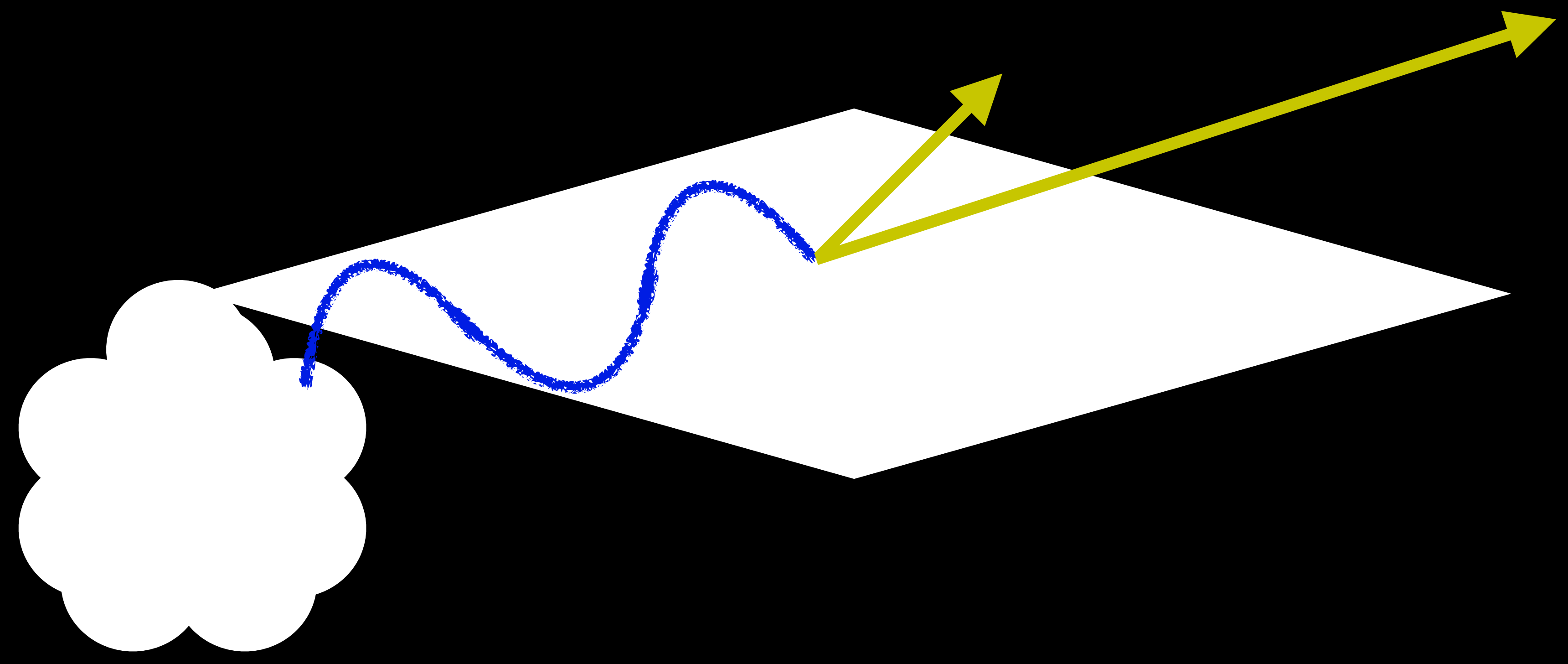
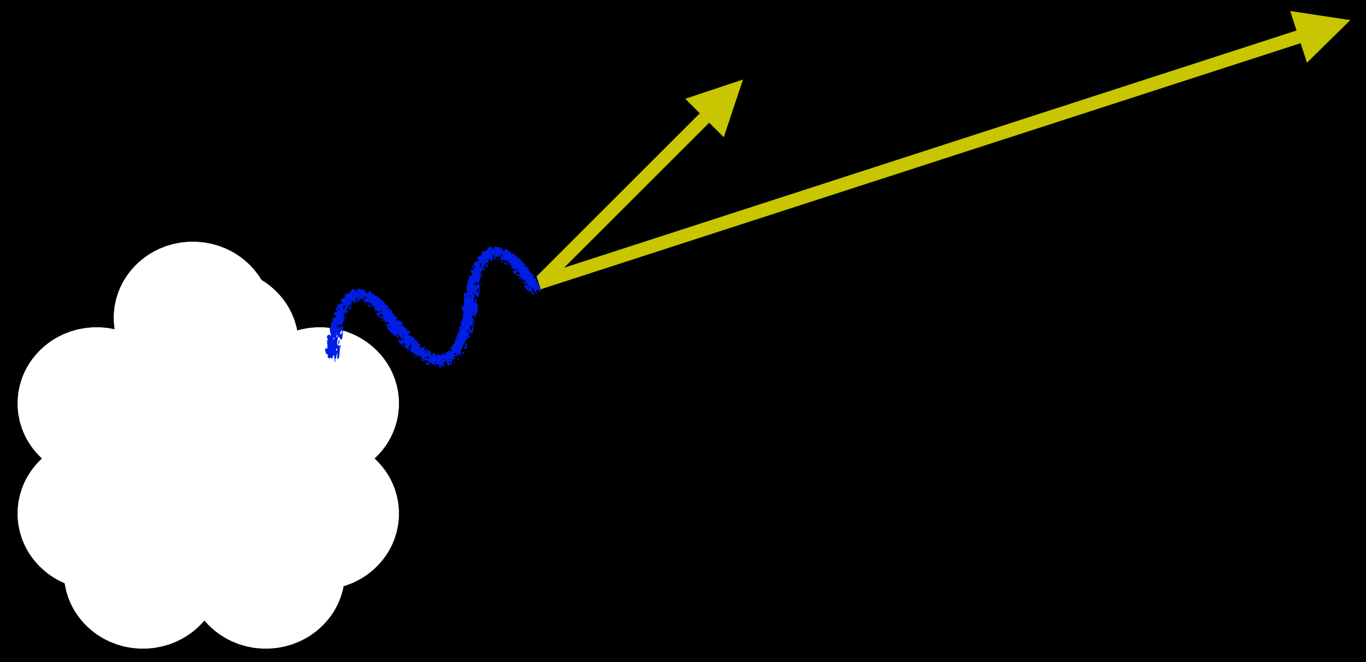




High energy positrons from RMC

(Internal*) conversion as external conversion

$$\mu^- \ \mathrm{Al} \to \nu_\mu \, \gamma^{(*)} \ \mathrm{Mg} \qquad\qquad \mu^- \ \mathrm{Al} \to e^+ \ \mathrm{Na}$$

Ryan Plestid | October 28th 2020 Mu2e-II Snowmass Meeting V

Intensity frontier fellow at FNAL | Postdoctoral Scholar at UKY

What this talk is about

- RMC is an important and poorly understood background for Mu2e.
- RMC on Al demands nuclear physics input (unlike muon DIO).
- Photon spectrum of RMC probes the relevant nuclear physics.
- The photon spectrum can be used to predict the positron/electron spectrum near the end-point (small parameter is $T_{e^\pm}/E_\gamma$)

arXiv:2010.09509

Available on the arXiv.

FERMILAB-PUB-20-525-T

The high energy spectrum of internal positrons from radiative muon capture on nuclei

Ryan Plestid[1,2,*] and Richard J. Hill[1,2,†]

[1] *Department of Physics and Astronomy, University of Kentucky Lexington, KY 40506, USA*
[2] *Theoretical Physics Department, Fermilab, Batavia, IL 60510,USA*

(Dated: October 20, 2020)

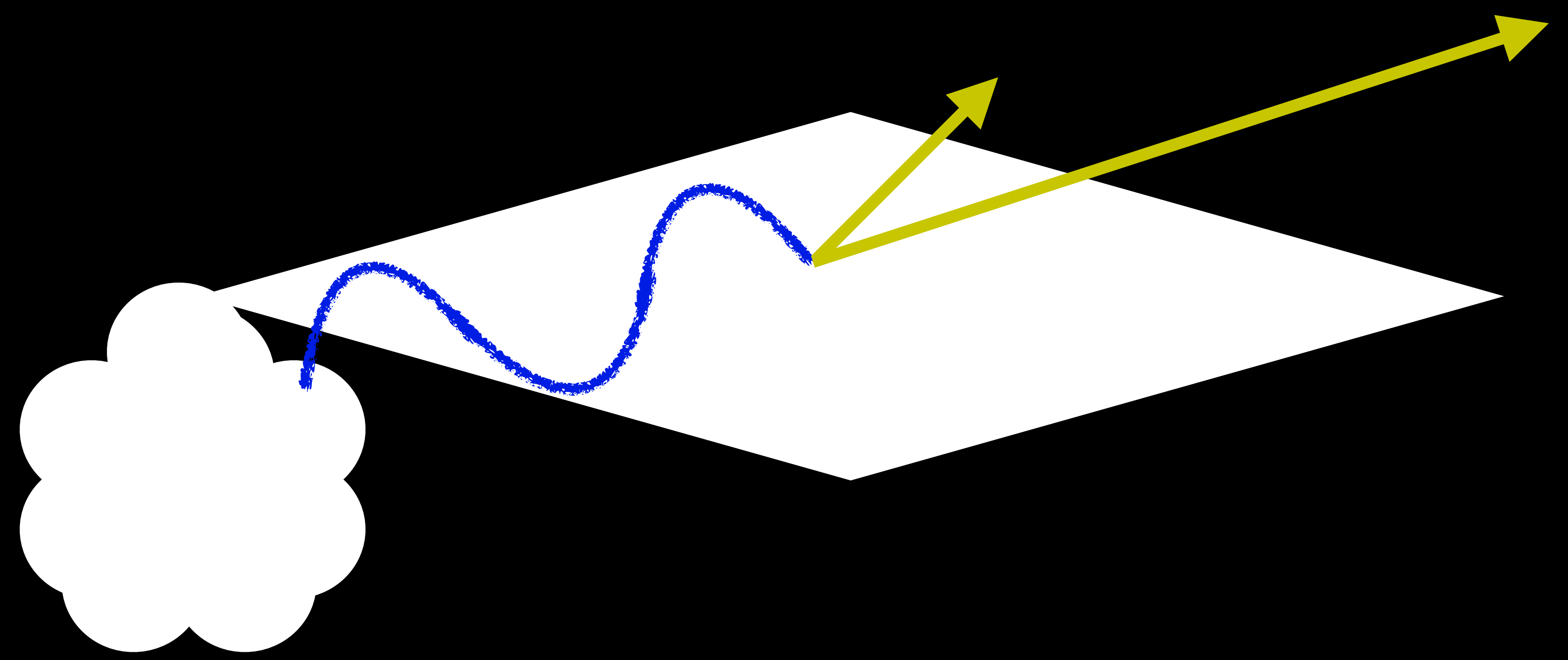
The Mu2e and COMET collaborations will search for nucleus-catalyzed muon conversion to positrons ($\mu^- \to e^+$) as a signal of lepton number violation. A key background for this search is radiative muon capture where either: 1) a real photon converts to an e^+e^- pair "externally" in surrounding material, or 2) a virtual photon mediates the production of an e^+e^- pair "internally". If the e^+ has an energy approaching the signal region then it can serve as an irreducible background. In this work we describe how the near end-point internal positron spectrum can be related to the real photon spectrum from the same nucleus, which encodes all non-trivial nuclear physics.

Work done in collaboration with Richard Hill.

RMC served two ways

arXiv:2010.09509

External

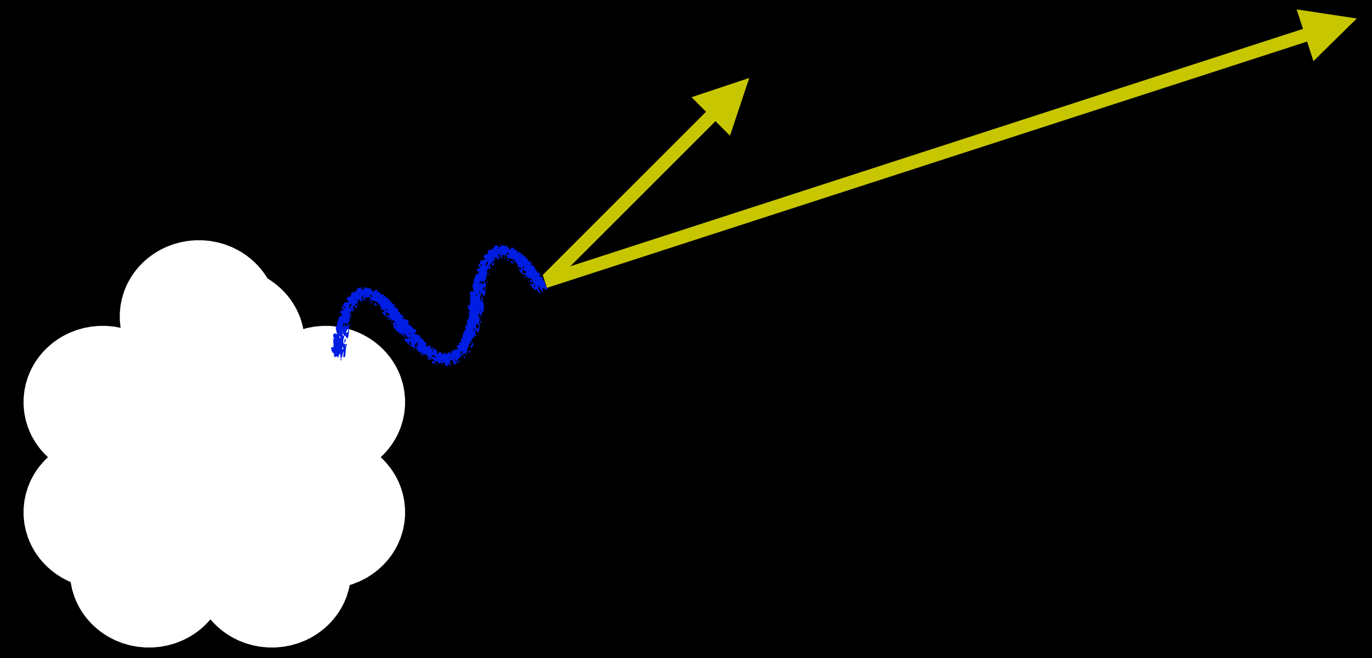


Compton scattering

In-medium pair production

Detector/target dependent

Internal

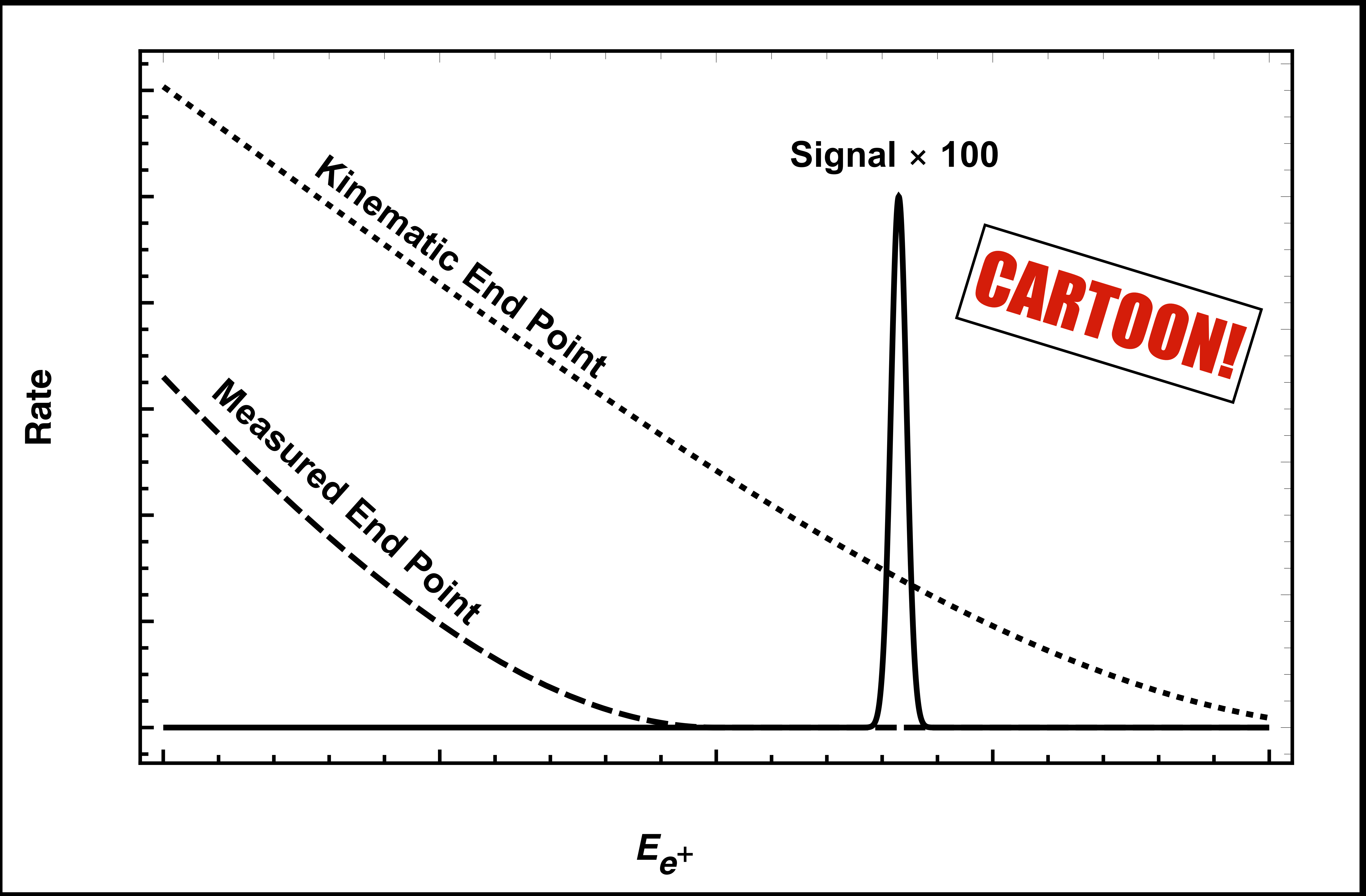


Virtual photon to $e^+ e^-$

Detector independent

arXiv:2010.09509

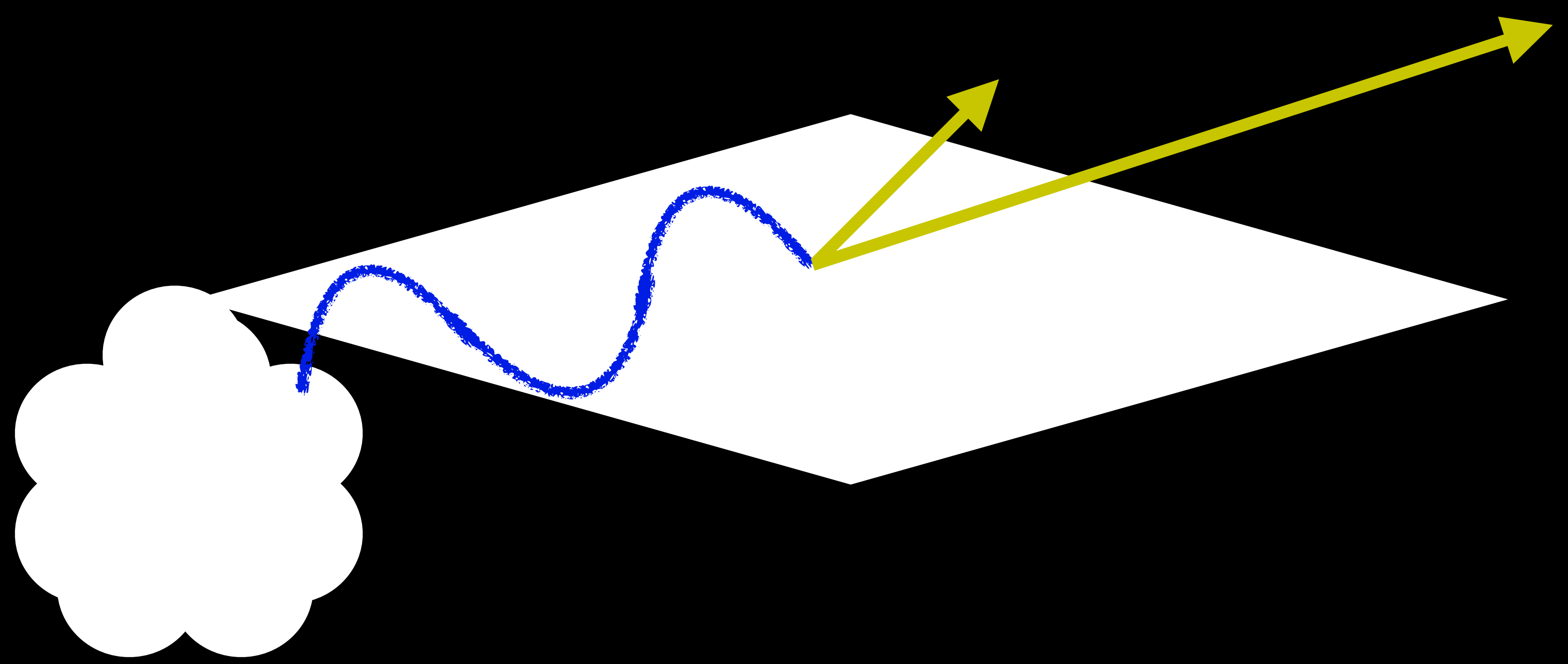
Motivation



Signal: Positron from muon capture on ^{27}Al

$$\mu^- \ \mathrm{Al} \to e^+ \ \mathrm{Na}$$

Background: positrons from high energy photons



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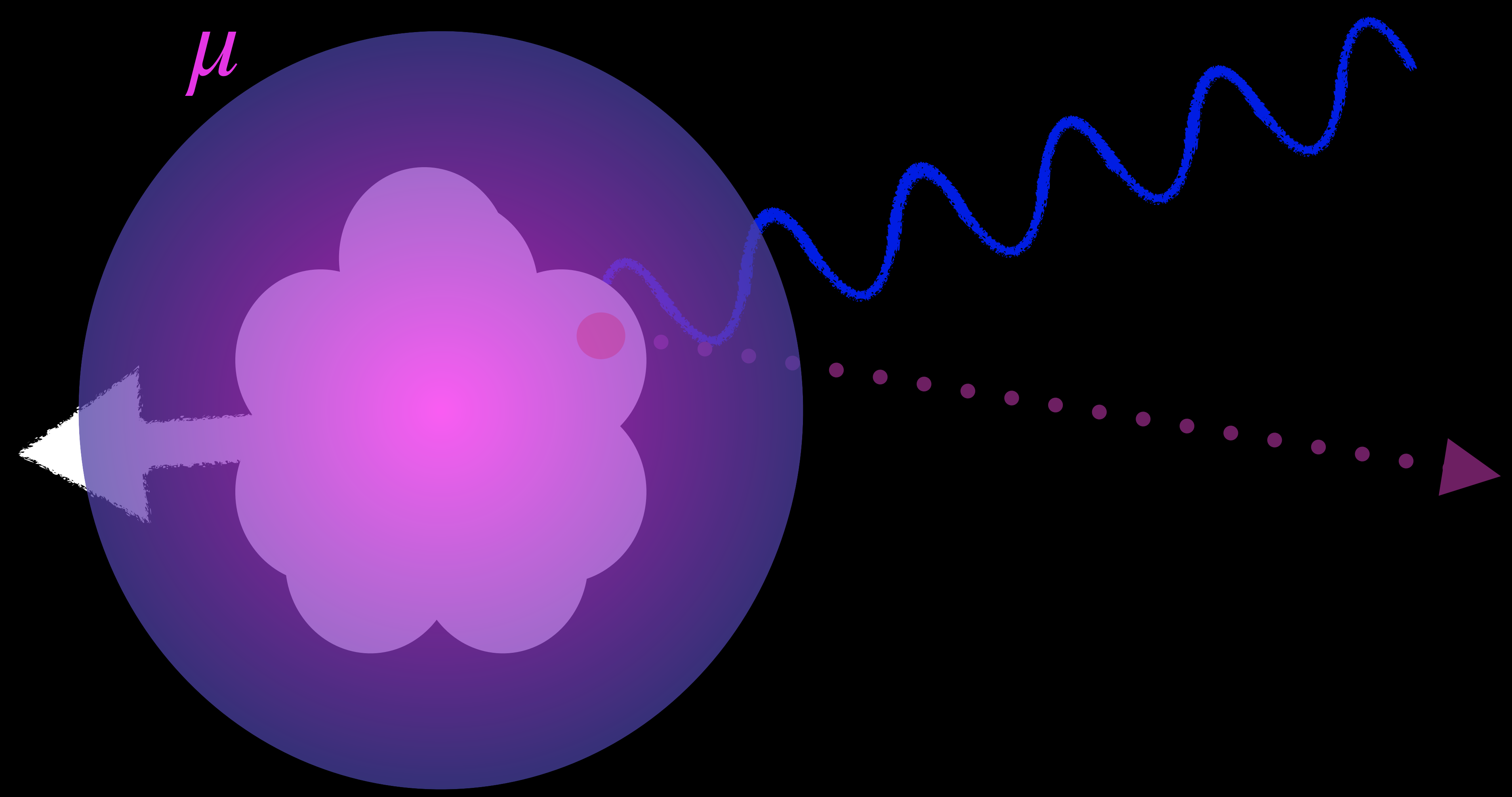
On-shell RMC



- Mass of the muon becomes available as energy for the photon.

- ~ 100 MeV photons are emitted.

- Photon amplitude is complicated by nuclear physics.

$$\mathscr{M}_0 = \epsilon_\mu \mathscr{J}_0^\mu$$

Photon on-shell

$$\mathscr{J}_0^\mu = \langle \mathrm{Mg}, \nu \,|\, \hat{J}^\mu_{\mathrm{EM}} \,|\, \mathrm{Al}, \mu_{1s} \rangle$$

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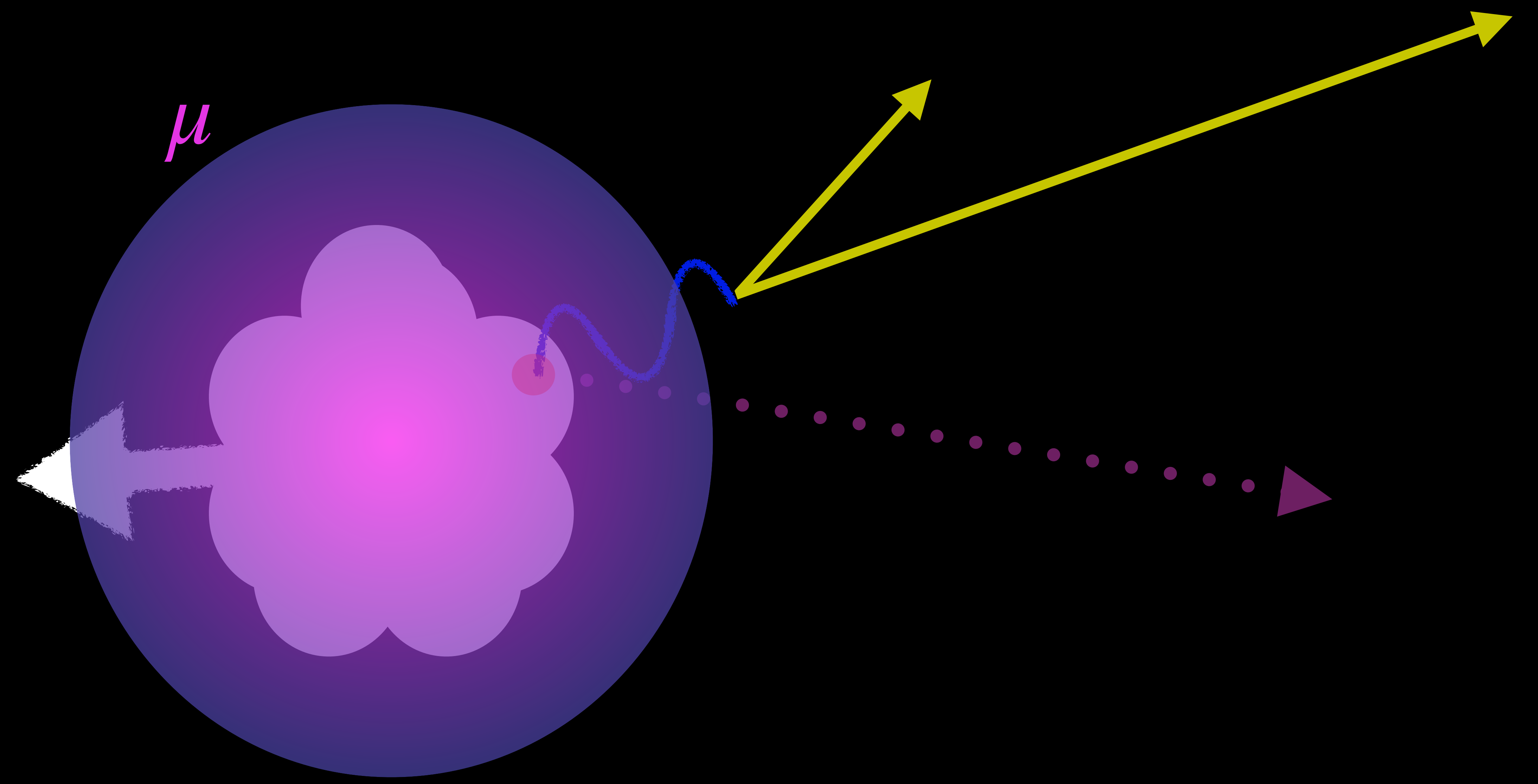
Internal conversion

- Calculable QED pair production amplitude.
- Must be contracted against the ***off-shell*** photon amplitude.
- Can depend on longitudinally polarized matrix elements.



$$\mathscr{M} = -\bar{u}\gamma_\mu v \times \frac{e}{q^2} \times \mathscr{J}^\mu_*$$

Photon off-shell

$$\mathscr{J}^\mu_* = \langle \mathrm{Mg}, \nu \,|\, \hat{J}^\mu_{\mathrm{EM}} \,|\, \mathrm{Al}, \mu_{1s} \rangle$$

arXiv:2010.09509

The trouble: Nuclear physics

Photon off-shell

$$\mathscr{J}^{\mu}_{*} = \langle \mathrm{Mg}, \nu \,|\, \hat{J}^{\mu}_{\mathrm{EM}} \,|\; \mathrm{Atom} \rangle$$

Photon on-shell

$$\mathscr{J}^{\mu}_{0} = \langle \mathrm{Mg}, \nu \,|\, \hat{J}^{\mu}_{\mathrm{EM}} \,|\; \mathrm{Atom} \rangle$$

Can this be reliably calculated? **Can this be reliably measured?**

arXiv:2010.09509

The trouble: Nuclear physics

Photon on-shell

$$\mathscr{J}_0^{\mu} = \langle \mathrm{Mg}, \nu \,|\, \hat{J}^{\mu}_{\mathrm{EM}} \,|\, \mathrm{Atom} \rangle$$

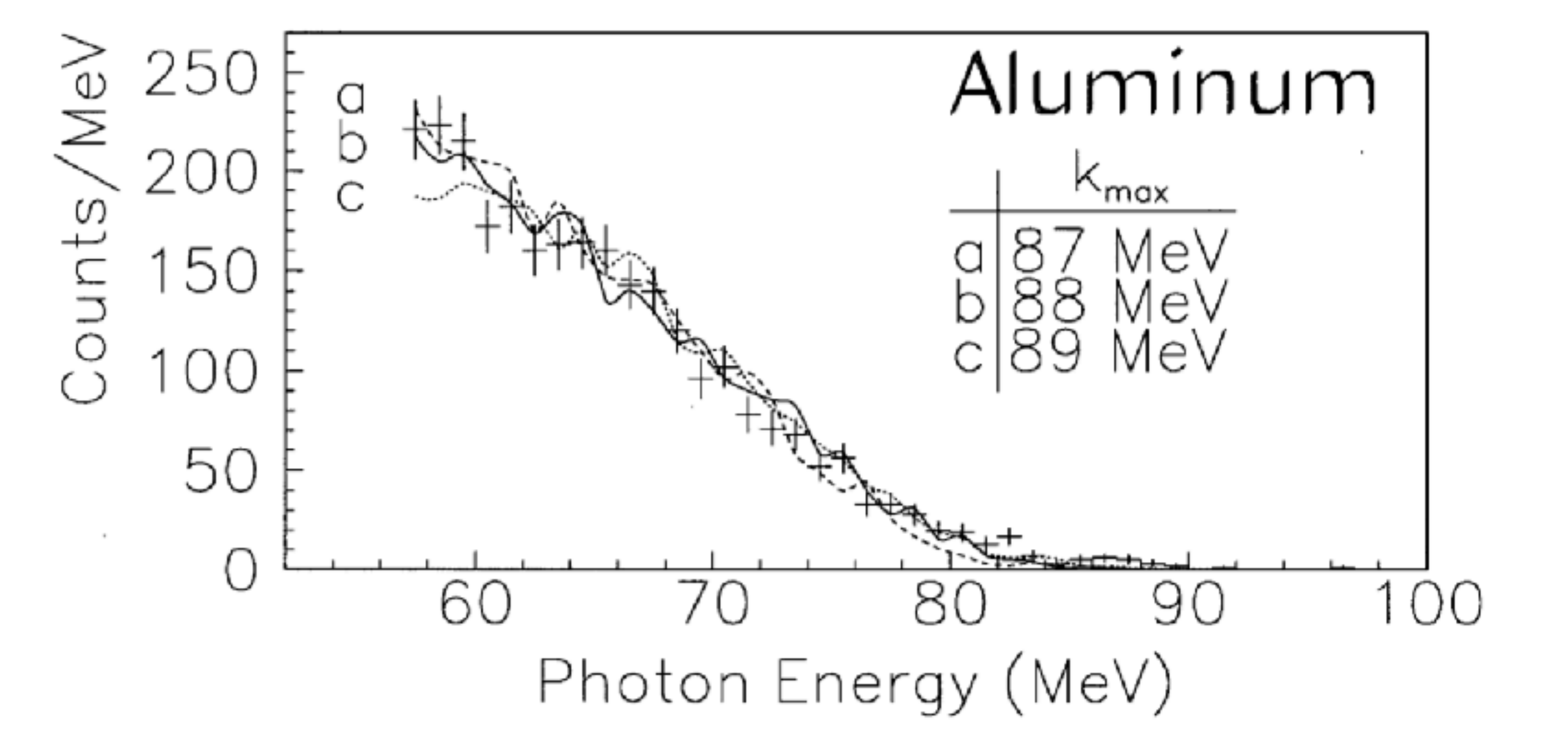


Figure 5.2: Comparison of the experimental RMC on Si and Al photon spectra with the closure spectral shapes given by Eq. (5.1) after these shapes have been convoluted by the spectrometer Monte Carlo, and analyzed by RMCOFIA. The solid line in each figure is the spectral shape for the best fit value of k_{max}.

Related to photon spectrum

Has been measured, but not at a high enough level of precision for Mu2e.

P.C. Bergbusch, M.Sc. Thesis, UBC, 1993

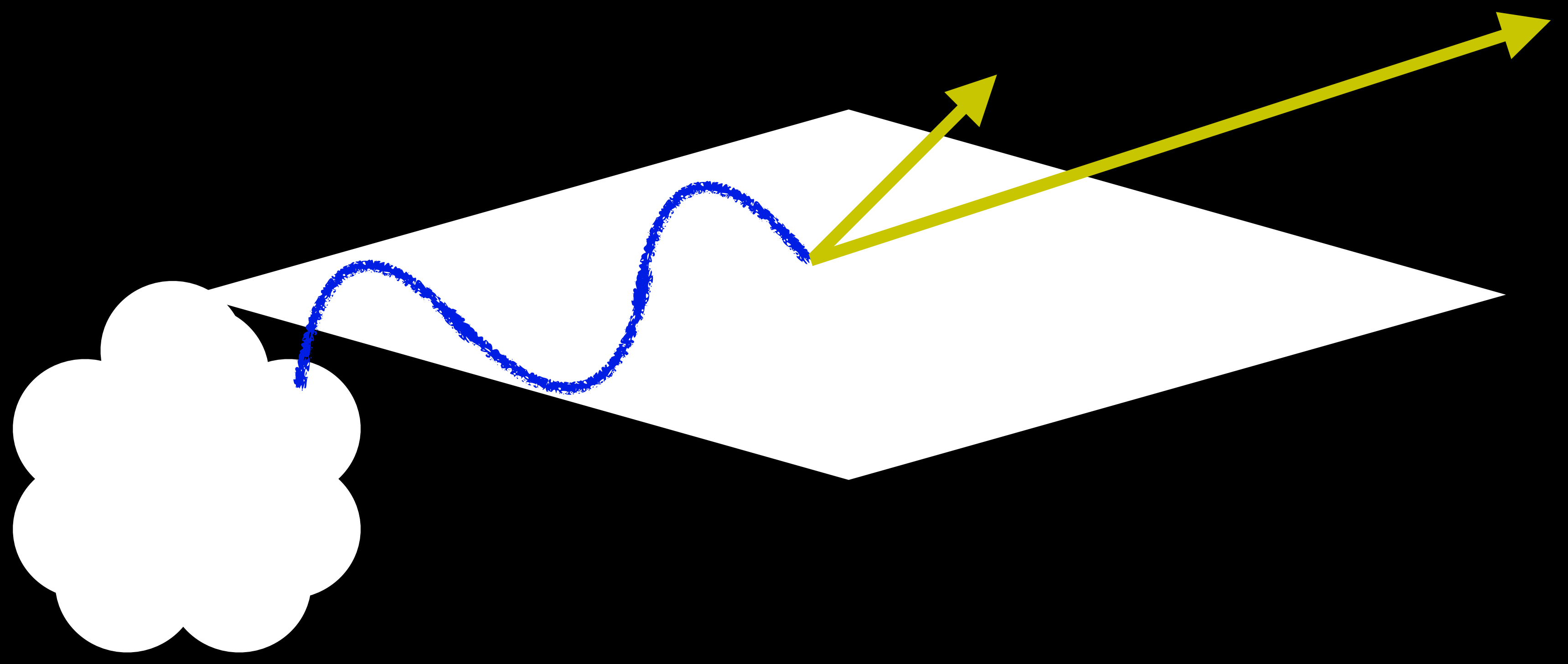
Suppose we measure the photon spectrum.

Then what?

arXiv:2010.09509

External conversion

- Monte Carlo simulations can be conducted with full detector setup in e.g. GEANT-4.
- Includes Compton scattering and pair production in surrounding detector material etc.
- For a spherically symmetric amplitude, a RMC independent probability can be constructed



$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_+} = \int \mathrm{d}E_\gamma P(E_+ \,|\, E_\gamma) \times \frac{\mathrm{d}\Gamma_{\mathrm{RMC}}}{\mathrm{d}E_\gamma}$$

$$P(E_+ \,|\, E_\gamma)$$

$$\langle |\mathcal{M}_0|^2 \rangle$$

What about internal conversion?

Can we relate it to photon measurements?

Yes!

But only in certain regions of phase space…

This work revisits an old topic covered in:

PHYSICAL REVIEW VOLUME 98, NUMBER 5 JUNE 1, 1955

Internal Pair Production Associated with the Emission of High-Energy Gamma Rays

NORMAN M. KROLL, *Columbia University, New York, New York*

AND

WALTER WADA, *Naval Research Laboratories, Washington, D. C.*

(Received January 10, 1955)

The theory of inner pair production associated with the radiative capture of π^- mesons and with the decay of the π^0 meson is discussed. Appropriate distribution functions are derived and compared with recently obtained experimental results. The weak dependence of the theoretical predictions upon the details of meson theory is emphasized. The possible utility of the double conversion process, in which the π^0 meson decays into two electron-positron pairs, for the determination of the π^0 parity is also discussed.

We disagree with this paper in a couple places. If interested see Appendix C of our paper arXiv:2010.09509

Basic Idea:

Decompose 4-body LIPS into 3-body LIPS

EM current matrix element

Standard QED Trace[spinors]

$$\mathrm{d}\Gamma_{e^+e^-} \sim \mathrm{d}\Phi_4 \mathcal{J}^{\mu\nu}_{*} L_{\mu\nu} \times \frac{4\pi\alpha}{m_*^4}$$

4-body PS

$$\mathrm{d}\Phi_4 = \mathrm{d}\Phi_{3*} \times \frac{\mathrm{d}m_*^2}{2\pi} \times \mathrm{d}\Phi_2(\gamma^* \to e^+e^-)$$

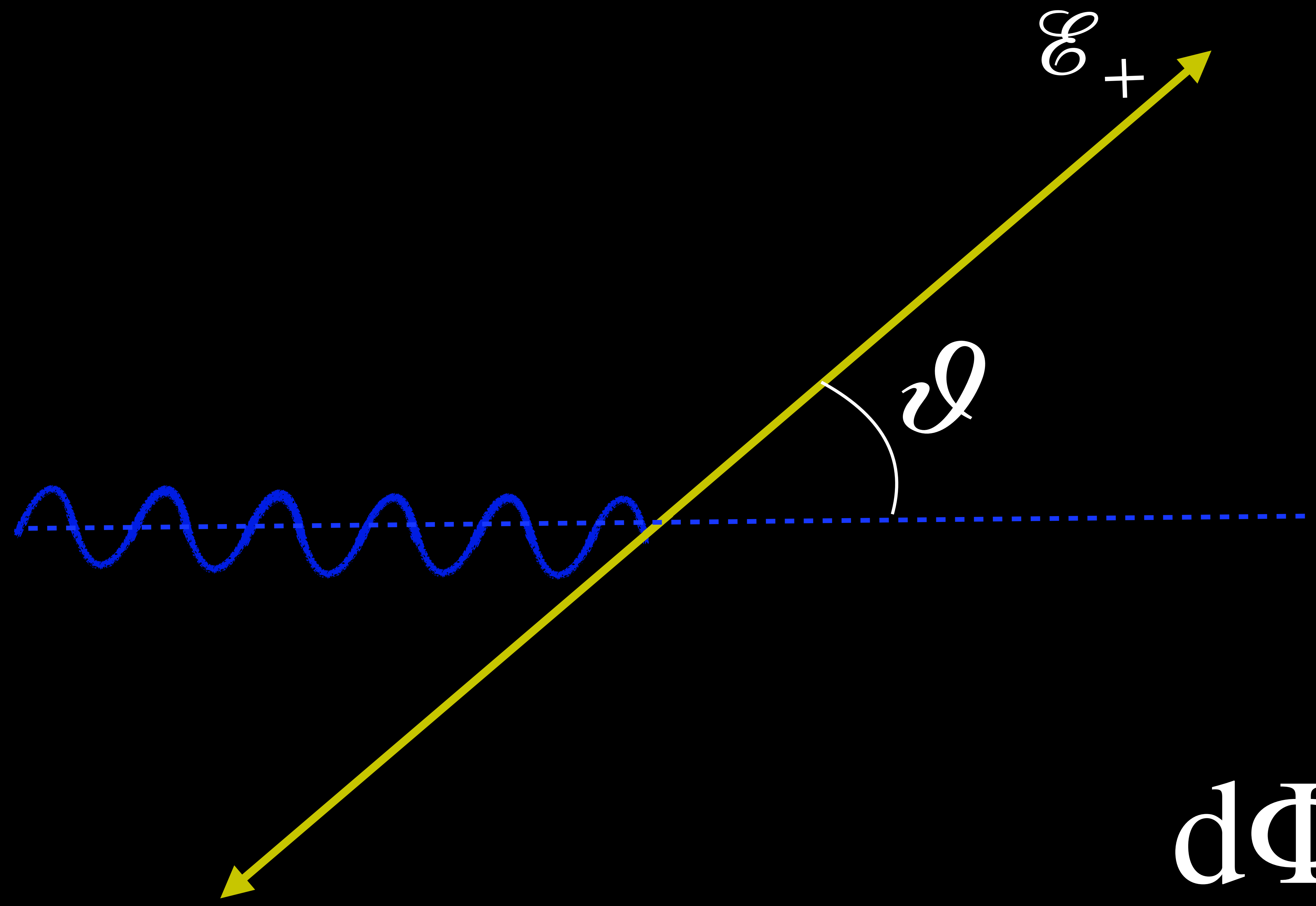
In rest frame of photon

$$\Rightarrow \quad \mathrm{d}\Phi_2 = \frac{\mathrm{d}\cos\vartheta \mathrm{d}\varphi}{32\pi^2}\beta_e$$

$$\beta_e = \sqrt{1 - \frac{4m_e^2}{m_*^2}}$$

arXiv:2010.09509

Photon rest frame



$$d\Phi_2 = \frac{d\cos\vartheta\, d\varphi}{32\pi^2}\beta_e$$

$$\beta_e = \sqrt{1 - \frac{4m_e^2}{m_*^2}}$$

Integrate over azimuthal angles

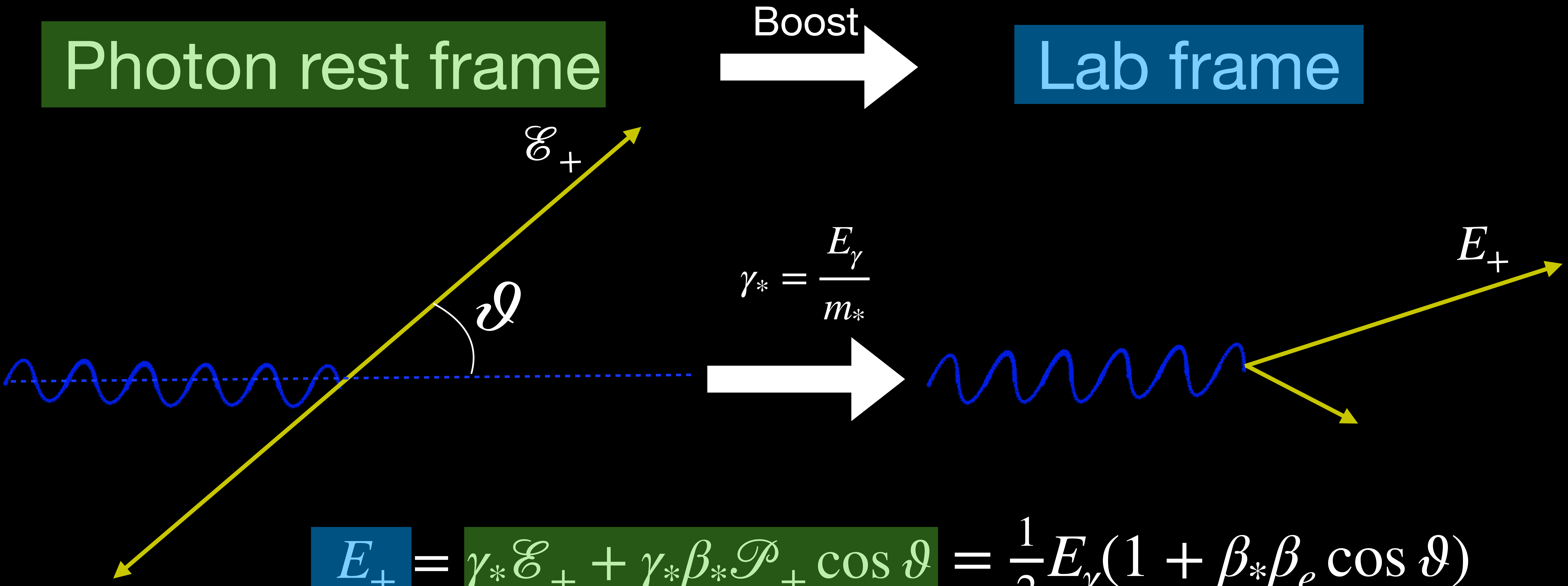
$$\frac{\mathrm{d}\cos\vartheta\,\cancel{\mathrm{d}\varphi}}{32\pi^2}\beta_e$$

$$\frac{\langle L_{\mu\nu}\rangle_\varphi}{m_*^2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & A & 0 \\ 0 & 0 & 0 & B \end{pmatrix}$$

$$\mathcal{J}_*^{\mu\nu} L_{\mu\nu}$$

$$= A(\mathcal{J}_*^{11} + \mathcal{J}_*^{22}) + B\mathcal{J}_*^{33}$$

$$A = 2 - (1 - \cos^2\vartheta)\beta_e^2 \qquad B = 2(1 - \beta_e^2\cos^2\vartheta)$$



$$\boxed{E_+} = \boxed{\gamma_*\mathscr{E}_+ + \gamma_*\beta_*\mathscr{P}_+\cos\vartheta} = \frac{1}{2}E_\gamma(1+\beta_*\beta_e\cos\vartheta)$$

Notice : Fixing E_+ is equivalent to fixing $\cos\vartheta$

$$\frac{\mathrm{d}\,\cancel{\cos\vartheta}\,\cancel{\mathrm{d}\varphi}}{32\pi^2}\beta_e$$

$$\frac{\mathrm{d}\Gamma_{ee}}{\mathrm{d}E_+} = \frac{1}{2M_{\mathrm{atom}}} \int \mathrm{d}\Phi_3 \underbrace{\frac{\alpha}{4\pi E_\gamma} \int_{m_*^-}^{m_*^+} \frac{\mathrm{d}m_*^2}{m_*^2} \mathcal{J}_*^{\mu\nu} \frac{\langle L_{\mu\nu} \rangle_\varphi}{m_*^2}}$$

$$\mathrm{d}\cancel{\Phi_2}$$

$$\overbrace{\mathcal{J}_0^{\mu\nu}[-g_{\mu\nu}] \quad P_{\mathrm{int}}(E_+ \,|\, E_\gamma, \Pi)}$$

Real photon amplitude

Internal conversion

Extra phase space variables

$$\frac{\langle L_{\mu\nu} \rangle_\varphi}{m_*^2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & A & 0 \\ 0 & 0 & 0 & B \end{pmatrix} \Bigg| \begin{array}{l} \mathcal{J}_*^{\mu\nu} L_{\mu\nu} \\ = A(\mathcal{J}_*^{11} + \mathcal{J}_*^{22}) + B\mathcal{J}_*^{33} \end{array}$$

$$A = 2 - (1 - \cos^2\vartheta)\beta_e^2 \qquad B = 2(1 - \beta_e^2 \cos^2\vartheta)$$

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$$\frac{\mathrm{d}\Gamma_{ee}}{\mathrm{d}E_+} = \int \mathrm{d}\Phi_3 \underbrace{\frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi_3}\, P_{\mathrm{int}}(E_+ \,|\, E_\gamma, \Pi)}$$

$$\overbrace{\mathcal{J}_0^{\mu\nu}[-g_{\mu\nu}] \quad P_{\mathrm{int}}(E_+ \,|\, E_\gamma, \Pi)}$$

Real photon amplitude

Internal conversion

Extra phase space variables

- The function $P_{\mathrm{int}}(E_+ \,|\, E_\gamma, \Pi)$ depends on matrix elements with off-shell photon kinematics.
- To be calculable we need to find a limit where we can approximate with on-shell photon kinematics.

Off-shell transverse

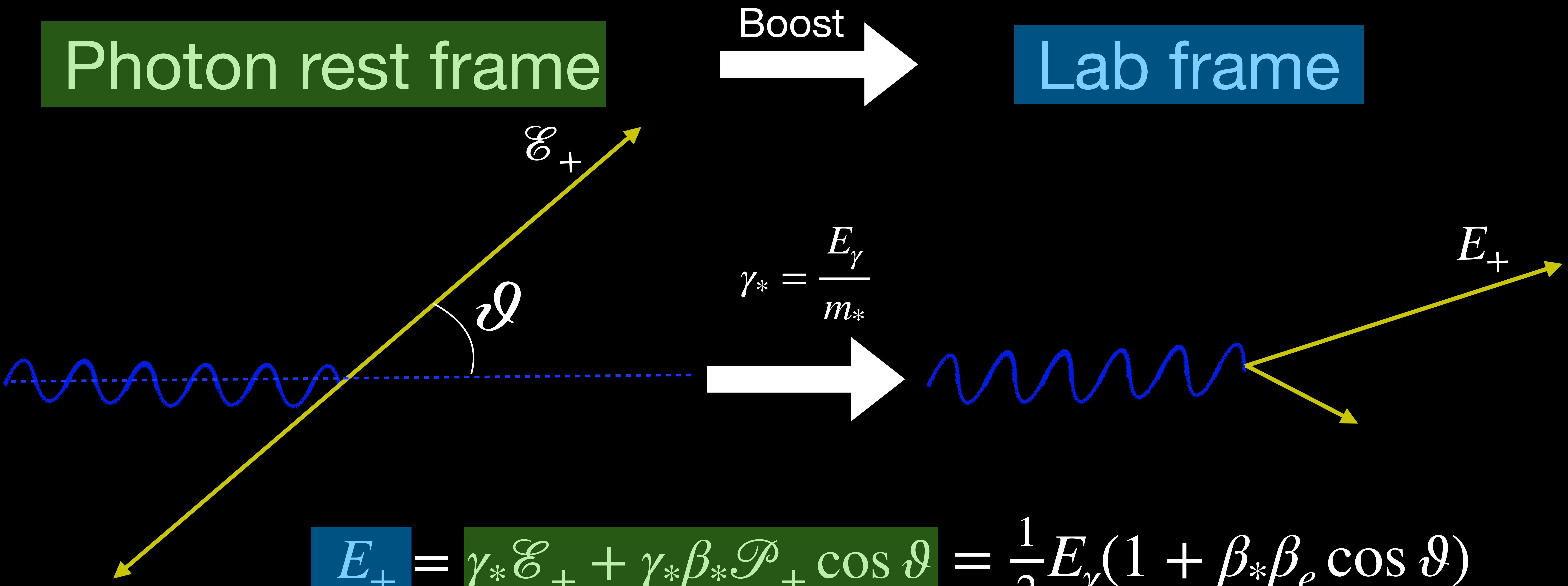
$$P_{\rm int}(E_+ \,|\, E_\gamma, \Pi) = \frac{\alpha}{2\pi E_\gamma}\int_{m_*-}^{m_*^+} \frac{\mathrm{d}m_*}{m_*}\Big\{ 2(1 - [1 - \cos^2\theta]\beta_e^2)(1 + T_*^2) + 2(1 - \cos^2\theta)\beta_e^2 L_*^2 \Big\}$$

Longitudinal

$$\frac{\mathrm{d}\Gamma_{ee}}{\mathrm{d}E_+} = \int \mathrm{d}\Phi_3 \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi_3} P_{\rm int}(E_+ \,|\, E_\gamma, \Pi)$$

- The function $P_{\rm int}(E_+ \,|\, E_\gamma, \Pi)$ depends on matrix elements with off-shell photon kinematics.
- To be calculable we need to find a limit where we can approximate with on-shell photon kinematics.

Endpoint Positrons



$$\boxed{E_+} = \boxed{\gamma_* \mathscr{E}_+ + \gamma_* \beta_* \mathscr{P}_+ \cos \vartheta} = \frac{1}{2} E_\gamma (1 + \beta_* \beta_e \cos \vartheta)$$

To approach maximum energy we need ***high energy*** (virtual) photons, and ***collinear*** positron production.

Integrate over azimuthal angles

$$\frac{\mathrm{d}\cos\vartheta\,\cancel{\mathrm{d}\varphi}}{32\pi^2}\beta_e$$

$$\mathcal{J}_*^{\mu\nu}L_{\mu\nu}$$

$$\frac{\langle L_{\mu\nu}\rangle_\varphi}{m_*^2} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & A & 0 \\ 0 & 0 & 0 & B \end{pmatrix} \qquad = A(\mathcal{J}_*^{11} + \mathcal{J}_*^{22}) + B\mathcal{J}_*^{33}$$

$$A = 2 - \boxed{(1-\cos^2\vartheta)\beta_e^2} \qquad B = 2\boxed{(1-\beta_e^2\cos^2\vartheta)}$$

→ **Collinear suppression**

Off-shell transverse

$$P_{\text{int}}(E_+ \,|\, E_\gamma, \Pi) = \frac{\alpha}{2\pi E_\gamma} \int_{m_{*-}}^{m_*^+} \frac{\mathrm{d}m_*}{m_*} \left\{ 2(1 - [1 - \cos^2\theta]\beta_e^2)(1 + T_*^2) + 2(1 - \beta_e^2 \cos^2\theta) L_*^2 \right\}$$

Longitudinal

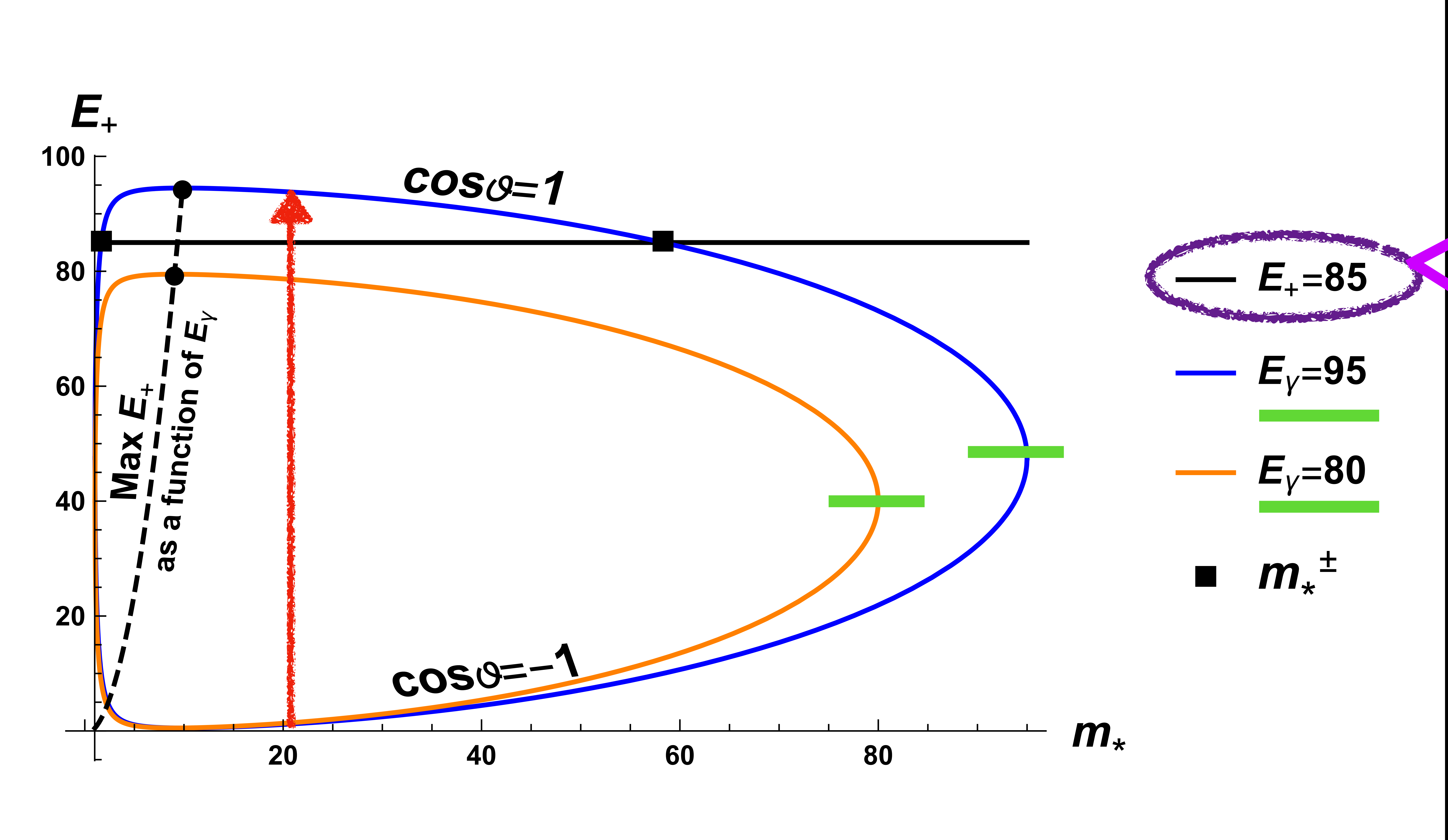
$$\frac{\mathrm{d}\Gamma_{ee}}{\mathrm{d}E_+} = \int \mathrm{d}\Phi_3 \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi_3} \, P_{\text{int}}(E_+ \,|\, E_\gamma, \Pi)$$

- The function $P_{\text{int}}(E_+ \,|\, E_\gamma, \Pi)$ depends on matrix elements with off-shell photon kinematics.
- To be calculable we need to find a limit where we can approximate with on-shell photon kinematics.

$$\sqrt{1 - m_*^2/E_\gamma^2}$$

$$E_+ = \frac{1}{2}E_\gamma(1 + \beta_*\beta_e\cos\vartheta)$$

$$\sqrt{1 - 4m_e^2/m_*^2}$$



$$\cos\theta \approx 1$$

$$m_*^2 \sim 2T_{e^-}E_\gamma$$

Off-shell transverse

$$P_{\rm int}(E_+ \,|\, E_\gamma, \Pi) = \frac{\alpha}{2\pi E_\gamma}\int_{m_*^-}^{m_*^+} \frac{\mathrm{d}m_*}{m_*}\left\{ 2(1-\cancel{[1-\cos^2\theta]\beta_e^2})(1+\cancel{T_*^2}) + 2\cancel{(1-\beta_e^2\cos^2\theta)}L_*^2 \right\}$$

Longitudinal

$$\frac{\mathrm{d}\Gamma_{ee}}{\mathrm{d}E_+} = \int \mathrm{d}\Phi_3 \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi_3}\, P_{\rm int}(E_+ \,|\, E_\gamma, \Pi)$$

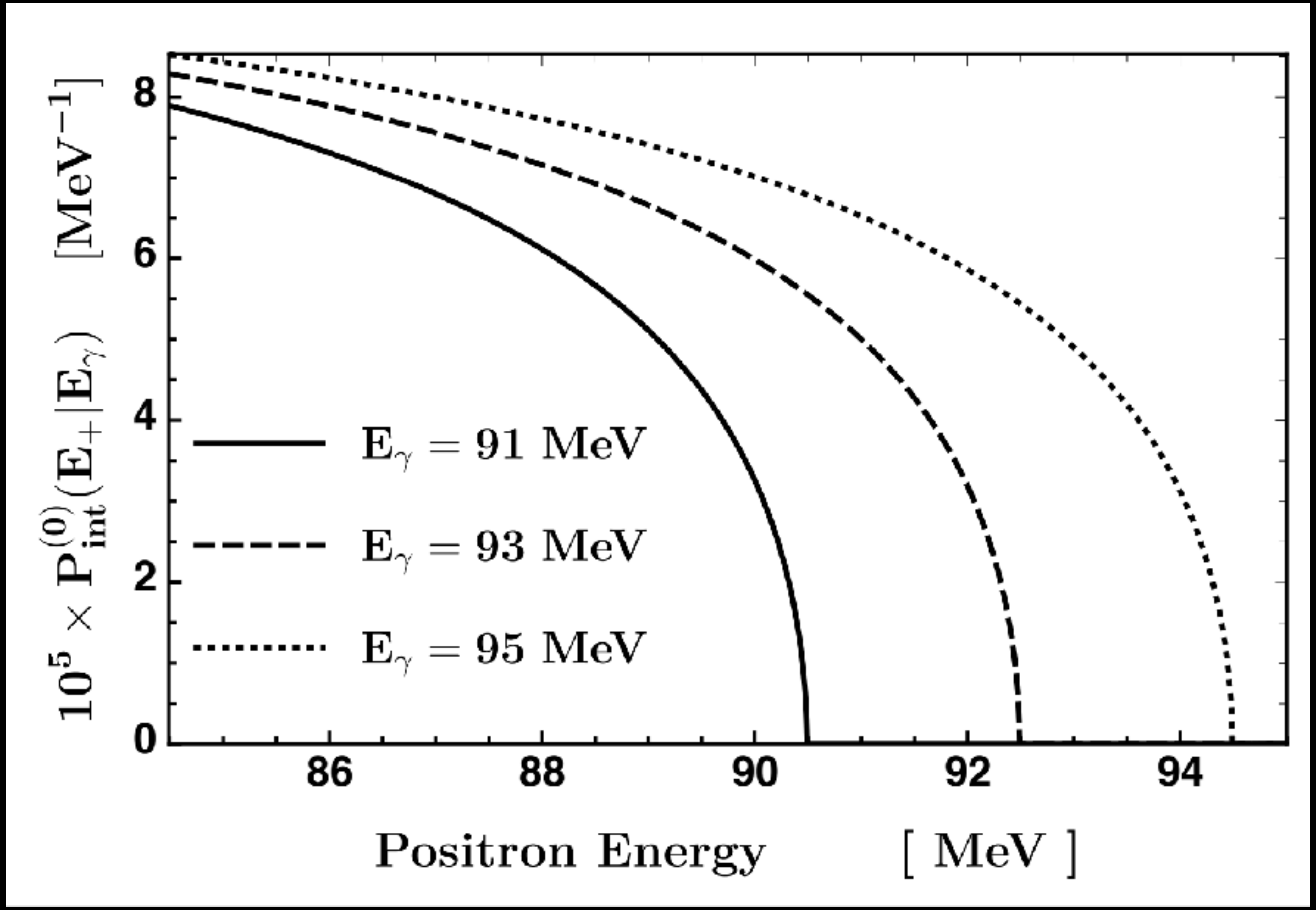
- The function $P_{\rm int}(E_+ \,|\, E_\gamma, \Pi)$ depends on matrix elements with off-shell photon kinematics.
- To be calculable we need to find a limit where we can approximate with on-shell photon kinematics.

arXiv:2010.09509

$$P_{\mathrm{int}}(E_+ \,|\, E_\gamma, \Pi) \approx \frac{\alpha}{2\pi E_\gamma}\int_{m_{*}^{-}}^{m_*^+} \frac{\mathrm{d}m_*}{m_*} = \frac{\alpha}{\pi E_\gamma}\log\left[\frac{m_*^+}{m_*^-}\right]$$

$$\frac{\mathrm{d}\Gamma_{ee}}{\mathrm{d}E_+} = \int \mathrm{d}E_\gamma \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma} P_{\mathrm{int}}(E_+ \,|\, E_\gamma, \Pi)$$

$$\frac{\mathrm{d}\Gamma_{ee}}{\mathrm{d}E_+} = \int \mathrm{d}\Phi_3 \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi_3} P_{\mathrm{int}}(E_+ \,|\, E_\gamma, \Pi)$$



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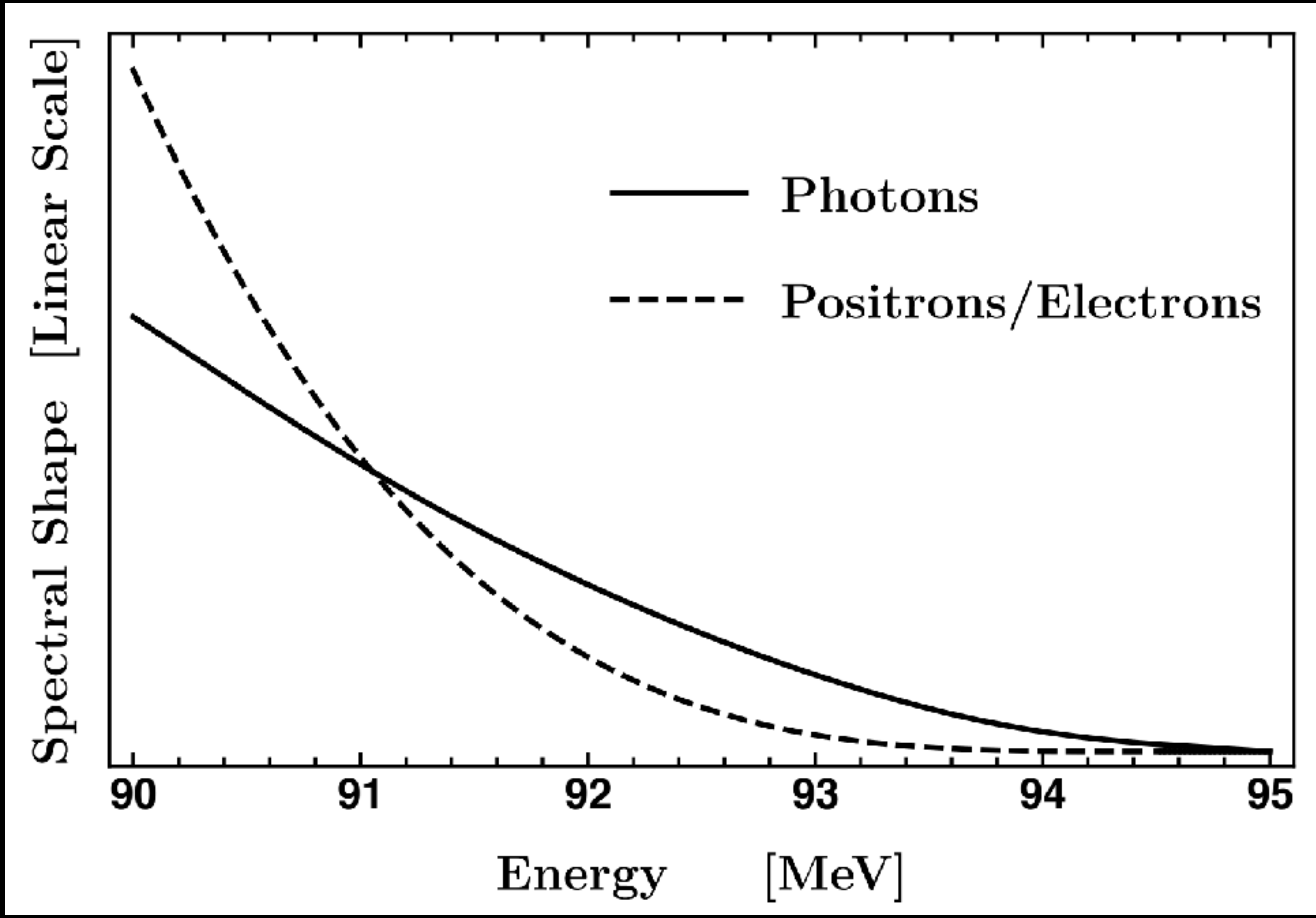
- This function + photon spectrum predicts the positron spectrum

- We provide error estimates in our paper



Main conclusions:

- Longitudinal polarizations are suppressed when the electron/positron is nearly collinear with the photon in the rest frame.

- Transverse matrix elements can be approximated by real photon matrix elements provided the virtuality is “small”.

- Both conditions are satisfied as $E_+ \to E_\gamma - m_e$ or equivalently as $T_- \to 0$. The small parameters we use are T_-/E_γ and m_e/E_γ.

- Near the end point there is a calculable function for internal conversion.

Ongoing work:

- RFG calculation of RMC on Al and Au.
- Coulomb corrections to internal conversion.
- Better understanding of sub-leading corrections to the internal-conversion probability

In collaboration with Richard Hill & Kaushik Borah

Many thanks to

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& Stefano Di Falco