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Estimating the size of the "missing uncertainty" from the flux shape

Stephen Dolan

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Overview

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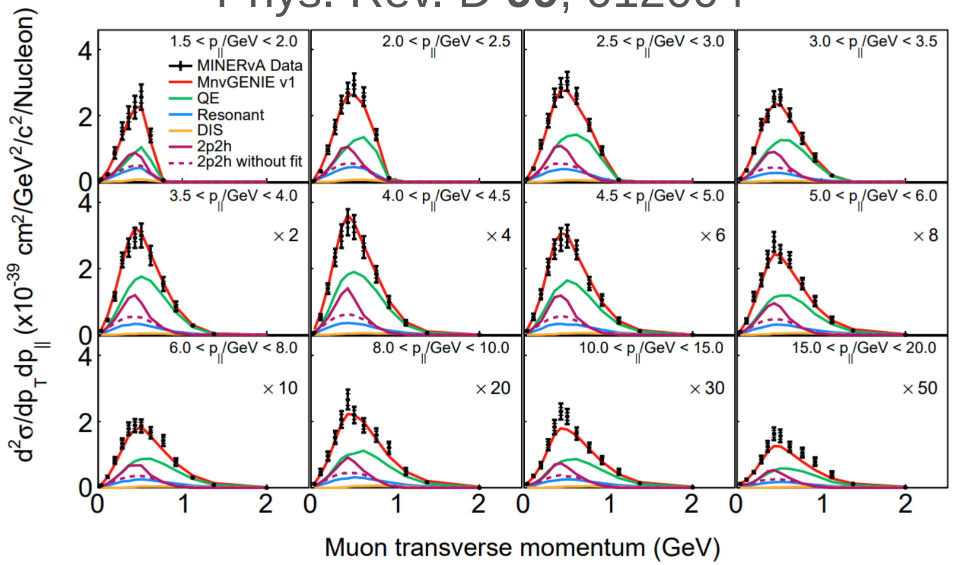
- Really we should be folding the model through an ensemble of possible real fluxes and considering the resultant spread as a systematic uncertainty.
- And really we also need to worry about how the reported cross-section uncertainty on the result correlates with the flux shape, but I'll largely ignore these until the end of the talk …

So let's do it!

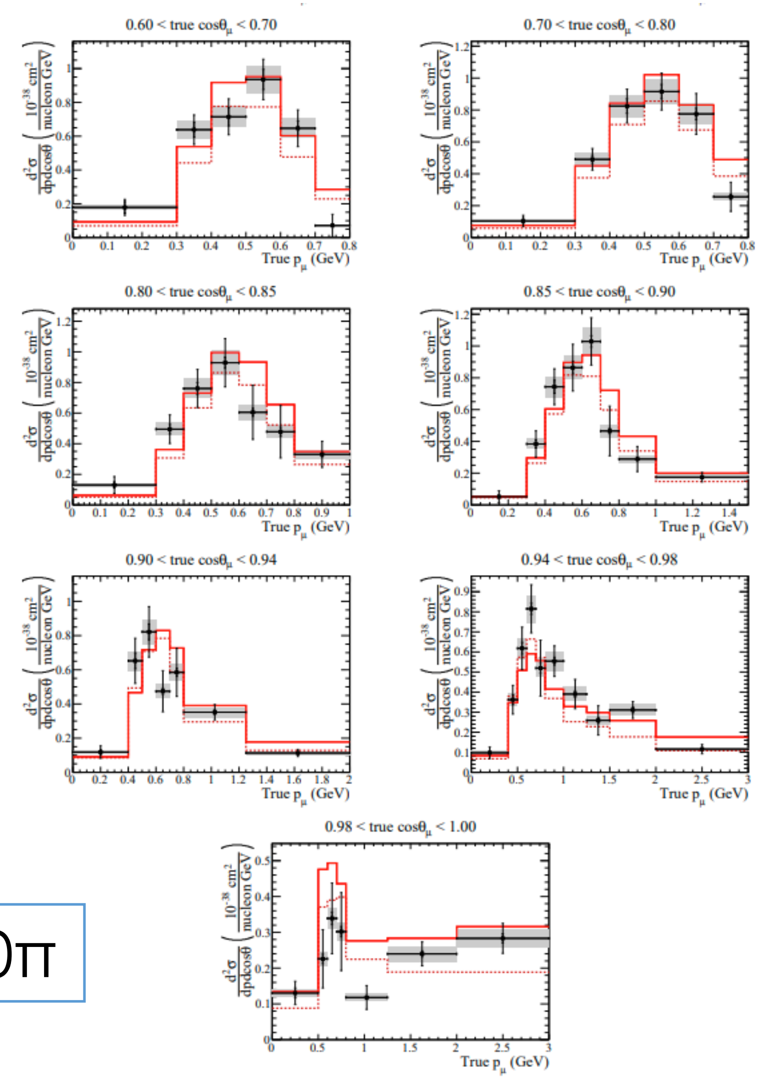
Choose some measurements:

Phys. Rev. D **99**, 012004



MINERvA 2019 CCQE-like

Phys. Rev. D **93**, 112012



T2K 2016 CC0π

So let's do it!

Choose some model

GENIE v3.00.06 CMC G18_10b_00_000

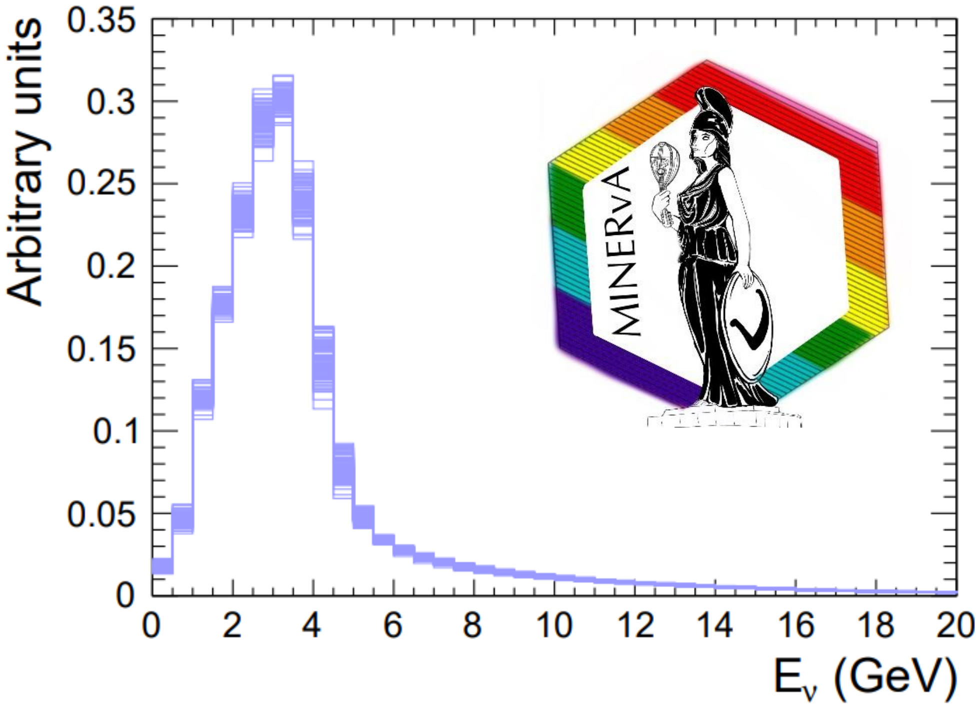
GENIE's more "theory-driven" model out of the box (no GENIE tuning)

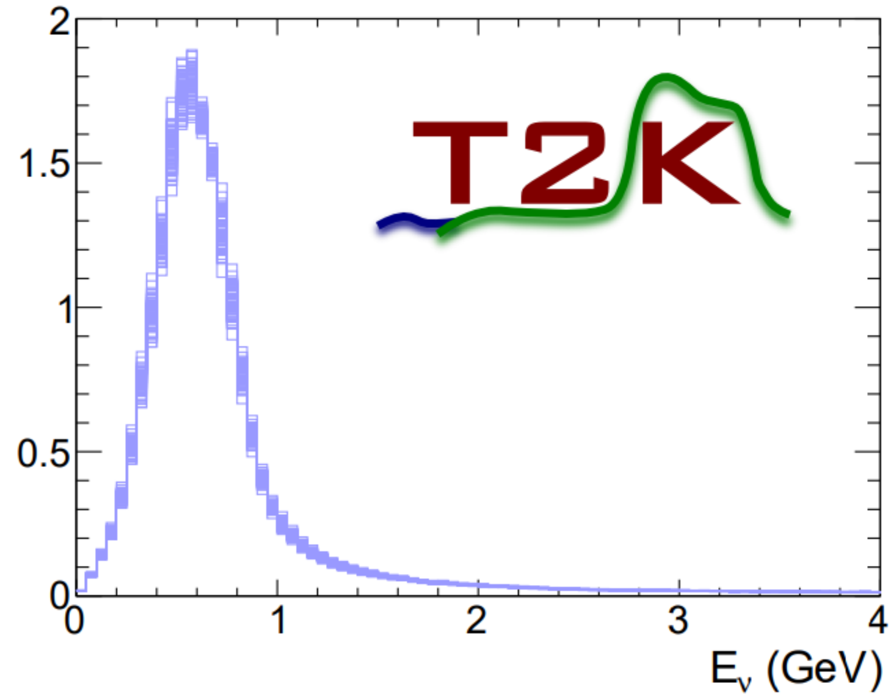
- 1p1h: Valencia, LFG
- 2p2h: Valencia
- RES: Berger Seghal
- FSI: hN (cascade)

So let's do it!

Build some flux shapes:

- Throw flux toys from the covariance matrices provided by the experiments, construct flux shapes.





- There certainly is a notable shape component to the flux uncertainties, but the mean flux remains fairly stable:
  - $\mathrm{std}(< E_\nu >)_{MINER\nu A} \sim 6.5$ MeV, $\mathrm{std}(< E_\nu >)_{T2K} \sim 5.5$ MeV

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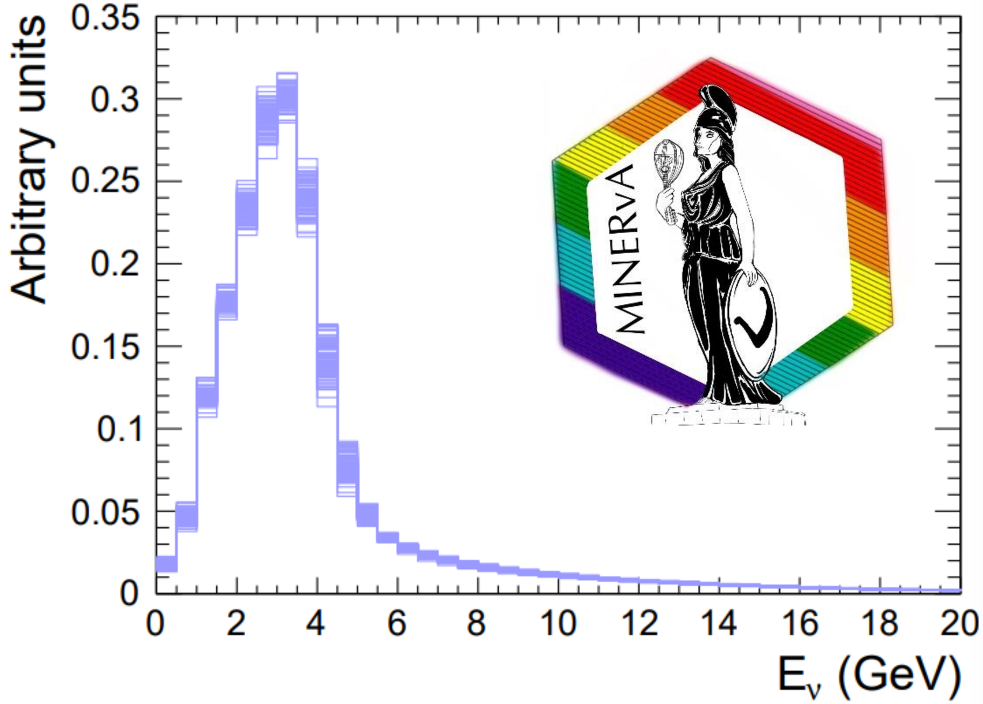

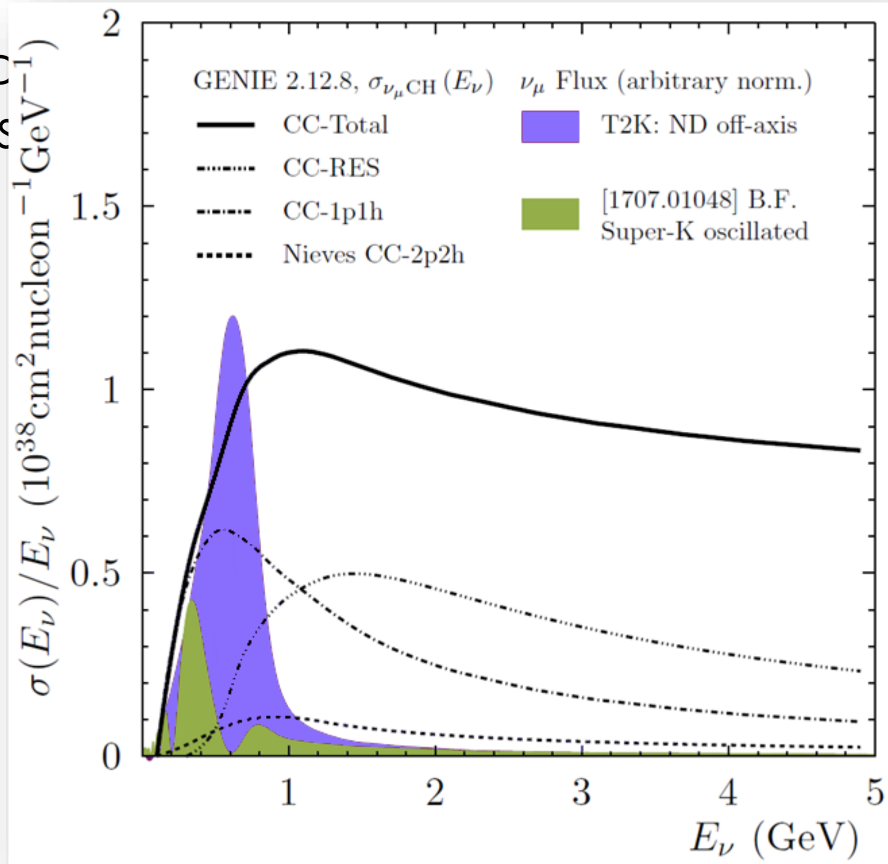

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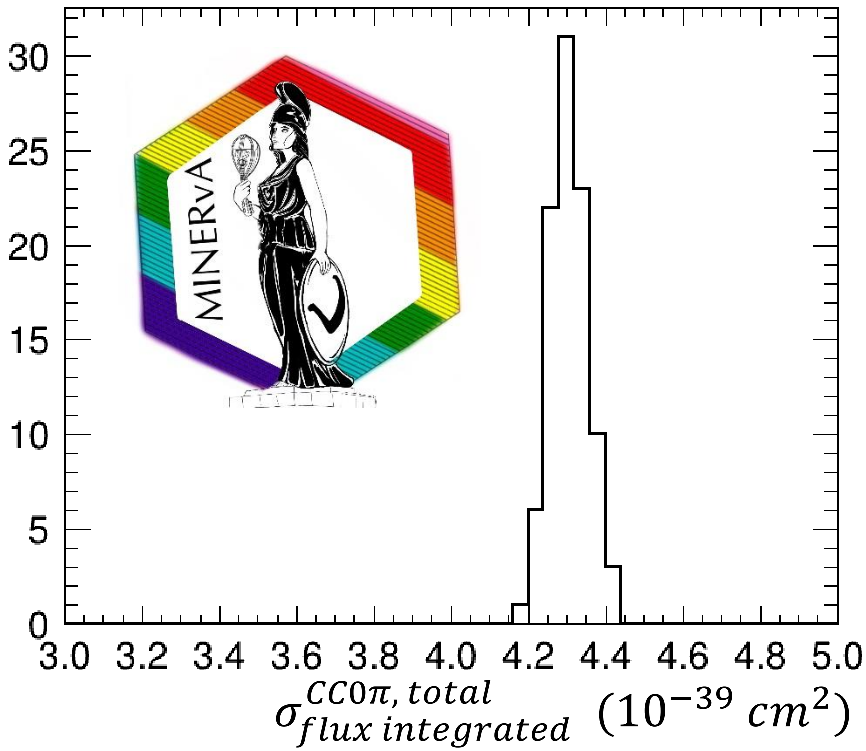
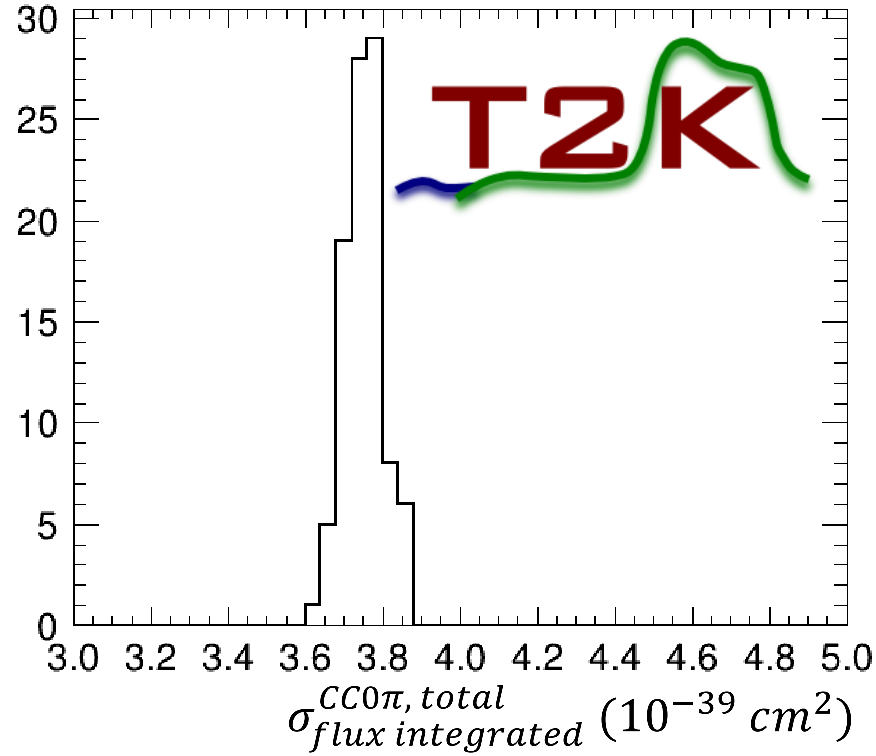
So let's do it!

Run the model through each of the fluxes

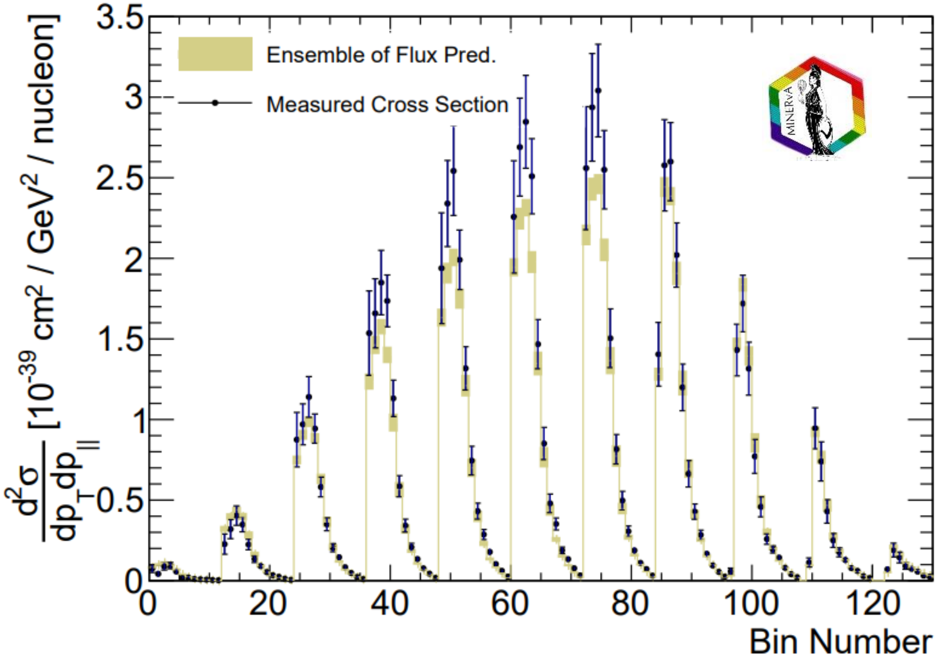


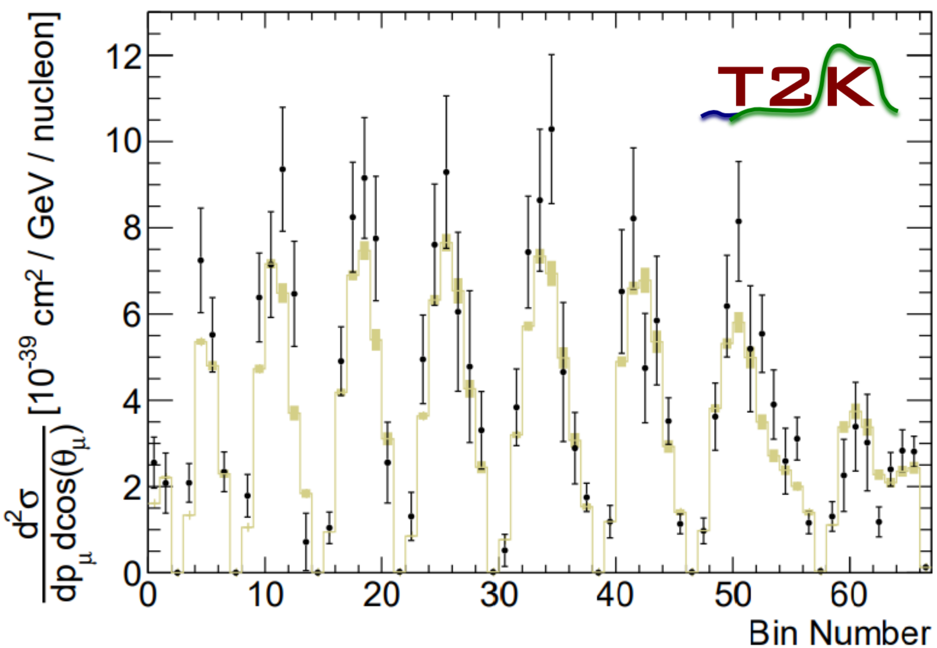


- The impact on the flux integrated cross section is small, not changing by much more than 1% across the ensemble of model predictions

Comparison of xsec predictions

Compute differential xsec for each flux folded model





- The cross section errors can be compared to the size of the standard deviation of the ensemble of predictions in each bin
- This is the spread of the model predictions that should be considered when comparing with a *first approach* measurement **if correlations between the flux shape and the cross section result are ignored** (more on this later)

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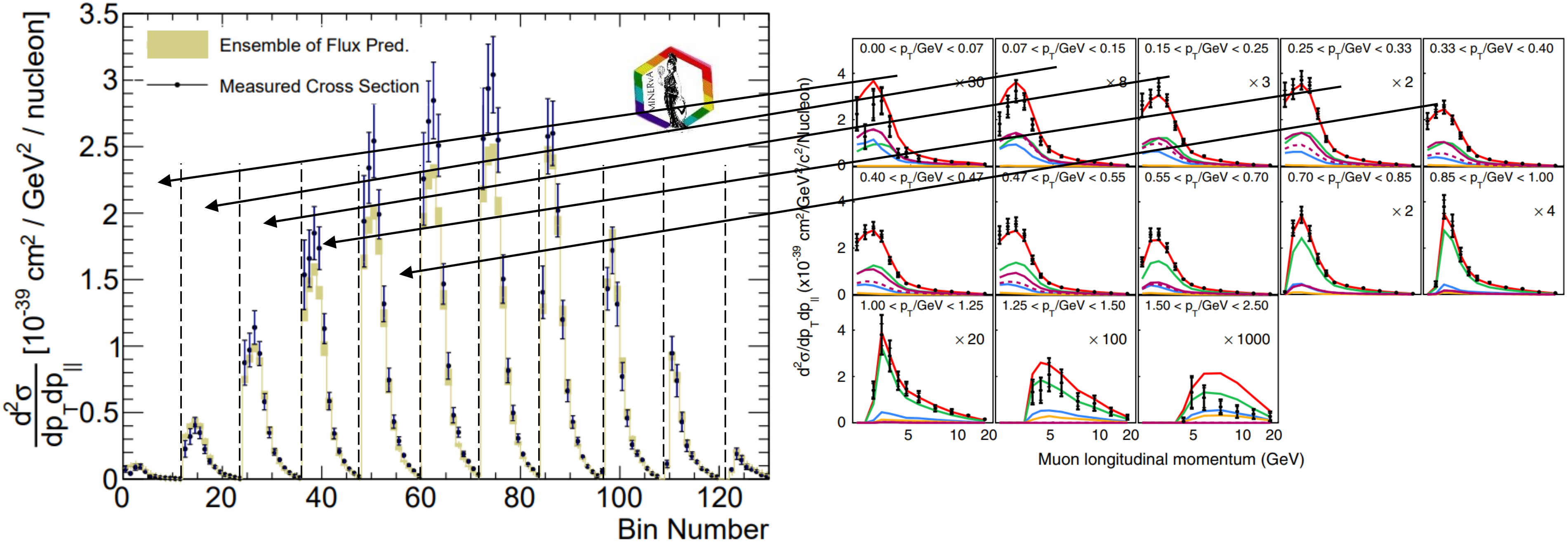

- Ordered in increasing *slices* of p_T, where each bin increases in $p_{||}$

Comparison of xsec predictions

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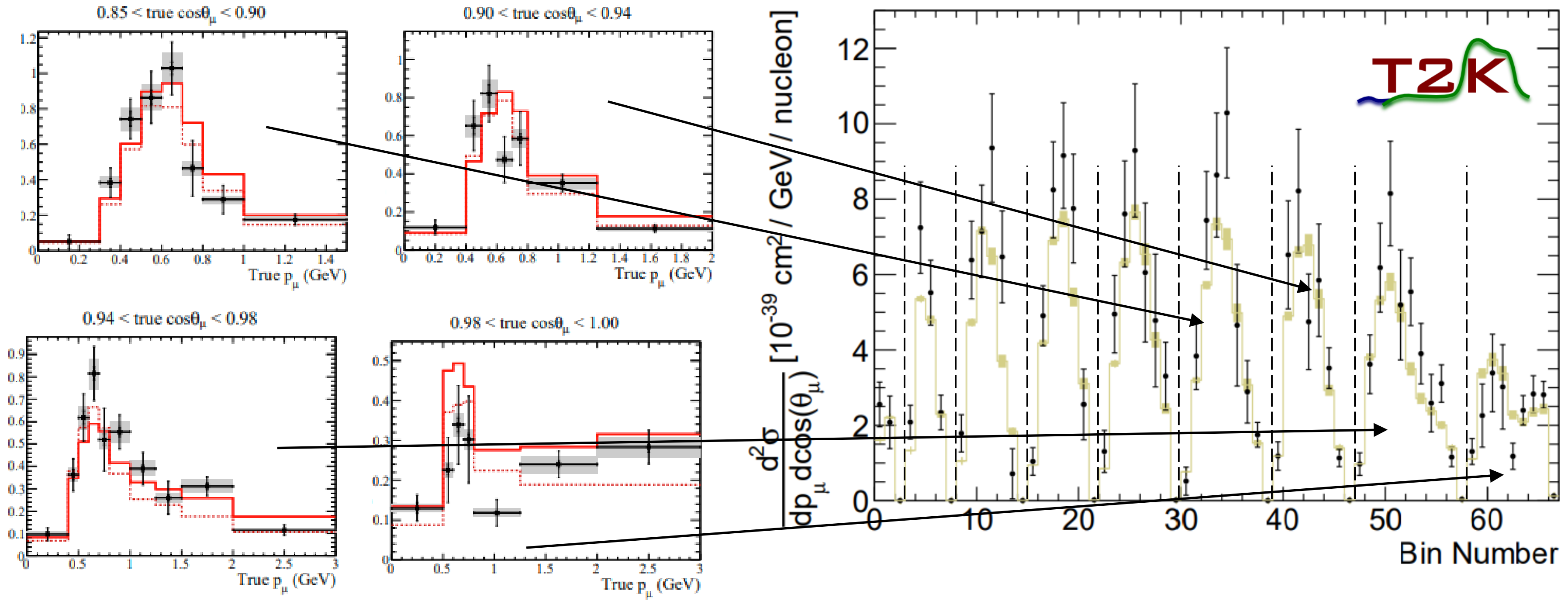
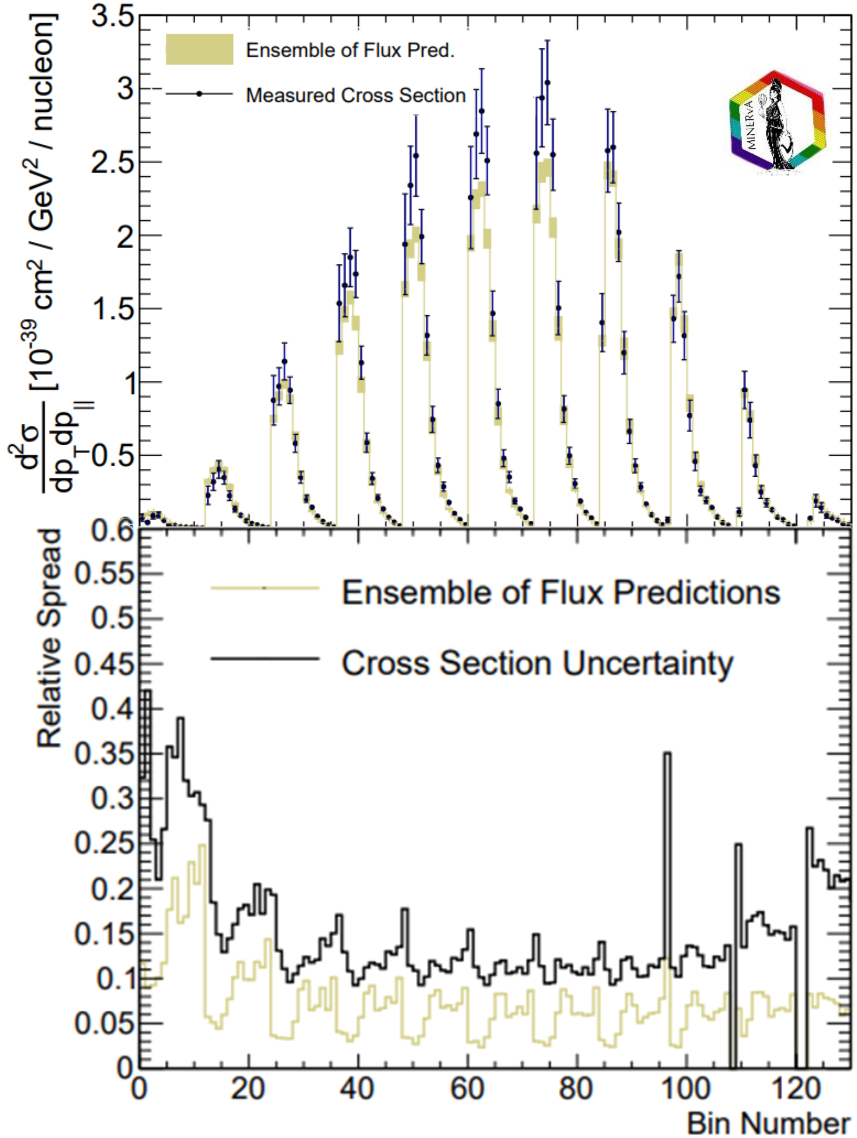
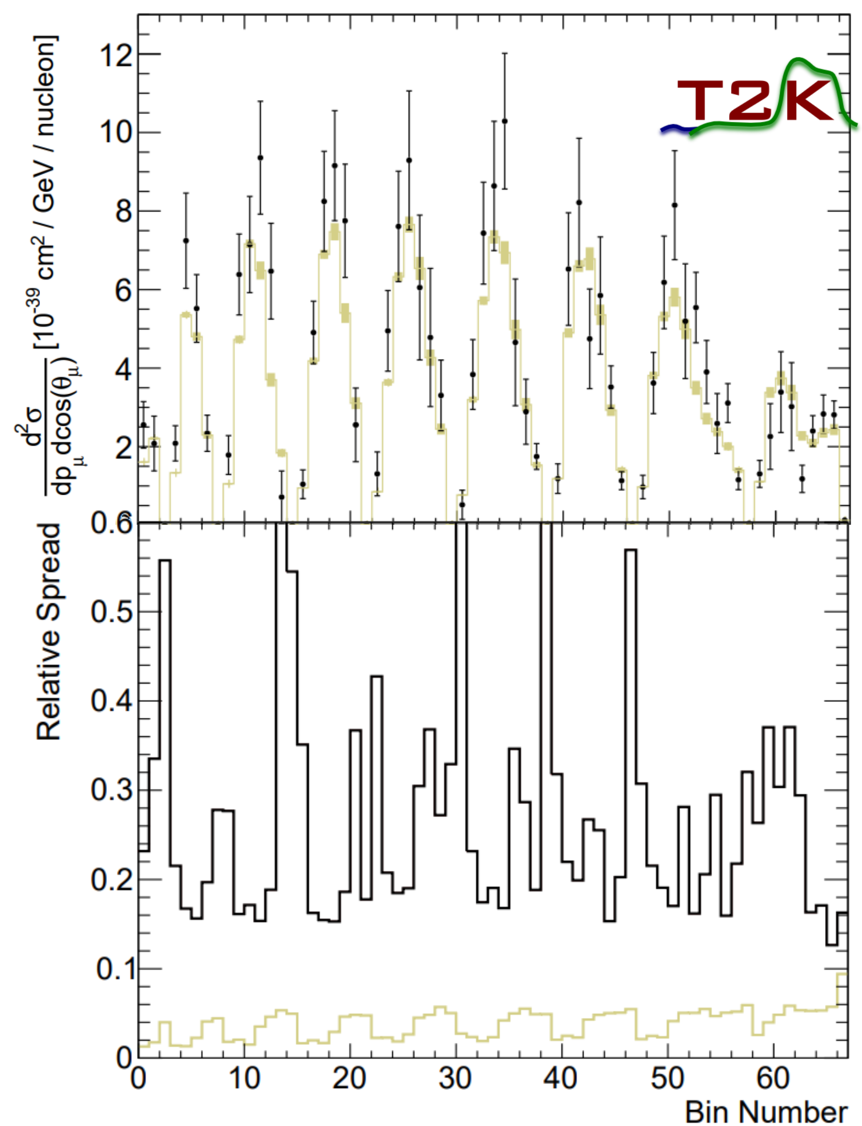


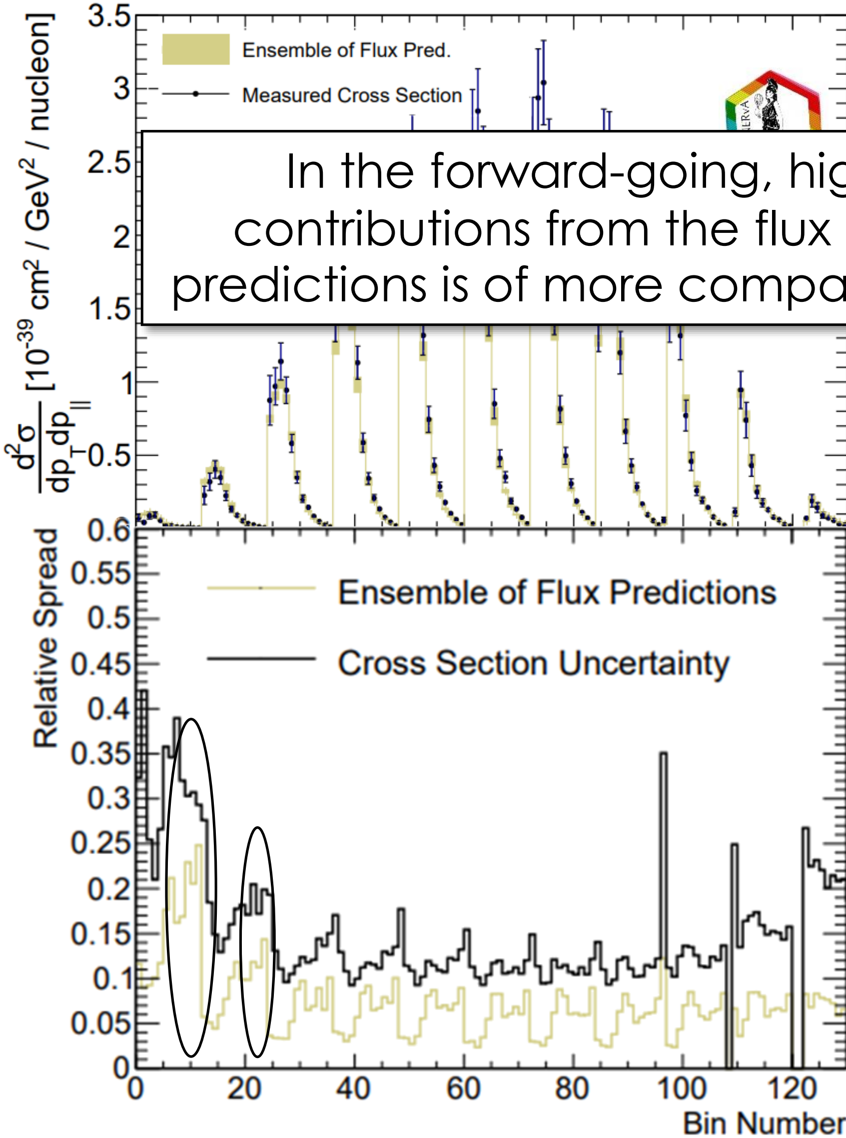
- Ordered in increasing *slices* of $\cos\theta_\mu$, where each bin increases in p_μ

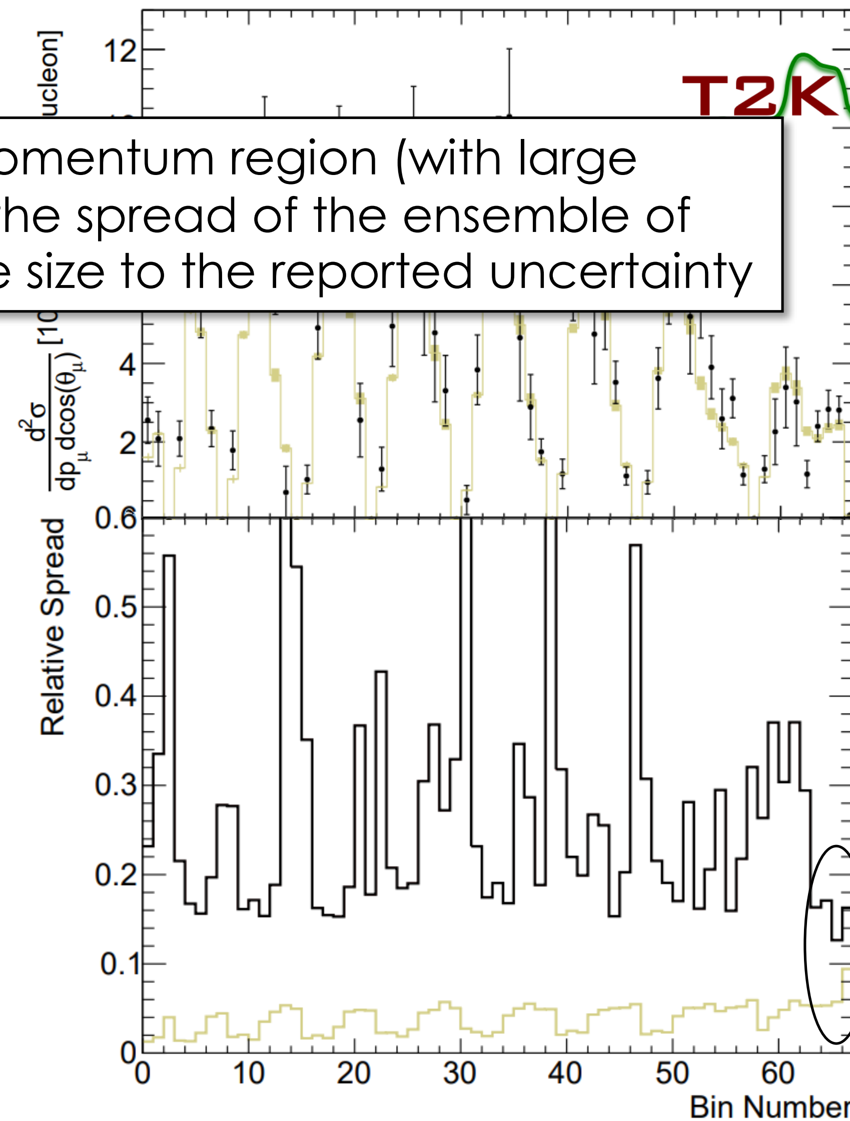
Comparison of xsec predictions





Comparison of xsec predictions

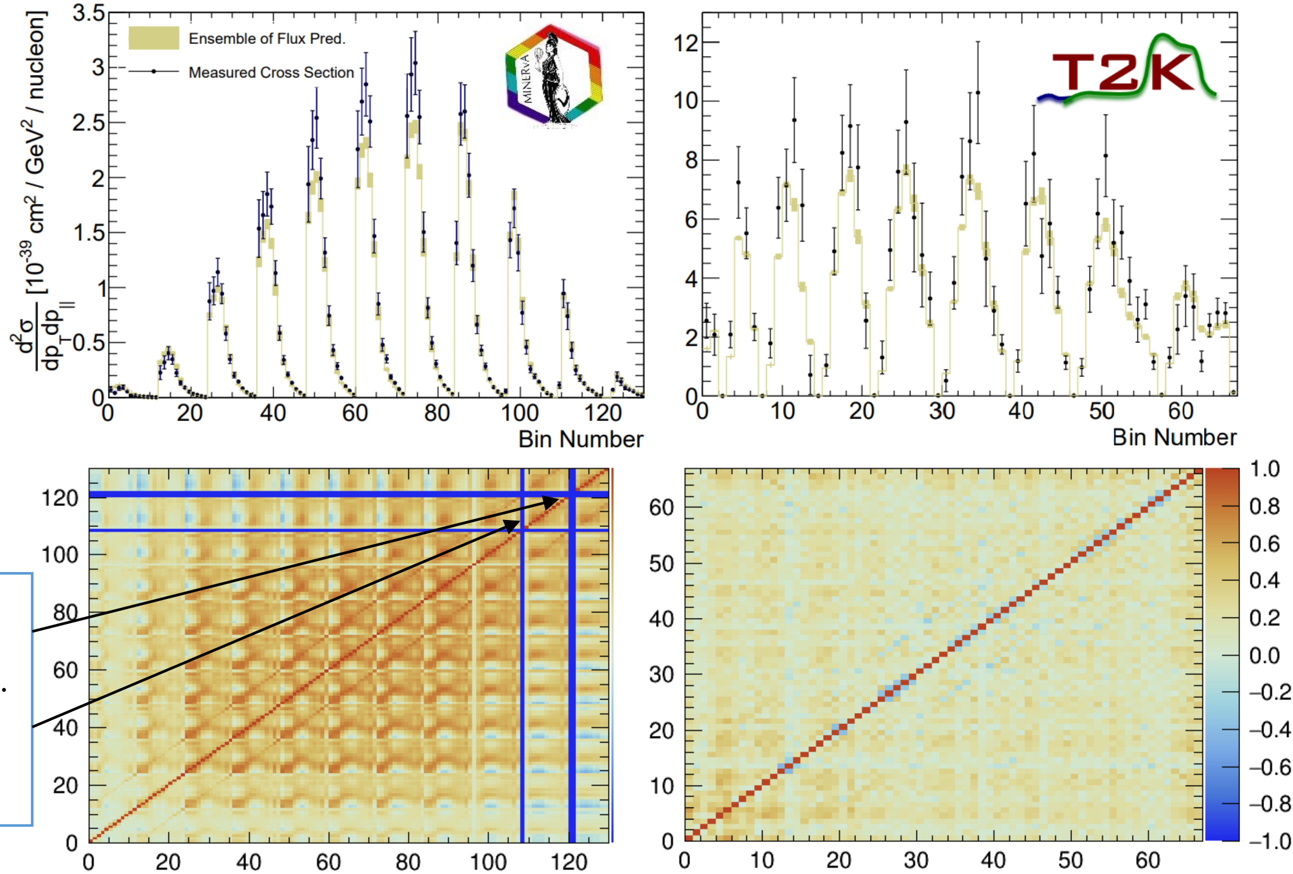




In the forward-going, high momentum region (with large contributions from the flux tail) the spread of the ensemble of predictions is of more comparable size to the reported uncertainty

Correlations between bins

- We shouldn't really compare the spread of the ensemble of flux shape varied predictions with the reported xsec uncertainty by eye
- Really each xsec bin is correlated with all the others, as reported by the published covariance matrices. The error bars do not fully describe the uncertainty!
- Even though the overall scale of the flux-shape variation is smaller, it's possible that the effect is not "covered" by the xsec uncertainties.



N.B. MINERvA's covariance has zero uncertainty in a few bins (which are outside of their angular acceptance). We follow MINERvA's approach and handle matrix inversion with an SVD approach

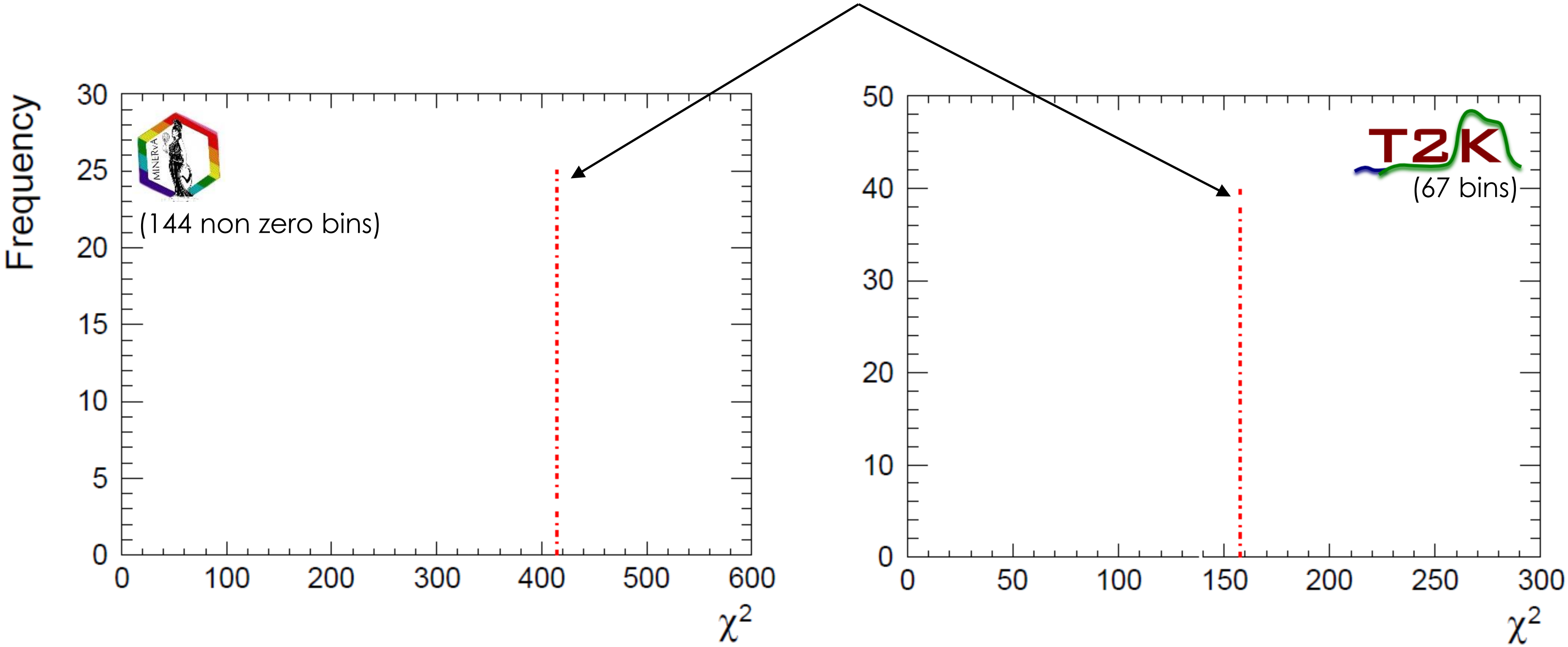
Calculating some χ^2

- Let's calculate something more quantitative:

$$\chi^2 = (\boldsymbol{\sigma}_{model} - \boldsymbol{\sigma}_{result})^T S^{-1}_{result}(\boldsymbol{\sigma}_{model} - \boldsymbol{\sigma}_{result})$$

What we usually do

- Calculate only the χ^2 for the model folded through the nominal flux



Calculating some χ^2

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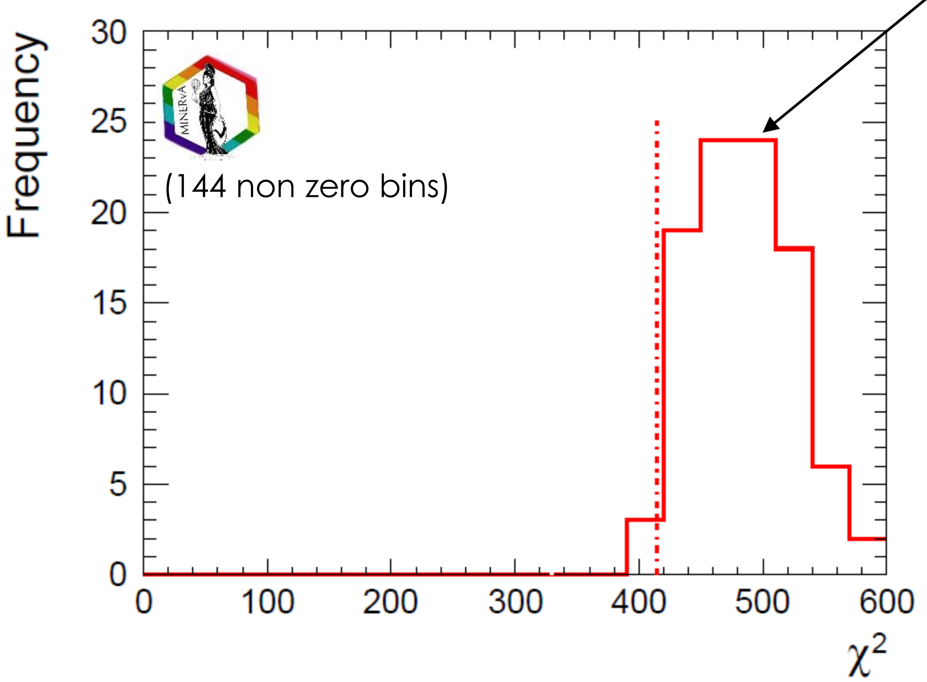
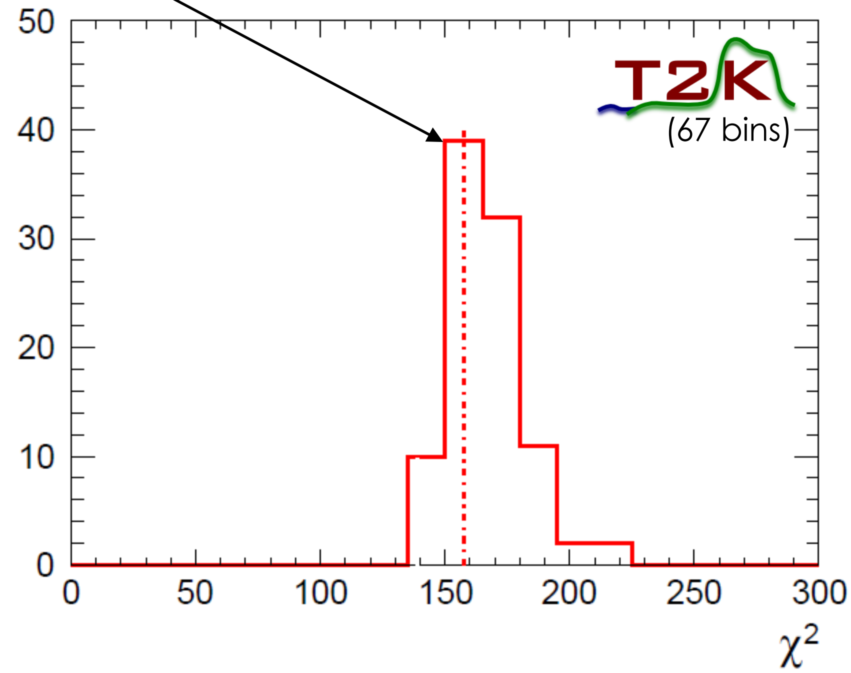
What we should do

- Calculate the χ^2 for the model folded through each of the flux shape variations



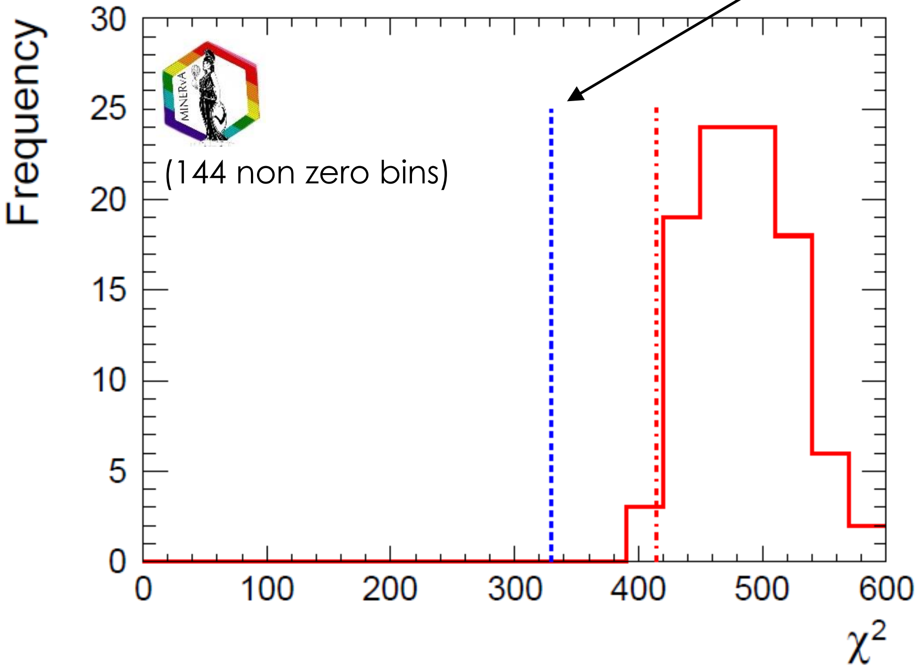


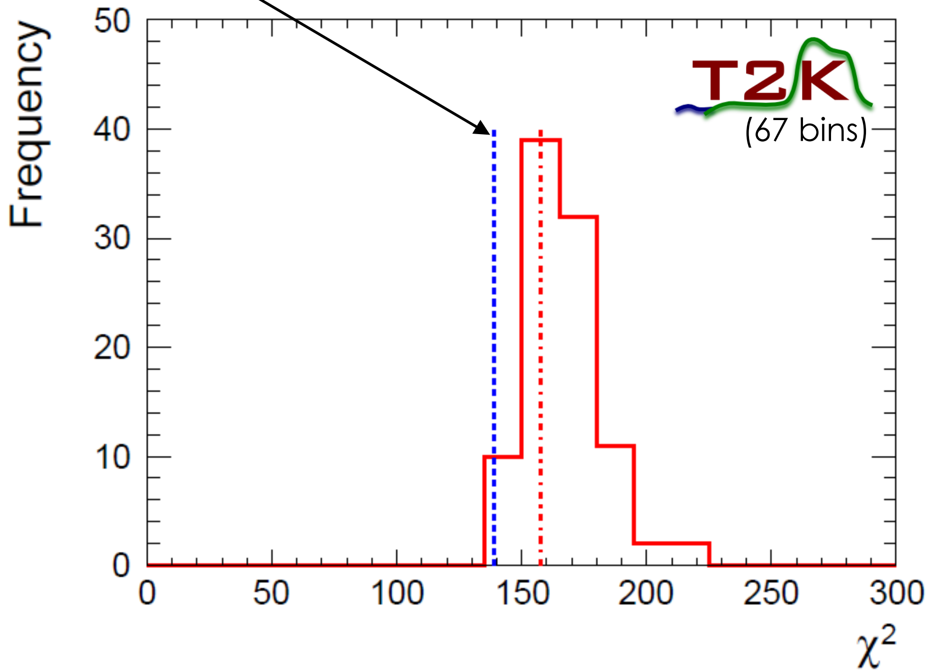
Calculating some χ^2

If the reported xsec result is not correlated with the flux shape

- We can simply include the flux shape error by adding the covariance from the ensemble of flux shape predictions to the reported covariance

$$\chi^2 = (\boldsymbol{\sigma}_{model} - \boldsymbol{\sigma}_{result})^T S^{-1}_{combined} (\boldsymbol{\sigma}_{model} - \boldsymbol{\sigma}_{result})$$



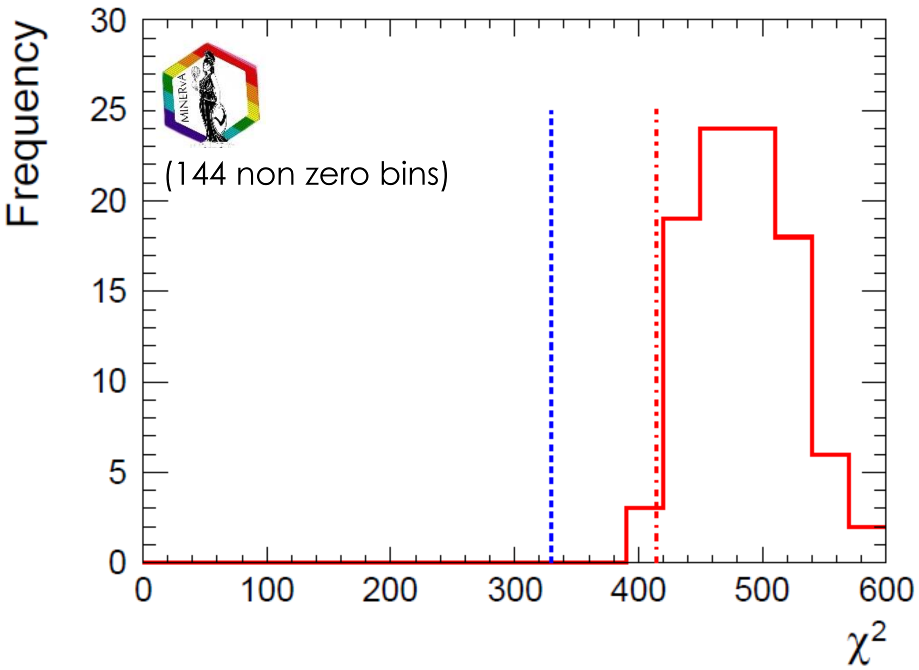


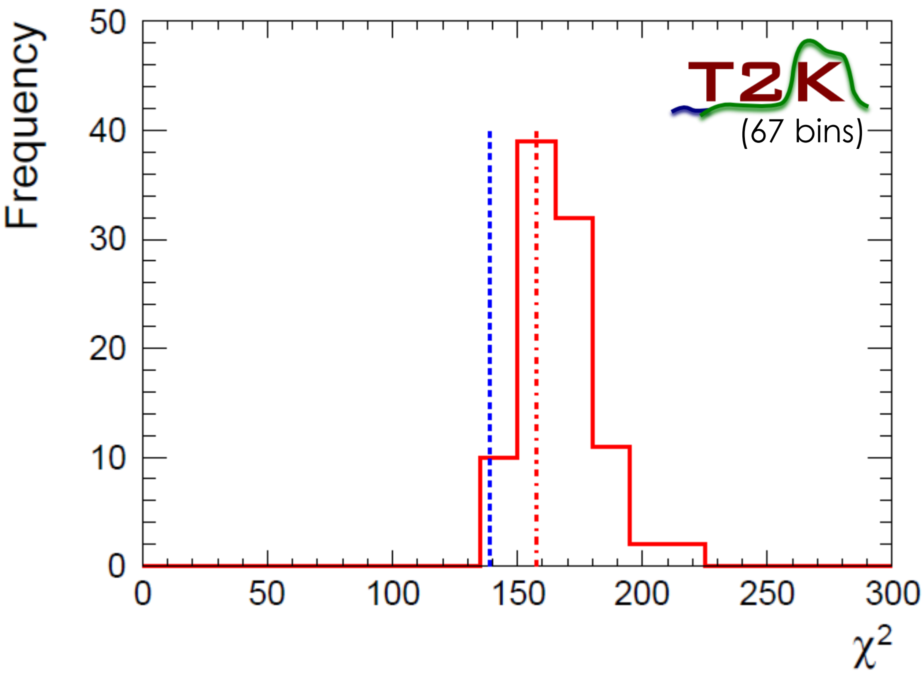
Calculating some χ^2

But what did we actually learn?

- The approximate (uncorrelated) inclusion of the flux shape uncertainty does change the reported χ^2.

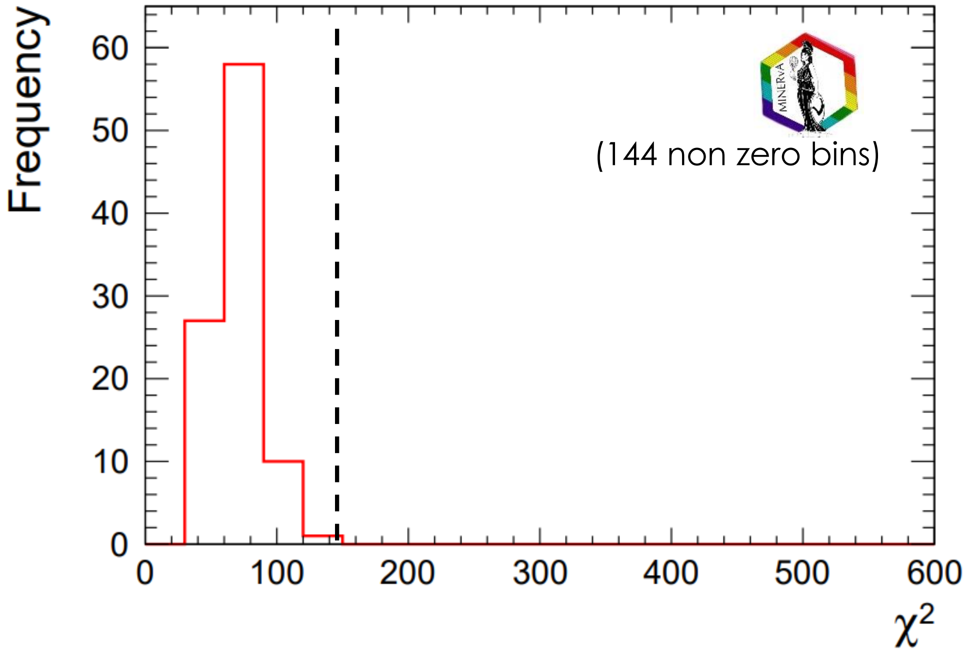
- But, the model's description of the data just goes from terrible to ever-so-slightly-less terrible! The physics conclusions remain practically unchanged.

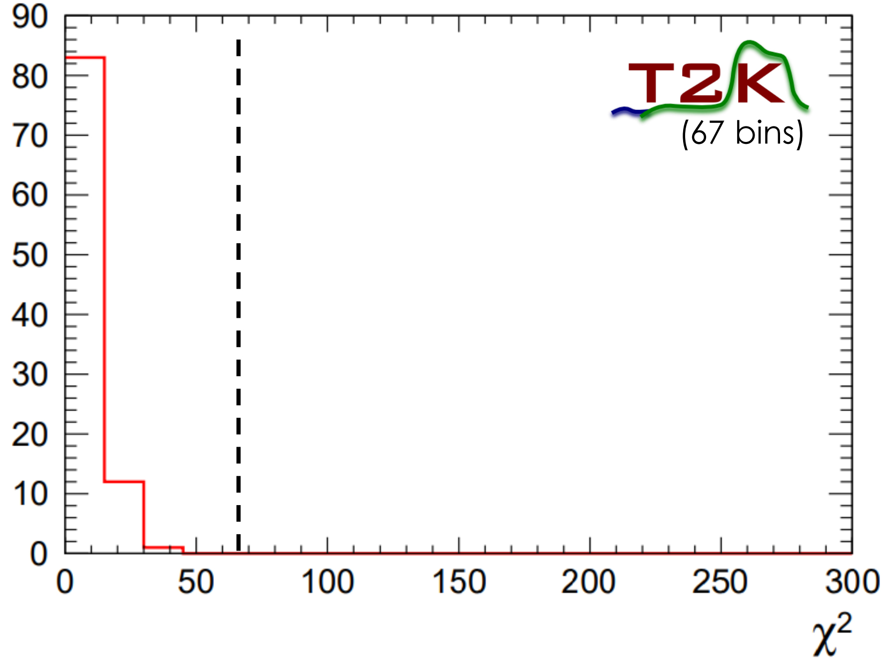




Now some make-believe

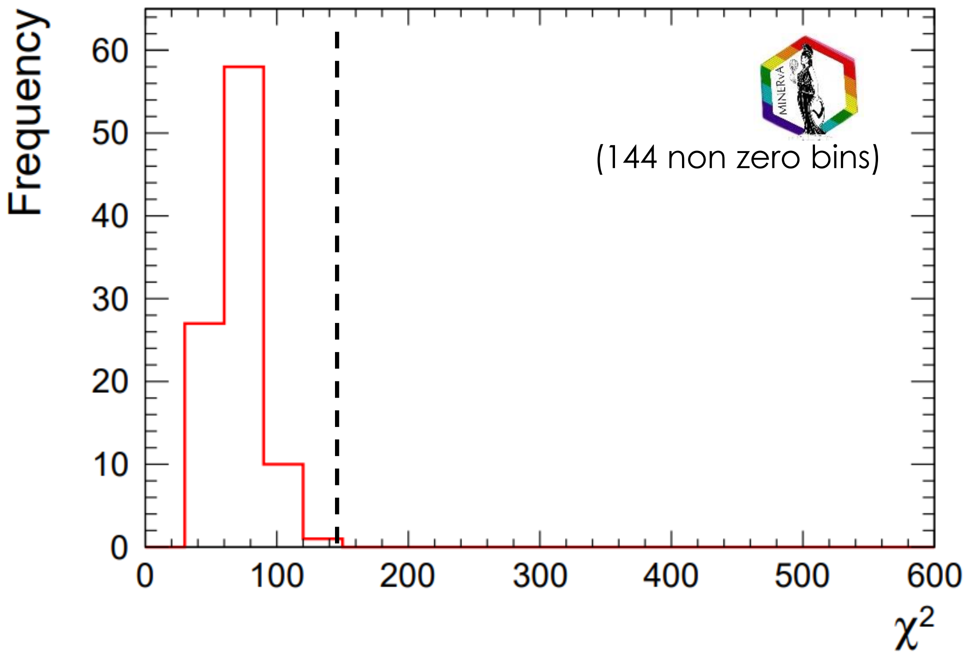
- Let's imagine a world where T2K and MINERvA had measured exactly the GENIE nominal prediction (the one using the nominal flux)
- And for simplicity let's say the reported covariance was unchanged*
- Now the spread of the predictions from the different flux shapes more directly tells us how "covered" they are by existing uncertainties**

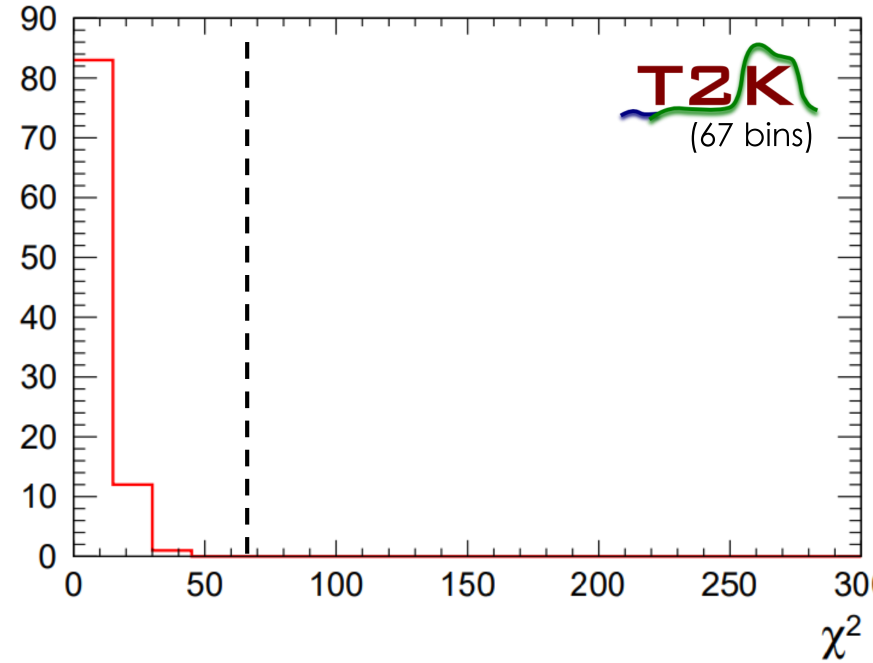




* This is clearly unrealistic, but the change is unlikely to be large enough to prevent this serving as an illustrative example
** Remember, we're also still neglecting the correlations between the reported uncertainties and the flux shape

Size of the missing uncertainty





- Keeping the caveats of this study in mind, it seems that the extra uncertainty from the flux shape variation is small relative to existing uncertainties, but non-negligible.

- As we gather more statistics and further mitigate systematics, the neglection of this uncertainty could become increasingly important.

Including the flux shape correlations

- One important caveat of the studies shown is that we have assumed there are no correlations between the result and the flux shape.

- But in reality we would expect certain flux shapes to have different permitted extracted cross sections. E.g.:
    - Consider a flux shape that moves strength into the flux tail in a CC0π analysis
    - With this flux, we would expect the observed event rate to have a larger contribution from the CCNπ background
    - In this case we would find this flux shape should be more compatible with extracted cross sections with a lower normalisation

- This should be taken into account.

- However, we know the flux shape uncertainty is relatively small and so the expected size of the correlations are unlikely to be large enough to invalidate the qualitative conclusions of the studies shown

Including the flux shape correlations

- Including the flux shape correlations is easy. When we propagate our cross section uncertainties through "many universes" we can store the flux (shape) in addition to the extracted cross section.
- If you have flux parameters that vary the normalisation in bins of E_ν it's as simple as storing the value of these for each universe.
- The covariance between these parameters and the extracted cross section can then be calculated

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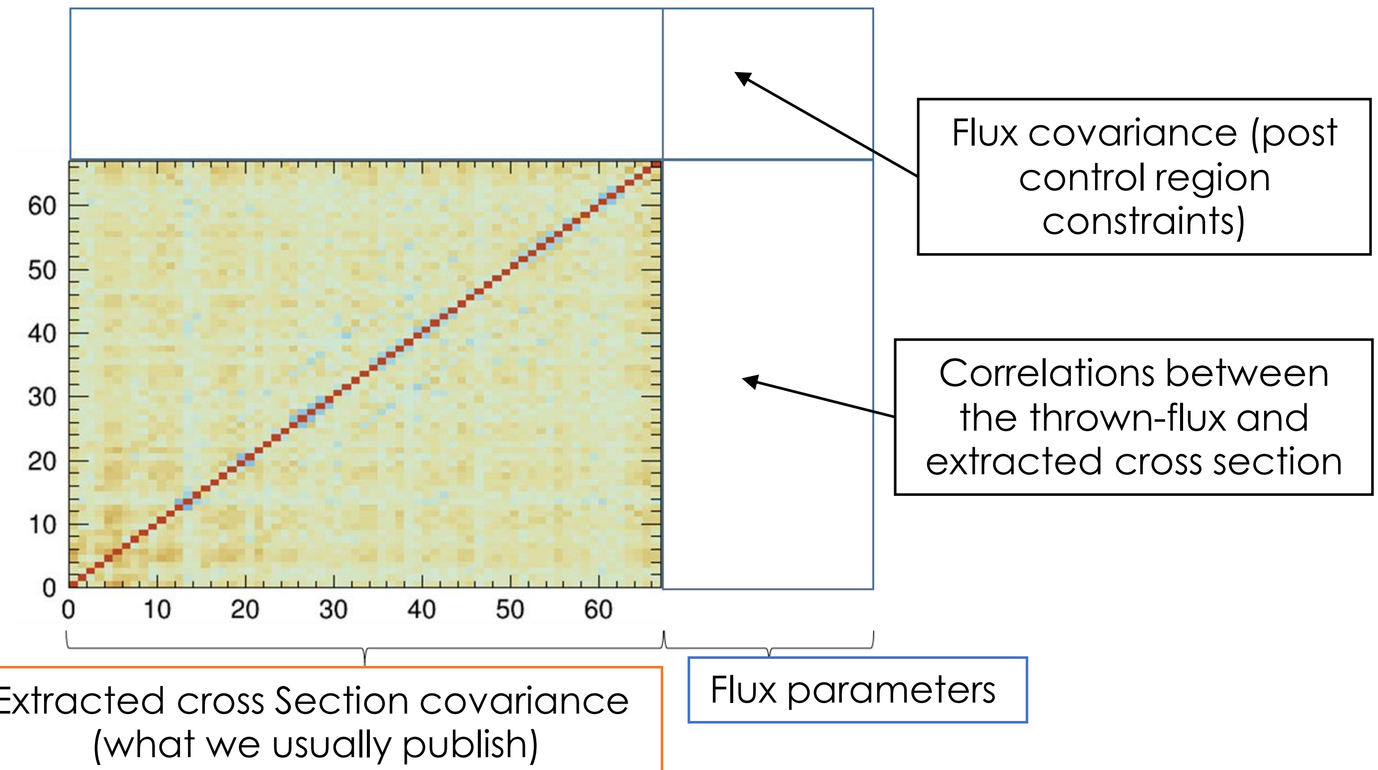
- If you have flux parameters that vary the normalisation in bins of E_ν it's as simple as storing the value of these for each universe.
- The covariance between these parameters and the extracted cross section can then be calculated
- A χ^2 can then be evaluated for a particular model folded through a particular flux



Summary

arXiv:2009.00552

- Comparing models to *first approach* measurements (most of them) using only a single reference flux misses part of the flux shape uncertainty.

- Ignoring correlations between the extracted cross section and the flux shape, it was found that this uncertainty was not dominant but also was not negligible for test T2K and MINERvA measurements.

- As we gather more statistics, this uncertainty may become critical for any quantitative model comparison.

- A complete model comparison with a *first approach* measurement requires experiments to publish the correlations between the flux shape and the extracted cross section.