# Exclusive semileptonic decays from lattice QCD

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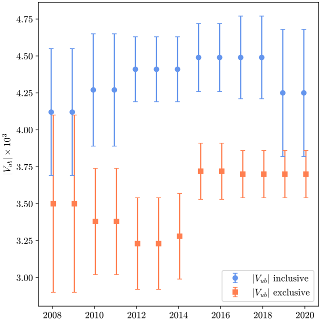
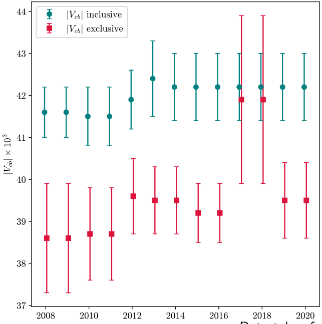
# Motivation: the CKM matrix

- CKM matrix elements are key in BSM physics explorations
- $|V_{ub}|$ and $|V_{cb}|$ controversial: tensions inclusive vs exclusive



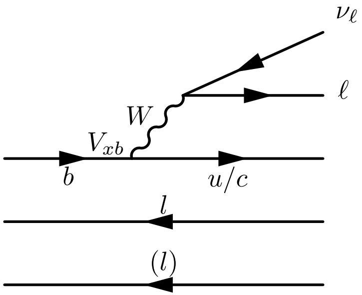




Data taken from the PDG reviews

- See talk by B. Grinstein (Monday)

# Lattice + exclusive determinations

- An exclusive determination focus on one process
- Theoretically very clean
  - Key theory input comes from the lattice (first principles)
- Needs experimental input




$$\underbrace{\frac{d\Gamma}{dq^2}}_{\text{Experiment}} = (\text{Known factors}) \times \underbrace{(\text{Form factor(s)})}_{\text{Lattice}} \times |V_{xb}|^2$$

# Fermionic regularizations

- Different regularizations have different systematics
- No 'perfect' regularization

| | Asqtad/HISQ | Clover | Domain-Wall |
|---|---|---|---|
| Disc. errors | $O(\alpha_s a^2)$ | $O(a^2)$ | $O(a^2)$ |
| Chiral properties | Good | Bad | Excellent |
| Allows 1-flavor? | With issues | Yes | Yes |
| Cost | Very cheap | Cheap | Expensive |
| Efficiency | Low | High | High |

- HISQ reduces the taste-breaking interactions wrt asqtad
- Baryons are difficult with asqtad/HISQ
- Spurious particles appear in asqtad/HISQ
- Clover requires fine tuning

# Heavy quarks in lattice QCD

Heavy quark treatment in Lattice QCD

- For light quarks ($m_l \lesssim \Lambda_{QCD}$), leading discretization errors $\sim \alpha_s^k (a\Lambda_{QCD})^n$
- For heavy quarks ($m_Q > \Lambda_{QCD}$), discretization errors grow as $\sim \alpha_s^k (am_Q)^n$
  - State-of-the-art calculations use $am_c \lesssim 1$, but $am_b > 1$

Need special actions and ETs to describe (at least) the bottom quark

- Relativistic HQ actions (FermiLab, Oktay-Kronfeld...)
- Non-Relativistic QCD (NRQCD)

If the action is improved enough, one can treat the bottom as a light quark

- Highly improved action AND small lattice spacing
- Use unphysical values for $m_b$ and extrapolate

# Parametrizations

- Theoretically motivated parametrizations give us a reasonable ansatz to extend lattice results to the whole $q^2$ range
- They are extremely useful to fit experimental and lattice data together
  - Lattice calculations are (usually) more accurate at large $q^2$
    - Many times they are restricted to $q^2 = q^2_{\mathrm{Max}}$
  - Experiments are (usually) more accurate at small $q^2$ (phase space suppression)
- Three parametrizations are normally used for these processes, all of them based on basic properties of the form factors and dispersion relations (i.e. analyticity, unitarity)

**BGL**

- General and model-independent parametrization
- One can add HQET input or use HQS to tighten the bounds

**CLN**

- Attempts to improve BGL by using HQET and HQS
- It's being deprecated by the community

**BCL**

- Simplification of BGL including explicit terms to address certain problems of BGL applied to heavy-to-light decays

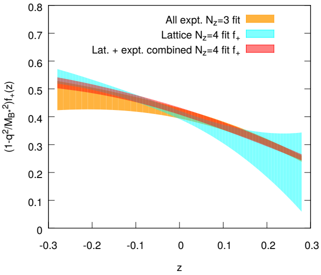
# $|V_{ub}|$ and $B_{(s,c)} \to \pi(K, D)\ell\nu$

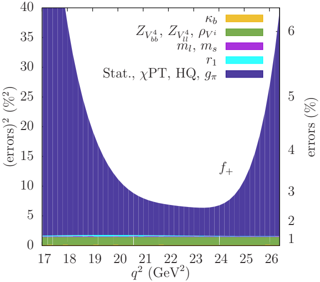
- The differential decay rate for the $B_{(s,c)} \to P\ell\nu$ ($P = \pi, K, D$) semileptonic decay is (neglecting $m_\ell$)

$$\frac{d\Gamma}{dq^2} = \eta_{\mathrm{EW}} \frac{G_F^2 |V_{ub}|^2}{24\pi^2} |\vec{p}_P|^3 |f_+(q^2)|^2$$

- The $|\vec{p}_P|^3$ factor kills the phase space at large $q^2 = (p_B - p_P)^2$
    - This is a feature of all the decays to a pseudoscalar ($\pi, K, D_{(s)}$...)
- Measurements of the differential decay rate at large $q^2$ are less accurate
- Lattice calculations at large $q^2$ display less discretization errors

- Sea sector: $N_F = 2+1$ asqtad fermions with lightest $m_\pi \approx 180$ MeV
- Valence sector: Asqtad for $l, s$, Fermilab HQ for $b$, mostly nonperturbative renormalization
- Lattice spacings: 0.12 fm, 0.09 fm, 0.06 fm and 0.045 fm
- Fourteen ensembles in total with typical $N_{Conf} \sim O(10^3)$ and $V_{\text{Max}} = 64^3 \times 192$

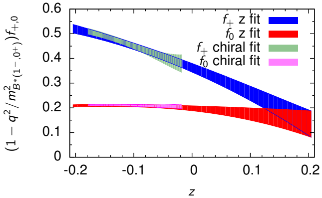


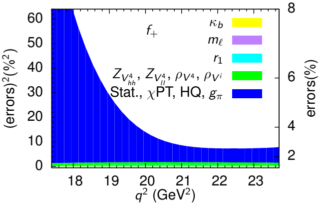




- Published results in 2015, $|V_{ub}| = 3.72(16) \times 10^{-3}$ using BCL and Belle and BaBar data Phys.Rev.D92, 014024 (2015) BaBar, Phys.Rev.D83, 032007 (2011), Phys.Rev.D86, 092004 (2012) Belle, Phys.Rev.D83, 071101 (2011), Phys.Rev.D88, 032005 (2013)

# $|V_{ub}|$ and $B_s \to K\ell\nu$: FNAL/MILC

- Sea sector: $N_F = 2+1$ asqtad fermions with lightest $m_\pi \approx 180$ MeV
- Valence sector: Asqtad for $l, s$, Fermilab HQ for $b$, mostly nonperturbative renormalization
- Lattice spacings: 0.12 fm, 0.09 fm and 0.06 fm
- Six ensembles in total with typical $N_{Conf} \sim O(10^3)$ and $V_{\rm Max} = 64^3 \times 144$

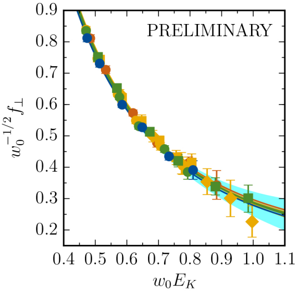


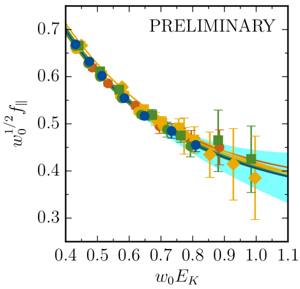



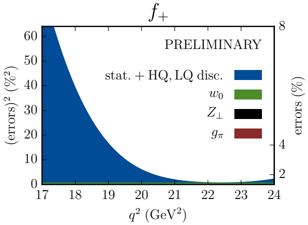

- Published results in 2015 using BCL, no experimental input for $B_s \to K\ell\nu$
  Phys.Rev.D100, 034501 (2019)

# $|V_{ub}|$ and $B_{(s)} \to \pi(K)\ell\nu$: FNAL/MILC

- Sea sector: $N_F = 2+1+1$ HISQ fermions with lightest $m_\pi \approx$ physical
- Valence sector: HISQ for $l, s$, Fermilab HQ for $b$, mostly nonperturbative renormalization
- Lattice spacings: 0.15 fm, 0.12 fm, 0.088 fm and 0.057 fm
- Seven ensembles in total with typical $N_{Conf} \sim O(10^3)$ and $V_{\text{Max}} = 96^3 \times 192$




- Currently work in progress, preliminary results for the chiral-continuum extrapolation

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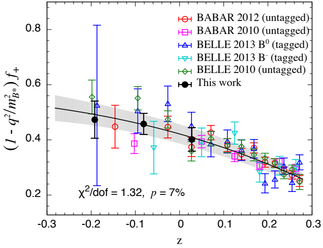
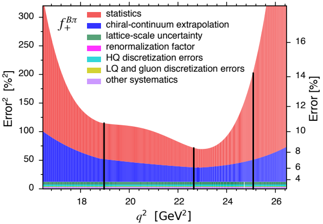
# $|V_{ub}|$ and $B \to \pi\ell\nu$: RBC/UKQCD

- Sea sector: $N_F = 2 + 1$ Domain-Wall fermions with lightest $m_\pi \approx 290$ MeV
- Valence sector: Domain-Wall fermions for $l, s$, relativistic HQ effective action for $b$, mostly nonperturbative renormalization
- Lattice spacings: 0.11 fm and 0.083 fm
- Five ensembles in total with typical $N_{Conf} \sim O(10^2)$ and $V_{\text{Max}} = 32^3 \times 64$

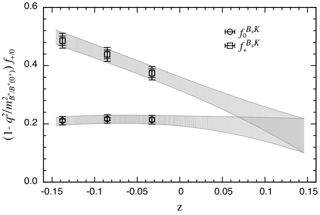


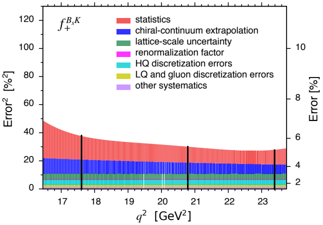




- Published results in 2015, $|V_{ub}| = 3.61(32) \times 10^{-3}$ using BCL and Belle and BaBar data Phys.Rev.D91, 074510 (2015) BaBar: Phys.Rev.D83, 032007 (2011); Phys.Rev.D86, 092004 (2012) Belle: Phys.Rev.D83, 071101 (2011); Phys.Rev.D88, 032005 (2013)

# $|V_{ub}|$ and $B_s \to K\ell\nu$: RBC/UKQCD

- Sea sector: $N_F = 2+1$ Domain-Wall fermions with lightest $m_\pi \approx 290$ MeV
- Valence sector: Domain-Wall fermions for $l, s$, relativistic HQ effective action for $b$, mostly nonperturbative renormalization
- Lattice spacings: 0.11 fm and 0.083 fm
- Five ensembles in total with typical $N_{Conf} \sim O(10^2)$ and $V_{\rm Max} = 32^3 \times 64$



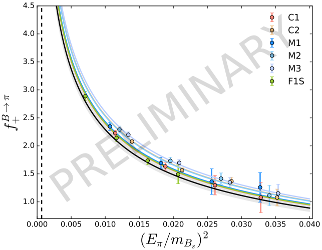



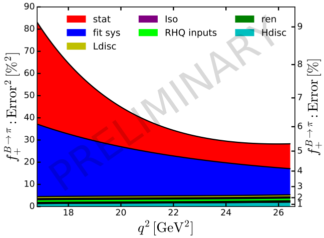

- Published results in 2015 using BCL, no experimental input for $B_s \to K\ell\nu$

Phys.Rev.D91, 074510 (2015)

# $|V_{ub}|$ and $B \to \pi \ell \nu$: RBC/UKQCD

- Sea sector: $N_F = 2+1$ Domain-Wall fermions with lightest $m_\pi \approx 267$ MeV
- Valence sector: Domain-Wall fermions for $l, s$, relativistic HQ effective action for the $b$ quark
- Lattice spacings: 0.11 fm, 0.083 fm and 0.071 fm
- Six ensembles in total with typical $N_{Conf} \sim O(10^2)$ and $V_{\mathrm{Max}} = 48^3 \times 96$




- Currently work in progress, preliminary results

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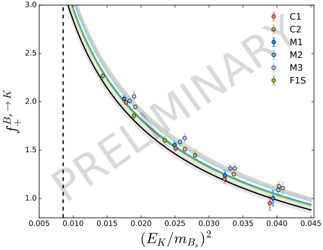
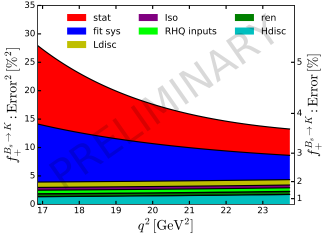
# $|V_{ub}|$ and $B_s \to K\ell\nu$: RBC/UKQCD

- Sea sector: $N_F = 2+1$ Domain-Wall fermions with lightest $m_\pi \approx 267$ MeV
- Valence sector: Domain-Wall fermions for $l, s$, relativistic HQ effective action for the $b$ quark
- Lattice spacings: 0.11 fm, 0.083 fm and 0.071 fm
- Six ensembles in total with typical $N_{Conf} \sim O(10^2)$ and $V_{\text{Max}} = 48^3 \times 96$




- Currently work in progress, at the $z$-expansion stage (BCL)

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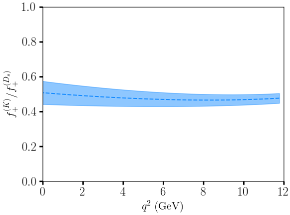
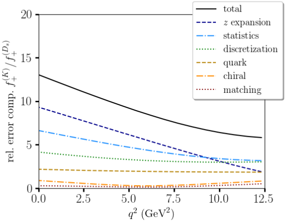
# $|V_{ub}/V_{cb}|$ from $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$: HPQCD

- Sea sector: $N_F = 2+1$ HISQ fermions with lightest $m_\pi \approx 260$ MeV
- Valence sector: HISQ fermions for $l, s$, NRQCD for the $b$ quark
- Lattice spacings: 0.12 fm and 0.083 fm
- Five ensembles in total with typical $N_{Conf} \sim O(10^3)$ and $V_{\text{Max}} = 28^3 \times 96$




- Published results in 2018 using BCL, no experimental input for $B_s \to K\ell\nu$
  Phys.Rev.D98, 114509 (2018)

# $|V_{cb}|$ and $B_{(s)} \to D^{(*)}_{(s)} \ell \nu$

- The differential decay rate for the $B_{(s)} \to P \ell \nu$ ($P = D_{(s)}$) semileptonic decay is (neglecting $m_\ell$)

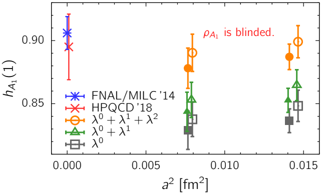
$$\frac{d\Gamma}{dw} = |\eta_{\mathrm{EW}}|^2 \frac{G_F^2 |V_{cb}|^2}{48\pi^2} M_B^5 (1+r)^2 (w^2-1)^{3/2} |\mathcal{G}(w)|^2$$

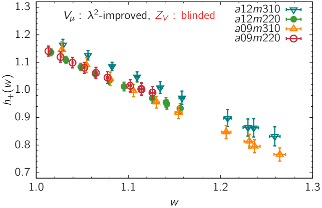
- The differential decay rate for the $B_{(s)} \to V \ell \nu$ ($P = D^*_{(s)}$) semileptonic decay is (neglecting $m_\ell$)

$$\frac{d\Gamma}{dw} = |\eta_{\mathrm{EW}}|^2 \frac{G_F^2 |V_{cb}|^2}{48\pi^2} M_B^5 (1-r)^2 \chi(w) (w^2-1)^{1/2} |\mathcal{F}(w)|^2$$

- The $(w^2-1)^{1/2,3/2}$ factor kills the phase space at small recoil $w = p_B p_D$ (large $q^2 = (p_B - p_P)^2$)
    - The decays to vector particles are much more efficient in experiments

- Sea sector: $N_F = 2+1+1$ HISQ fermions with lightest $m_\pi \approx 220$ MeV
- Valence sector: HISQ fermions for $l, s$, OK for $c, b$
- Lattice spacings: 0.12 fm, 0.09 fm and 0.06 fm
- Six ensembles in total with typical $N_{Conf} \sim O(10^3)$ and $V_{\rm Max} = 64^3 \times 144$




- Currently work in progress
- There are plans to add another 0.06 fm ensemble with $V = 96^3 \times 192$, and to include corrections $O(\lambda^3)$ to the currents

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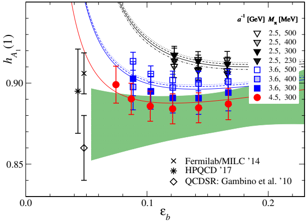
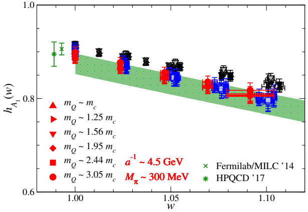
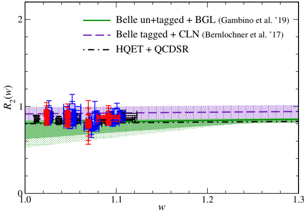
# $|V_{cb}|$ and $B \to D^{(*)}\ell\nu$: JLQCD

- Sea sector: $N_F = 2+1$ Domain-Wall fermions with lightest $m_\pi \approx 230$ MeV
- Valence sector: Domain-Wall fermions for all the quarks, unphysical $m_b$
- Lattice spacings: 0.08 fm, 0.055 fm and 0.044 fm
- Eight ensembles in total with typical $N_{Conf} \sim O(10^2)$ and $V_{\text{Max}} = 64^3 \times 128$




- Currently work in progress

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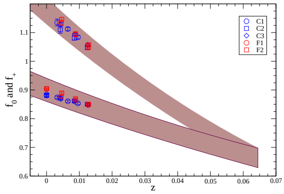
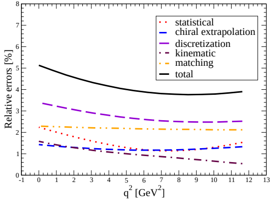
# $|V_{cb}|$ and $B \to D\ell\nu$: HPQCD

- Sea sector: $N_F = 2+1$ asqtad fermions with lightest $m_\pi \approx 274$ MeV
- Valence sector: HISQ for $l, s, c$, NRQCD for $b$
- Lattice spacings: 0.12 fm and 0.09 fm
- Five ensembles in total with typical $N_{Conf} \sim O(10^3)$ and $V_{\rm Max} = 28^3 \times 96$

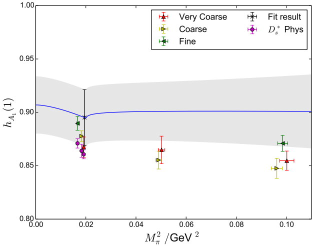



- Published results in 2015, $|V_{cb}| = 40.2(2.1) \times 10^{-3}$ using BCL and BaBar data Phys.Rev.D92, 054510 (2015) BaBar, Phys.Rev.Lett.104, 011802 (2010)

# $|V_{cb}|$ and $B_{(s)} \to D^*_{(s)} \ell \nu$: HPQCD

- Sea sector: $N_F = 2 + 1 + 1$ HISQ fermions with lightest $m_\pi \approx$ physical
- Valence sector: HISQ for $l, s, c$, NRQCD for the $b$ quark
- Lattice spacings: 0.09 fm, 0.06 fm and 0.044 fm
- Eight ensembles in total with typical $N_{Conf} \sim O(10^3)$ and $V_{\text{Max}} = 64^3 \times 96$



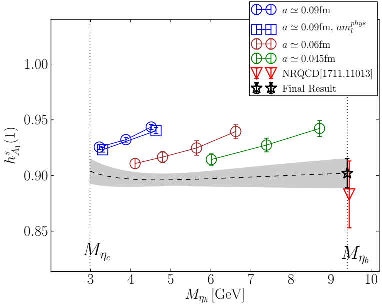

| Uncertainty | $h_{A_1}(1)$ | $h^s_{A_1}(1)$ | $h_{A_1}(1)/h^s_{A_1}(1)$ |
|---|---|---|---|
| $\alpha_s^2$ | 2.1 | 2.5 | 0.4 |
| $\alpha_s \Lambda_{\text{QCD}}/m_b$ | 0.9 | 0.9 | 0.0 |
| $(\Lambda_{\text{QCD}}/m_b)^2$ | 0.8 | 0.8 | 0.0 |
| $a^2$ | 0.7 | 1.4 | 1.4 |
| $g_{D^* D\pi}$ | 0.2 | 0.03 | 0.2 |
| Total systematic | 2.7 | 3.2 | 1.7 |
| Data | 1.1 | 1.4 | 1.4 |
| Total | 2.9 | 3.5 | 2.2 |

- Published results in 2017, $|V_{cb}| = 41.3(2.2) \times 10^{-3}$ using BGL and Belle data (unfolded)

Phys.Rev.D97, 054502 (2017) Belle, arXiv:1702.01521

# $|V_{cb}|$ and $B_s \to D_s^* \ell \nu$: HPQCD

- Sea sector: $N_F = 2+1+1$ HISQ fermions with lightest $m_\pi \approx$ physical
- Valence sector: HISQ for $l, s, c, b$ with unphysical $m_b$
- Lattice spacings: 0.09 fm, 0.06 fm and 0.044 fm
- Four ensembles in total with typical $N_{Conf} \sim O(10^2)$ and $V_{\mathrm{Max}} = 64^3 \times 192$



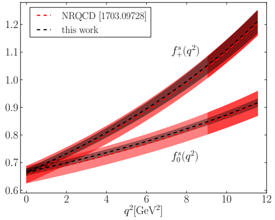

| Source | % Fractional Error |
|---|---|
| Statistics $+Z_A$ | 1.06 |
| $a \to 0$ | 0.73 |
| $m_h \to m_b$ | 0.69 |
| mass mistuning | 0.20 |
| Total | 1.45 |

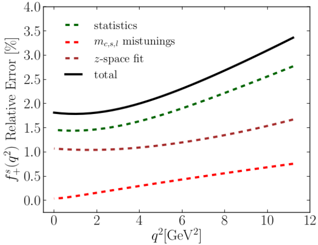
- Published results in 2019, improved 2017 results by using a HISQ $b$ quark and larger volumes

Phys.Rev.D99, 114512 (2019)

# $|V_{cb}|$ and $B_s \to D_s \ell \nu$: HPQCD

- Sea sector: $N_F = 2+1+1$ HISQ fermions with lightest $m_\pi \approx$ physical
- Valence sector: HISQ for $l, s, c, b$ with unphysical $m_b$
- Lattice spacings: 0.09 fm, 0.06 fm and 0.044 fm
- Four ensembles in total with typical $N_{Conf} \sim O(10^2)$ and $V_{\text{Max}} = 64^3 \times 192$



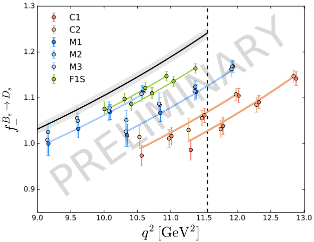



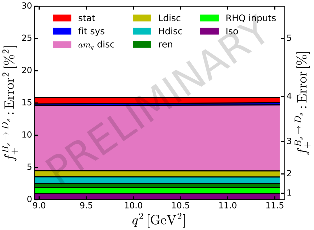

- Published results in 2019

Phys.Rev.D101, 074513 (2019)

# $|V_{cb}|$ and $B_s \to D_s \ell\nu$: RBC/UKQCD

- Sea sector: $N_F = 2+1$ Domain-Wall fermions with lightest $m_\pi \approx 267$ MeV
- Valence sector: Domain-Wall fermions for $l, s, c$, relativistic HQ effective action for the $b$ quark
- Lattice spacings: 0.11 fm, 0.083 fm and 0.071 fm
- Six ensembles in total with typical $N_{Conf} \sim O(10^2)$ and $V_{\rm Max} = 48^3 \times 96$




- Currently work in progress, at the $z$-expansion stage (BGL)

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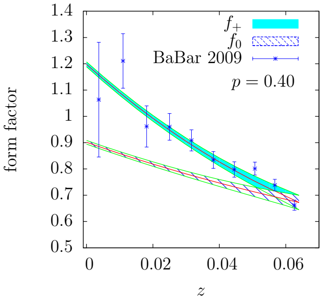
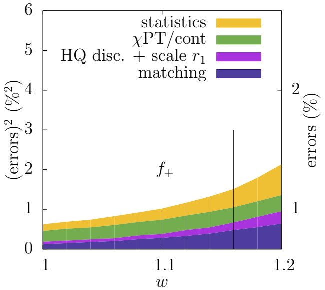
# $|V_{cb}|$ and $B \to D\ell\nu$: FNAL/MILC

- Sea sector: $N_F = 2+1$ asqtad fermions with lightest $m_\pi \approx 180$ MeV
- Valence sector: Asqtad for $l, s$, Fermilab heavy quarks for $c, b$, mostly nonperturbative renormalization
- Lattice spacings: 0.045 fm, 0.06 fm 0.09 fm and 0.12 fm
- Fourteen ensembles in total with typical $N_{Conf} \sim O(10^3)$ and $V_{\text{Max}} = 64^3 \times 192$



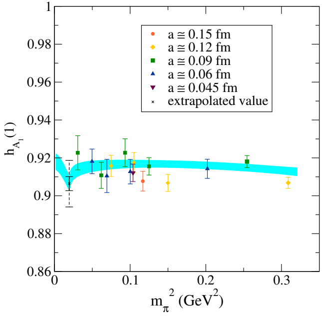

- Published results in 2015, $|V_{cb}| = 39.6(1.7) \times 10^{-3}$ using BGL and BaBar data

Phys.Rev.D92, 034506 (2015)

BaBar, Phys. Rev. Lett.104, 011802 (2010)

# $|V_{cb}|$ and $B \to D^*\ell\nu$: FNAL/MILC

- Sea sector: $N_F = 2+1$ asqtad fermions with lightest $m_\pi \approx 180$ MeV
- Valence sector: Asqtad for $l, s$, Fermilab heavy quarks for $c, b$, mostly nonperturbative renormalization
- Lattice spacings: 0.045 fm, 0.06 fm 0.09 fm, 0.12 fm and 0.15 fm
- Fourteen ensembles in total with typical $N_{Conf} \sim O(10^3)$ and $V_{\text{Max}} = 64^3 \times 192$



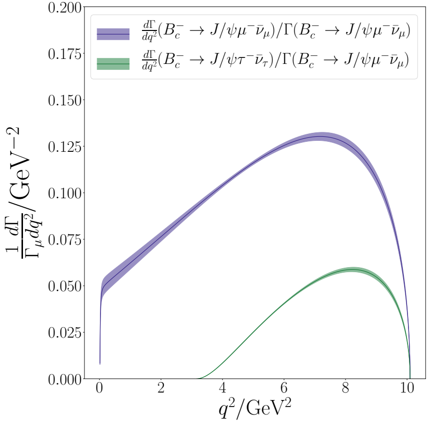

| Uncertainty | $h_{A_1}(1)$ |
|---|---|
| Statistics | 0.4% |
| Scale $(r_1)$ error | 0.1% |
| $\chi$PT fits | 0.5% |
| $g_{D^*D\pi}$ | 0.3% |
| Discretization errors | 1.0% |
| Perturbation theory | 0.4% |
| Isospin | 0.1% |
| Total | 1.4% |

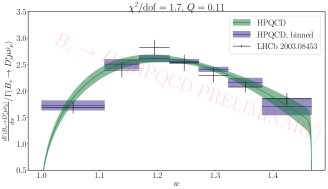
- Published results in 2014, $|V_{cb}| = 39.04(75)$ using BGL and Belle, BaBar and CLEO data

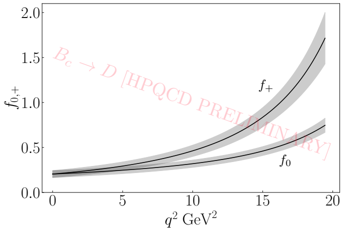
Phys.Rev.D89, 114504 (2014) HFAG, arXiv:1207.1158

# New channels from HPQCD

- More interesting projects from HPQCD
  - $B_c \to J/\psi \ell \nu$ and $B_s \to D_s^* \ell \nu$, see J. Harrison's talk
  - $B_c \to D \ell \nu$ see L. Cooper talk

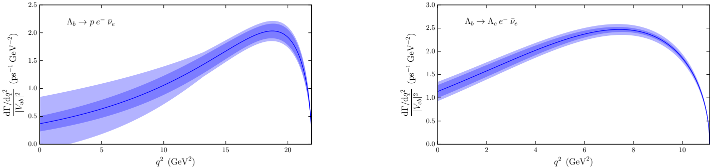


# $|V_{ub}/V_{cb}|$ and $\Lambda_b \to p, \Lambda_c \ell \nu$

- Sea sector: $N_F = 2+1$ Domain-Wall fermions with lightest $m_\pi \approx 230$ MeV
- Valence sector: Domain-Wall fermions for $l, s$, relativistic HQ effective action for $c, b$
- Lattice spacings: 0.11 fm and 0.083 fm
- Six ensembles in total with typical $N_{Conf} \sim O(10^2)$ and $V_{\rm Max} = 32^3 \times 64$



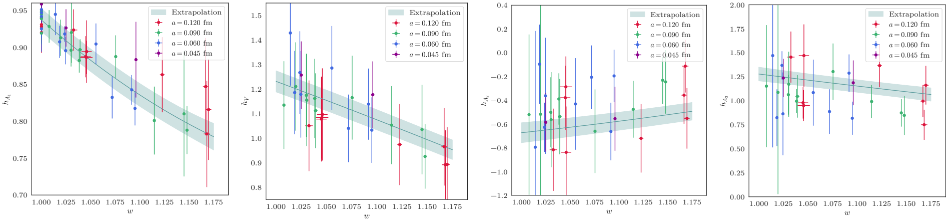

- Published results in 2015, $|V_{ub}/V_{cb}| = 0.083(6)$ using BCL and LHCb data W. Detmold *et al*, Phys.Rev.D92, 034503 (2015) LHCb, Nature Physics 10 (2015) 1038

# $|V_{ub}/V_{cb}|$ and $\Lambda_b \to p, \Lambda_{(c)}\ell\nu$

- Specific plans to improve the calculation and reduce systematic errors
  - Include new ensembles with finer lattice spacing (down to 0.073 fm), larger volumnes (up to $48^3 \times 96$) and ligther pion masses (physical)
  - Better tuning of the HQ actions
  - Use AMA to increase statistics
  - Use a physical $m_s$ for $\Lambda_b \to \Lambda$
  - Non-perturbative renormalization

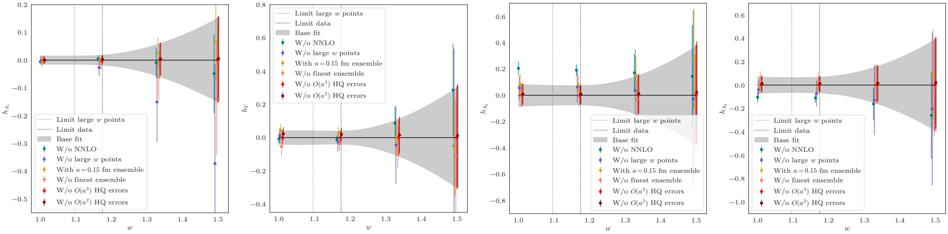
# $|V_{cb}|$ and $B \to D^* \ell \nu$: FNAL/MILC

- Sea sector: $N_F = 2 + 1$ asqtad fermions with lightest $m_\pi \approx 180$ MeV
- Valence sector: Asqtad for $l, s$, Fermilab heavy quarks for $c, b$, mostly nonperturbative renormalization
- Lattice spacings: 0.045 fm, 0.06 fm 0.09 fm, 0.12 fm and 0.15 fm
- Fifteen ensembles in total with typical $N_{Conf} \sim O(10^3)$ and $V_{\text{Max}} = 64^3 \times 192$



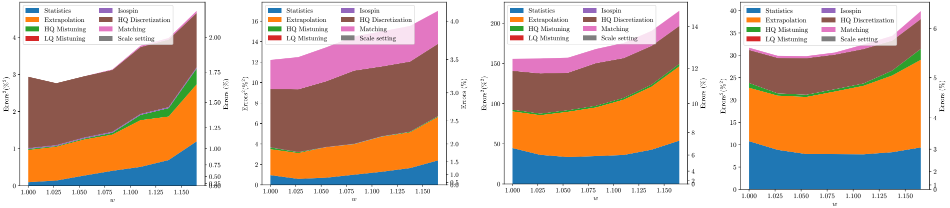
# $|V_{cb}|$ and $B \to D^*\ell\nu$: FNAL/MILC

- Chiral-continuum extrapolation quite stable under modifications




- Largest deviations: removing NNLO terms

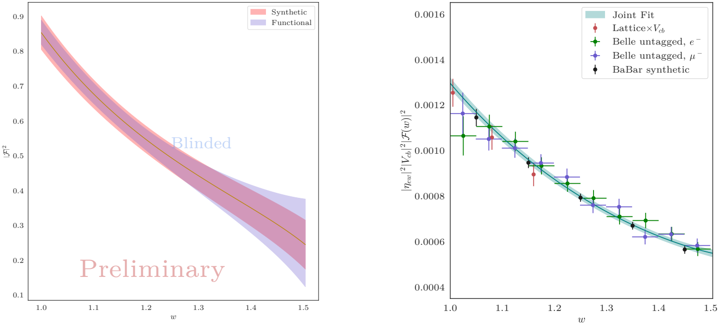
- Main contributions to the errors are HQ discretization and the chiral-continuum extrapolation, with a sizable contribution from matching




- From left to right: $h_{A_1}$, $h_V$, $h_{A_2}$ and $h_{A_3}$

# $|V_{cb}|$ and $B \to D^*\ell\nu$: FNAL/MILC

- We use BGL for the $z$-expansion




- Blinded results (close to unblinding)
- Expecting $\approx 2\%$ error in $|V_{cb}|$
- There is a roadmap to improve the $B \to D^{(*)}\ell\nu$ calculations

# Summary

- Lattice QCD is a key ingredient in high-precision, exclusive determinations of $|V_{xb}|$ (see Th. Mannel comments on Monday)
  - Offers fully first-principles, non-perturbative calculations of the form factors
- The lattice community is deeply involved in exclusive determinations of CKM matrix elements
  - Large variety of channels
  - Several groups target the same decay with different approaches
  - Good communication with experimentalist
- Lattice groups have established roadmaps to improve their calculations and reduce theoretical errors

- The solution of the $|V_{xb}|$ puzzles will necessarily involve the lattice