How can we improve $|V_{xb}|$ Determinations?

Snowmass 2021: Theory meets experiment on $|V_{ub}|$ and $|V_{cb}|$

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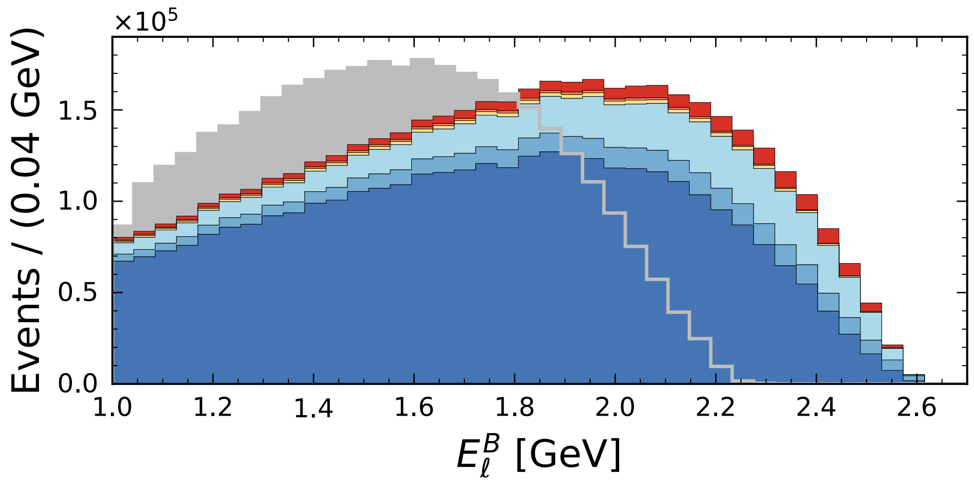
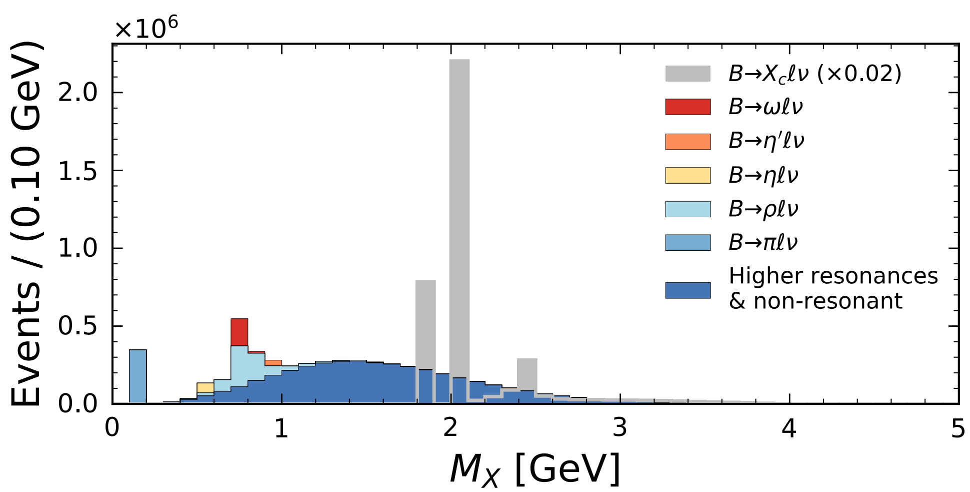
Caveats on inclusive $|V_{ub}|$

Phase-Space Coverage





$$\left|V_{ub}\right| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u \ell \bar{\nu}_\ell)}{\tau\,\Delta\Gamma(B \to X_u \ell \bar{\nu}_\ell)}}$$

Cut on E_ℓ^B, M_X

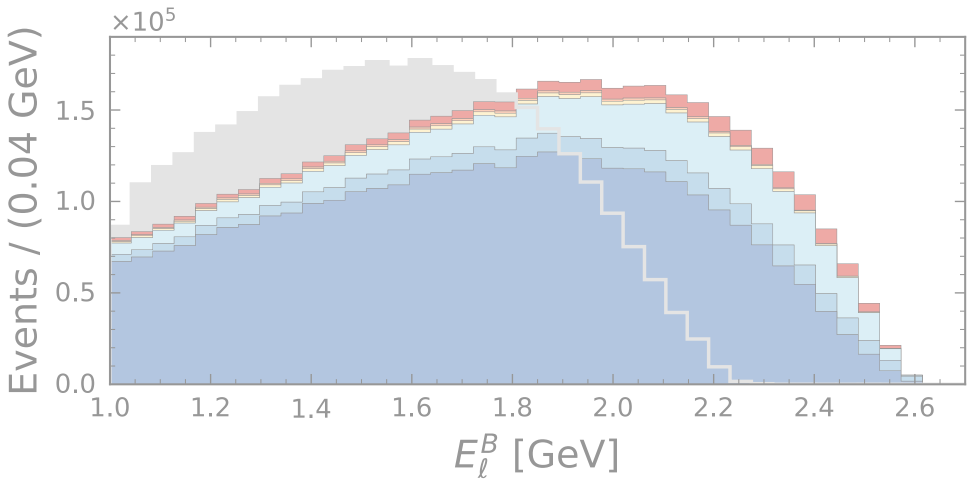
Theory error gets large
Experimental uncert. small

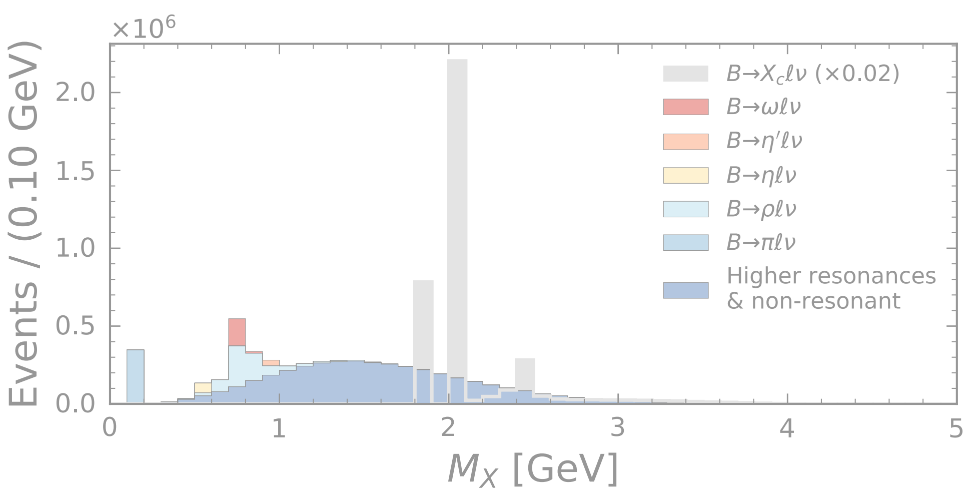
high cut

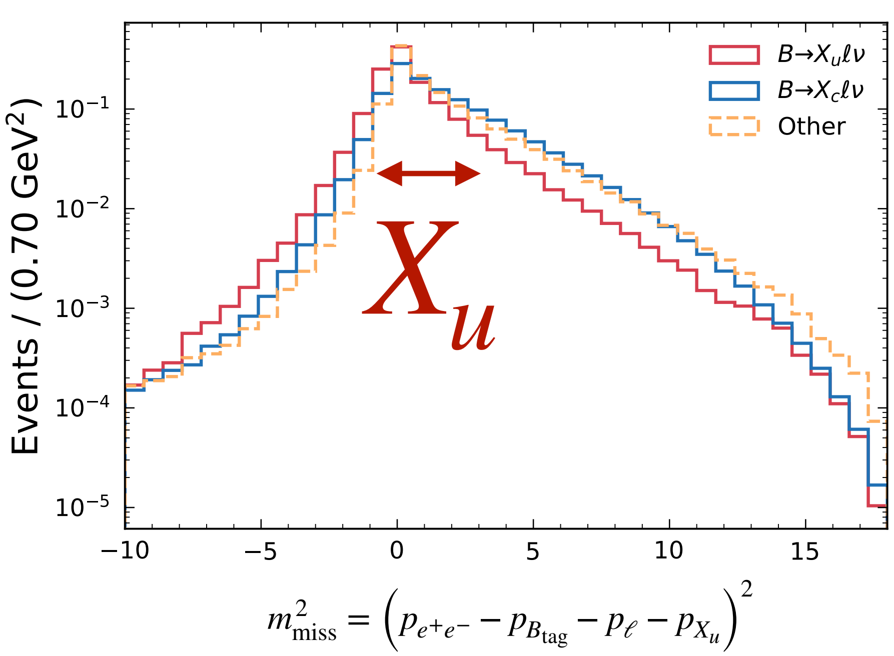
Theory error gets small
Experimental uncert. large

low cut

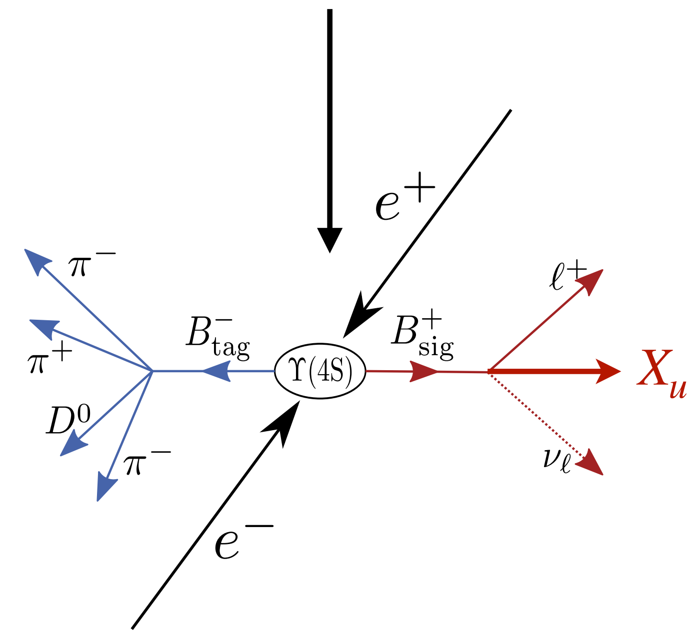
Phase-Space Coverage







Clear separation of $b \to u \ell \bar{\nu}_\ell$ from $b \to c \ell \bar{\nu}_\ell$ only possible in corners of phase-space



(Often) use hadronic tagging & multivariate (or regular) background suppression

Can exploit that there are differences:

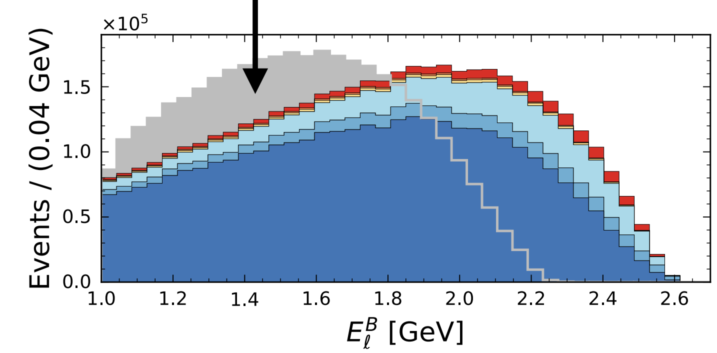
$X_u \longleftrightarrow X_c$

Higher multiplicity
Often come with charged and neutral **Kaons**
D* decays (**slow pions**)
(**Slightly lower** E_ℓ)

Direct cuts on m_X, E_ℓ **problematic**
(i.e. direct shape-function dependence)

Ok, but what's the problem?

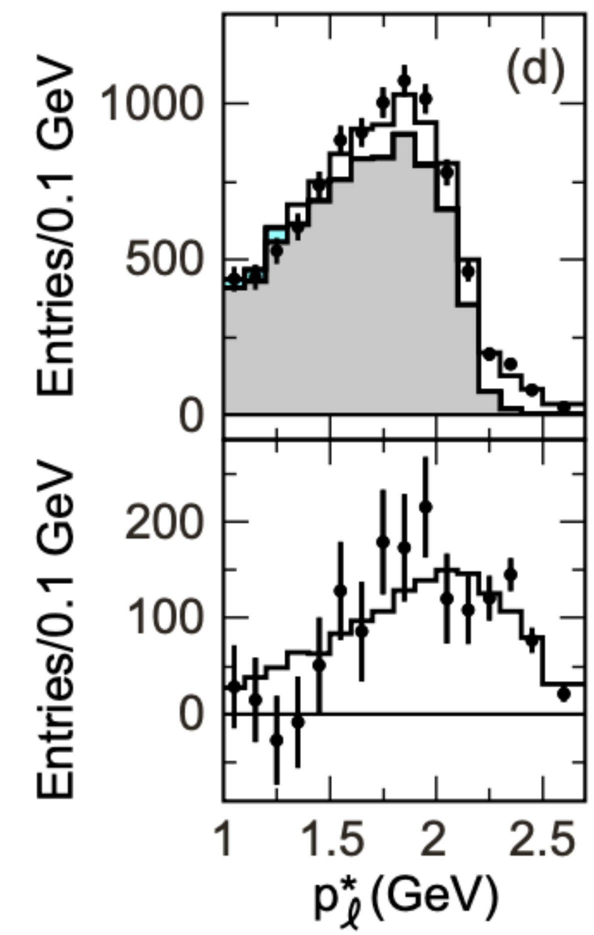
reduced
to an acceptable level



Abstract

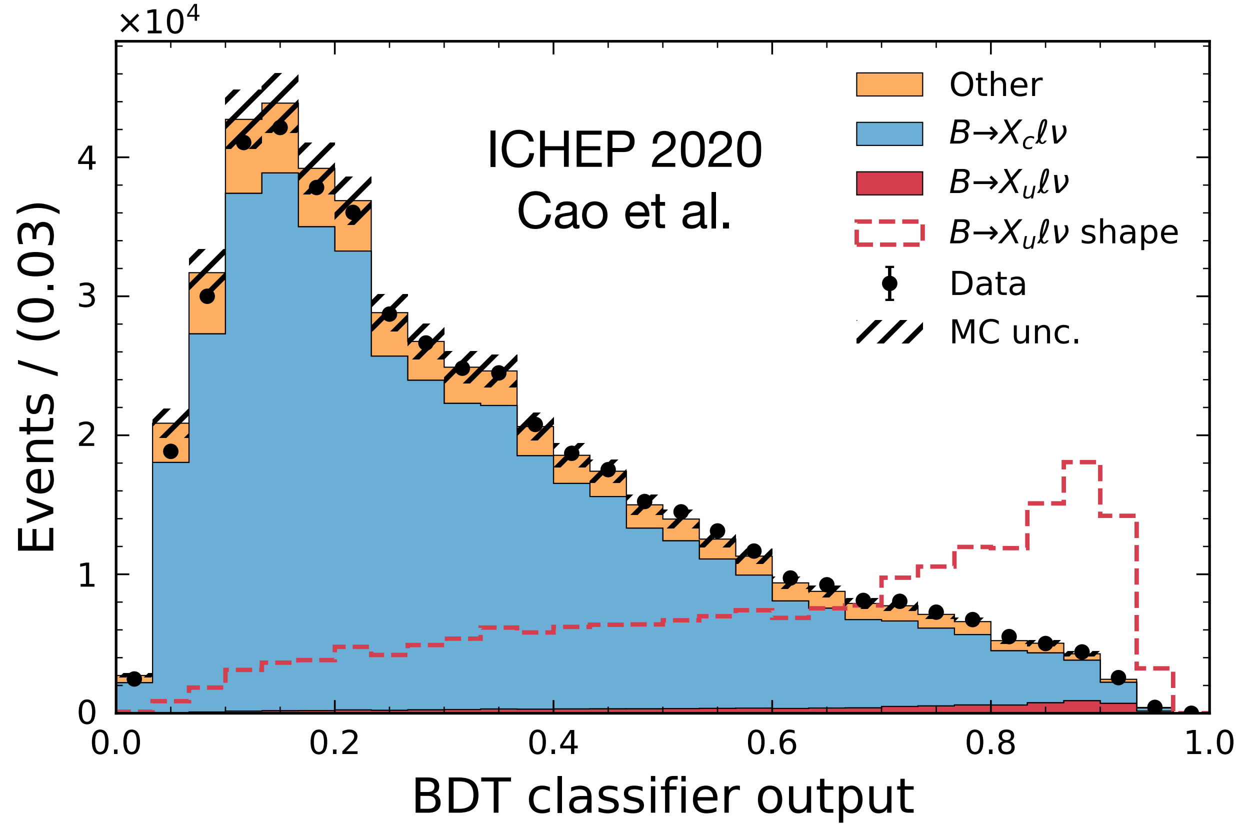
We present the partial branching fraction for inclusive charmless semileptonic B decays and the corresponding value of the CKM matrix element $|V_{ub}|$, using a multivariate analysis method to access $\sim$90% of the $B \to X_u \ell \nu$ phase space. This approach dramatically reduces the theoretical uncertainties from the $b-$quark mass and non-perturbative QCD compared to all previous inclusive measurements. The results are based on a sample of 657 million $B\bar{B}$ pairs collected with the Belle detector. We find that $\Delta\mathcal{B}(B \to X_u \ell \nu; p_\ell^{*B} > 1.0\ \mathrm{GeV}/c) = 1.963 \times (1 \pm 0.088_{\mathrm{stat.}} \pm 0.081_{\mathrm{sys.}}) \times 10^{-3}$. Corresponding values of $|V_{ub}|$ are extracted using several theoretical calculations.

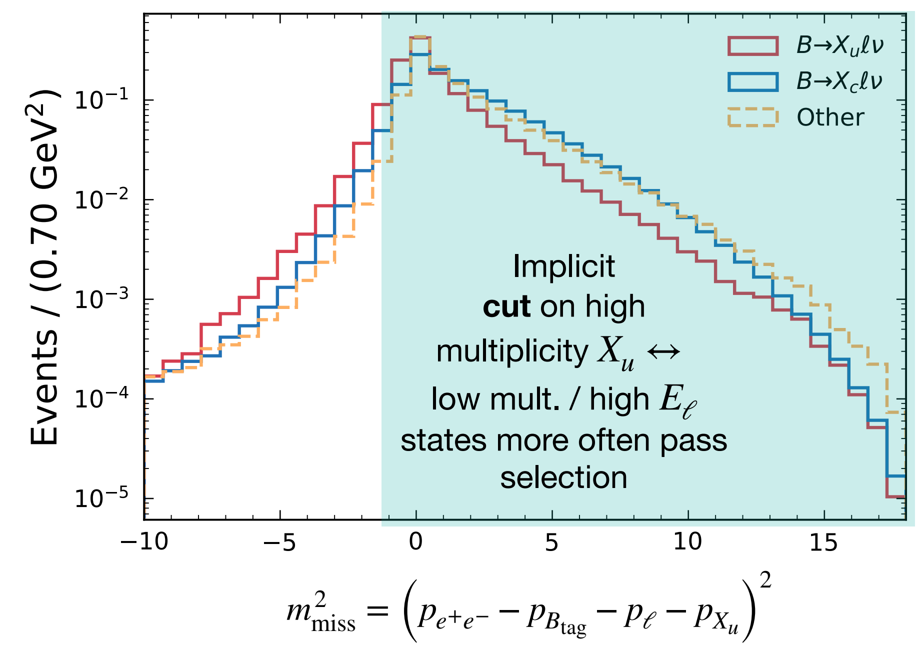
We report measurements of partial branching fractions for inclusive charmless semileptonic B decays $\overline{B} \to X_u \ell \bar{\nu}$, and the determination of the CKM matrix element $|V_{ub}|$. The analysis is based on a sample of 467 million $\Upsilon(4S) \to B\overline{B}$ decays recorded with the *BABAR* detector at the PEP-II e^+e^- storage rings. We select events in which the decay of one of the B mesons is fully reconstructed and an electron or a muon signals the semileptonic decay of the other B meson. We measure partial branching fractions $\Delta\mathcal{B}$ in several restricted regions of phase space and determine the CKM element $|V_{ub}|$ based on different QCD predictions. For decays with a charged lepton momentum $p_\ell^* > 1.0$ GeV in the B meson rest frame, we obtain $\Delta\mathcal{B} = (1.80 \pm 0.13_{\mathrm{stat.}} \pm 0.15_{\mathrm{sys.}} \pm 0.02_{\mathrm{theo.}}) \times 10^{-3}$ from a fit to the two-dimensional $M_X - q^2$ distribution. Here, M_X refers to the invariant mass of the final state hadron X and q^2 is the invariant mass squared of the charged lepton and neutrino. From this measurement we extract $|V_{ub}| = (4.33 \pm 0.24_{\mathrm{exp.}} \pm 0.15_{\mathrm{theo.}}) \times 10^{-3}$ as the arithmetic average of four results obtained from four different QCD predictions of the partial rate. We separately determine partial branching fractions for $\overline{B}^0$ and B^- decays and derive a limit on the isospin breaking in $\overline{B} \to X_u \ell \bar{\nu}$ decays.

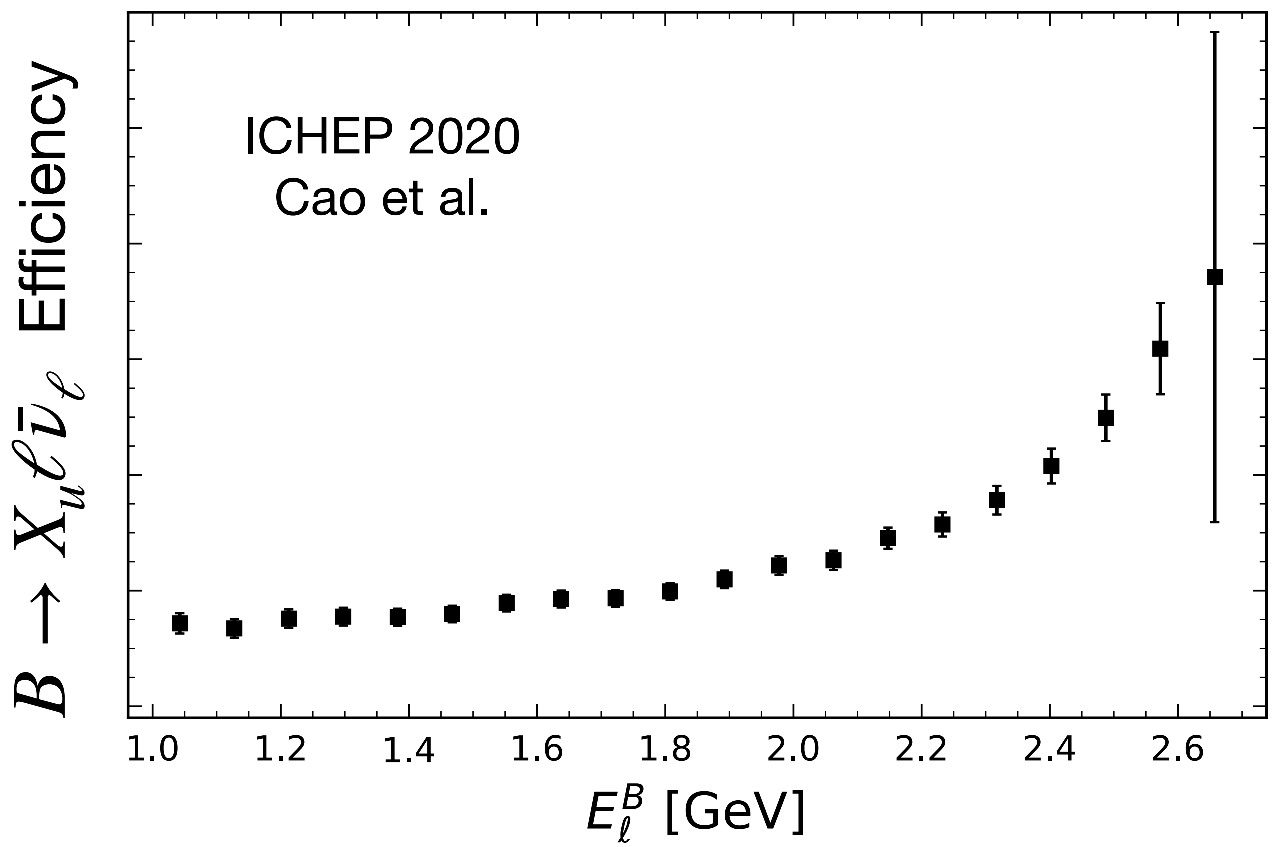


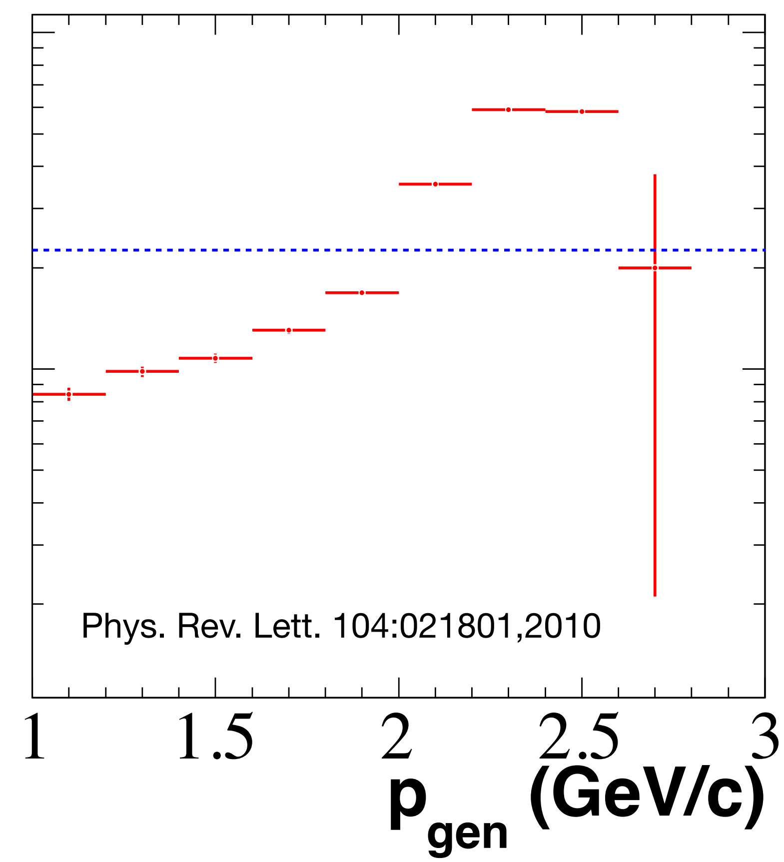
Comes at a cost

The cost: model dependence









Similar for Phys.Rev. D86 (2012) 032004

Estimated by variations of **underlying theory assumptions** and **Hybrid** model parameters used to determine (and correct for) selection efficiencies

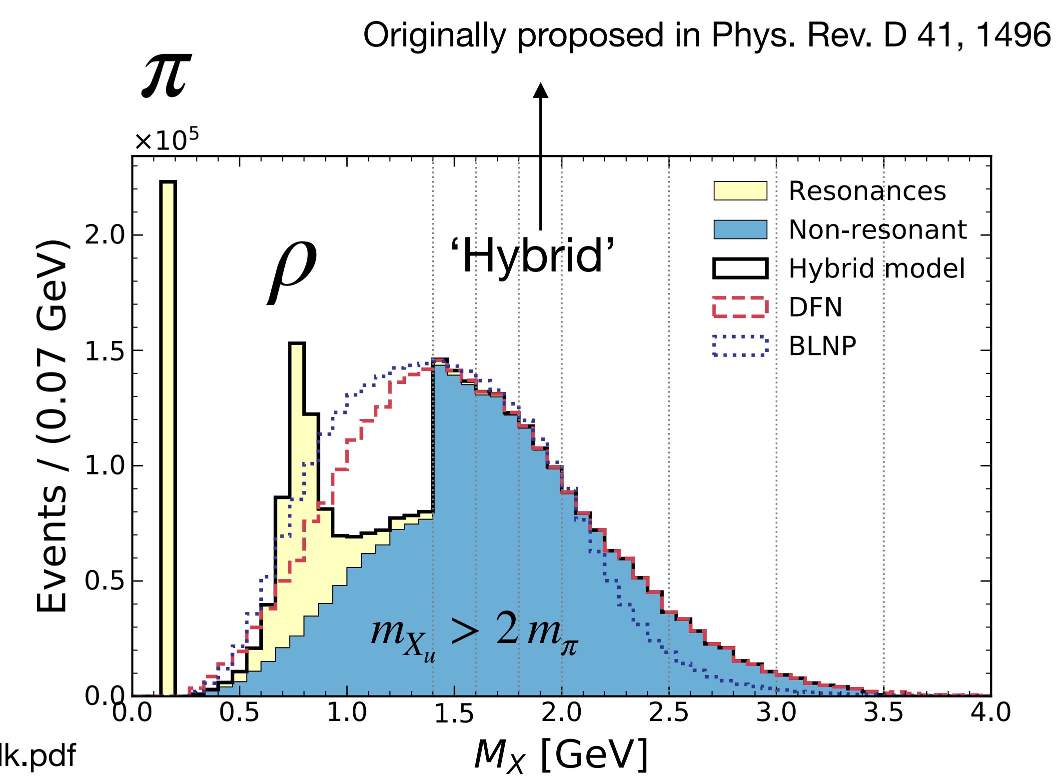
Tables from Phys. Rev. Lett. 104:021801,2010 and Phys.Rev. D86 (2012) 032004

Phase space restriction	$M_X - q^2$
Data statistical uncertainty	7.1
MC statistical uncertainty	1.1
Track efficiency	0.7
Photon efficiency	1.0
π^0 efficiency	0.9
Particle identification	2.3
K_L production/detection	1.6
K_S production/detection	1.2
Shape function parameters	5.4
Shape function form	1.5
Exclusive $\overline{B} \to X_u \ell \bar{\nu}$	1.9
$s\bar{s}$ production	2.7
B semileptonic branching ratic	1.0
D decays	1.1
$B \to D\ell\nu$ form factor	0.4
$B \to D^*\ell\nu$ form factor	0.7
$B \to D^{**}\ell\nu$ form factor	0.9
$B \to D^{**}$ reweighting	1.9
m_{ES} background subtraction	1.9
combinatorial backg.	1.0
Total semileptonic BF	1.4
Total systematic uncertainty	8.4
Total experimental uncertaint	11.0

$p_\ell^{*B} > 1.0$ GeV	$\Delta\mathcal{B}/\mathcal{B}$ (%)
$\mathcal{B}(D^{(*)}\ell\nu)$	1.2
$(D^{(*)}\ell\nu)$ form factors	1.2
$\mathcal{B}(D^{**}e\nu)$ & form factors	0.2
$B \to X_u\ell\nu$ (SF)	3.6
$B \to X_u\ell\nu$ $(g \to s\bar{s})$	1.5
$\mathcal{B}(B \to \pi/\rho/\omega\ell\nu)$	2.3
$\mathcal{B}(B \to \eta,\ \eta'\ell\nu)$	3.2
$\mathcal{B}(B \to X_u\ell\nu)$ un-meas.	2.9
Cont./Comb.	1.8
Sec./Fakes/Fit.	1.0
PID/Reconstruction	3.1
BDT	3.1
Systematics	8.1
Statistics	8.8

MC Mix of res. and non-resonant processes

non-resonant X_u fragmented via JETSET / Pythia



More about how this is made:

https://indico.fnal.gov/event/44316/contributions/190792/attachments/132360/162611/Talk.pdf

Future directions:

Focus on experimental **most sensitive region** (high E_ℓ^B)

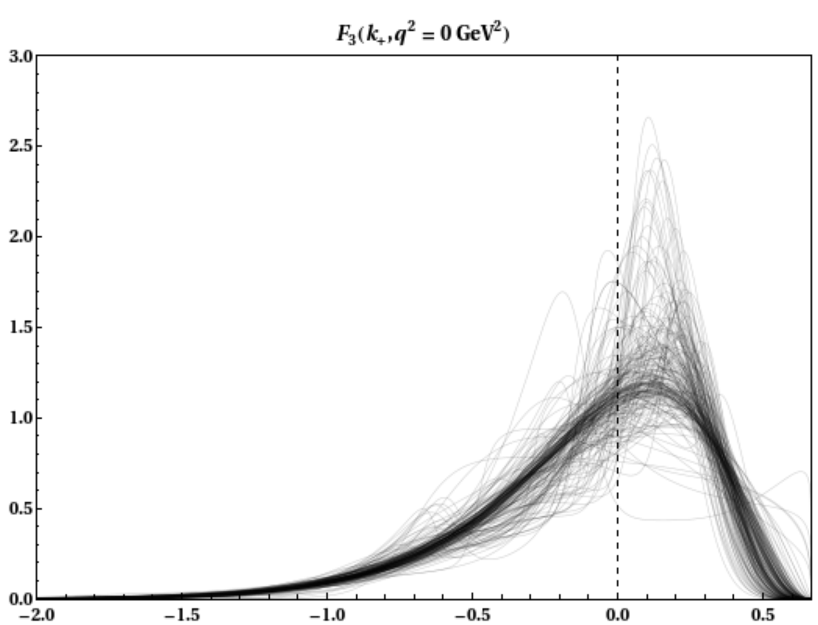
Determine Shape-Function in a data-driven way

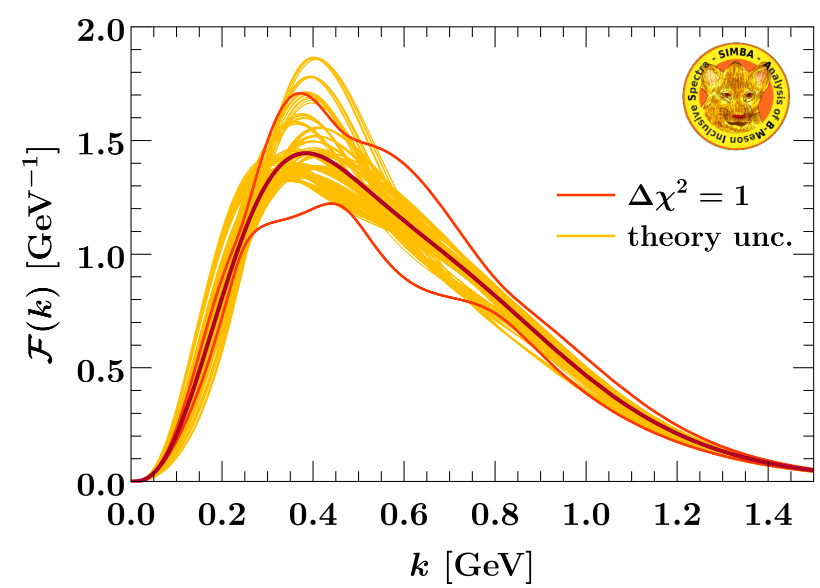
Note: this is not the NNVub Logo

NNVub

P. Gambino, K. Healey, C. Mondino, Phys. Rev. D 94, 014031 (2016), [arXiv:1604.07598]

F. Bernlochner, H. Lacker, Z. Ligeti, I. Stewart, F. Tackmann, K. Tackmann
Submitted to PRL
[arXiv:2007.04320]

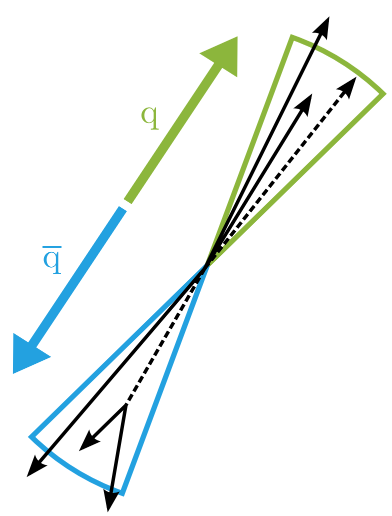


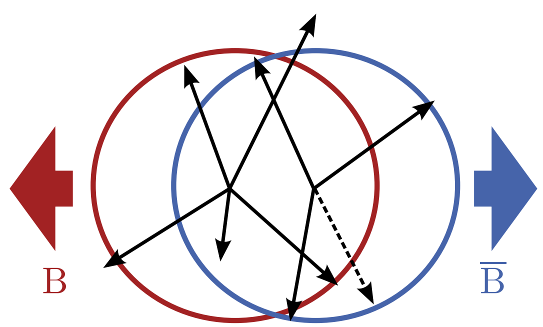


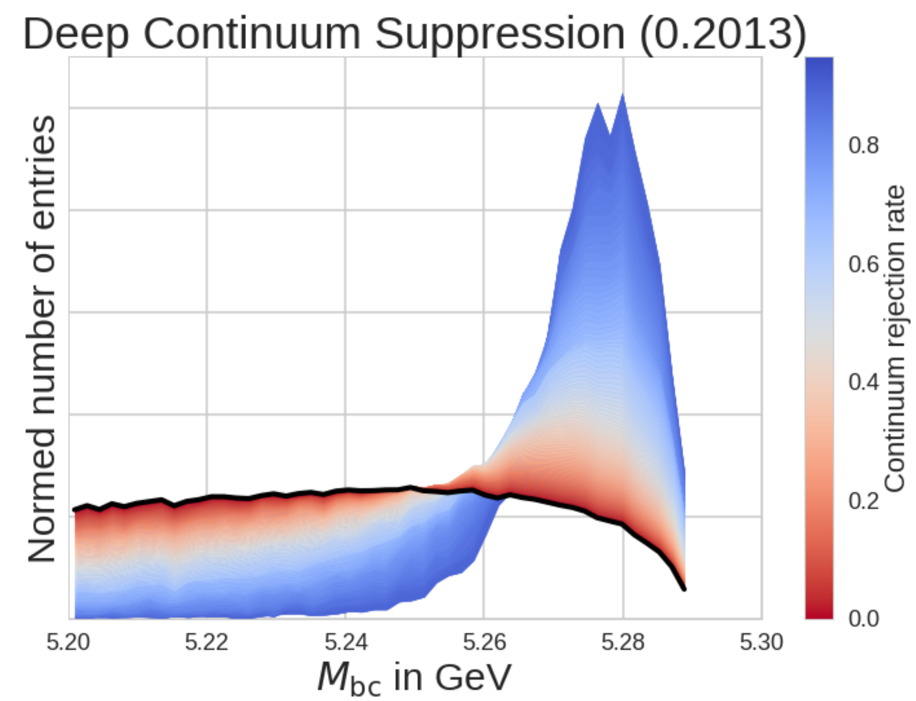
Future directions:

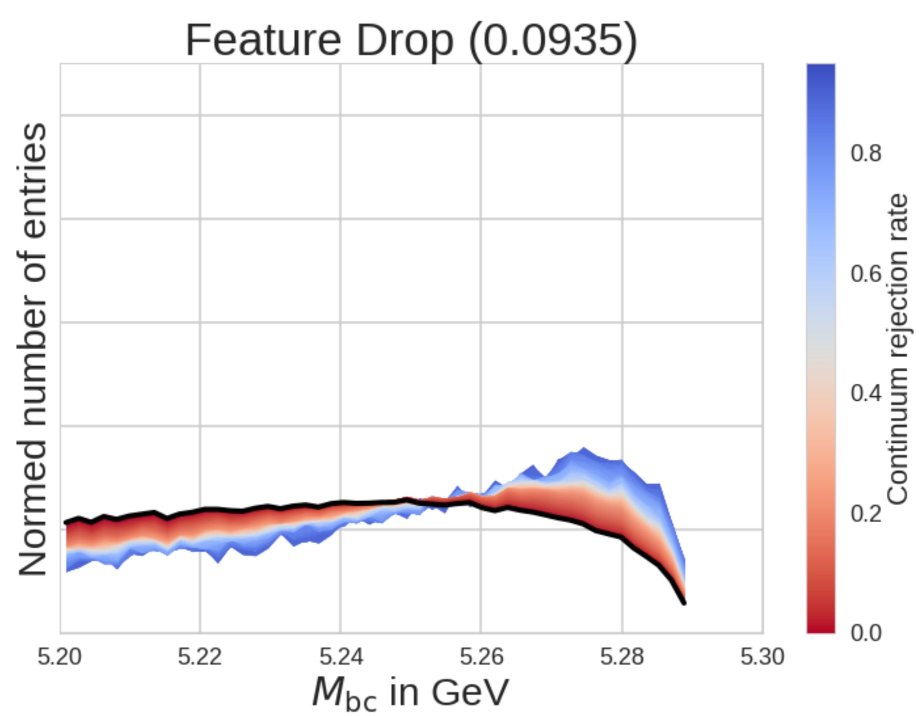
$$M_{\mathrm{bc}} = \sqrt{E_{\mathrm{beam}}^2 - p_B^2}$$

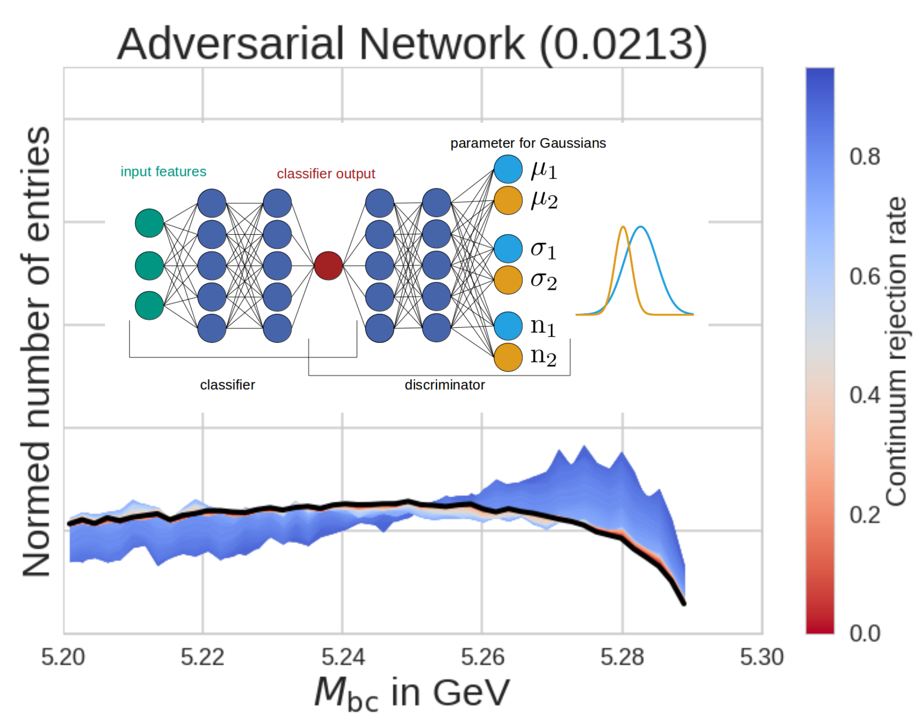
Avoid Efficiency shaping





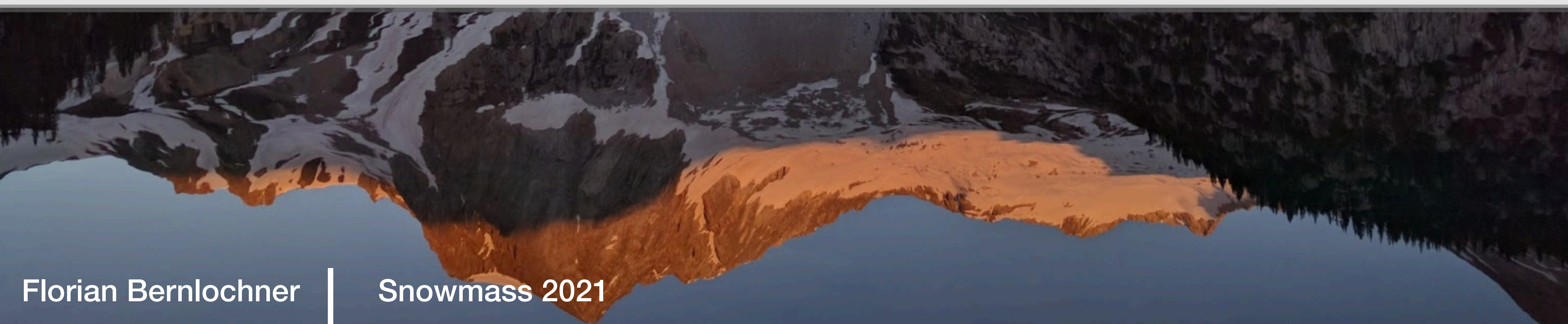




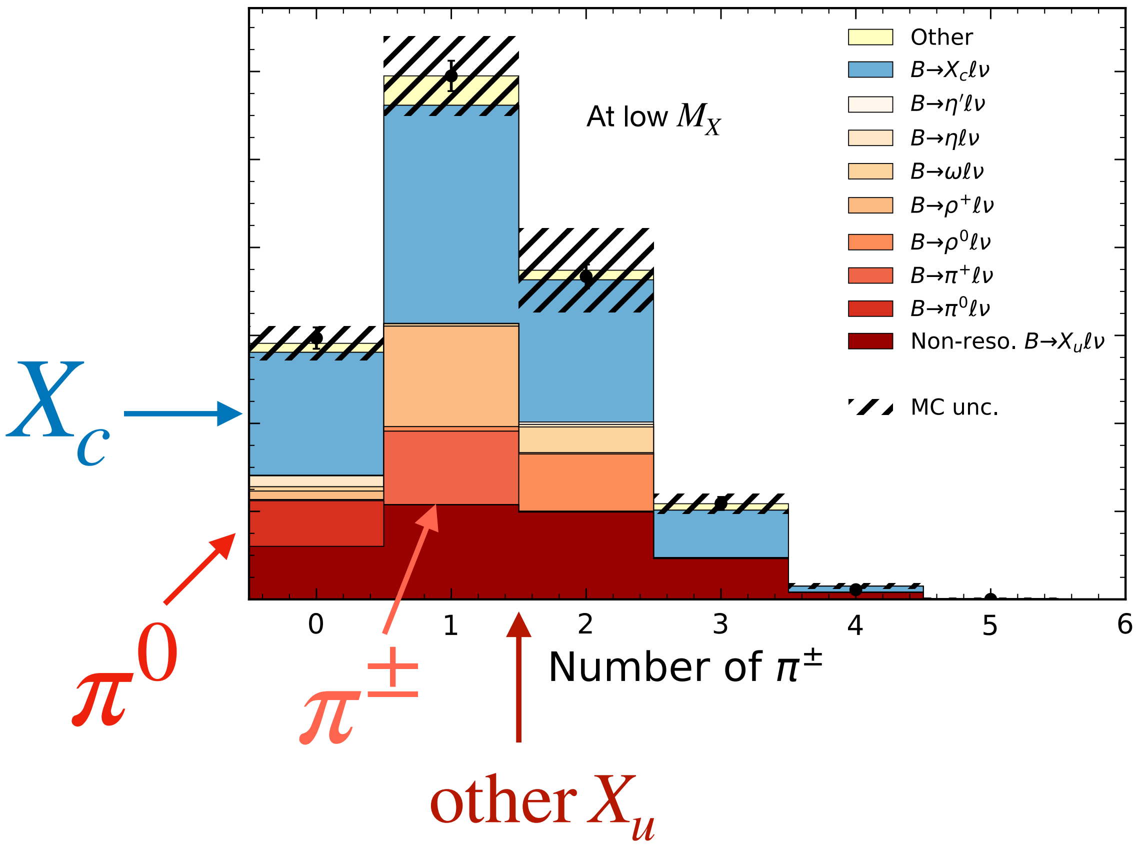


https://indico.cern.ch/event/655447/contributions/2742185/attachments/1552413/2439489/adversarial_networks_in_belle2-iml.pdf

Combined incl. and excl. $|V_{ub}|$



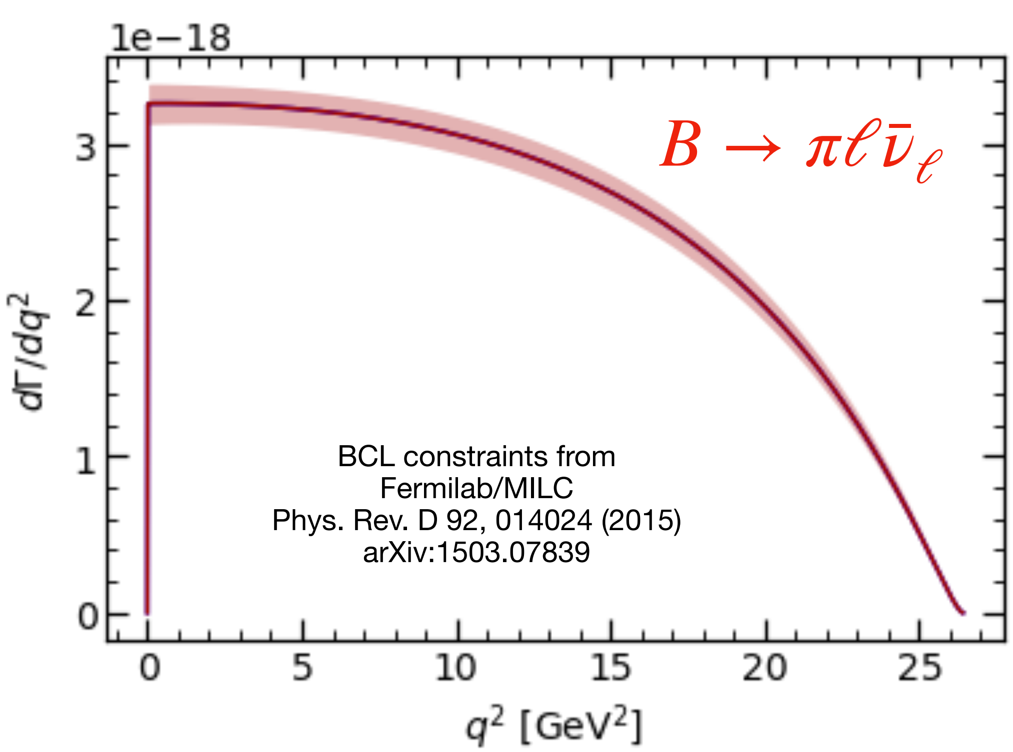
Can we measure both at the same time?



$$\Delta\mathscr{B}(B \to X_u \ell \bar{\nu}_\ell) = \Delta\mathscr{B}(B \to \pi \ell \bar{\nu}_\ell) + \Delta\mathscr{B}(B \to X_u^{\text{other}} \ell \bar{\nu}_\ell)$$

$$\mathscr{B}(B \to \pi \ell \bar{\nu}_\ell)$$

$$\frac{|V_{ub}^{\text{excl.}}|}{|V_{ub}^{\text{incl.}}|} = \frac{\sqrt{\dfrac{\mathscr{B}(B \to \pi \ell \bar{\nu}_\ell)}{\Gamma(B \to \pi \ell \bar{\nu}_\ell)}}}{\sqrt{\dfrac{\Delta\mathscr{B}(B \to X_u \ell \bar{\nu}_\ell)}{\Delta\Gamma(B \to X_u \ell \bar{\nu}_\ell)}}}$$



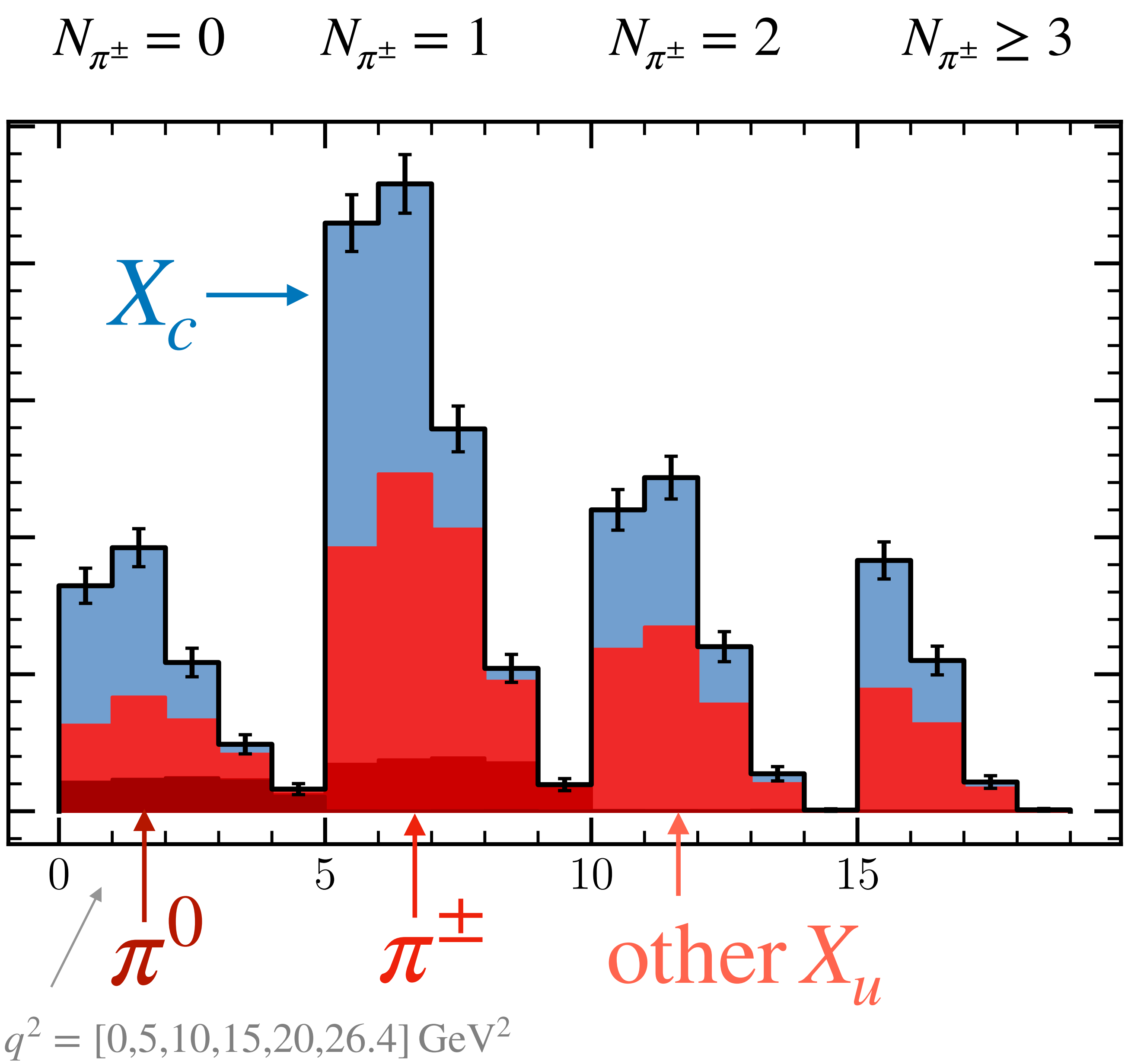
$\longrightarrow$

Use $q^2 : N_{\pi^\pm}$ to separate

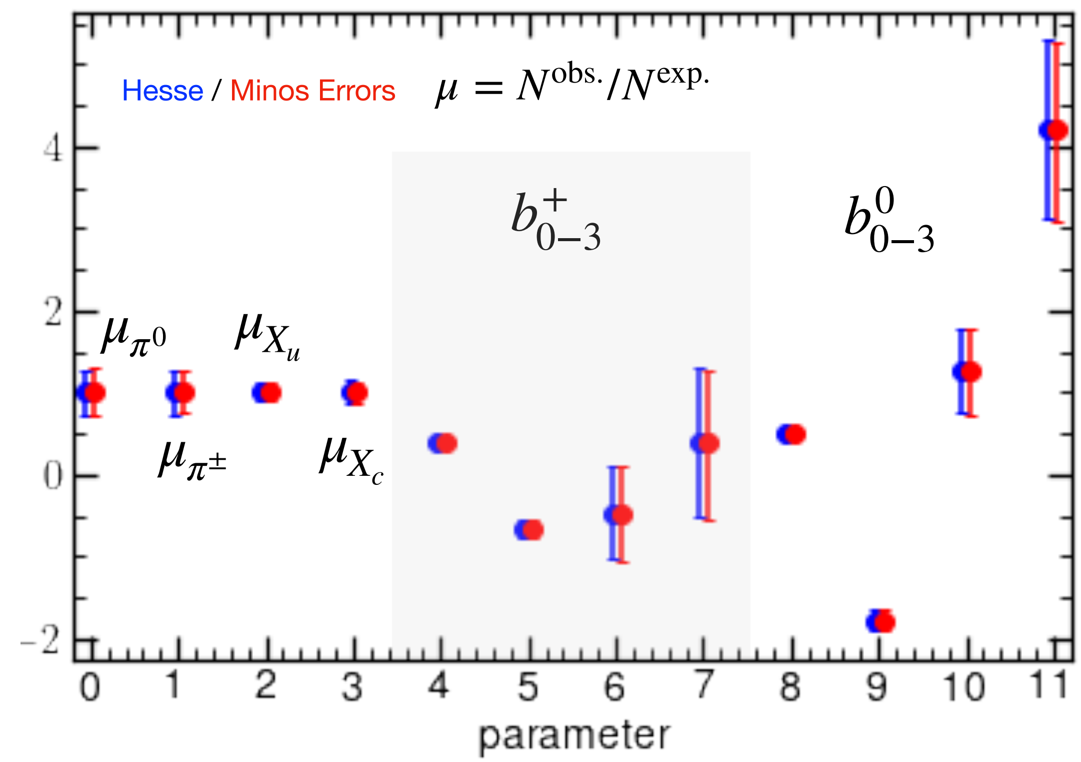
$$B \to \pi^0 \ell \bar{\nu}_\ell$$
$$B \to \pi^\pm \ell \bar{\nu}_\ell$$
$$B \to X_u^{\text{other}} \ell \bar{\nu}_\ell$$
$$B \to X_c \ell \bar{\nu}_\ell + \text{other Bkg}\,.$$

Asimov Fit

Fermilab/MILC
Phys. Rev. D 92, 014024 (2015)
arXiv:1503.07839



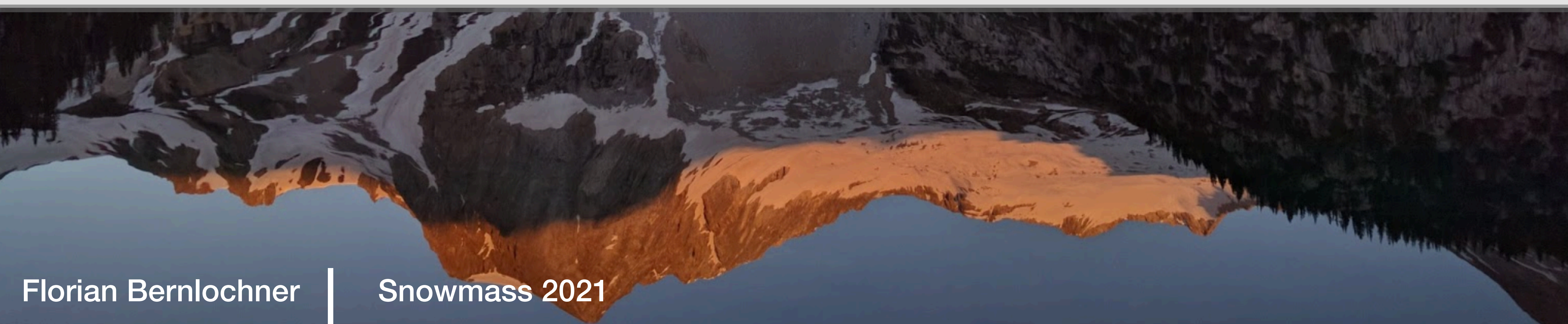
Fit Setup: $\chi^2 = \chi^2_{\mathrm{exp}} + \chi^2_{\mathrm{FNAL}}$



μ_{π^0}	$\mu_{\pi^\pm}$	μ_{X_u}	μ_{X_c}	b^+_{0-3}				b^0_{0-3}			
1.00	0.51	0.39	-0.61	0.13	0.55	0.63	0.57	0.23	0.54	0.64	0.43
0.51	1.00	0.10	-0.40	0.17	0.58	0.58	0.51	0.22	0.52	0.62	0.41
0.39	0.10	1.00	-0.89	0.03	0.35	0.53	0.49	0.18	0.40	0.50	0.34
-0.61	-0.40	-0.89	1.00	-0.09	-0.54	-0.70	-0.65	-0.25	-0.57	-0.69	-0.46
0.13	0.17	0.03	-0.09	1.00	0.35	-0.10	-0.16	0.24	0.20	0.10	0.04
0.55	0.58	0.35	-0.54	0.35	1.00	0.44	0.29	0.21	0.60	0.65	0.44
0.63	0.58	0.53	-0.70	-0.10	0.44	1.00	0.97	0.29	0.66	0.87	0.62
0.57	0.51	0.49	-0.65	-0.16	0.29	0.97	1.00	0.30	0.65	0.80	0.47
0.23	0.22	0.18	-0.25	0.24	0.21	0.29	0.30	1.00	0.24	0.07	-0.10
0.54	0.52	0.40	-0.57	0.20	0.60	0.66	0.65	0.24	1.00	0.60	0.08
0.64	0.62	0.50	-0.69	0.10	0.65	0.87	0.80	0.07	0.60	1.00	0.70
0.43	0.41	0.34	-0.46	0.04	0.44	0.62	0.47	-0.10	0.08	0.70	1.00

Individual components
seem to separate well in Asimov with
made-up (but semi-realistic) distributions

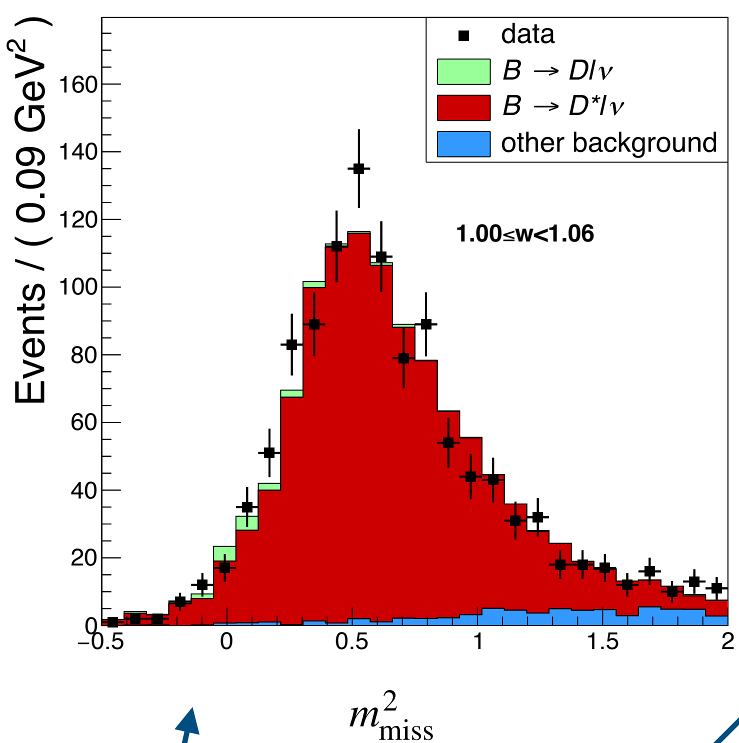
Exclusive $|V_{ub}|$ and $|V_{cb}|$

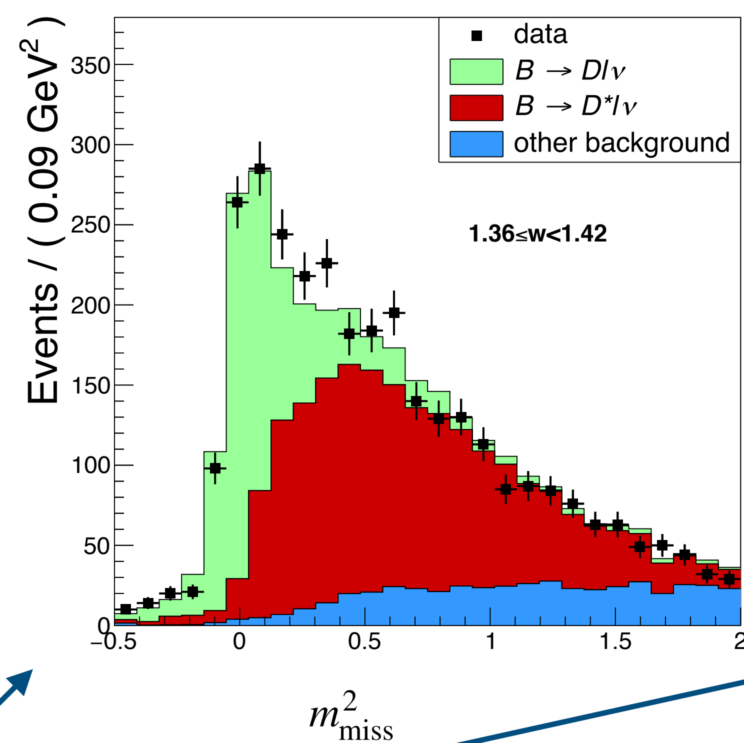


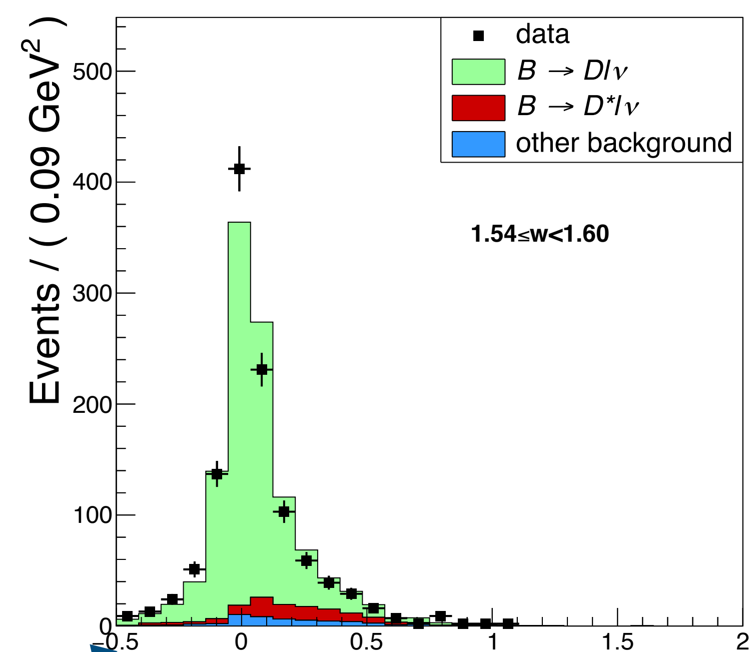
Combined Measurements of $B \to D^{(*)}\ell\bar{\nu}_\ell$

(Tagged) Measurements of $B \to D\ell\bar{\nu}_\ell$ suffer from large down-feed from $B \to D^*\ell\bar{\nu}_\ell$

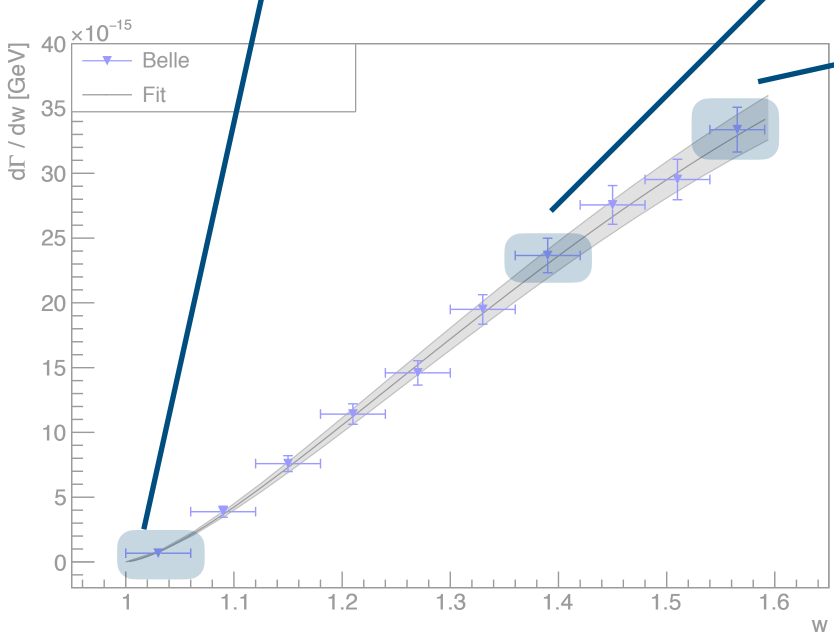
Phys. Rev. D 93, 032006 (2016), arXiv:1510.03657





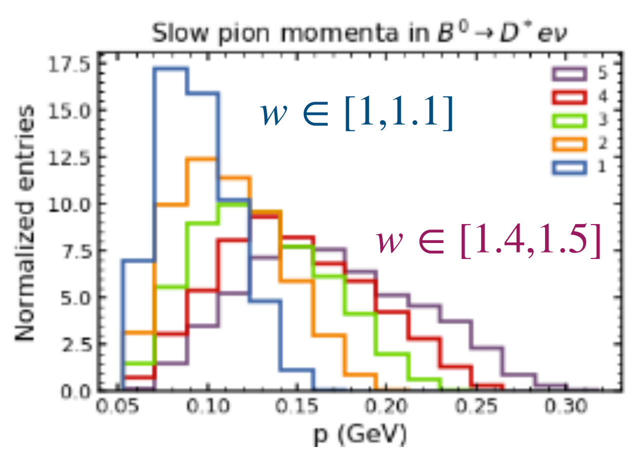


$$m^2_{\text{miss}} = \left(p_{e^+e^-} - p_{\mathrm{B_{tag}}} - p_\ell - p_D\right)^2$$

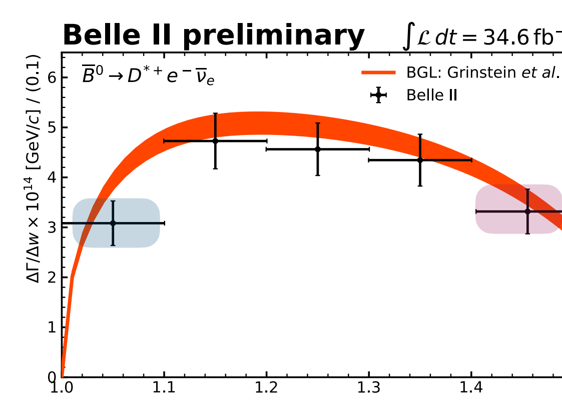


Thus far **unexploited** data set to measure w spectrum of $B \to D^*\ell\bar{\nu}_\ell$

- w spectrum encodes most sensitivity of all projected spectra
- no slow pion reconstruction necessary



Especially the region of low w (important for $|V_{cb}|$) has very soft π_s: $p_\pi \approx 70\,\text{MeV}$



Combined Fits of $B \to D^{(*)}\ell\bar{\nu}_\ell$

Phys. Rev. D 95, 115008 (2017), [arXiv:1703.05330]

Interesting if heavy quark symmetry inspired Form Factors are used:

$$\hat{h}(w) = h(w)/\xi(w) \longleftarrow$$ Leading Isgur-Wise function

$B \to D\ell\bar{\nu}_\ell$

$$\hat{h}_+ = 1 + \hat{\alpha}_s\Big[C_{V_1} + \frac{w+1}{2}\,(C_{V_2} + C_{V_3})\Big] + (\varepsilon_c + \varepsilon_b)\,\hat{L}_1\,,$$
$$\hat{h}_- = \hat{\alpha}_s\,\frac{w+1}{2}\,(C_{V_2} - C_{V_3}) + (\varepsilon_c - \varepsilon_b)\,\hat{L}_4\,,$$

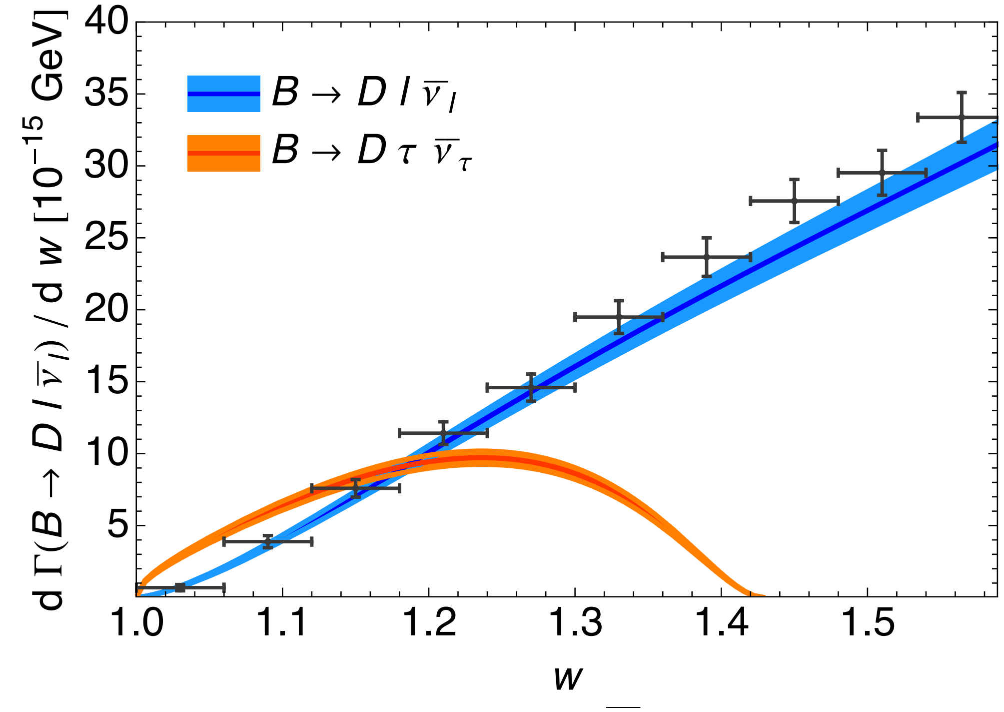
$B \to D^*\ell\bar{\nu}_\ell$

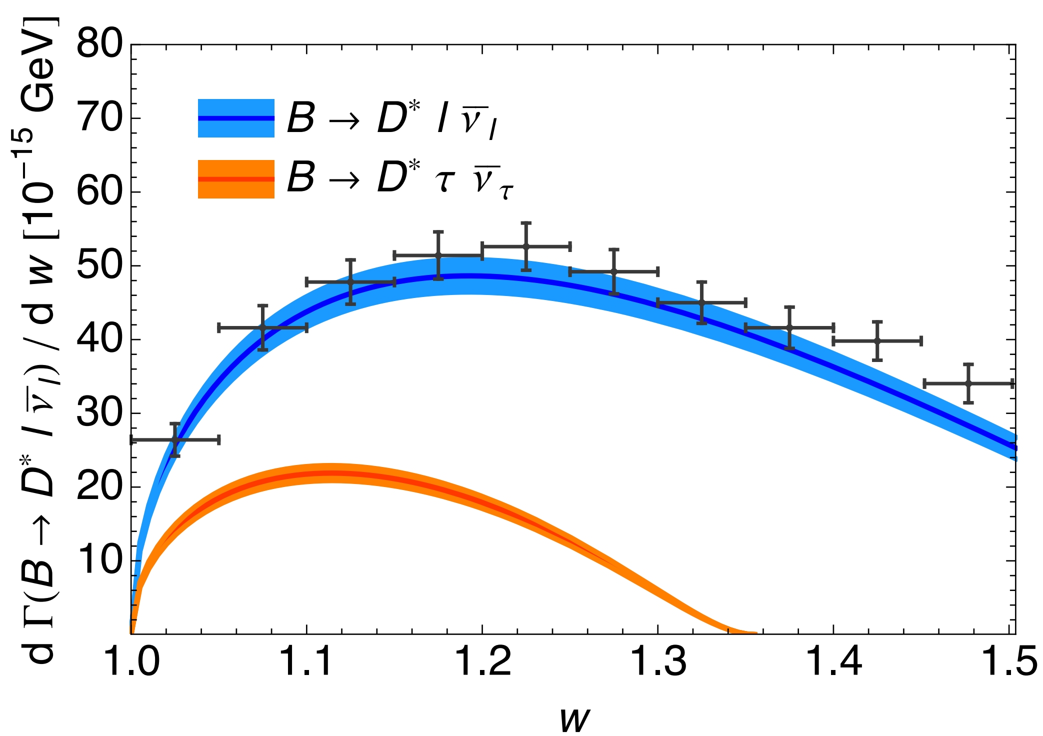
$$\hat{h}_V = 1 + \hat{\alpha}_s\,C_{V_1} + \varepsilon_c\big(\hat{L}_2 - \hat{L}_5\big) + \varepsilon_b\big(\hat{L}_1 - \hat{L}_4\big)\,,$$
$$\hat{h}_{A_1} = 1 + \hat{\alpha}_s\,C_{A_1} + \varepsilon_c\left(\hat{L}_2 - \hat{L}_5\,\frac{w-1}{w+1}\right) + \varepsilon_b\left(\hat{L}_1 - \hat{L}_4\,\frac{w-1}{w+1}\right),$$
$$\hat{h}_{A_2} = \hat{\alpha}_s\,C_{A_2} + \varepsilon_c\big(\hat{L}_3 + \hat{L}_6\big)\,,$$
$$\hat{h}_{A_3} = 1 + \hat{\alpha}_s\big(C_{A_1} + C_{A_3}\big) + \varepsilon_c\big(\hat{L}_2 - \hat{L}_3 + \hat{L}_6 - \hat{L}_5\big) + \varepsilon_b\big(\hat{L}_1 - \hat{L}_4\big)\,,$$

This links dynamics of $B \to D\ell\bar{\nu}_\ell$ & $B \to D^*\ell\bar{\nu}_\ell$

Example fit for leading IW function and sub-leading parameters

$\lvert V_{cb}\rvert \times 10^3$	38.8 ± 1.2
$\mathcal{G}(1)$	1.055 ± 0.008
$\mathcal{F}(1)$	0.904 ± 0.012
$\bar{\rho}_*^2$	1.17 ± 0.12
$\hat{\chi}_2(1)$	-0.26 ± 0.26
$\hat{\chi}_2'(1)$	0.21 ± 0.38
$\hat{\chi}_3'(1)$	0.02 ± 0.07
$\eta(1)$	0.30 ± 0.04
$\eta'(1)$	0 (fixed)
m_b^{1S} [GeV]	4.70 ± 0.05
δm_{bc} [GeV]	3.40 ± 0.02





Careful with unitarity constraints in experimental Fits

Unitarity constraints are interesting ingredients to incorporate into fits, but one has to be careful

$$g(z) = \frac{1}{P_V(z)\phi_g(z)} \sum_n a_n^g z^n \,, \qquad \sum_n |a_n^g|^2 \leq 1 \,,$$

$$F_A(z) = \frac{1}{P_A(z)\phi_{F_A}(z)} \sum_n a_n^{F_A} z^n \,, \qquad \sum_{F_A,n} |a_n^{F_A}|^2 \leq 1 \,,$$

Two problems:

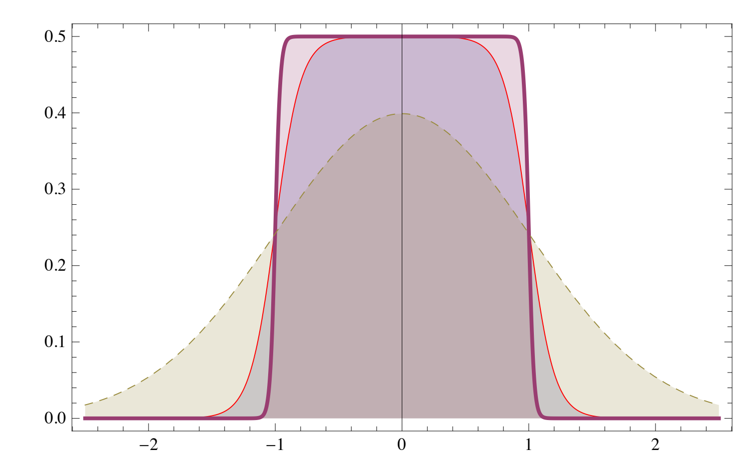
1) If included, they can strongly constrain higher order terms (a priori fine); but one has to be careful as the uncertainties on these will then **highly depend** on the **prior probability.**

   At best this introduces an **undesired dependence on prior**, **at worst** it could **bias** results.

2) If one averages several results, such UT constraints should be included **only once** (as otherwise one starts to use this prior *n* times if one averages *n* measurements). **Safest way** is if **measurements** provide **results** always (also) **without** UT constraints applied to keep them **"future proof"**

Possible prior choices to enforce that the quadratic sum of parameters remains smaller than unity

Commonly used constraints are: Gaussian, Double Fermi Dirac (DFD) with $w = 10$, DFD with $w = 50$

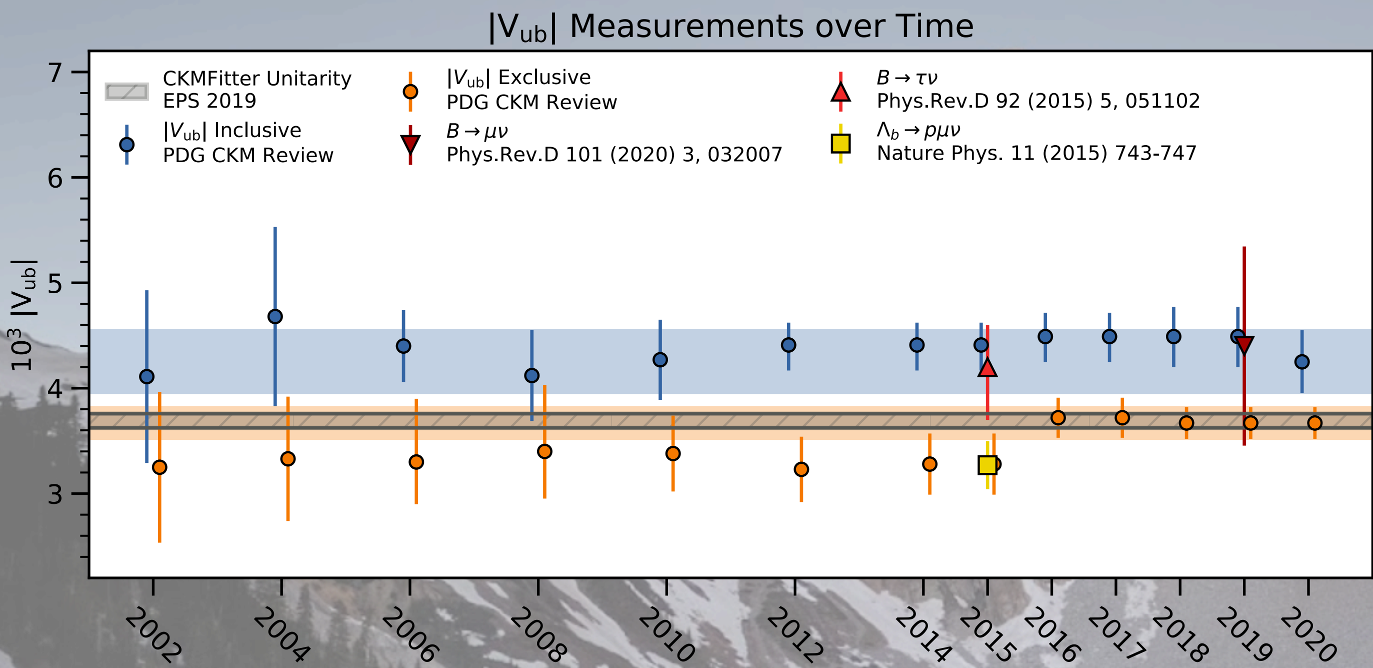


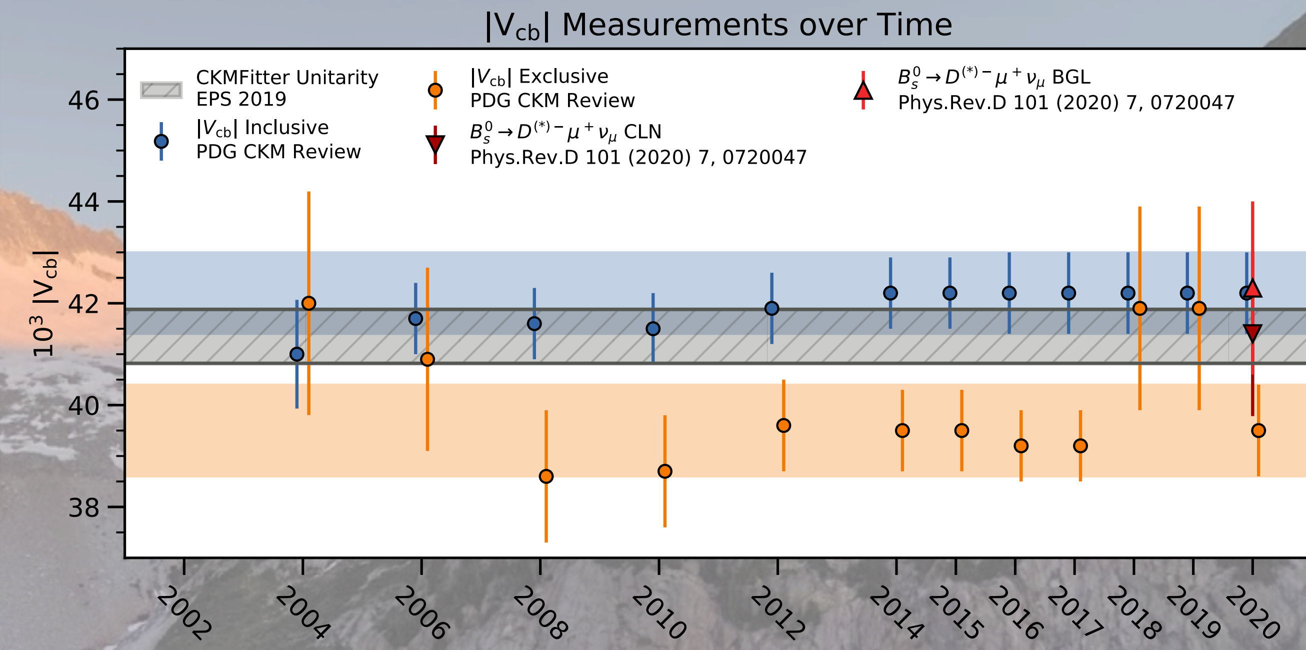
$$DFD(x, w) = 1/\left(2\left(1+e^{w(x-1)}\right)\left(1+e^{-w(x+1)}\right)\right)$$

→ Nice features of DFD: Pull on NP within one-sigma with negligible penalization in probability; pulls larger than one sigma penalized heavily

Wrap-Up

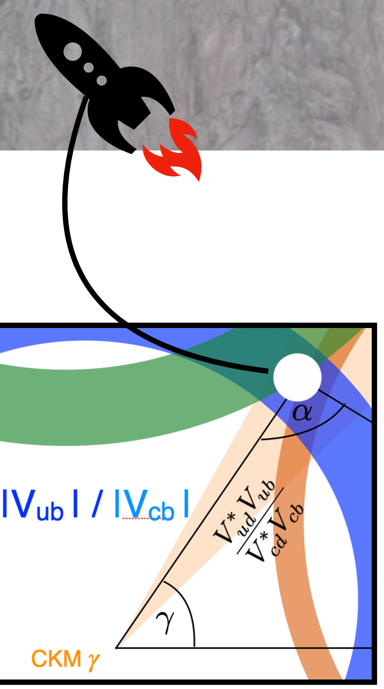
Vxb over time: Markus Prim





LHCb and Belle II will record unprecedented data sets in the next decade

This will allow many new directions; we should carefully rethink the established methods



Example implementation for $b \to u \ell \bar{\nu}_\ell$ Hybrid
https://github.com/b2-hive/eFFORT

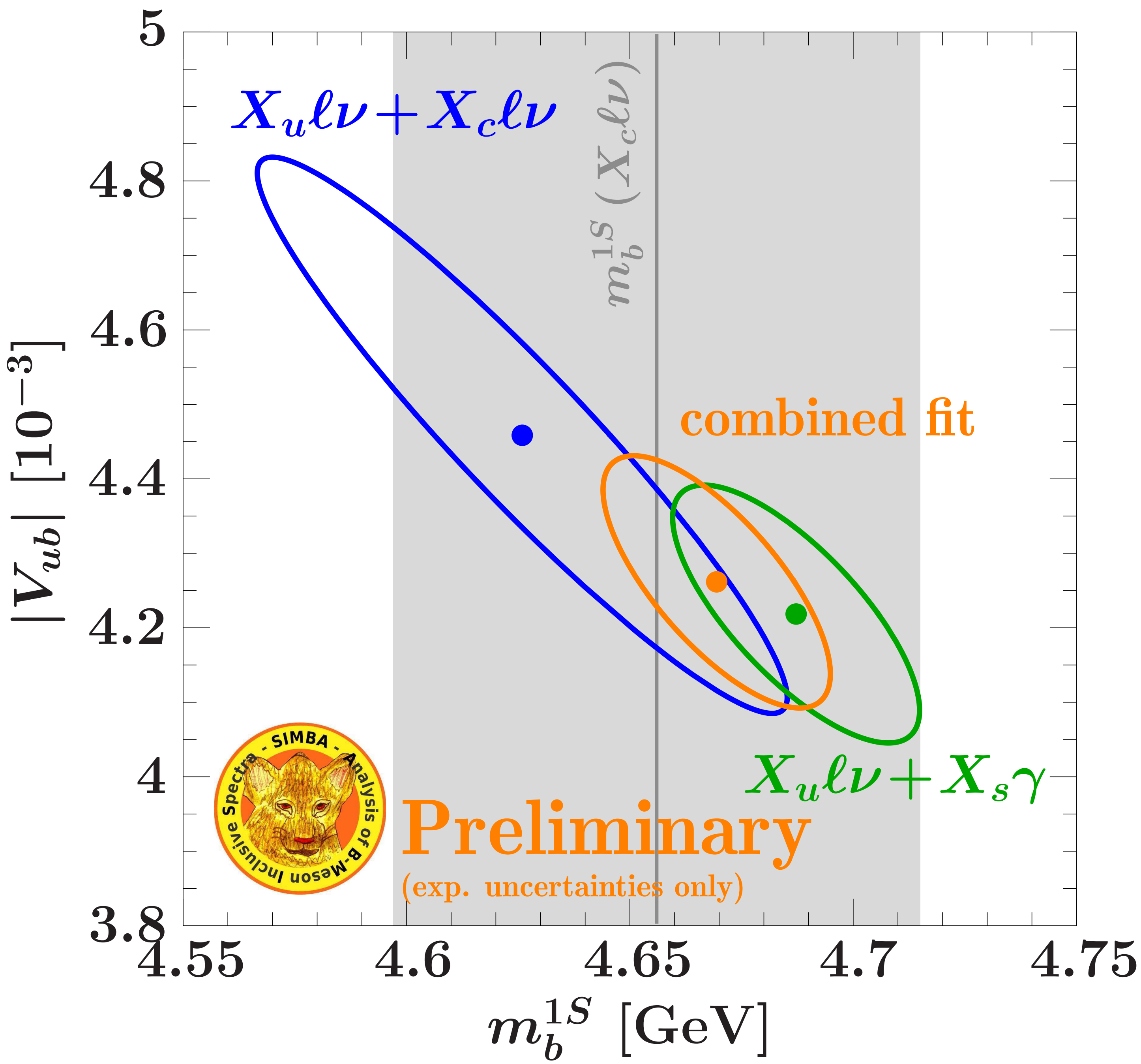
Helicity Amplitude Module for Matrix Element Reweighting

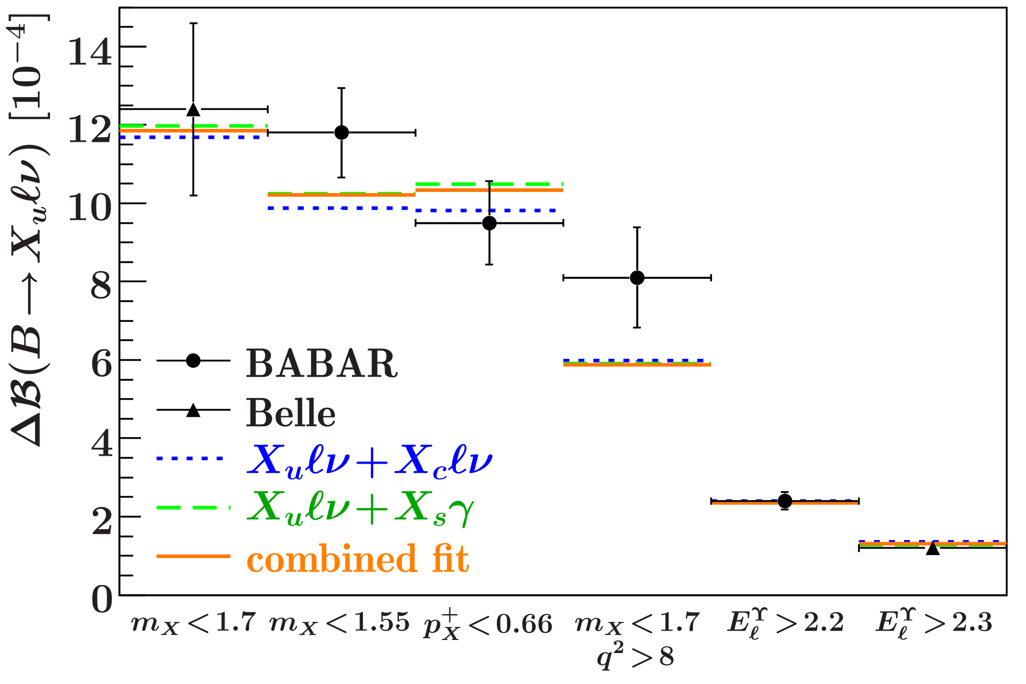
Example implementation for HQET FFs:
https://hammer.physics.lbl.gov/

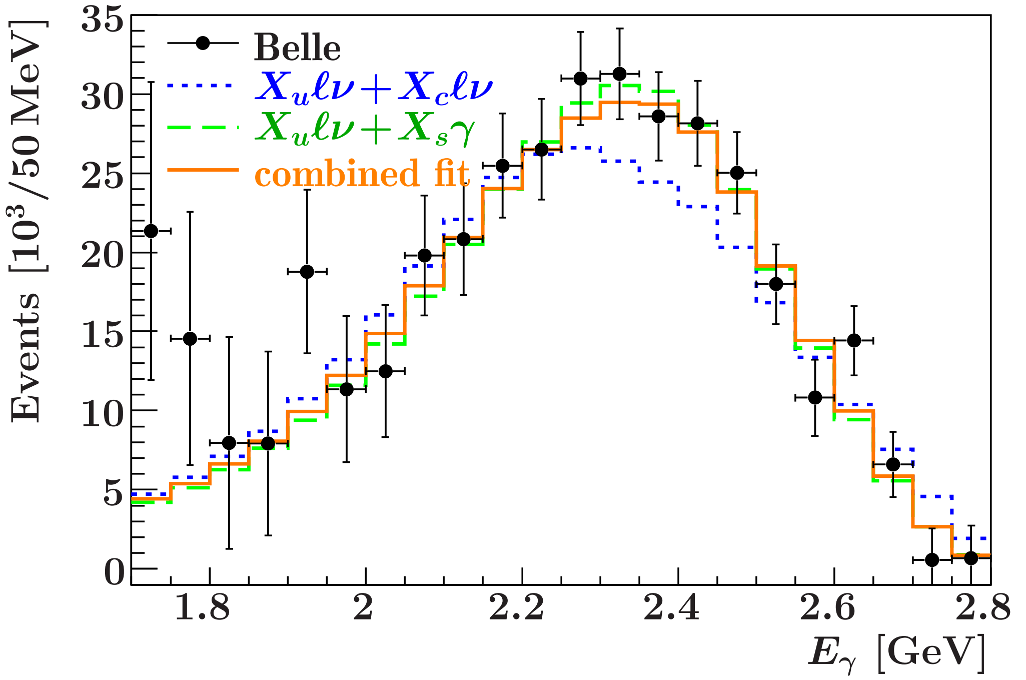
Also check out RooHammerModel:
https://arxiv.org/abs/2007.12605

More Information

Proof-of-Concept: Vxb Workshop 2009

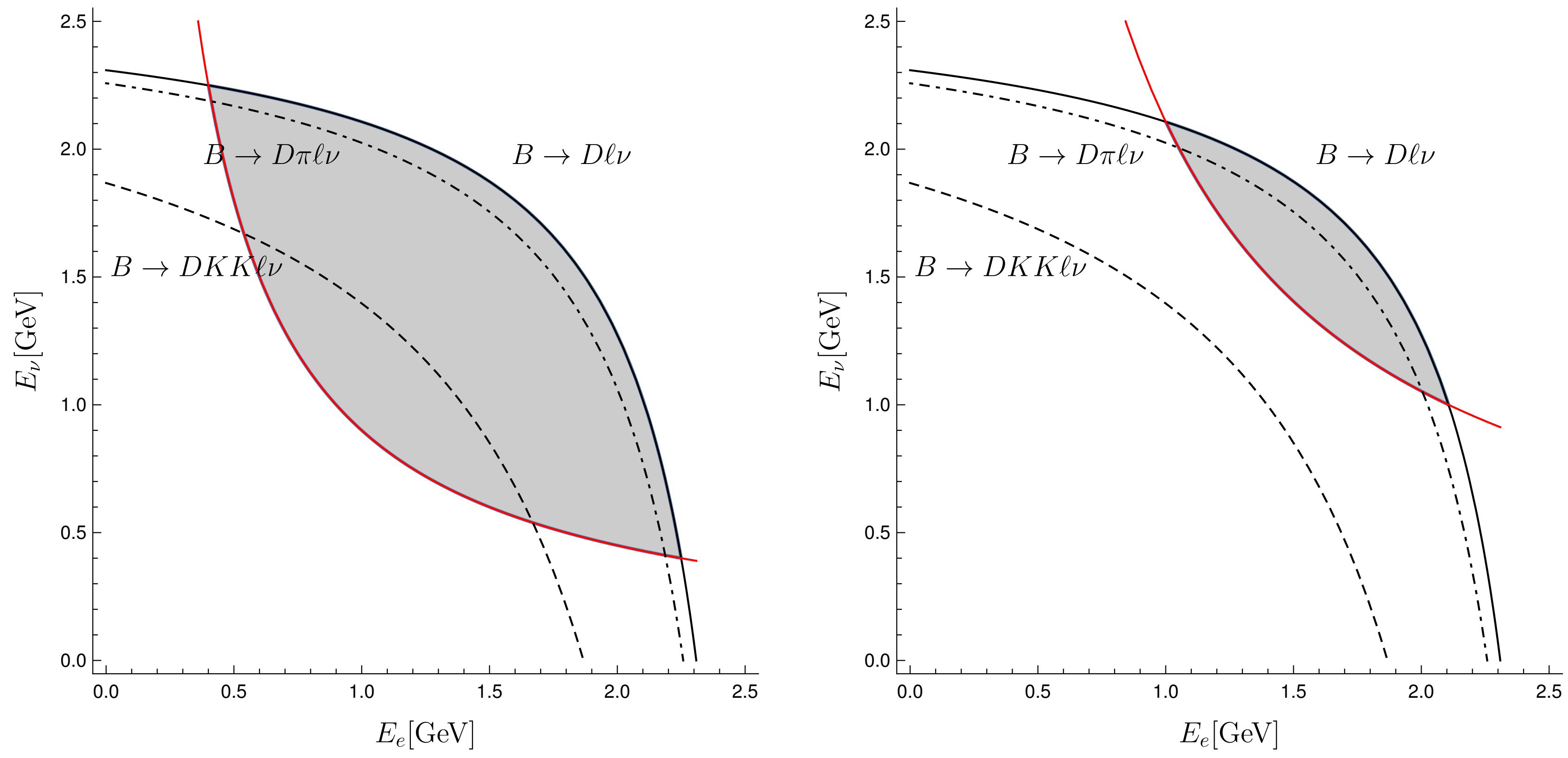






- No theory uncertainties yet

Wrong E_γ spectrum without $B \to X_s\gamma$



M. Fael, T. Mannel, K. Vos, JHEP 2019, Article number: 177 (2019), [arXiv:1812.07472]

New results from Belle II expected this summer; first time $|V_{cb}|$ from q^2-Moments

Inclusive $|V_{cb}|$