# $V_{cb}$ and $V_{ub}$

# Continuum QCD Theory Overview

**Thomas Mannel**

**Theoretische Physik I** **Universität Siegen**

Particle Physics Phenomenology after the Higgs Discovery

Collaborative Research Center TRR 257

**Virtual Snowmass, January 11th, 2021**

# Menue

# Introduction

Two Roads to $V_{xb}$ from semileptonic $b \to x\ell\bar{\nu}$ transitions

- Exclusive Channels
  - Nonperturbative Input: Form Factors

  $\to$ ... dominated by lattice (Talk by A. Vaquero)
- Inclusive decays
  - Heavy Quark Expansion

  $\to$ Local OPE for $V_{cb}$
  HQE parameters as nonperturbative inputs

  $\to$ Light Cone OPE for $V_{ub}$
  Shape functions = Light-Cone distributions as nonperturbative inputs

# A Word on Exclusive Decays

Form Factors are clearly the domain of lattice QCD, however:

- Restrictions to the region close to maximal $q^2$
- Extrapolation to all values of $q^2$ needed
- QCD sum rule estimates still can help at small $q^2$
- In addition: Zero recoil sum rules close to maximal $q^2$

# Inclusive Determination of $V_{cb}$

- Standard tool: Heavy Quark Expansion
- Structure of the expansion (@ tree):

$$\begin{aligned} d\Gamma &= d\Gamma_0 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 d\Gamma_2 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 d\Gamma_3 + \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^4 d\Gamma_4 \\ &+ d\Gamma_5 \left(a_0 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^5 + a_2 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2\right) \\ &+ \ldots + d\Gamma_7 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^4 \end{aligned}$$

- Power counting $m_c^2 \sim \Lambda_{\rm QCD} m_b$

- $\Gamma_0$ is the decay of a free quark ("Parton Model")
- $\Gamma_1$ vanishes due to Heavy Quark Symmetries
- $\Gamma_2$ is expressed in terms of two parameters

$$\begin{aligned} 2M_H\mu_\pi^2 &= -\langle H(v)|\bar{Q}_v(iD)^2 Q_v|H(v)\rangle \\ 2M_H\mu_G^2 &= \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(iD^\nu)Q_v|H(v)\rangle \end{aligned}$$

$\mu_\pi$: Kinetic energy and $\mu_G$: Chromomagnetic moment

- $\Gamma_3$ two more parameters

$$\begin{aligned} 2M_H\rho_D^3 &= -\langle H(v)|\bar{Q}_v(iD_\mu)(ivD)(iD^\mu)Q_v|H(v)\rangle \\ 2M_H\rho_{LS}^3 &= \langle H(v)|\bar{Q}_v\sigma_{\mu\nu}(iD^\mu)(ivD)(iD^\nu)Q_v|H(v)\rangle \end{aligned}$$

$\rho_D$: Darwin Term and $\rho_{LS}$: Spin-Orbit Term

- $\Gamma_4$ and $\Gamma_5$ have been computed Bigi, Uraltsev, Turczyk, TM, ...

## Present state of the $b \to c$ semileptonic Calculations

- Tree level terms up to and including $1/m_b^5$ known
  Bigi, Zwicky, Uraltsev, Turczyk, TM, ...
- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the partonic rate known
  Melnikov, Czarnecki, Pak
- New: First results for the partonic $\alpha_s^3$ contributions are known
  Fael, Schoenwald, Steinhauser,
- $\mathcal{O}(\alpha_s)$ for the full $1/m_b^2$ is known
  Becher, Boos, Lunghi, Gambino, Pivovarov, Rosenthal, Alberti
- Relatively New: First results for $\alpha_s/m_b^3$ ThM., Moreno, Pivovarov

**We are getting at a TH-uncertainty of 1% in $V_{cb,incl}$!**

# Strategy for the $V_{cb}$ Extraction

- Based on the HQE for the inclusive rates and for moments of spectra
- (Cut) moments of the charged lepton energy, hadronic energy and hadronic invariant mass spectra
- Extract the HQE parameters from this data
- Obtain $V_{cb}$ from the total semileptonic rate

Problem: Number of HQE parameters in higher orders!

- 4 up to $1/m^3$
- 13 up to $1/m^4$ (tree level)
- 31 up to order $1/m^5$ (tree level)
- Factorial Proliferation

# What is the quark mass?

Hadron mass vs. quark mass: $m_H = m_Q + \bar{\Lambda} + \mathcal{O}(1/m_Q)$

Quark mass is not a physical parameter!

Standard starting point: Pole mass scheme: ($m_0$: "bare" mass)

$$S(p) = \frac{-iZ_2^{\rm OS}}{\not p - m_0 + \Sigma(p, m_Q^{\rm Pole})} \longrightarrow \frac{-i}{\not p - m_Q^{\rm Pole}} \quad \text{as } p^2 \to (m_Q^{\rm Pole})^2 \qquad m_0 = Z_m^{\rm OS} m_Q^{\rm Pole} = \left(1 + \sum_{n=1}^{\infty} c_n \left(\frac{\alpha_s}{\pi}\right)^n\right) m_Q^{\rm Pole}$$

Alternatively: Use dim-reg and subtract only $1/\epsilon$ (and some constant) terms

$$m_0 = Z_m^{\overline{\rm MS}} m_Q^{\overline{\rm MS}} = \left(1 + \sum_{n=1}^{\infty} b_n \left(\frac{\alpha_s}{\pi}\right)^n\right) m_Q^{\overline{\rm MS}}$$

and so (with finite $a_n$!)

$$m_Q^{\rm Pole} = z^{{\rm Pole}\to\overline{\rm MS}}\, m_Q^{\overline{\rm MS}} = \frac{Z_m^{\overline{\rm MS}}}{Z_m^{\rm OS}} m_Q^{\overline{\rm MS}} \qquad \text{with} \qquad z^{{\rm Pole}\to\overline{\rm MS}} = 1 + \sum_{n=1}^{\infty} a_n \left(\frac{\alpha_s}{\pi}\right)^n$$

What does this mean for the rates?
Calculate in the Pole scheme:

$$d\Gamma \sim G_F^2 |V_{\rm CKM}|^2 (m_Q^{\rm Pole})^5 \left(1 + \frac{\alpha_s}{\pi} r_1 + \left(\frac{\alpha_s}{\pi}\right)^2 r_2 + \cdots \right)$$

Change the mass scheme to eg. the $\overline{\rm MS}$ scheme:

$$\begin{aligned} d\Gamma &\sim G_F^2 |V_{\rm CKM}|^2 (m_Q^{\overline{\rm MS}})^5 (z^{{\rm Pole}\to\overline{\rm MS}})^5 \left(1 + \frac{\alpha_s}{\pi} r_1 + \left(\frac{\alpha_s}{\pi}\right)^2 r_2 + \cdots \right) \\ &= G_F^2 |V_{\rm CKM}|^2 (m_Q^{\overline{\rm MS}})^5 \left(1 + \frac{\alpha_s}{\pi} (r_1 + 5a_1) + \cdots \right) \end{aligned}$$

The size of the radiative corrections depends on the mass definition!

Invent a clever scheme, suited to the HQE. This means:

- Good convergence of the perturbative series for the rates and moments (technically: Absence of renormalon problems )
- A way to get a precise value of the mass in the corresponding definition
- (Mass must be usable for scales below $m$)

Two schemes are frequently used:

- Kinetic (mass) scheme (Shifman, Voloshin, Uraltsev, ...)
- $\Upsilon(1S)$ scheme (Hoang, Manohar, Ligeti, ...)

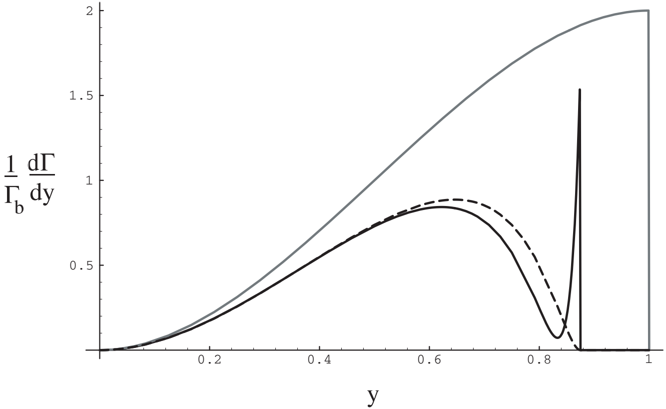
Involves a calculation of the relation betessn pole mass and the chosen mass
(See also the next talk by Matthias Steinhauser)

We will see the current fits in the comming talks. PDG2020 quotes:

$$V_{cb}^{\mathrm{incl}} = (42.2 \pm 0.8) \times 10^{-3}$$

# Inclusive Determination of $V_{ub}$

The local OPE breaks down in certain corners of phase space:



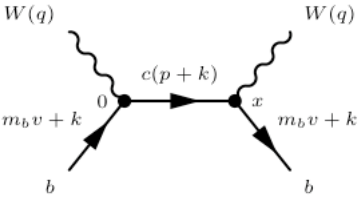
- Endpoint region: $\rho = m_c^2/m_b^2$, $y = 2E_\ell/m_b$

$$\frac{d\Gamma}{dy} \sim \Theta(1-y-\rho)\left[2 - \frac{\mu_\pi^2}{(m_b(1-y))^2}\left(\frac{\rho}{1-y}\right)^2\left\{3 - 4\frac{\rho}{1-y}\right\}\right]$$

$$\frac{d\Gamma}{dy} = \frac{G_F^2 m_b^5 |V_{ub}|^2}{192\pi^3}\left[\theta(2E - m_b)y\left\{(3-2y)y - \frac{5y^2}{3}\frac{\mu_\pi^2}{m_b^2} + \frac{y}{3}(6+5y)\frac{\mu_G^2}{m_b^2}\right\}\right.$$
$$\left. + \frac{\mu_\pi^2 - 11\mu_G^2}{6m_b^2}\delta(1-y) + \frac{\mu_\pi^2}{6m_b^2}\delta'(1-y)\right] \quad \text{for} \quad \rho \to 0$$

- The expansion parameter is actually $1/[m_b(1-y)]$
- Cutting away the $b \to c$ contriution by an energy cut forces us into the "endpoint region" with $y \sim 1$, where the standard HQE breaks down
- Instead of the Standard HQE: Light-Cone OPE
- Instead of HQE parameters we have light-cone distributions

# Heavy Quark Expansion in a Nutshell




Intermediate Propagator is taken in the external gluon field: $\frac{1}{\not{Q} + i\not{D} - m_c}$

Standard HQE Power Counting: $Q = m_b v - q \sim m_b$, $m_c \sim m_b$ and $iD \sim \Lambda_{\rm QCD}$

$$\frac{1}{\not{Q} + i\not{D} - m_c} = \frac{1}{\not{Q} - m_c} - \frac{1}{\not{Q} - m_c} i\not{D} \frac{1}{\not{Q} - m_c} + \frac{1}{\not{Q} - m_c} i\not{D} \frac{1}{\not{Q} - m_c} i\not{D} \frac{1}{\not{Q} - m_c} + \cdots$$

... yields the standard $1/m_b$ expansion.

# Light-Cone Expansion in a Nutshell

Modified power Counting for the endpoint region: $Q^2 \sim \mathcal{O}(m\Lambda_{\rm QCD})$
(c.f. resonance region with $Q^2 \sim \mathcal{O}(\Lambda_{\rm QCD}^2)$)
Light-Cone Vectors: $n^2 = \bar{n}^2 = 0$, $v = (n+\bar{n})/2$ and $n\bar{n} = 2$ with

$$Q = \frac{1}{2}((nQ)\bar{n} + (\bar{n}Q)n) \qquad (\bar{n}Q) \sim \mathcal{O}(m) \qquad (nQ) \sim \mathcal{O}(\Lambda_{\rm QCD}) \qquad iD \sim \Lambda_{\rm QCD}$$

Expand $\dfrac{1}{\not{Q} + i\not{D}}$

$$\frac{1}{\not{Q} + i\not{D}} = \frac{1}{2}\not{n}\frac{1}{(nQ) + (inD)} + \cdots = \frac{1}{2}\not{n}\int d\omega \frac{1}{(nQ) - \omega}\delta(\omega + inD) + \cdots$$

can be pushed to subleading order (Bauer, Luke ThM)

# Shape Functions

- Formal Definition:

$$2M_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + inD) b_v |B(v)\rangle$$

- Moment Expansion:

$$f(\omega) = \delta(\omega) + \frac{\mu_\pi^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{18m_b^3}\delta'''(\omega) + \cdots$$

- Relates the standard HQE to the Light-Cone OPE
- QCD Corrections need “Soft Collinear Effective Theory”
- Status:
  - NLO QCD Corrections are known at NLO for leading term (SCET Calculation)
  - First Subleading terms have been identified

# Implementations

Problem: What do we know about the shape functions (leading and subleading)?

- Obtaining the Shape functions:
  - From Comparison with $B \to X_s \gamma$
  - From the knowledge of (a few) moments
  - From modeling
- QCD based:
  - BLNP (Bosch, Lange, Neubert, Paz)
  - GGOU (Gambino, Giordano, Ossola, Uraltsev)
  - SIMBA (Tackmann, Tackmann, Lacker, Liegti, Stewart ...)
- “QCD inspired”:
  - Dressed Gluon Exponentiation (Andersen, Gardi)
  - Analytic Coupling (Aglietti et al.)
- Attempts to avoid the shape functions (Bauer Ligeti, Luke ...)

# QCD Based Approaches

BLNP (Bosch, Lange, Neubert, Paz)

- Leading order including $\mathcal{O}(\alpha_s)$
- Subleading order at tree level
- Leading and subleading shape functions ansätze with correct moments
- Update in progress to include more infomation on higher moments

GGOU (Gambino, Giordano, Ossola, Uraltsev)

- Leading order including $\mathcal{O}(\alpha_s)$
- Subleading orders included by a parameter dependend shape function
- Leading and subleading shape functions ansätze with correct moments

# QCD Insprired Approaches

DGE: Dresses Gluon Exponentiation (Andersen, Gardi)

- Partonic calculation including QCD corrections
- Identification of “large terms” (Sudakov terms)
- Resummation (“Exponentiation”) of these terms
- Constraints on this procedure from “Renormalon cancellations”
- Allows a calculation of the Shape Functions under these assumptions

PDG2020 quotes:

$$V_{ub}^{\mathrm{incl}} = (4.25 \pm 0.12^{+0.15}_{-0.14} \pm 0.23) \times 10^{-3}$$

# Perspectives for inclusive $V_{cb}$

- Reduce the number of independent HQE parameters by studying "Lorentz invariant / Reparametrization invariant" observables (Fael, ThM, Vos, ...)
- Improvement of quark masses: Kinetic Mass at $1/m^3$ etc., Update of the 1S scheme? other mass definitions?
- At sub percent level, ancient deamons my return: Duality Violations: Convergence of HQE, Contributions missed by the HQE ...

# Perspectives for inclusive $V_{ub}$

- Update of BLNP:
  - Include higher known moments into the shape functions
  - Include the full $1/m$ contributions at $\alpha_s$
- Update of GGOU (?)
- Shape function independent methods?

Definitively some more work has to be done here!