Inclusive semileptonic B decays and lattice QCD

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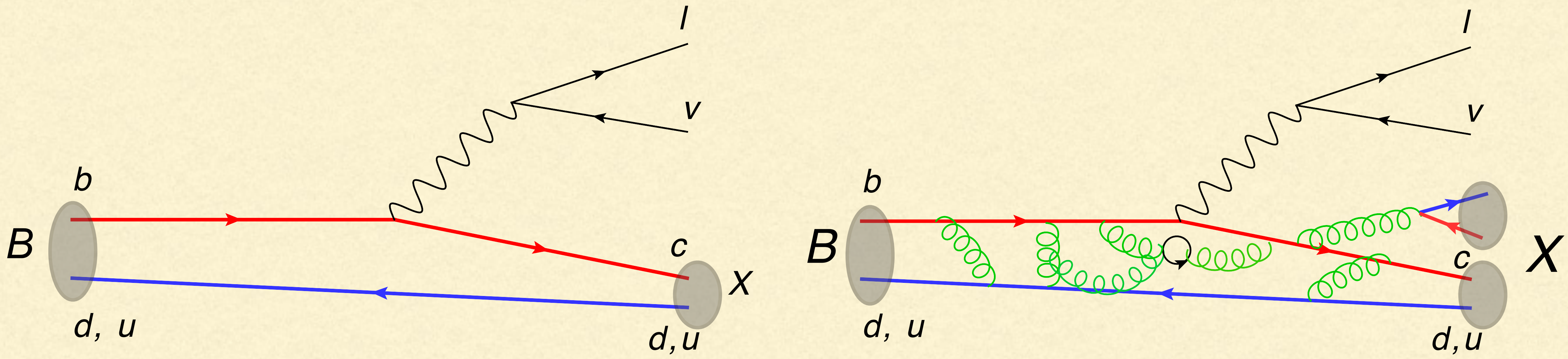
Theory meets experiment on $|V_{ub}|$ and $|V_{cb}|$, 11-12 January 2021

INCLUSIVE DECAYS: BASICS



- ***Simple idea:*** inclusive decays do not depend on final state, long distance dynamics of the B meson factorizes. An OPE allows us to express it in terms of B meson matrix elements of local operators, embodying *quark-hadron duality*

- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: ***double series in α_s, Λ/m_b***

- Lowest order: decay of a free *b*, linear Λ/m_b absent. Depends on $m_{b,c}$, 2 parameters at $O(1/m_b^2)$, 2 more at $O(1/m_b^3)$...

INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in Λ/m_b and α_s

$$M_i = M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)}\right)\frac{\mu_\pi^2}{m_b^2}$$
$$+ \left(M_i^{(G,0)} + \frac{\alpha_s}{\pi} M_i^{(G,1)}\right)\frac{\mu_G^2}{m_b^2} + M_i^{(D,0)}\frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)}\frac{\rho_{LS}^3}{m_b^3} + \ldots$$

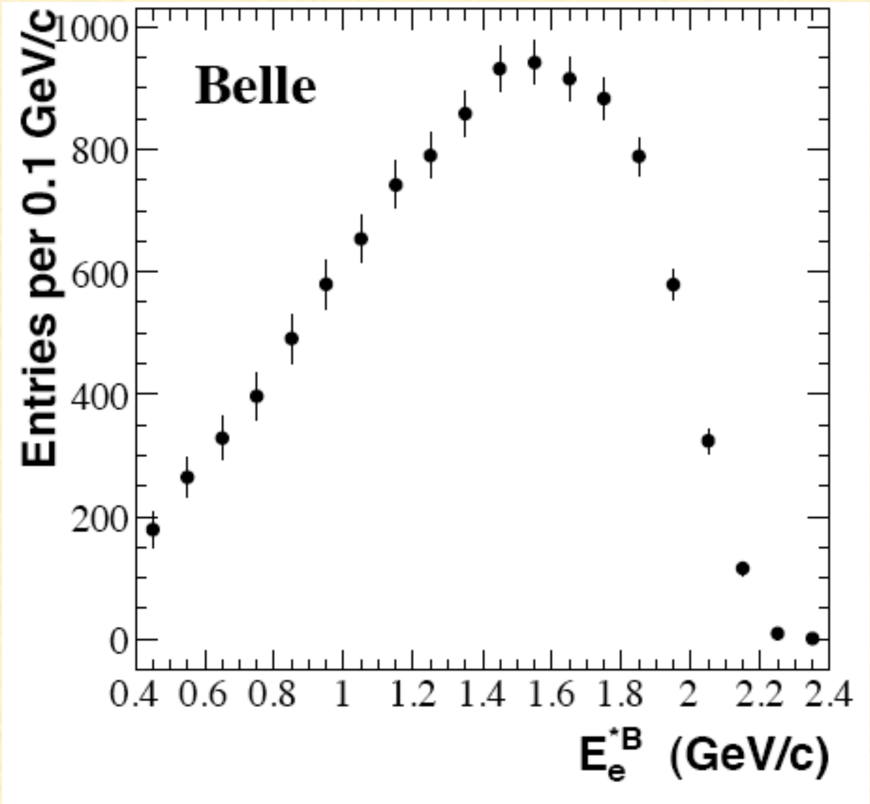
$$\mu_\pi^2(\mu) = \frac{1}{2M_B}\langle B|\bar{b}_v(i\vec{D})^2 b_v|B\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2M_B}\langle B|\bar{b}_v \frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b_v|B\rangle_\mu$$

Reliability of the method depends on our control of higher order effects. Quark-hadron duality violation would manifest as inconsistency in the fit.

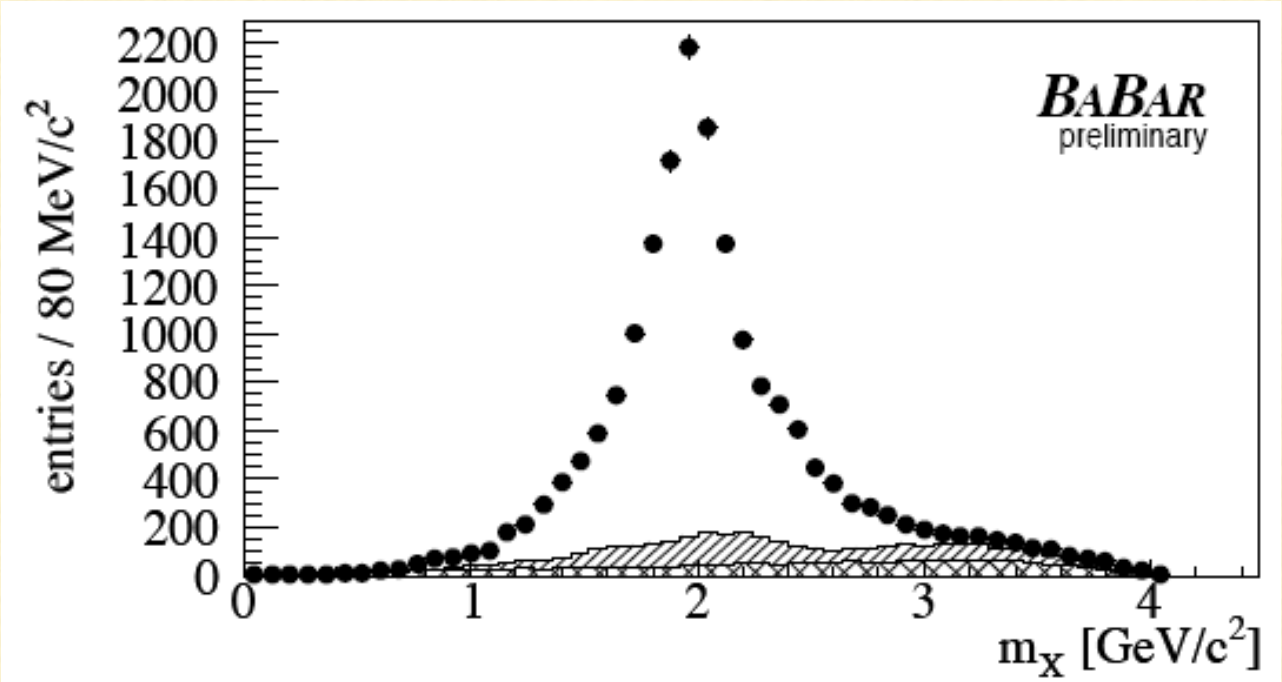
Current HFLAV **kinetic scheme** fit includes all corrections $O(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3)$, m_c constraint from sum rules/lattice

EXTRACTION OF THE OPE PARAMETERS

E_l spectrum



hadronic mass spectrum



Global shape parameters (first moments of the distributions, various lower cut on E_l) tell us about m_b, m_c and the B structure, total rate about $|V_{cb}|$

OPE parameters describe universal properties of the B meson and of the quarks → useful in many applications (rare decays, V_{ub},...)

FIT RESULTS

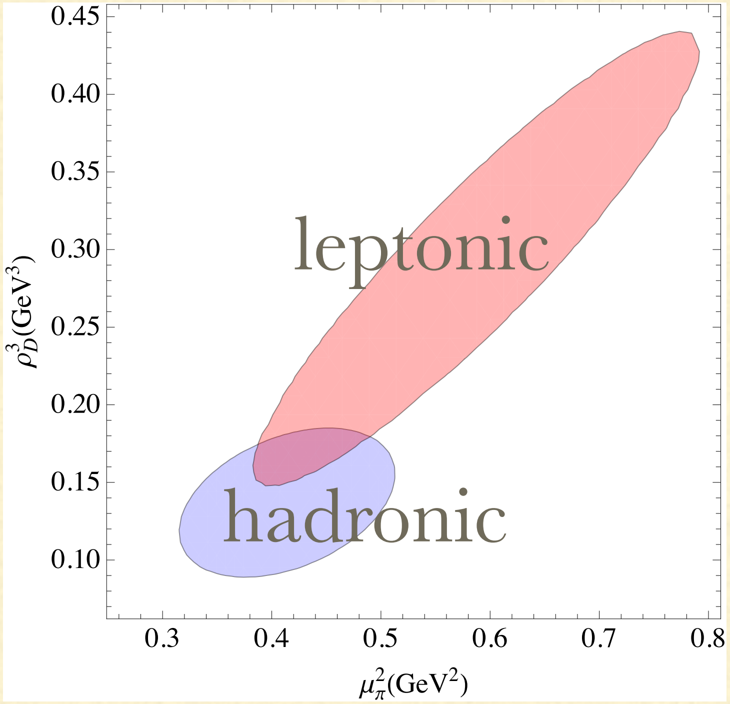
m_b^{kin}	$\overline{m}_c(3\,\mathrm{GeV})$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.553	0.987	0.465	0.170	0.332	-0.150	10.65	42.21
0.020	0.013	0.068	0.038	0.062	0.096	0.16	0.78

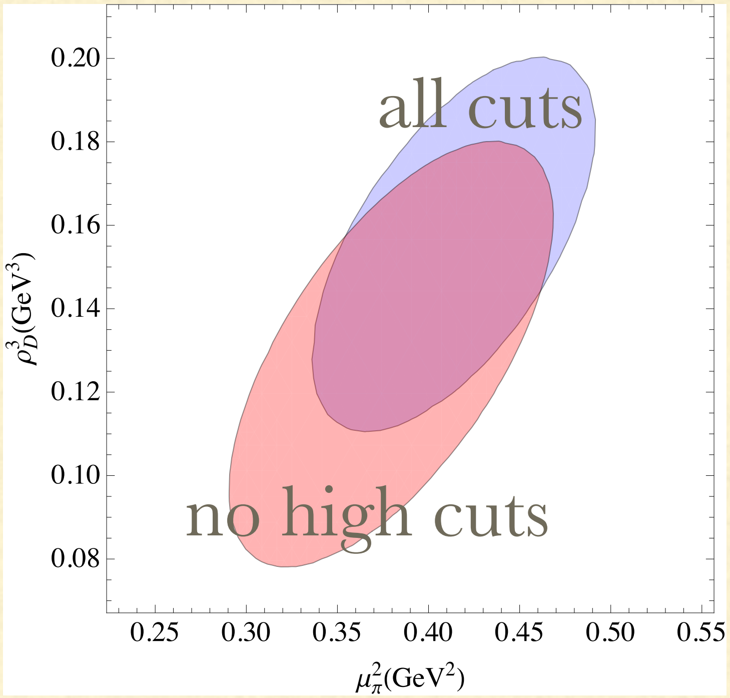
this is HFLAV fit

Alberti, Healey, Nandi, PG, 1411.6560

Without mass constraints $m_b^{kin}(1\mathrm{GeV}) - 0.85\,\overline{m}_c(3\mathrm{GeV}) = 3.714 \pm 0.018\,\mathrm{GeV}$

- results depend little on assumption for correlations and choice of inputs, 1.8% determination of V_{cb}
- 20-30% determination of the OPE parameters
- b mass determination in agreement with recent lattice and sum rules results





HIGHER POWER CORRECTIONS

Proliferation of non-pert parameters starting $1/m^4$: 9 at dim 7, 18 at dim 8

In principle relevant: HQE contains $O(1/m_b^n 1/m_c^k)$

Mannel,Turczyk,Uraltsev 1009.4622

Lowest Lying State Saturation Approx (LLSA) truncating

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

see also Heinonen,Mannel 1407.4384

and relating higher dimensional to lower dimensional matrix elements, e.g.

$$\rho_D^3 = \epsilon\,\mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon\,\mu_G^2 \qquad \epsilon \sim 0.4\mathrm{GeV}$$

excitation energy to P-wave states. LLSA might set the scale of effect, but large corrections to LLSA have been found in some cases 1206.2296

We use LLSA as loose constraint or priors (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in a fit including higher powers. The rest of the fit is unchanged, with slightly smaller theoretical errors

$$\boxed{|V_{cb}| = 42.00(64) \times 10^{-3}}$$

Healy, Turzcyk, PG 1606.06174

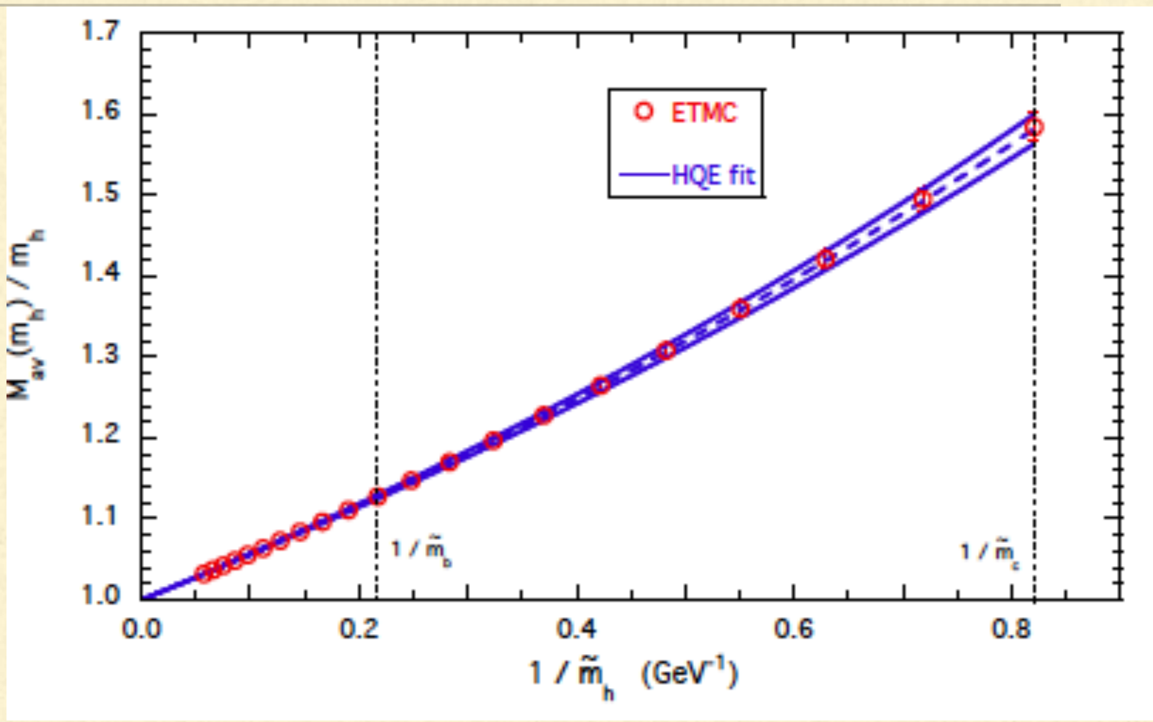
PROSPECTS for INCLUSIVE V_{cb}

- Theoretical uncertainties generally larger than experimental ones
- $O(\alpha_s/m_b^3)$ calculation completed for width (Mannel, Pivovarov) in progress for the moments (S. Nandi, PG)
- 3loop relation between MS and kin scheme just completed 2005.06487
  It can be used to improve the precision of the m_b input
- $O(\alpha_s^3)$ corrections to total width just completed by Fael, Schoenwald, Steinhauser 2011.13654: towards 1% uncertainty
- Electroweak (QED) corrections require attention
- New observables in view of Belle-II: FB asymmetry proposed by S.Turczyk could be measured already by Babar and Belle now, q^2 moments (Fael, Mannel, Vos)…
- **Lattice QCD** is the next frontier

MESON MASSES FROM ETMC

Melis, Simula, PG 1704.06105

$$M_{H_Q} = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - a_H \mu_G^2}{2m_Q} + \ldots$$



- on the lattice one can compute mesons for arbitrary quark masses
  see also Kronfeld & Simone hep-ph/0006345, 1802.04248
- We used both pseudoscalar and vector mesons
- Direct 2+1+1 simulation, a=0.62-0.89 fm, m_π=210-450 MeV, heavy masses from m_c to $3m_c$, ETM ratio method with extrapolation to static point.
- Kinetic scheme with cutoff at 1GeV, good sensitivity up to $1/m^3$ corrections
- Results consistent with s.l. fits, improvements under way, also following new 3loop calculation of pole-kinetic mass relation

INCLUSIVE DECAYS ON THE LATTICE

- Inclusive processes nearly impossible to treat directly on the lattice
- However, vacuum current correlators can be computed in euclidean space-time and related to $e^+e^- \to$ hadrons or τ decay via analyticity
- In our case the correlators have to be computed in the B meson
  Hashimoto 1703.01881
- Analytic continuation more complicated: two cuts, decay occurs only on a portion of the physical cut.
- While the calculation of the spectral density of hadronic correlators is an ill-posed problem, it is accessible after smearing, as provided by phase-space integration Hansen, Meyer, Robaina, Lupo, Tantalo, Bailas, Hashimoto, Ishikawa

A NEW APPROACH

Hashimoto, PG 2005.13730

$$\frac{d\Gamma}{dq^2 dq^0 dE_\ell} = \frac{G_F^2 |V_{cb}|^2}{8\pi^3} L_{\mu\nu} W^{\mu\nu}$$ triple diff distribution B_s decays

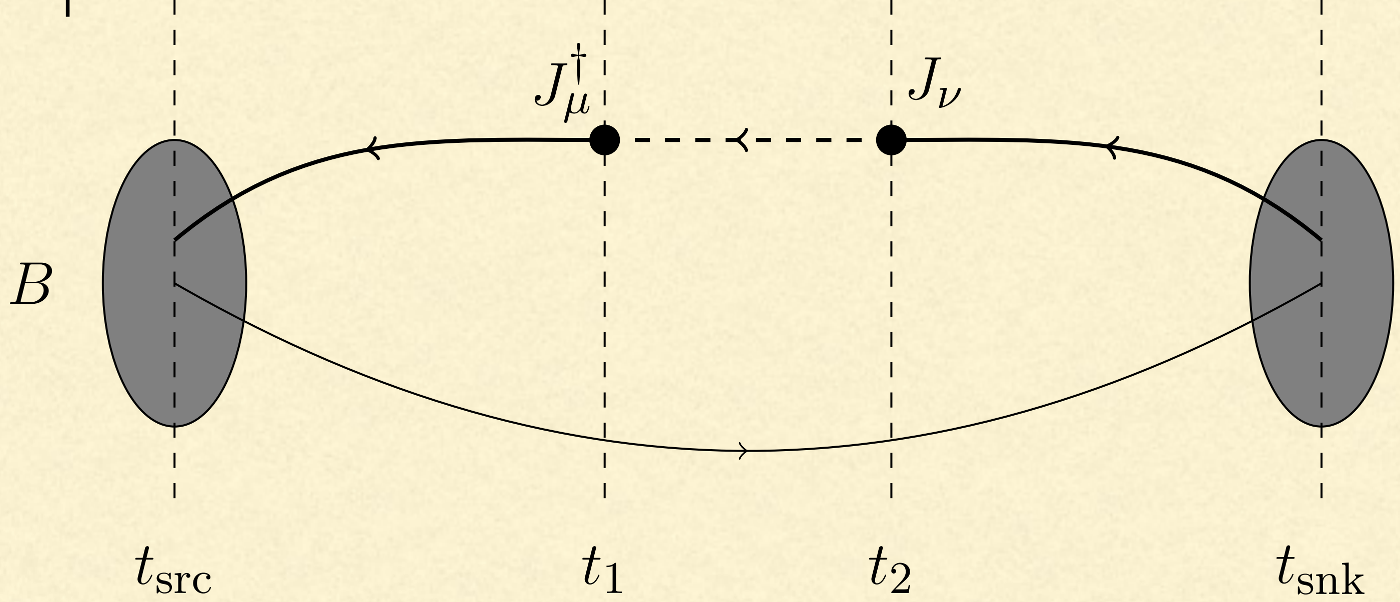
$$W^{\mu\nu} \sim \sum_{X_c} \frac{1}{2E_{B_s}} \langle \bar{B}_s(\boldsymbol{p}) | J^{\mu\dagger} | X_c(\boldsymbol{r})\rangle \langle X_c(\boldsymbol{r}) | J^\nu | \bar{B}_s(\boldsymbol{p})\rangle \sim Im\, i \int d^4x e^{-iq.x} \langle B_s | T J^{\mu\dagger}(x) J^\nu(0) | B_s \rangle$$

after integration over E_l

$$\Gamma = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \int_0^{\boldsymbol{q}^2_{\max}} d\boldsymbol{q}^2 \sqrt{\boldsymbol{q}^2} \sum_{l=0}^{2} \bar{X}^{(l)} \qquad \bar{X}^{(l)} \equiv \int_{\sqrt{m_{D_s}^2 + \boldsymbol{q}^2}}^{m_{B_s} - \sqrt{\boldsymbol{q}^2}} d\omega\, X^{(l)} = \int K(\omega, \mathbf{q})_{\mu\nu} \mathbf{W}^{\mu\nu} \mathbf{d}\omega$$

where ω hadr. energy, $X^{(l)}$ linear combinations of $W^{\mu\nu}$.

4point functions on the lattice are related to the hadronic tensor in euclidean



$$\sim \langle B_s | J^{\mu\dagger}(\mathbf{x}, t) J^\nu(\mathbf{0}, 0) | B_s \rangle$$

A NEW APPROACH

Hashimoto, PG 2005.13730

$$\sum_{\boldsymbol{x}} e^{i\boldsymbol{q}\cdot\boldsymbol{x}} \frac{1}{2m_{B_s}} \langle B_s(\mathbf{0})|J_\mu^\dagger(\boldsymbol{x},t)J_\nu(\mathbf{0},0)|B_s(\mathbf{0})\rangle \sim \langle B_s(\mathbf{0})|\tilde{J}_\mu^\dagger(-\boldsymbol{q})e^{-\hat{H}t}\tilde{J}_\nu(\boldsymbol{q})|B(\mathbf{0})\rangle$$

$\tilde{J}$ FT of J

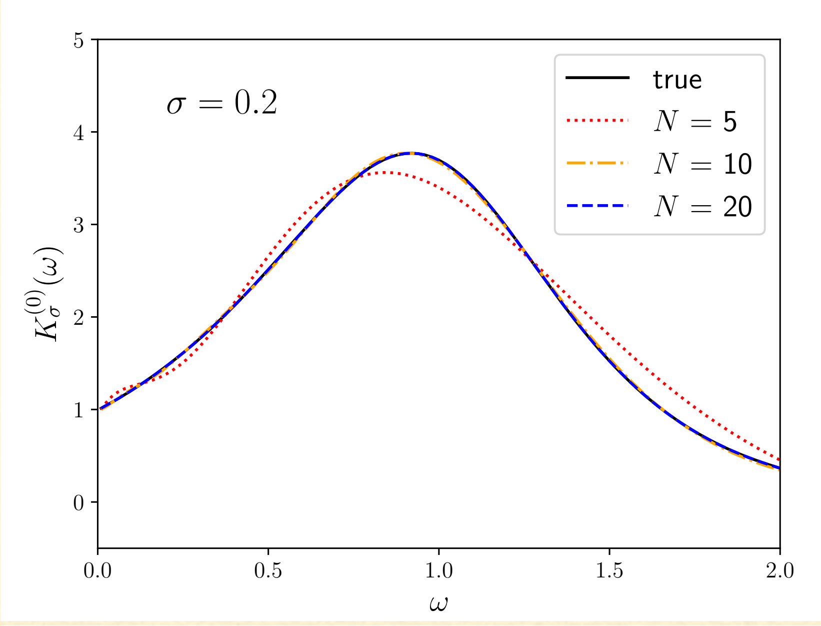
integral over ω becomes

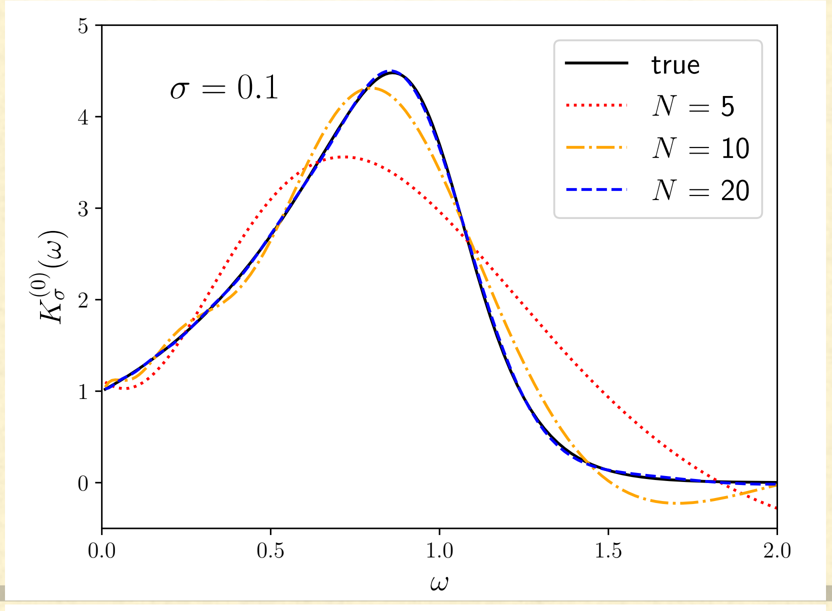
$$\int_0^\infty d\omega\, K(\omega,\boldsymbol{q})\langle B_s(\mathbf{0})|\tilde{J}_\mu^\dagger(-\boldsymbol{q})\delta(\hat{H}-\omega)\tilde{J}_\nu(\boldsymbol{q})|B_s(\mathbf{0})\rangle$$
$$= \langle B_s(\mathbf{0})|\tilde{J}_\mu^\dagger(-\boldsymbol{q})K(\hat{H},\boldsymbol{q})\tilde{J}_\nu(\boldsymbol{q})|B_s(\mathbf{0})\rangle$$

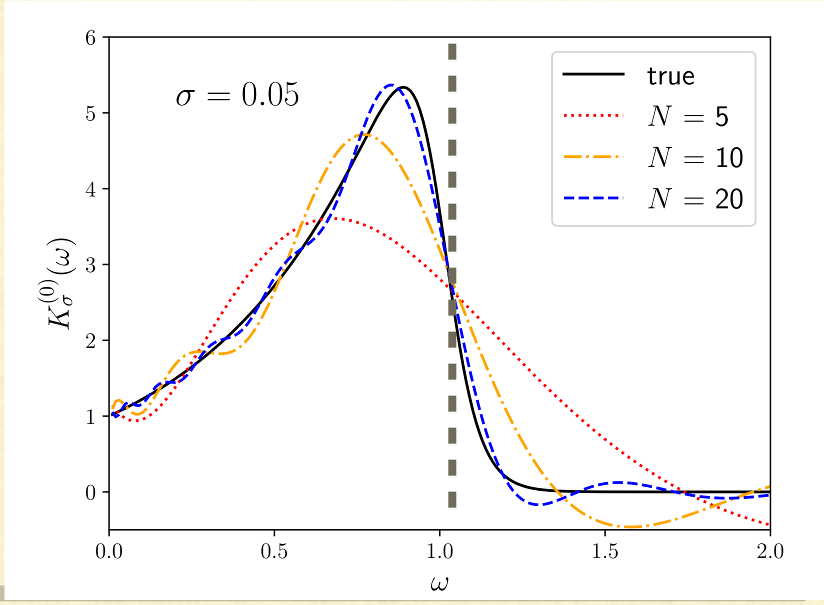
K approximated by polynomials $K(\hat{H},\boldsymbol{q}) = k_0(\boldsymbol{q}) + k_1(\boldsymbol{q})e^{-\hat{H}} + \cdots + k_N(\boldsymbol{q})e^{-N\hat{H}}$

K has a sharp hedge: sigmoid $1/(1+e^{x/\sigma})$ used to replace kinematic $\theta(x)$ for $\sigma \to 0$

Larger number N of Chebyshev polynomials needed for small σ

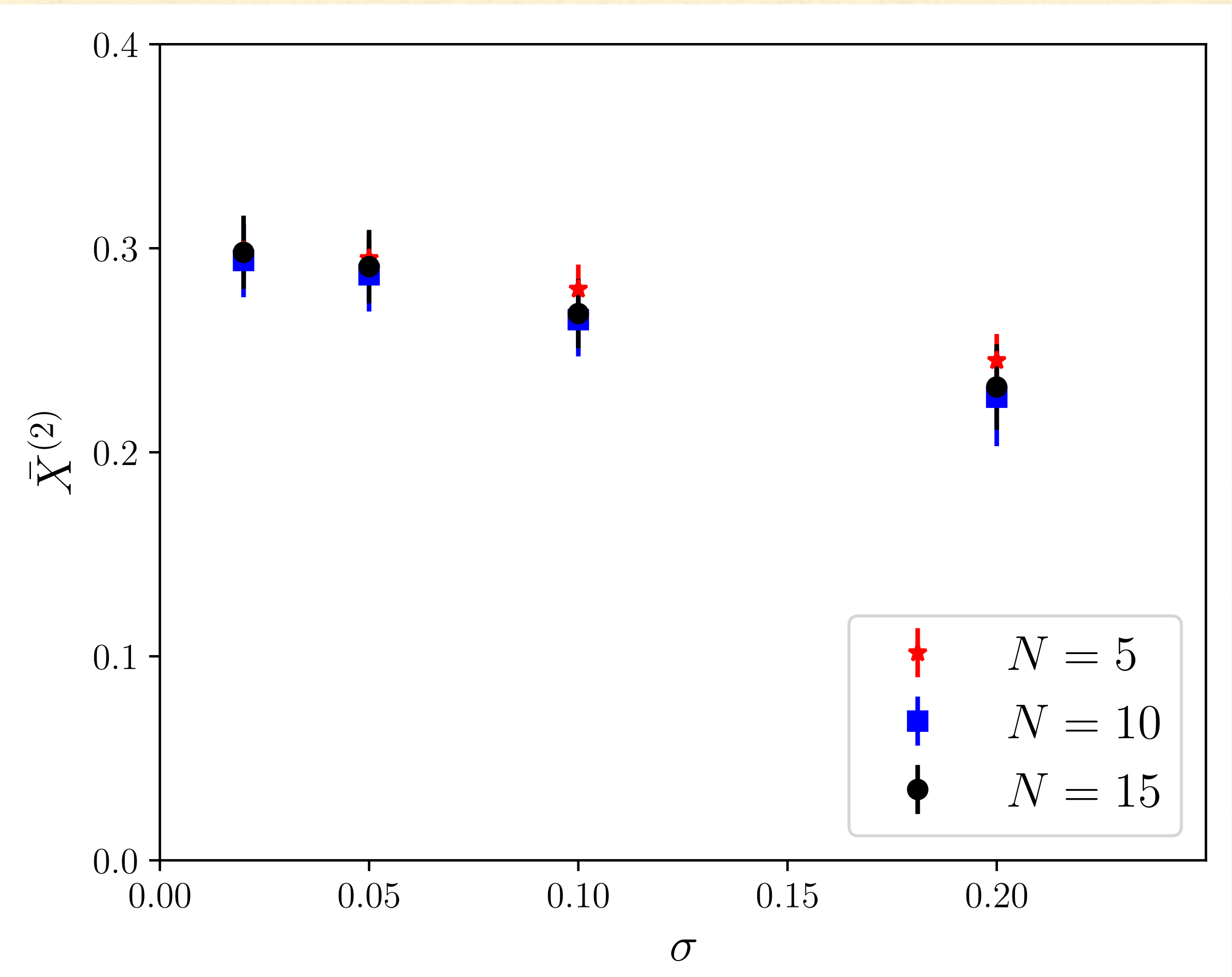






A PILOT NUMERICAL STUDY

Hashimoto, PG 2005.13730



Smeared spectral functions can be computed on the lattice in JLQCD setup, see 1704.08993

2+1 flavours of Moebius domain wall fermions with $1/a$=3.610(9)GeV on 48^3x96
M_{Bs}=3.45 GeV, i.e. $m_b^{kin}(1GeV)$≈2.70GeV
physical charm mass $m_c^{MS}(3GeV) = 1.00$GeV

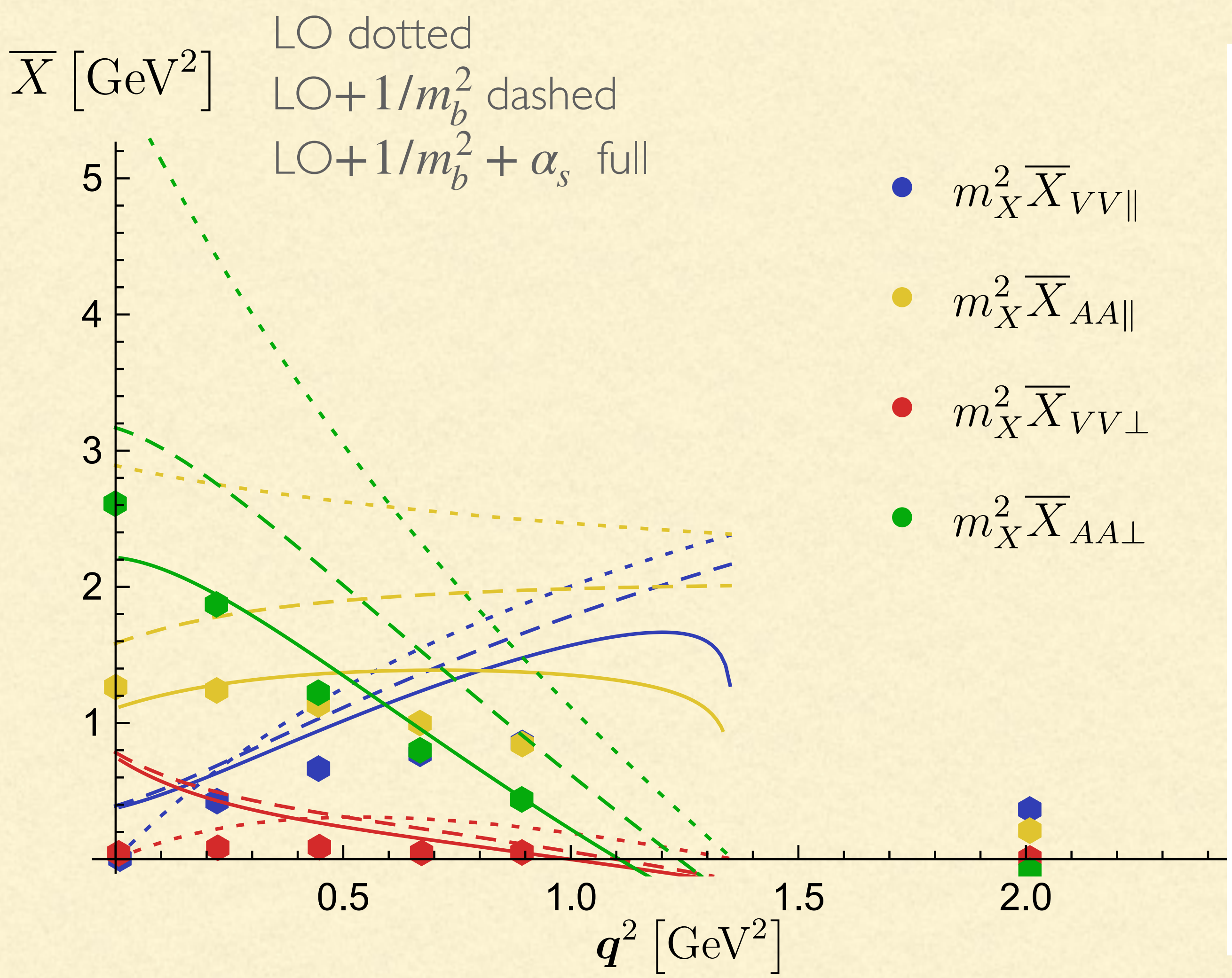
m_b-m_c~1.7GeV only, $\mathbf{q}^{max} \sim 1.16$GeV

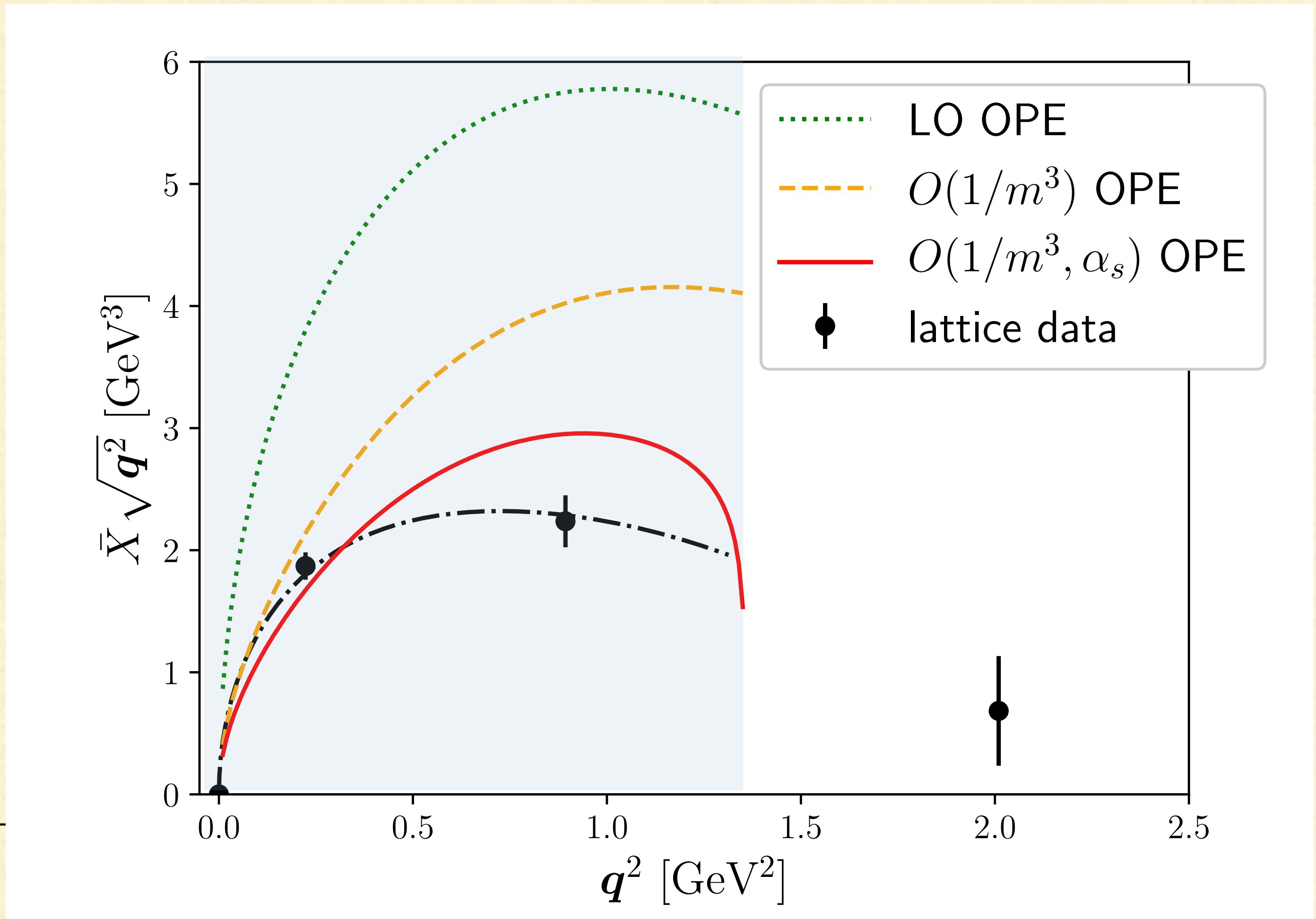
NB $m_b^{lat} = 2.44 m_c^{lat}$: we don't know it precisely…

Extrapolation to $\sigma \to 0$ possible, but error due to finite *N* must be estimated

COMPARISON WITH OPE

OPE matrix elements from fits, sizeable power and pert corrections!



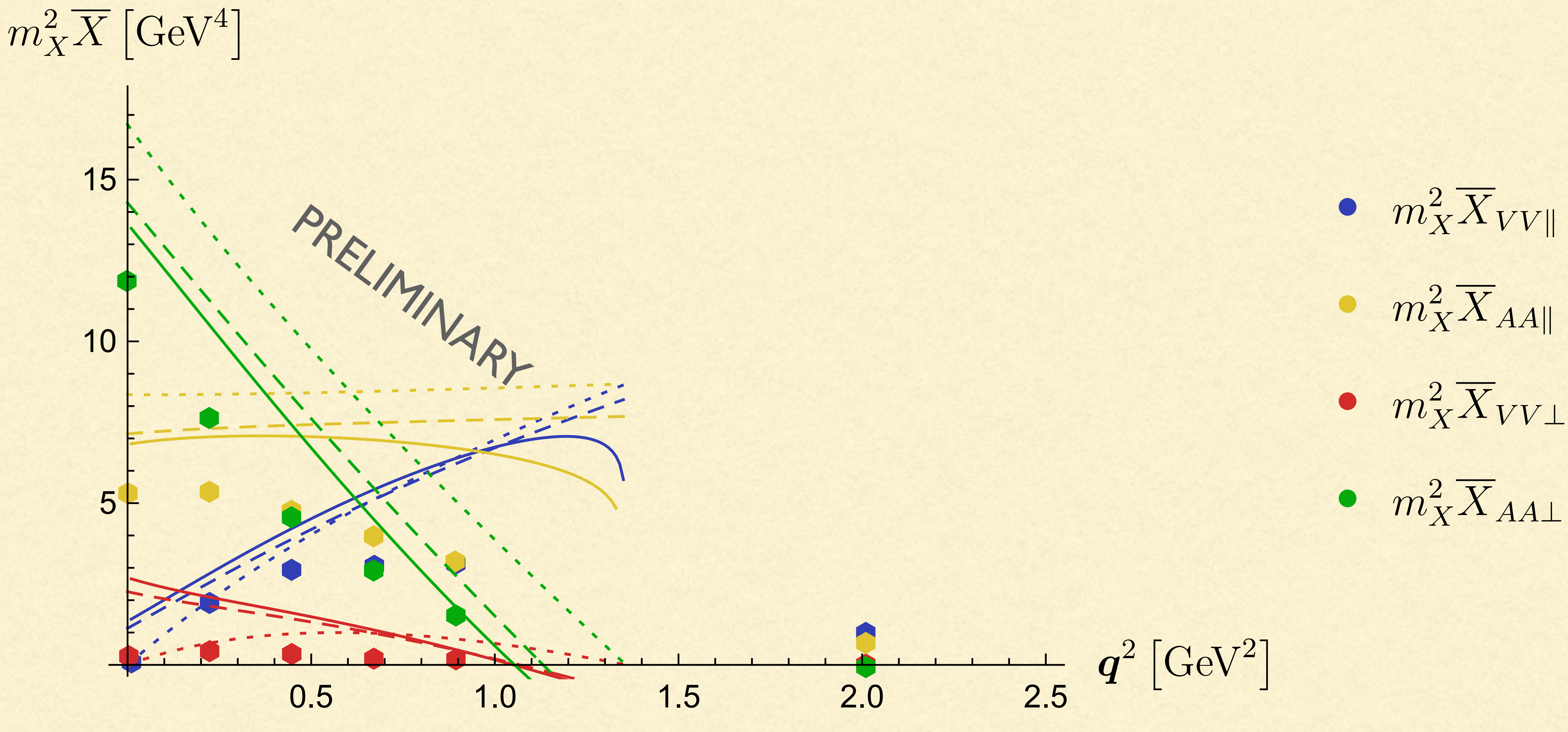


$\Gamma/|V_{cb}|^2 = 4.9(6)\ 10^{-13}\,\mathrm{GeV}$ Lattice

$\Gamma/|V_{cb}|^2 = 5.4(8)\ 10^{-13}\,\mathrm{GeV}$ OPE including $O(\alpha_s^2, 1/m_b^3)$

OPE uncertainty: "*b*" mass error (dominant), higher orders, matrix elements

HADRONIC MOMENTS



Hashimoto, Maechler, PG in progress

WHAT NEXT?

- Leptonic, hadronic energy moments, SV sum rules with existing data
- D inclusive semileptonic decays vs Cleo-c data for widths and lepton spectra (validation of the method, study of lattice systematics such as finite volume effects and disconnected diagrams, …)
- Towards the physical *b* mass (ratio method, step scaling, …): large recoil momentum $\mathbf{q}$ problematic
- Smooth cuts on experimental and OPE side?
- $B \to X_u \ell \nu, B \to X_s \ell^+ \ell^-$: kinematic cuts can *in principle* be implemented
- Extension of the method to low energy l-N inelastic scattering
  Hashimoto et al., 2010.01253 [hep-lat]

STARTING A COLLABORATION

- Shoji Hashimoto **KEK**
- Marco Panero, Sandro Maechler, Antonio Smecca, PG **Turin**
- Nazario Tantalo, Agostino Patella **Roma Tor Vergata**
- Silvano Simula, Francesco Sanfilippo **INFN Roma Tre**

CONCLUSIONS

- Inclusive s.l. B decays are in a good shape: consistent fit, new higher order calculations and future data from Belle II give hope for smaller uncertainties, but tension with $B \to D^* \ell \nu$ persists

- New lattice method allows for fully non-pert calculation of inclusive observables (widths, moments with arbitrary kinematic cuts) potentially validating OPE. Promising pilot computation at $m_b \sim 2.7\,GeV$ in good agreement with OPE.

- Lattice can also act as a *virtual lab,* computing obs we cannot access experimentally (or not precisely), which may enhance OPE predictivity, and observing the onset of duality