Time resolution and efficiency of SPADs and SiPMs for photons and charged particles

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What can you expect?

An analytic treatment of a single SPAD cell

Questions such as

Important physical effects that determine time resolution?

Ultimate limits of timing capabilities?

SPADs as timing detectors for charged particles?

answered by a first-principles calculation

(Quenching, electronics, noise, etc. not covered in this talk!)

Contains approximations; complementary to simulations, but useful for their validation
A simple SPAD

SPAD = 1D silicon device with

low-field region
(“conversion region”)

and

high-field region
(“gain region”)

n-in-p SPAD

p  p-epi  p⁺  n⁺

Conversion region  Gain region

| E | 10 – 100 μm  0.5 – 2 μm
SPAD signal formation (*photon detection*)

(1a) Photon absorbed in conversion region (> 750 nm), electron drifts & diffuses towards conversion region.

n-in-p SPAD

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(1b) Photon absorbed directly in gain region (< 500 nm)
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(1b) Photon absorbed directly in gain region (< 500 nm)

(2) Electrons, holes multiply in gain region, induce current

(3) Signal crosses threshold → Time resolution

(4) Avalanche is quenched and current decays
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Photon absorption in conversion layer

Probability for photon to create e/h pair at $x_0$

$$p(x_0) \, dx_0 \sim e^{-x_0/l_a} \, dx_0 \quad l_a \ldots \text{absorption length}$$

Absorption length is strongly wavelength-dependent

Conversion layer important for efficient detection of long wavelengths

Drift, diffusion in conversion region

Drift and diffusion define arrival time distribution $\rho(t)$ of conversion electron in gain region

Variance of arrival time: $\sigma^2_{t,\text{arr}}$

Overall timescale set by drift time

$$T = \frac{w}{v_e}, \quad v_e \approx 0.1 \mu m/ps$$

Drift limit:

$$l_a \gg w$$

$$\sigma_{t,\text{arr}} = \frac{T}{\sqrt{12}}$$

Typical values: $\sigma_{t,\text{arr}} \approx 2.89 / 28.9 / 289$ ps

for $w = 1 / 10 / 100$ $\mu m$

Diffusion limit:

$$l_a \ll w$$

$$\sigma_{t,\text{arr}} = \frac{\sqrt{2DT}}{v_e}$$

Typical values: $\sigma_{t,\text{arr}} \approx 1.6 / 8.37 / 26.46$ ps

for $w = 1 / 10 / 100$ $\mu m$
SPAD signal formation (photon detection)

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Electron-hole avalanches in silicon

Gain layer operated “above breakdown”: individual charge carriers can trigger diverging avalanche

Multiplication of charge carriers driven by impact ionisation

\[ \alpha, \beta \ldots \text{impact ionisation coefficients for electrons / holes} \]

\[ \text{[prob. / unit length for ionisation]} \]

Current induced on electrodes:

\[ I_{\text{ind}} \sim e_0 (N_e v_e + N_h v_h) \]

Avalanche grows stochastically

Contribution to time resolution:

\[ \sigma_{t, \text{av}} \]

\text{variance of threshold-crossing time}

Avalanche fluctuations at early times determine time resolution

\[ \sigma_{t, \text{av}} \]

Fluctuations visible

Smooth exponential growth

Threshold
Average evolution of avalanche

To compute **time resolution**, need to understand **average evolution** of avalanche and **fluctuations around average**

Avalanche grows exponentially on average \( I_{\text{ind}} \sim h(x_0) e^{\gamma v^* t} \)

\( \gamma v^* \) … asymptotic growth rate of avalanche

(from transcendental equation)

\( h(x_0) \) … overall normalisation depends on the position and type of the initial charge (analytic expression for constant fields)
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\[ h(x_0) \sim N_e^0 u_e(x_0) + N_h^0 u_h(x_0) \]

Si, \( d = 1 \, \mu m \)

\[ E = 4.5 \cdot 10^5 \, V/m \]
For large thresholds \((I_{th} \to \infty)\), the time resolution from avalanche fluctuations is

\[
\sigma_{t,av}^2 = \frac{\sigma^2(\ln I^{\text{ind}})}{(\gamma v^*)^2}
\]
Time resolution from avalanche fluctuations

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\[
\sigma_{t,av}^2 = \frac{\sigma^2(\ln I^{\text{ind}})}{(\gamma v^*)^2} \approx \frac{A^{-2} + A^{-1}}{(\gamma v^*)^2}
\]

Avalanche parameter: 
(dependent of initial position \(x_0\))

\[
A \approx \frac{\alpha v_e N_e^0 + \beta v_h N_h^0}{\alpha v_e + \beta v_h}
\]

**Approximation:** applicable when initial charge carrier has larger impact ionisation coeff., e.g. an initial electron for silicon. Otherwise: \(A \rightarrow A_{\text{eff}}\) (computed numerically).
Time resolution from avalanche fluctuations

For large thresholds \((I_{th} \rightarrow \infty)\), time resolution from avalanche fluctuations is

\[
\sigma_{t,av}^2 = \frac{\sigma^2 (\ln I^{\text{ind}})}{(\gamma v^*)^2} \approx \frac{A^{-2} + A^{-1}}{(\gamma v^*)^2} \sim \frac{\sigma(1)}{(\gamma v^*)^2}
\]

Avalanche parameter:
(independent of initial position \(x_0\))

\[
A \approx \frac{\alpha v_e N_e^0 + \beta v_h N_h^0}{\alpha v_e + \beta v_h}
\]

Comparison with avalanche MC for a realistic field distribution in silicon
Conversion in gain layer

For **photoabsorption** in the **gain region**, conversion point \( x_0 \) fluctuates → additional contribution to time resolution

\[
\sigma_t^2 = \sigma_{t,av}^2 + \sigma_{t,\text{pos}}^2
\]

\[
\sigma_{t,\text{pos}}^2 = \frac{\sigma^2(\ln h(x_0))}{(\gamma v^*)^2}
\]

Variance taken over distribution of initial charge carrier, \( p(x_0) \)

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Variance taken over distribution of initial charge carrier, \( p(x_0) \)

Average avalanche

\[
I_{\text{ind}}^\text{avg} \sim h(x_0) e^{\gamma v^* t}
\]
Conversion in gain layer

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\]

Variance taken over distribution of initial charge carrier, \( p(x_0) \)

**Similar to contribution from avalanche fluctuations**

(depending on electric field and thickness)
Single-photon time resolution

Total time resolution with contributions from drift & diffusion, avalanche fluctuations, and position fluctuations

\[ \sigma_t^2 = \sigma_{t,av}^2 + \sigma_{t,arr}^2 + \sigma_{t,pos}^2 \]

- Drift + diffusion
- Avalanche fluctuations
- Avalanche pos. dependence

for absorption in conversion region

for absorption in gain region
Single-photon time resolution

Total time resolution with contributions from drift & diffusion, avalanche fluctuations, and position fluctuations

\[
\sigma_t^2 = \sigma_{t,av}^2 + \left\{ \begin{array}{ll}
\sigma_{t,arr}^2 \\
\sigma_{t,pos}^2
\end{array} \right.
\]

for absorption in conversion region

for absorption in gain region

\[
\sigma_{t,avg}, \sigma_{t,pos} \approx \frac{c_0}{\gamma v^*}, \quad c_0 \approx 0.5 - 2
\]

\[\lesssim 5 \text{ ps} \quad \text{at high electric fields}\]

Strongly dependent on wavelength!

\[\approx 3 - 30 \text{ ps}\]

Single-photon time resolutions of a few ps seem achievable in principle!
(1) MIP produces multiple clusters throughout sensor

(2) Each cluster in gain region immediately starts independent avalanche

(3) Induced current crosses threshold
   → Time resolution, efficiency

(5) Avalanche is quenched and current decays
Efficiency for charged particles

MIP produces multiple clusters throughout sensor

Average cluster spacing for silicon: $\lambda \approx 0.2 \, \mu m$

$$
\varepsilon = 1 - \exp \left( -\frac{1}{\lambda} \left( d - \sum_{n=1}^{\infty} p_{\text{clu}}(n) f_n(E) \right) \right)
$$

Expect high efficiency!

$p_{\text{clu}}(n)$ … cluster size distribution
(computed with PAI model; see backup)

Efficiency above breakdown limited by single-interaction probability

Thin gain layer alone is sufficient for MIP detection

Width $d$ of gain region
Time resolution for MIPs determined by avalanche …
(fluctuations + position dependence)

… but more complicated initial conditions: MIP produces clusters of e/h pairs along its track

As before: \( \sigma_t \approx \frac{c_0}{\gamma v^*} \approx \mathcal{O}(1) \)

Time resolution for MIPs and single photons expected to be similar!
SPADs are efficient detectors for individual photons and individual MIPs

Timing performance ultimately determined by

arrival time distribution of conversion electrons

(for photon detection)

and formation of avalanche in gain region

For both single-photon and charged particle detection, time resolution of few ps should in principle be achievable

More information, (many) more details, and more “rules of thumb”:

“Time resolution and efficiency of SPADs and SiPMs for photons and charged particles” [arXiv:2102.00091]


[both submitted to Nucl. Instrum. Meth. A]
Backup
SPAD equivalent electrical circuit

Fig. 7. Equivalent electrical circuit of the single photon avalanche diode (SPAD) with integrated quenching resistor.

Fig. 5. Effect of acquisition bandwidth on signal shape, measured with a $1 \times 1$ mm$^2$ SiPMs (FBK NUV-HD 2018) with 40 $\mu$m pitch.
From SPADs to SiPMs

Additional contributions to time resolution

SPAD pixel

2D-dependence of electric field across pixel

(\textit{guard rings etc.})

Different $\gamma_v^*$ depending on position

Which parallel connection scheme to use?

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image}
\caption{Normalized transit time map, obtained by thresholding the average signals, measured in $11 \times 11$ positions (step 0.25 mm), on the $3 \times 3 \text{ mm}^2$ SiPM with 1 bonding wire. Measurement performed at room temperature, at an excess bias of 7 V.}
\end{figure}

F. Acerbi \textit{et al} 2015 \textit{JINST} \textbf{10} P07014
Electron-hole avalanches are complicated!

Electrons and holes travel in opposite directions: spatial development of avalanche is nontrivial (correlations!)

But: can exploit analogies between classical stochastic systems and many-body quantum mechanics

A charge avalanche is “almost” like a quantum mechanical system, only need to replace

amplitudes $\rightarrow$ probabilities, $i \rightarrow 1$

Time evolution of the avalanche determined by “Schroedinger’s equation”:

$$\frac{d}{dt} |\psi> = \hat{H} |\psi>$$

$\hat{H} = e^\alpha h + h^\beta e$

“State” = complete description of the avalanche

$\alpha, \beta$ ... “coupling constants”
Cluster statistics in silicon

Cluster size distribution in silicon and in a gas

Distribution of total number of charges cluster size distribution * poisson

Computed with HEED using photoabsorption ionisation (PAI) model
Charge carrier dynamics in silicon

Impact ionisation coefficients vs. electric field

Charge carrier drift velocity vs. electric field

Scaling of time resolution

\[ \sigma_t \approx \frac{1}{\gamma v^*} \]

\[ v^* = \frac{2v_e v_h}{v_e + v_h} \]

\[ \gamma = \frac{2}{d} \left( \frac{\alpha + \beta}{2} + \frac{\lambda}{d} \right) \]

\[ \lambda + \sqrt{\alpha \beta d^2 - \lambda^2 \cot \sqrt{\alpha \beta d^2 - \lambda^2}} = 0 \]
Average avalanche from first principles

\[
\frac{\partial}{\partial t} \langle n_e(x) \rangle + \frac{\partial}{\partial x} v_e(x) \langle n_e(x) \rangle = \alpha(x)v_e(x) \langle n_e(x) \rangle + \beta(x)v_h(x) \langle n_h(x) \rangle
\]

\[
\frac{\partial}{\partial t} \langle n_h(x) \rangle - \frac{\partial}{\partial x} v_h(x) \langle n_h(x) \rangle = \alpha(x)v_e(x) \langle n_e(x) \rangle + \beta(x)v_h(x) \langle n_h(x) \rangle
\]
Avalanche fluctuations with more precision

\[ A = \frac{\alpha v_e N_e^0 + \beta v_h N_h^0}{\alpha v_e + \beta v_h} \]

\[ \psi_1(A) \approx A^{-2} + A^{-1} \]

\[ \frac{1}{A_{\text{eff}}} = \lim_{t \to \infty} \epsilon \left( 1 + \frac{\sigma(N)^2}{<N>^2} \right) \]

\[ N = N_e + N_h \]

(can be computed numerically)
Why electron-hole avalanches are complicated

What happens here …

… is determined by the electrons and holes in the interior of the past light cone

Past light cones intersect: nontrivial correlations between different positions (!!)

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Past light cones intersect: nontrivial correlations between different positions (!!)
Efficiency to trigger a diverging avalanche

\[
\frac{d P_e(x)}{d x} = -\alpha(x)[1 - P_e(x)] [P_e(x) + P_h(x) - P_e(x)P_h(x)]
\]

\[
\frac{d P_h(x)}{d x} = \beta(x)[1 - P_h(x)] [P_e(x) + P_h(x) - P_e(x)P_h(x)]
\]