Beam dynamics with a crab cavity

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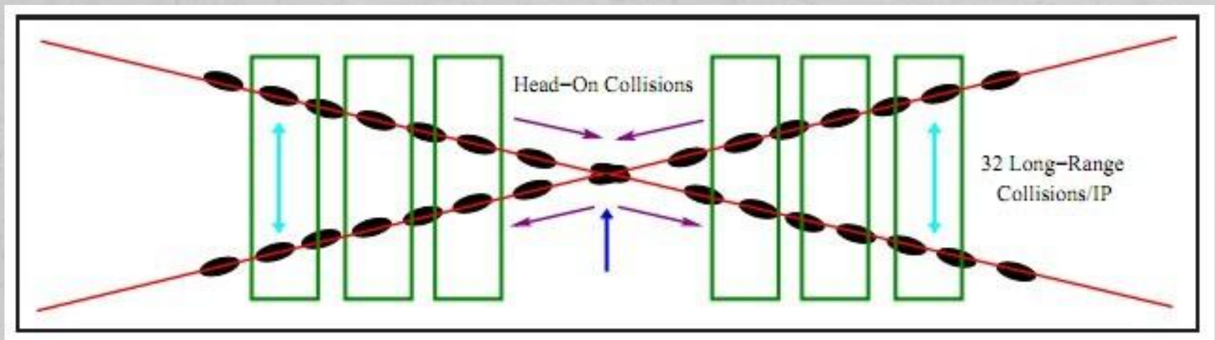
Why crab cavity?

Long range interaction without crab cavity causes geometrical luminosity loss:



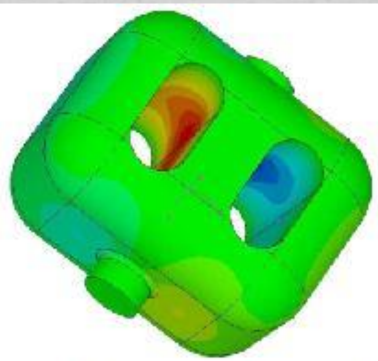


FIG. 14. (Color) Surface electric field in a parallel-bar structure with rounded rectangular bars operating in the π mode.

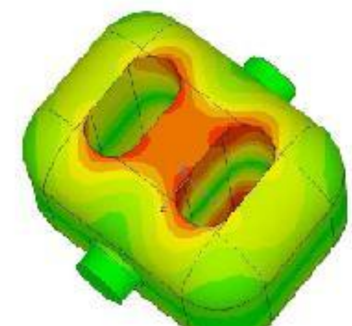


FIG. 15. (Color) Surface magnetic field in a parallel-bar structure with rounded rectangular bars operating in the π mode.

A crab cavity deflects the beams transversely and can increase luminosity, but nonlinear EM fields in the cavity may cause accidental beam loss.

We study the effects of a crab cavity by analyzing data from a J lab model.

Picture (above) from Calage et al. "LHC Crab Cavity Aspects and Stategy"
(below) from Delayen & Wang "New compact TEM-type deflecting and crabbing rf structure"

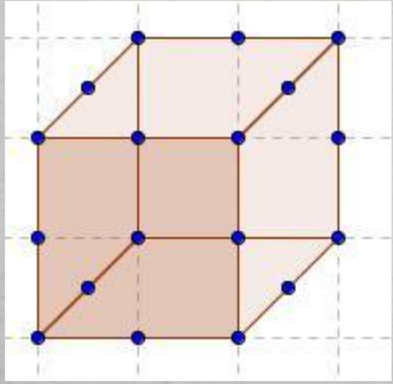
Beam dynamics

Using Lorentz's equation of motion **F**=q[**E**+**v**×**B**] to predict the beam trajectories in a crab cavity, we obtain

$$\frac{dp_x}{dt} = \frac{q}{p_0}E_x\left(x,y,\beta ct\right)\sin\left(\omega t\left(-\frac{z}{\beta c}\right)\right) - \frac{q\beta c}{p_0}\frac{1}{\mu_0}H_y\left(x,y,\beta ct\right)\cos\left(\omega\left(t-\frac{z}{\beta c}\right)\right),$$
$$\frac{dp_y}{dt} = \frac{q}{p_0}E_y\left(x,y,\beta ct\right)\sin\left(\omega\left(t-\frac{z}{\beta c}\right)\right) + \frac{q\beta c}{p_0}\frac{1}{\mu_0}H_x\left(x,y,\beta ct\right)\cos\left(\omega\left(t-\frac{z}{\beta c}\right)\right),$$
$$\frac{dp_z}{dt} = \frac{q}{p_0}E_\sigma\left(x,y,\beta ct\right)\sin\left(\omega\left(t-\frac{z}{\beta c}\right)\right).$$

from which we can calculate the crab cavity kicks.

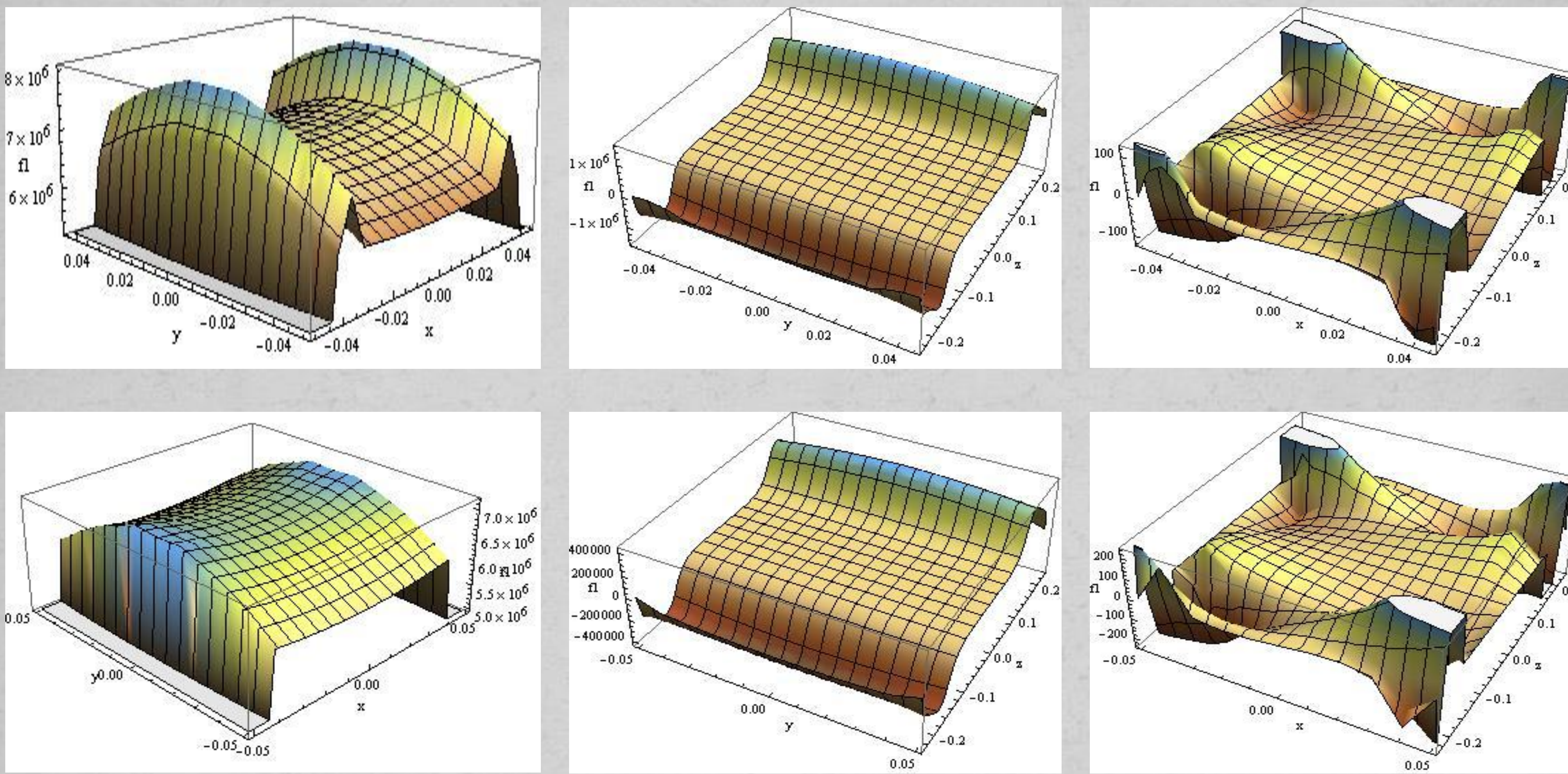
To obtain continuous fields, we use a 3D quadratic interpolation method described in Dhatt & Touzot, *The Finite Element Method Displayed*:



1. Cover the available grids with cubes of side length = 2* grid spacing
2. Select the vertices and midpoint of each edge (20 points)
3. Use basic polynomials $N_i(x,y,z)$ as given in the Dhatt& Touzot
4. The interpolation at any point (x,y,z) is given by $f(x,y,z)=\Sigma c_{i*}N_i(x,y,z)$ where $c_i = f(x_i, y_i, z_i)/N_i$

Results and further studies

Interpolation results of some field components (above) compared to field plots (below):



E_x vs (x,y) E_z vs (y,z) H_x vs (x,z)

Interpolation roughly matches the plot of field data except Ex and Hy.

We will improve the program for a better fit if possible and apply the interpolation results to calculate the actual crab cavity kicks.