Toward the N3LO accuracy of parton distribution functions

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The role of PDFs in precision QCD studies. An overwhelming number of theoretical predictions for hadron colliders requires parton distribution functions (PDFs) [1–8], the nonperturbative functions quantifying probabilities for finding quarks and gluons in hadrons in high-energy scattering processes. We witness a revolution in computing hard scattering cross sections in perturbative QCD to a high accuracy, achieved by including radiative contributions up to the second and third order in the small coupling constant, or N2LO and N3LO. In this letter, we emphasize importance of determination of parton distributions to accuracy that is comparable to those of N2LO/N3LO hard cross sections. Obtaining such accurate PDFs necessitates continued advancements in the areas of quantum field theory, experimental measurements, and statistical methods.

Progress in understanding of PDFs beyond the current level is critical for realizing the physics programs of the high-luminosity runs of the Large Hadron Collider (HL-LHC). Limitations in the knowledge of the PDFs constrain the accuracy of measurements of the Higgs boson couplings and electroweak parameters in the key channels at the HL-LHC [9, 10]. By knowing the PDFs for the gluon and other flavors approximately to 1-2% accuracy, one greatly reduces the total uncertainties on the Higgs couplings in gluon-gluon fusion and electroweak boson fusion. The energy reach in searches for very massive new particles at the HL-LHC can be extended by better knowing the PDFs at the largest momentum fractions, $x > 0.1$, and by pinning down the flavor composition of the partonic sea. As interest grows in hadron scattering at very small partonic momentum fractions, $x < 10^{-5}$, at hadron colliders (HL-LHC, LHeC, FCC-hh) as well as in the astrophysics experiments, one must include effects of small-x resummation and saturation in QCD theory and, when warranted, in the PDFs [11].

PDFs contribute to precise measurements of the QCD coupling constant, heavy-quark masses, weak boson mass, and electroweak flavor-mixing parameters. This requires continuous benchmarking and improvements of the theoretical framework in the perturbative approach [12, 13]. As lattice QCD techniques advance in computations of PDFs from the first principles, unpolarized phenomenological PDFs serve as important benchmarks for testing the lattice QCD methods [14, 15].

Path toward N3LO PDFs. In turn, determination of PDFs at the N2LO/N3LO accuracy expected for the HL-LHC requires coordinated advancements in several areas. These include implementation of novel accurate measurements, development of fast interfaces for N2LO/N3LO computations, implementations of electroweak contributions and QCD resummations, detailed studies of experimental and theoretical systematic uncertainties, and distribution of final PDF parametrizations in a convenient form for across-the-board applications, such as the PDF4LHC'2015 format [12] for combined PDFs used at the LHC Run-2. Efforts are underway to compute cross sections [16–22] and QCD evolution equations [23, 24] at N^{3}LO accuracy. Before the N3LO cross sections become available for all fitted processes, uncertainties in the ultimate precision predictions associated with the use of the mix of N2LO and N3LO cross sections need to be estimated. Theory uncertainties are important even in NNLO fits [20, 25, 26].

Future non-LHC experiments. Lepton-hadron scattering and production of Drell-Yan pairs on nuclear targets have been used to constrain the differences between up and down (anti-)quark PDFs in the proton under the assumption of charge symmetry. In spite of their importance for constraining the PDFs, these measurements have limited accuracy and are sensitive to nuclear effects. It is thus desirable to accurately remeasure the relevant PDFs directly on proton targets. In this regard, it is important to compare the potentials of proton scattering experiments at the HL-LHC and future lepton-hadron (EIC, LHeC) and fixed-target (AMBER, LHCb Spin) facilities that may operate concurrently, or in a short succession, with the HL-LHC.

PDF analyses as a part of theory infrastructure. Accurate determination of PDFs constitutes a critical component of theory infrastructure for future hadronic experiments, together with the development of Monte-Carlo event generators and multi-loop calculations in QFT [27]. The global QCD analysis of the PDFs is an exciting research area at the intersection of frontier experiments and state-of-the-art theory. It increasingly incorporates newest methods from multivariate data science, artificial intelligence, and high-performance computing. It presents ample opportunities for training of students and postdocs in mathematical and theoretical skills applicable in many areas of science and industry.

Precision PDFs in the United States. Among several groups (ABM, CTEQ-TEA, HERAPDF, MMHT, NNPDF) working on the determination of general-purpose NNLO PDFs, one group (CTEQ-TEA [5, 7, 28–33])

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Topic	Status, 2013	Status and plans, 2020
Benchmarking of PDFs for the LHC	Before PDF4LHC'2015 recommendation	In progress toward PDF4LHC'2X recommendation
PDFs with NLO EW contributions	MSTW'04 QED, NNPDF2.3 QED	Needs an update using LuXQED and other photon PDFs; PDFs with leptons and massive bosons
PDFs with resummations	Small x (in progress)	Needs an update for PDFs with small-x and threshold resummations
Parton luminosities at 14, 33, 100 TeV	CT10, MSTW2008, NNPDF2.3 Update at 100 TeV in CERN YR (1607.01831)	Need an update based on the latest PDFs
LHC processes to measure PDFs	W/Z, single-incl. jet, high-p_T Z, $t\bar{t}$, $W+c$ production	updates on these processes + $Q\bar{Q}$, dijet, $\gamma/W/Z$ +jet, low-Q DY, …
Future experiments to probe PDFs	LHC Run-2 DIS: LHeC	LHC Run-3 DIS: EIC, LHeC, …

NEW TASKS in THE HL-LHC ERA:		
Obtain complete NNLO and N3LO predictions for PDF-sensitive processes	Improve models for correlated systematic errors	Find ways to constrain large-x PDFs without relying on nuclear targets
Develop and benchmark fast NNLO interfaces	Estimate NNLO theory uncertainties	Develop an agreement on comparing and combining PDF fits

TABLE I. PDF-related topics in Snowmass'2013 [36] and '2021 studies.

is currently based in the US. Each general-purpose global analysis of PDFs is a major undertaking, involving significant investment in development, testing, and tuning of theoretical and computational frameworks. Recall that it took more than ten years from the publication of NNLO DGLAP equations [34, 35] to the release of NNLO PDF parametrizations with benchmarked accuracy [12]. Further advancements require support for the critical mass of the personnel with the specialized expertise. These advancements greatly benefit from the collaborations between experimentalists and theorists, and from international collaborations. Since the Electron-Ion Collider can provide powerful new constraints on large-x PDFs, it makes sense to forge novel collaborations between the HEP and nuclear physics communities in the US.

Computational needs. CPU power for generating higher-order ApplGrid and FastNLO tables, replacing K-factor tables in the fits is indispensable as we push to higher orders. Docker and Singularity tools are also key, as they allow us to distribute uniform environments for the analysis purposes. The AI/ML techniques increasingly are powerful alternatives to traditional fits.

Plans for Snowmass. Our group will explore opportunities for determination of the PDFs and implications for future studies explored by the Snowmass Frontiers. We plan to update some comparisons of the PDFs presented in the Snowmass 2013 report [36]. PDF-related topics in the Snowmass'2013 report, and their possible updates in the Snowmass'2021 report, are listed in the upper part of Table I. The lower part of the Table lists some of the new issues that must be addressed to achieve the targeted accuracy of the PDFs in the HL-LHC era. This contribution will complement dedicated physics studies of PDFs described in the companion LOI's [10, 37–41], as well as the concurrent efforts by the PDF4LHC working group and Les Houches workshop.

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