Flux shape uncertainties in cross-section data & model comparisons

CEWG Meeting

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Status quo

- Cross section measurements are "flux integrated"
  - Measured cross sections are valid only for a specific neutrino flux

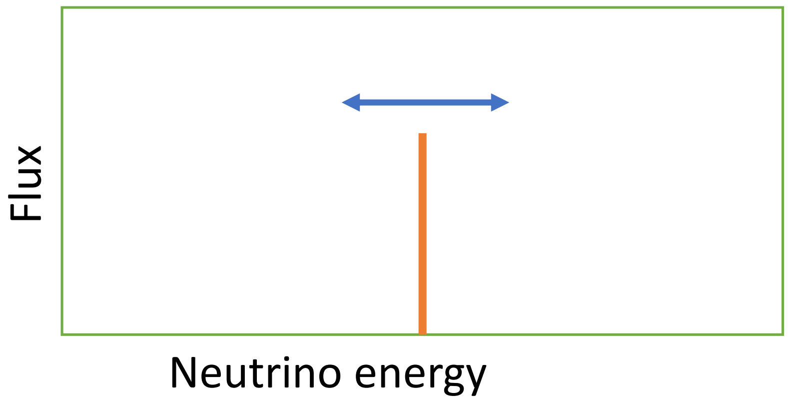
$$\sigma = \frac{N}{\epsilon \Phi T}$$

- Unfolding procedure uses flux uncertainty to evaluate effect on results → part of covariance matrix
- Models use nominal flux for cross-section predictions
  - As far as I know
- χ2 is calculated using nominal model prediction and covariance matrix
  - Assumption: Flux uncertainties are included in covariance matrix

Problem: Flux shape uncertainty

- Claim: Flux shape uncertainty is not (fully) included in the covariance matrix of the unfolded result when doing model comparisons
- Instructive example: "perfect" 1-bin measurement
  - All efficiencies perfectly flat
  - No background
  - Monochromatic neutrino beam with perfectly known intensity
  - Only systematic uncertainty is neutrino energy

$$\sigma = \frac{N}{\epsilon \Phi T}$$

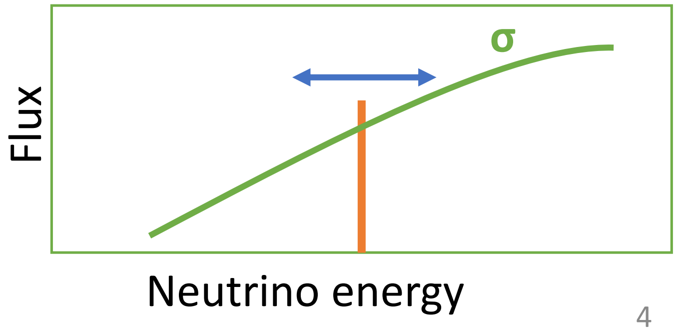


Problem: Flux shape uncertainty

- Variation of beam energy does not vary the result!

$$\sigma = \frac{N}{\epsilon \Phi T} \overset{syst.var.}{=} const$$

  - Systematic uncertainty of result = 0
  - Result is still correct
    - We know the flux integrated cross section very well
    - We just do not know the flux shape very well
- When using only nominal flux for model comparison the flux shape uncertainty is ignored
  - Simple example: cross section proportional to E
  - Should introduce an additional uncertainty proportional to neutrino energy uncertainty



Re-phrasing the problem

- Measurement provides best guess at cross-section integrated over the *real* flux profile
- Model predictions are calculated using the *nominal* flux profile
- Difference between nominal and real flux shapes is not taken into account when comparing the two
  - What we measure and what we compare it to are different things!
- Perfect monochromatic beam example:
  - Measurement: $\sigma(E_{real})$, well known
  - Model: $\sigma(E_{nominal})$, perfectly known
  - $\Delta E = E_{real} - E_{nominal}$, not well known, ignored in comparison

Two possible solutions

- 1st Approach
  - Measure XSEC in (unknown) *real* flux
  - Provide flux & covariance w/ XSEC correlations
  - Propagate shape uncertainties in model predictions
  - Extra work at point of model comparison
  - ☹

$$\sigma = \frac{N(\boldsymbol{\theta}, \boldsymbol{\phi})}{T(\boldsymbol{\theta})\Phi(\boldsymbol{\phi})}$$

- 2nd Approach
  - Measure XSEC in fixed *reference* flux
  - Use varied assumed real flux to extrapolate measurement to reference flux
    - Needs a XSEC model to do so
  - Degrades discrimination power of measurement
    - By covering different E-dependences
  - Also ☹

$$\sigma' = \frac{N(\boldsymbol{\theta}, \boldsymbol{\phi}')}{T(\boldsymbol{\theta})\Phi(\boldsymbol{\phi}')}$$

$$\sigma' = \frac{N(\boldsymbol{\theta}, \boldsymbol{\phi})}{T(\boldsymbol{\theta})\Phi(\boldsymbol{\phi}')} \frac{N_{\mathrm{MC}}(\boldsymbol{\theta}, \boldsymbol{\phi}')}{N_{\mathrm{MC}}(\boldsymbol{\theta}, \boldsymbol{\phi})}$$

What to do and what not

- Comparing a first-approach result with a model at only a single flux (as you would do with a second-approach measurement) is wrong!
  - It ignores flux shape errors
  - Amount of wrongness depends on size of flux shape effect compared to all other uncertainties → non-negligible!
  - Extra ☹ ☹ ☹ ☹
- Tried to figure out how to do first-approach and second-approach measurements with fitted and "classical" unfolding schemes
  - → [arXiv:2009.00552]←
  - Includes "recipes" we hope can be adapted to many different experiments

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Future XSEC releases

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- Pretty confident about the how
  - → [arXiv:2009.00552]←
- Now need to decide on the what
- What kind of results do we want to release?
- What kind of results do our “customers” need/want?
- Four kinds of unfolded results to choose from:
  - Regularised vs unregularized
  - 1st approach vs 2nd approach

4 kinds of XSEC results 1/2

- Regularised
  - Introduces some bias
    - Depending on regularisation strength
  - Allows (some) visual interpretation of results
- Unregularised
  - Unbiased
  - Visual interpretation of result often not reliable
    - Strong bin-to-bin anticorrelations
  - Need to use provided covariance matrix to draw conclusions

4 kinds of XSEC results 2/2

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- 1st approach (XSEC in real flux)
  - No model-dependent flux corrections
  - Allows direct comparison of *measurements* in same flux
  - Model predictions need flux uncertainty
    - E.g. calculate prediction for many flux throws
    - Would need to be correlated to XSEC covariance!
      - At least if there is considerable flux shape error in the result
- 2nd approach (XSEC extrapolated to reference flux)
  - Less work for model builders
    - Need only one prediction in single reference flux
  - Model-dependent flux extrapolation
    - Uses neutrino-energy dependence of the model
  - Reduced statistical power
    - Need to cover different possible energy-dependencies
    - Any model only has one specific dependency
    - The others add “unnecessary” contributions to the covariance

4 kinds of XSEC results summary

	1st approach	2nd approach
Unregularised	Least bias Best power No chi-by-eye Most difficult to use and interpret	No regularisation bias No chi-by-eye Flux extrapolation bias Diminished power
Regularised	Good power No extrapolation bias Regularisation bias Models need flux uncertainty	Easiest to use and interpret (by eye) Most bias Least power

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How almost all results are treated

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Most old results

How almost all results are treated

Most of our recent results

- What do we want to provide in the future?
  - More than 2 versions of same measurement too confusing?
  - One pretty and one technical result? R2 and U1?
    - U1 too complicated? U2 instead?

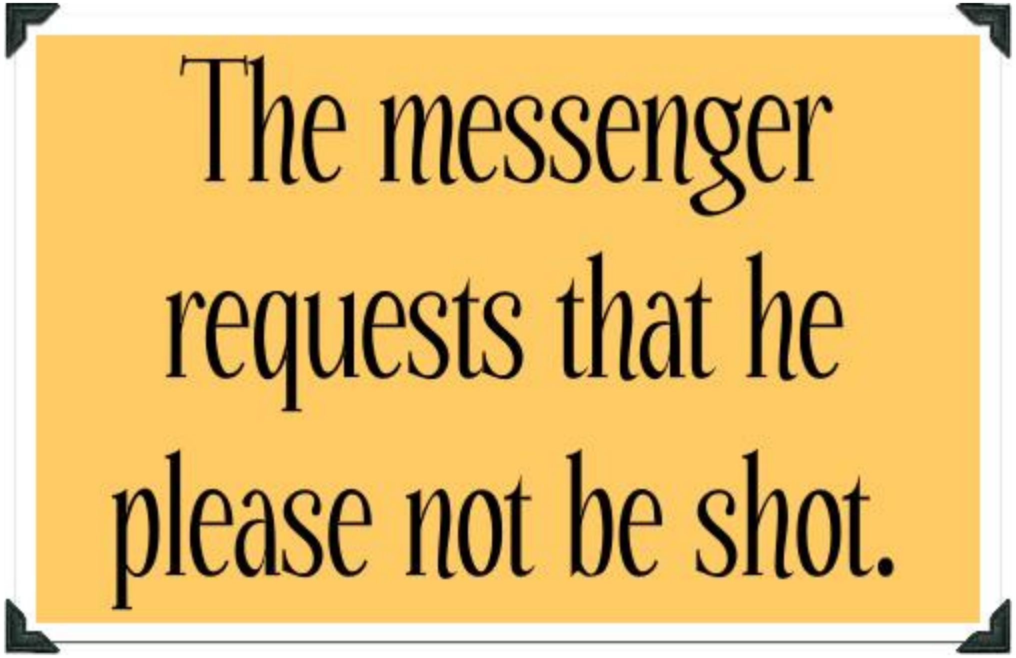
Flux releases

- Both approaches need a corresponding flux release
  - Either “best fit” parameters or nominal
- 1st approach measurements also need a public flux covariance matrix
  - Either “post fit” or “pre fit” (nominal)
  - Needs to be correlated with XSEC result!
    - Each XSEC result needs to provide its own XSEC-flux covariance matrix

Reality check

- “All our model comparisons are wrong”
  - But *how* wrong?
- Flux shape has some influence on result
  - Efficiencies are not perfectly flat
  - BG depends on flux shape
  - Adds “something” to covariance matrix
- Flux shape is not dominant systematic (probably?)
  - Has flux shape effect on model predictions been tested?
- Reality somewhere between “effect of flux shape is completely negligible” and “our $\chi 2$ are completely wrong”
  - How do we know where we are?
  - See Stephen’s presentation

Questions?



Comments?

Backup

Another instructive example

- Perfect two-bin measurement
  - 2 flux bins, 2 corresponding signal bins, no smearing, no inefficiency
  - Only flux and template weights

$$N_j = N_j^0 \phi_j c_j \qquad\qquad j = 1,2$$

- Fitter will adjust weights to make N_j fit the data
  - Every change in flux weight can be compensated by template weights → flux and template weights are anti-correlated
  - Constraint of weights comes only from flux prior
- Resulting best fit point and covariance describe what parameter combinations are compatible with the data

Current XSEC extraction

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- Flux integrated XSEC extracted by drawing from post-fit parameters and calculating

$$\sigma_j = \frac{N_j}{T\Phi} = \frac{N_j^0 \phi_j c_j}{T(\Phi_1^0 \phi_1 + \Phi_2^0 \phi_2)}$$

  - Each throw corresponds to one possible reality or universe
- Number of events N_j is almost constant by construction
- If total flux Φ is also constant, error on XSEC is small(→0)
  - σ_j ≔ Flux integrated XSEC in real flux
  - Flux and template weight uncertainties cancel
  - Don't care what real flux actually is
  - Correct result, but cannot be compared to model without model flux variations
  - Problem: How to not double count flux variations?

Change what we measure

- $\sigma_j \to \sigma_j' \coloneqq$ Flux integrated XSEC in best fit flux
  - Cross section in specific flux "once removed from reality"
  - Do care about what the real flux actually is
  - Would allow direct comparisons of models @ best fit flux
- For each throw (possible reality) calculate XSEC at that flux

$$\sigma_j' = \frac{N_j'}{\Phi'} = \frac{N_j^0 \phi_j' c_j}{\Phi_1^0 \phi_1' + \Phi_2^0 \phi_2'} = \sigma_j \frac{\Phi}{\Phi'} \frac{\phi_j'}{\phi_j}$$

  - Ignore thrown flux weights ϕ
  - Set flux weight in calculation to best fit value ϕ'
- Best fit point is identical by definition $(\phi' = \hat{\phi})$

$$\hat{\sigma}_j' = \hat{\sigma}_j$$

- Covariance is different, as flux and template weights no longer cancel

$$cov(\sigma') \neq cov(\sigma)$$

Could go further

- $\sigma_j' \to \sigma_j'' :=$ Flux integrated XSEC in nominal flux
  - Cross section in specific flux "twice removed from reality"
  - Do care about what the real flux actually is
  - Would allow direct comparisons of models @ nominal flux
- For each throw (possible reality) calculate XSEC at that flux

$$\sigma_j'' = \frac{N_j''}{\Phi''} = \frac{N_j^0 \phi_j'' c_j}{\Phi_1^0 \phi_1'' + \Phi_2^0 \phi_2''} = \sigma_j \frac{\Phi}{\Phi''} \frac{\phi_j''}{\phi_j}$$

  - Ignore thrown flux weights ϕ
  - Set flux weight in calculation to nominal value ϕ''
- Best fit point is different from other results ($\phi'' \neq \hat{\phi}$)

$$\hat{\sigma}_j'' \neq \hat{\sigma}_j' = \hat{\sigma}_j$$

- Covariance is different

$$cov(\sigma'') \neq cov(\sigma') \neq cov(\sigma)$$

- Are fit results for parameters still valid there?

Summary

- Fitter is doing what it is supposed to:
  - Finding parameter sets that are compatible with reality
- Each post-fit throw of the fit parameters corresponds to one possible/plausible reality
- Currently we calculate the flux integrated XSEC as it would be in each reality, i.e. with that reality's flux
  - Good for finding the real flux integrated XSEC
  - Bad for comparing with the flux integrated XSEC at a specific flux
- We should calculate the flux integrated XSEC at a specific flux, i.e. extrapolate from those realities' fluxes to the specific one
  - Can be done by using specific flux parameters in XSEC calculation
- That specific flux should probably be the best fit flux, as that is the point where the parameters and covariance are valid

Subtleties

- Fixing the flux parameters in the XSEC calculation is not the same as saying "we assume this to be the real flux"
- Former:
  - We are as ignorant about the real flux as ever
  - We want to calculate the XSEC as it would be in the specific flux
  - Base for extrapolation are the possible fluxes in each throw
- Latter:
  - We assume we know our real flux
  - Would also fix the template weights as they are anticorrelated
  - Nothing gained in terms of flux error

What about other measurements?

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- The conceptual difference between the presented XSEC definitions is subtle
  - Can be easily confused when not being very careful
- How do we now what our old measurements did?
- How can we know what other experiments did?
- First rule of thumb test:
  - Do they calculate the XSEC using the varied flux in each toy/throw/reality/universe directly?
    → Probably affected by this
  - Do they extrapolate from the varied flux to a specific flux?
    → Probably not affected by this

More realism

- In a realistic measurement there is smearing and other systematic parameters
- Average analysis bin weight becomes function of underlying parameters (detector, model, flux, …)

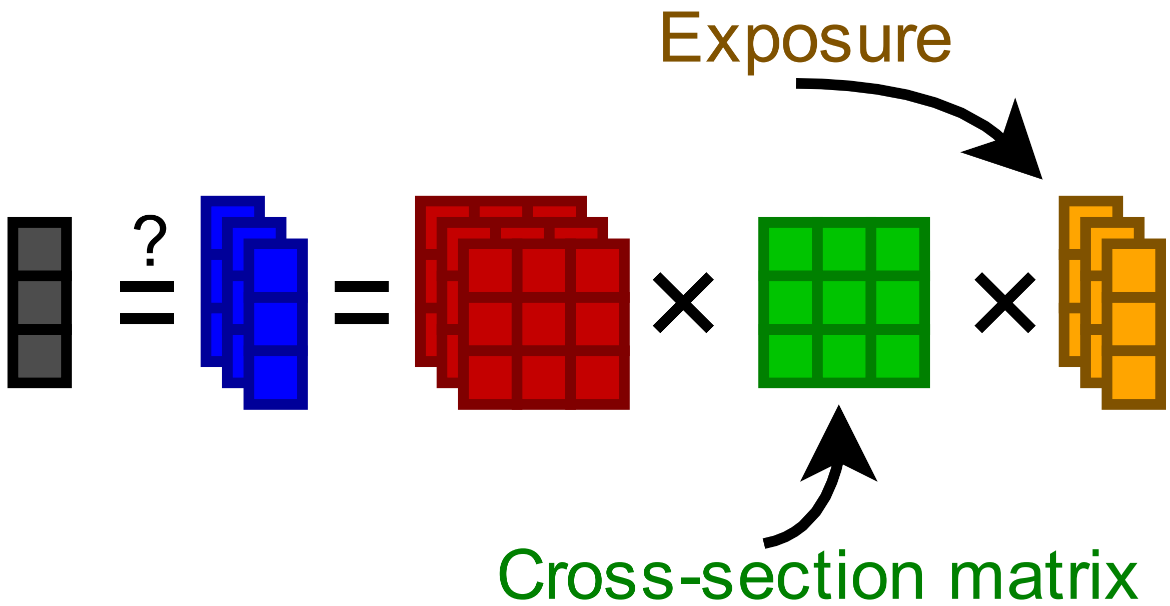
$$N_j = N_j^0 w_j(d_a, m_b, \phi_c) c_j$$

- Can no longer scale results with flux weights only

$$\sigma_j = \frac{N_j}{\epsilon_j T \Phi} = \frac{N_j^0 w_j(d_a, m_b, \phi_c) c_j}{\epsilon_j(d_a, m_b, \phi_c) T \Phi(\phi_c)}$$

$$\sigma'_j = \frac{N'_j}{\epsilon'_j T \Phi'} = \frac{N_j^0 w'_j(d_a, m_b, \phi'_c) c_j}{\epsilon'(d_a, m_b, \phi'_c) T \Phi'(\phi'_c)} = \sigma_j \frac{\Phi}{\Phi'} \frac{w'_j}{w_j} \frac{\epsilon_j}{\epsilon'_j}$$

Flux forward folding



- Model predicts cross section for each flux bin
- Provide set of flux exposures according to uncertainties
  - Exposure = flux × time × target mass
- Flux and detector uncertainties can be correlated
  - Make one response matrix correspond to one exposure vector