
- ...

The systematics related to this procedure will exist even in the case of perfect modelling of $e^+e^- \to V_1V_2$ processes.

Methodology of measurements of QCD observables: $e^+e^-\gamma$ vs. dedicated runs

$e^+e^-\gamma$

- Measure γ energy.
- Calculate the CM boost assuming γ comes from ISR.
- Alternatively to the points above do a kinematic fit of the hadronic final state to gen the energy of γ.
- Boost the event to the calculated CM.
- Calculate observables from the boosted hadronic final state.

Dedicated

- Make sure the CM energy is close to nominal using cuts.
- Calculate observables from hadronic final state.

The measurement of γ and the boost procedure bring additional uncertainties. The performance of these methods could be insufficient for the desired accuracy of the measurements.

FCC detector for $e^+e^-\gamma$

While at previous e^+e^- experiment the $e^+e^-\gamma$ events produced much less precise data sets for QCD measurements, the FCC-*ee* detectors would be a major improvement.

- The low angle limit for detector acceptance can be lowered to much lower angle than at LEP: the detector/machine interface has been set at 100mrad, so tracking and e/gamma acceptance should be good down to about 10 degrees or even less.
- Modern vertex detectors should ensure superior reconstruction of the event kinematics.

A dedicated study is needed!

$e^+e^-\gamma$ vs. dedicated runs: Point 0

- . . .

- Even if one registers hard γ the $e^+e^-\gamma$ process cannot be described in theory as γ plus $e^+e^- \to$ *hadrons* at lower scale. This is a significant theoretical distinction. To be on pair with the dedicated runs, $\alpha_s^3 \times \alpha_{EW}$ calculations are needed.

$e^+e^-\gamma$ vs. dedicated runs: Point 1

- It will take time to change the beam energy for dedicated runs.

- True. But it is acceptable to sacrifice a tiny fraction of running time to take a better data and better physics.

Need input from accelerator physicists and engineers.

$e^+e^-\gamma$ vs. dedicated runs: Point 2

- The $\sqrt{s} = 20 - 91\,\text{GeV}$ data can be taken during high energy runs using $e^+e^-\gamma$ anyway.

- The sys. uncertainties of such data will be much higher.
- Will take much more time to collect.
- Adjusting detector/reconstruction for such data could take even more time.
- Potential problems with acceptance of highly boosted events.
- Such data are not suitable for many analyses and calibration.
- If there will be two e^+e^- colliders in the future, the project with dedicated runs will be able to get the precious data much faster.

$e^+e^-\gamma$ vs. dedicated runs: Point 3

- There will be enough data from $e^+e^-\gamma$ anyway.

- Not really and not of good quality, see L3 [8] and OPAL [9] at LEPI:

Type	$\sqrt{s}$, GeV	$\langle\sqrt{s}\rangle$, GeV	Int. Lumi (*pb*)	Selection Eff.(%)	Purity(%)	Sel. Events
Reduced Centre-of-Mass Energy	30–50	41.4	142.4	48.3	68.4	1247
	50–60	55.3	142.4	41.0	78.0	1047
	60–70	65.4	142.4	35.2	86.0	1575
	70–80	75.7	142.4	29.9	89.0	2938
	80–84	82.3	142.4	27.4	90.5	2091
	84–86	85.1	142.4	27.5	87.0	1607
Z pole	91.2	91.2	8.3	98.5	99.8	248100

$$\alpha_S(M_Z)_{41\ \mathrm{GeV}} = 0.1418 \pm 0.0053(\mathrm{stat.}) \pm 0.0030(\mathrm{exp.syst.}) \pm 0.0055(\mathrm{hadr.}) \pm 0.0085(\mathrm{theory.})(NLO)$$

$$\alpha_S(M_Z)_{55\ \mathrm{GeV}} = 0.1260 \pm 0.0047(\mathrm{stat.}) \pm 0.0056(\mathrm{exp.syst.}) \pm 0.0066(\mathrm{hadr.}) \pm 0.0062(\mathrm{theory.})(NLO)$$

. . . V.S.

$$\alpha_S(M_Z)_{91\ \mathrm{GeV}} = 0.1210 \pm 0.0008(\mathrm{stat.}) \pm 0.0017(\mathrm{exp.syst.}) \pm 0.0040(\mathrm{hadr.}) \pm 0.0052(\mathrm{theory.})(NLO)$$

E_γ [GeV]	Events	$\sqrt{s'}_{Mean}$[GeV]	Background [%]		
			Non-rad. MH		$\tau\tau$
			Likelihood	Isolated tracks	
10-15	1560	78.1± 1.7	6.0± 0.7	6.2± 0.9	0.9± 0.2
15-20	954	71.8± 1.9	3.1± 0.5	4.9± 0.8	1.0± 0.3
20-25	697	65.1± 2.0	2.6± 0.6	6.3± 1.1	0.9± 0.4
25-30	513	57.6± 2.3	5.1± 1.1	7.9± 1.4	1.1± 0.5
30-35	453	49.0± 2.6	4.5± 1.1	9.6± 1.6	0.7± 0.4
35-40	376	38.5± 3.5	5.2± 1.2	13.1± 1.9	0.8± 0.5
40-45	290	24.4± 5.3	10.4± 2.3	12.9± 1.7	0.8± 0.5

$$\alpha_S(M_Z)_{comb} = 0.1182 \pm 0.0015(\mathrm{stat.}) \pm 0.0038(\mathrm{exp.syst.}) \pm 0.0070(\mathrm{hadr.}) \pm 0.0062(\mathrm{theory.})(NLO)$$

+specific problems: hadronization, systematics, statistics.

$e^+e^-\gamma$ vs. dedicated runs: Point 4

It is interesting to admit the differences between the hadronization uncertainties of results from OPAL [9]

$0.1182 \pm 0.0015(\mathrm{stat.}) \pm 0.0038(\mathrm{exp.syst.}) \pm 0.0070(\mathrm{hadr.}) \pm 0.0062(\mathrm{theory.})(NLO)$

and JADE [10]:

$0.1172 \pm 0.0006(\mathrm{stat.}) \pm 0.0020(\mathrm{exp.syst.}) \pm 0.0035(\mathrm{hadr.}) \pm 0.0030(\mathrm{theory.})(NNLO + NLLA)$

	Year	Type	$\sqrt{s}$	Hadr. unc.	Exp. syst. unc .
JADE	2008	Low energy	12-46	0.0035	0.0020
OPAL	2007	Radiative	10-45	0.0070	0.0038

$e^+e^-\gamma$ vs. dedicated runs: Scaling the L3 $e^+e^-\gamma$ case to FCC-*ee* $e^+e^-\gamma$

L3 [8]:

Type	$\sqrt{s}$, GeV	$\langle\sqrt{s}\rangle$, GeV	Lumi (pb)	Selection Eff.(%)	Purity(%)	Sel. Events	FCC $e^+e^-\gamma$
Reduced	30–50	41.4	142.4	48.3	68.4	1247	2.8×10^9
Centre-	50–60	55.3	142.4	41.0	78.0	1047	2.4×10^9
of-	60–70	65.4	142.4	35.2	86.0	1575	3.6×10^9
Mass	70–80	75.7	142.4	29.9	89.0	2938	6.7×10^9
Energy	80–84	82.3	142.4	27.4	90.5	2091	3.7×10^9
	84–86	85.1	142.4	27.5	87.0	1607	3.6×10^9
Z pole	91.2	91.2	8.3	98.5	99.8	248100	10^{12}

With a tighter selection from OPAL, the number of FCC $e^+e^-\gamma$ events would be order of magnitude smaller.
The dedicated runs could obtain such amount of data in some days or even hours.

Results from $e^+e^- \to hadrons$

Determination[2]	Type	Data and procedure	Ref.
0.1175 ± 0.0025	Non-global	ALEPH 3-jet rate (NNLO+MChad)	[12]
0.1199 ± 0.0059	fit	JADE 3-jet rate (NNLO+NLL+MChad)	[13]
0.1224 ± 0.0039	+MChad	ALEPH event shapes (NNLO+NLL+MChad)	[14]
0.1172 ± 0.0051		JADE event shapes (NNLO+NLL+MChad)	[10]
0.1189 ± 0.0041		OPAL event shapes (NNLO+NLL+MChad)	[15]
$0.1164\,^{+0.0028}_{-0.0026}$	Global fit	Thrust (NNLO+NLL+anlhad)	[16]
$0.1134\,^{+0.0031}_{-0.0025}$	+anlhad	Thrust (NNLO+NNLL+anlhad)	[17]
0.1135 ± 0.0011		Thrust (SCET NNLO+$\mathrm{N^3LL}$+anlhad)	[18]
0.1123 ± 0.0015		C-parameter (SCET NNLO+$\mathrm{N^3LL}$+anlhad)	[19]
0.11750 ± 0.00287	Global fit	EEC (NNLO+$\mathrm{N^2LL}$+MChad+NLO_{m_b})	[2]
0.11881 ± 0.00131	+MChad	2-jet rate ($\mathrm{N^3LO}$+$\mathrm{N^3LL}$+MChad+$\mathrm{N^2LO}_{m_b}$)	[1]

Global fits and wide $\sqrt{s}$ range → best precision.
The discrepancy between the analytic and MC hadronization should be clarified.

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[2] Credits to Ref. [11]

The full list of side benefits

The data collected at $\sqrt{s} = 20 - 91\,\text{GeV}$ would be:

- Perfect data for hadronization studies and other Monte Carlo studies/tuning.
- Additional data for electroweak fits and other analyses.
- Perfect data for detector calibration, e.g.
  - of jet energies using $e^+e^- \to 2jets$.
  - of lepton energies using $e^+e^- \to \mu^+\mu^-$ processes.
  - Particle Identification, e.g. using samples of particles with lower energies.
- **Super-bonus: b and c quark masses extraction and direct check of quark mass running using excellent b/c tagging.**

The number of side benefits alone makes the data taking below Z peak attractive enough.
The checks of quark mass running could deserve an own LOI.

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Materials

Figure 9 from https://www.comsol.com/blogs/exploiting-maximum-principles-to-save-time-and-resources/