Beam dynamics with a crab cavity

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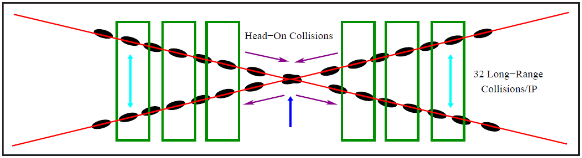
Motivation



Schematic of the LHC interaction region triplets to depict the crossing scheme required to minimize parasitic collisions with reducing β^*.

Inefficient overlap

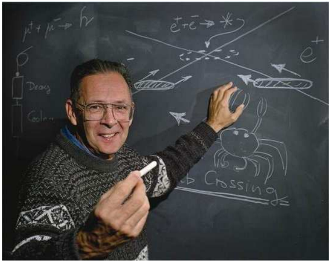
Crab cavity for CERN luminosity upgrade

Two bunches form a angle near IP to prevent parasitic collisions.
Without a crab cavity, it leads to geometrical luminosity loss due to decreased inter-sectional area.
A crab cavity deflects the beams transversely to compensate the geometric luminosity loss.

[0] Picture from Calaga et al. *LHC crab-cavity aspects and strategy*

Crab crossing design evolution

Crab crossing concept is first proposed by R. Palmer at 1988 for LC.



[0] K. Ohmi, *Crab crossing at KEKB*, Beam-beam workshop, SLAC 2007

Crab crossing design evolution

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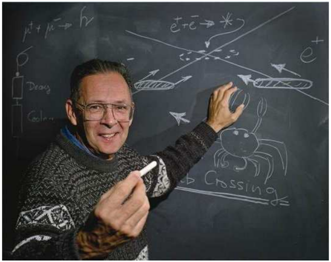
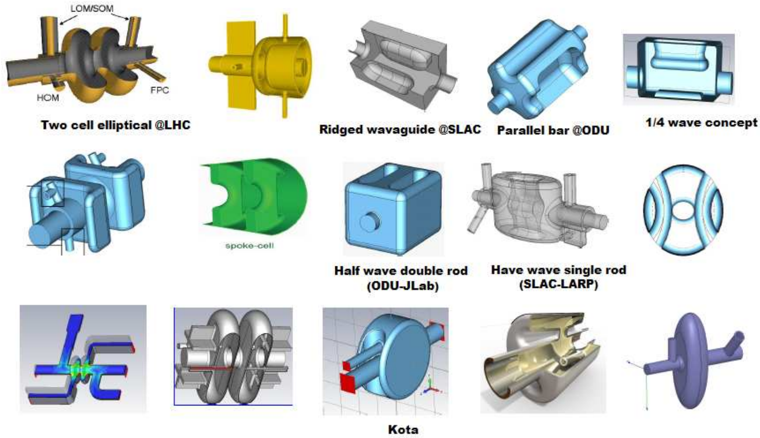


Successfully produced in Feb. 2007 at KEKB.

[0] K. Ohmi, *Crab crossing at KEKB*, Beam-beam workshop, SLAC 2007

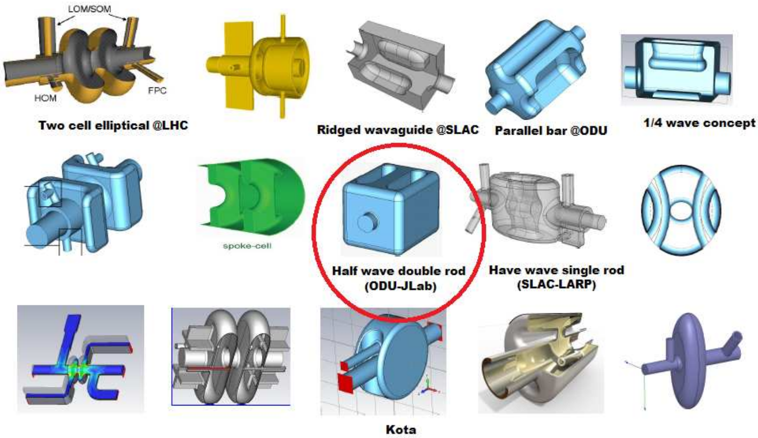
Crab cavity design evolution



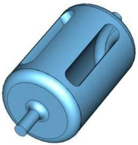
Focusing on compact cavity models.

[0] R. Calaga, *Crab Crossing For LHC Upgrade*, SRF July 2011

Crab cavity design evolution

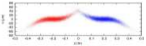


We are doing simulations with the ODU-JLab model.



Crab cavity specifications

	Baseline	Unit	LHC	KEK-B
RF	Frequency	MHz	400 (800)	509
RF	Deflecting Voltage	MV/Cav	5	2.0 (0.9-1.5)
RF	Peak E-field	MV/m	< 45	28
RF	Peak B-field	mT	< 80 mT	82 mT
Geometrical	Aperture (diameter)	mm	84	130
Geometrical	Cav Outer Envelope	mm	< 150	866/483
Geometrical	Module length	m	~ 1m	1.5 m
Geometrical	HV crossing	-	Desirable	N/A
Optics	β* (IR1/IR5)	cm	15-25	63/0.7
Optics	β crab	km	~ 5	0.2/0.04
Optics	Non-linear harmonics	Units $[10^{-4}]$	2-3	N/A
	Impedance Budget	Longitudinal, Transverse	60kΩ, 2.5MΩ/m	-



$\sigma_z = 7.55$ cm

Beam-beam separation

[0] R. Calaga, *Crab Crossing For LHC Upgrade*, SRF July 2011

Goals

To study the possible negative effects of a crab cavity on the tune footprint, dynamic aperture and emittance of the beam.

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To study the possible negative effects of a crab cavity on the tune footprint, dynamic aperture and emittance of the beam.

- Interpolate the field at any point
- Calculate crab cavity kicks
- Evaluate the impact by comparing simulation results with or without crab cavity

Calculate the parallel bar crab cavity kicks

EM fields in a TEM resonance structure are

$$\mathbf{E}(x, y, \sigma, t) = \mathbf{E}(x, \sigma) \cos\left(\frac{2\pi y}{\lambda}\right) \sin(\omega t),$$

$$\mathbf{B}(x, y, \sigma, t) = \frac{\mathbf{E}(x, \sigma)}{Z_0} \times \hat{y} \sin\left(\frac{2\pi y}{\lambda}\right) \cos(\omega t)$$

where $Z_0 = \sqrt{\epsilon/\mu}$.
Assuming two infinite rods parallel to the y-axis with uniform charge density q, and crossing the (x, σ) plane at $x = \pm a, \sigma = 0$. The potential is given by

$$V(x, \sigma) = \frac{q}{4\pi\epsilon_0} \ln\left(\frac{r_-^2}{r_+^2}\right),$$

where

$$r_-^2 = (x - a)^2 + \sigma^2, \; r_+^2 = (x + a)^2 + \sigma^2.$$

The electric fields are

$$E_x(x, \sigma) = -\frac{\partial V}{\partial x} = -\frac{aq}{\pi\epsilon_0}\left[\frac{x^2 - a^2 - \sigma^2}{r_-^2 r_+^2}\right]$$

$$E_\sigma(x, \sigma) = -\frac{\partial V}{\partial \sigma} = -\frac{aq}{\pi\epsilon_0}\left[\frac{2x\sigma}{r_-^2 r_+^2}\right]$$

Calculate the parallel bar crab cavity kicks

Using Lorentz's EOM $\mathrm{d}\mathbf{p}/\mathrm{d}t = \frac{1}{p_0} q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ and $\mathbf{v} = \beta c \hat{\sigma}$ we obtain

EOM of a particle with longitudinal distance z from the synchronous particle

$$
\begin{aligned}
\frac{\mathrm{d}p_x}{\mathrm{d}t} &= \frac{q}{p_0} E_x(x, \beta ct + z) \cos(ky) \sin\left(\omega\left(t - \frac{z}{\beta c}\right)\right) \\
\frac{\mathrm{d}p_y}{\mathrm{d}t} &= -\frac{q}{p_0}\frac{\beta c}{Z_0} E_\sigma(x, \beta ct + z) \sin\left(\frac{2\pi y}{\lambda}\right) \cos\left(\omega\left(t - \frac{z}{\beta c}\right)\right) \\
\frac{\mathrm{d}p_z}{\mathrm{d}t} &= -\frac{q}{p_0} E_\sigma(x, \beta ct + z) \cos(ky) sin\left(\omega\left(t - \frac{z}{\beta c}\right)\right).
\end{aligned}
$$

The reference particle passes through the cavity gap in time $t \in nT_0 + (-L_\sigma/2\beta c, L_\sigma/2\beta c)$, where L_σ is the cavity gap width along the σ direction.

No available analytical formula for crab cavity kicks. We have to obtain it via numerical integration.

The actual fields in use are simulated based on CSD Microwave Studio's numerical model of the cavity.

Symmetry of field components along z axis

E_x	E_y	E_z	H_x	H_y	H_z
S	S	A	A	A	S

Interpolation algorithm

Interpolation is a method of constructing new data points within the range of a discrete set of known data points. This algorithm is a slight variation of quadratic polynomial interpolation.

Polynomial interpolation

The idea is that any $n+1$ known data points uniquely determine a n-th polynomial. The value at any other points can be predicted by the polynomial. Given a discrete set of points, we usually pick the $n+1$ nearest points to the point of interpolation to construct the polynomial.

Pros:

- Fast
- Easy to implement

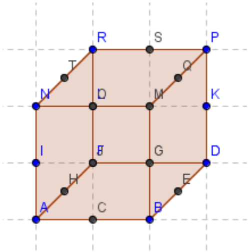
Cons:

- Only has C^0 continuity (does not have continuous derivatives)
- Large oscillations near endpoints (therefore interpolation order > 5 is rarely used)

Interpolation algorithm

Variation of 3D quadratic interpolation

Note: This algorithm requires uniform grid spacing along each direction.
1. Cover the domain with cubes with a side length of $2\times$ grid spacing.
2. Pick the 20 points on the vertices and edges. Discard points at the center of faces and in the center of the cube.
3. $f(x,y,z) = \sum_{i=1}^{20} c_i N_i(x,y,z,\xi_i,\eta_i,\zeta_i)$
where c_i are found from
$f(x_i,y_i,z_i) = c_i N_i(x_i,y_i,z_i,\xi_i,\eta_i,\zeta_i)$ and N_i's are polynomial functions which change from site to site.



Interpolation algorithm

Variation of 3D quadratic interpolation

- Nodes at the vertices:

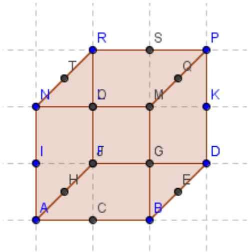
Node i	1	3	5	7	13	15	17	19
ξ_i	-1	1	1	-1	-1	1	1	-1
η_i	-1	-1	1	1	-1	-1	1	1
ζ_i	-1	-1	-1	-1	1	1	1	1

$$N_i = \frac{1}{8}(1+\xi_i x)(1+\eta_i y)(1+\zeta_i z)(-2+\xi_i x+\eta_i y+\zeta_i z)$$

-Nodes on the yz-plane:

Node i	2	6	14	18
ξ_i	0	0	0	0
η_i	-1	1	-1	1
ζ_i	-1	-1	1	1

$$N_i = \frac{1}{4}(1-x^2)(1+\eta_i y)(1+\zeta_i z)$$



Interpolation algorithm

Variation of 3D quadratic interpolation

-Nodes on the *xy*-plane:

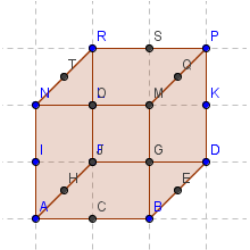
Node i	4	8	16	20
ξ_i	1	-1	1	-1
η_i	0	0	0	0
ζ_i	-1	-1	1	1

$$N_i = \frac{1}{4}(1 + \xi_i x)(1 - y^2)(1 + \zeta_i z)$$

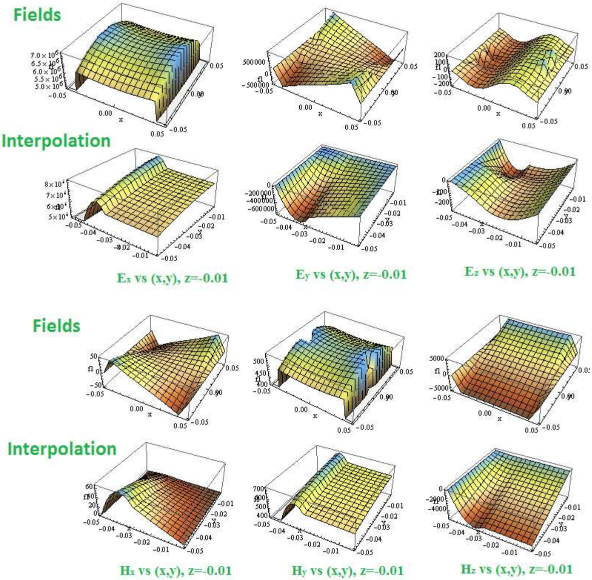
-Nodes on the *xz*-plane:

Node i	9	10	11	12
ξ_i	-1	1	1	-1
η_i	-1	-1	1	1
ζ_i	0	0	0	0

$$N_i = \frac{1}{4}(1 + \xi_i x)(1 + \eta_i y)(1 - z^2)$$



Interpolation results in comparison with Mathematica interpolation



Simulation

BB Simulation with crab cavity

Tracking particles through a model of SPS with all linear focusing fields and nonlinear fields.
A crab cavity will first be tested at SPS.
Crab cavity paremeters:

energy(GeV)	voltage(GV)	frequency(MHz)	radius(m)
26	13×10^{-4}	400	0.433

Looking for impacts on tune footprint, dynamic aperture and emittance.

TM mode: tune footprint

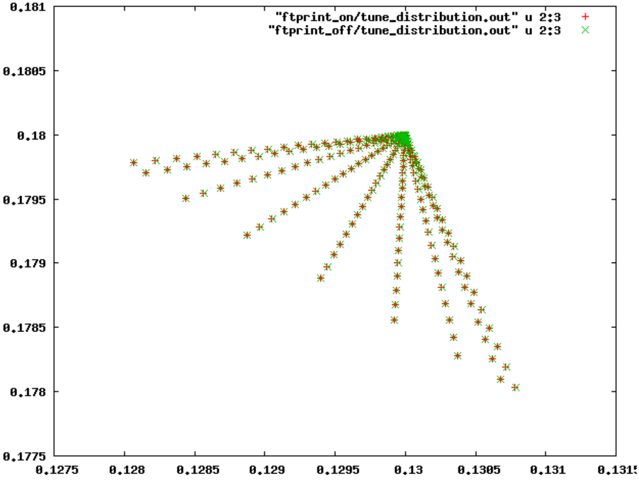


Figure: Tune footprint with crab cavity on (red) and off (green).

TM mode: dynamic aperture

Dynamic aperture specifies the maximal range below which particles are stable. Particles outside of the dynamic aperture will be lost.

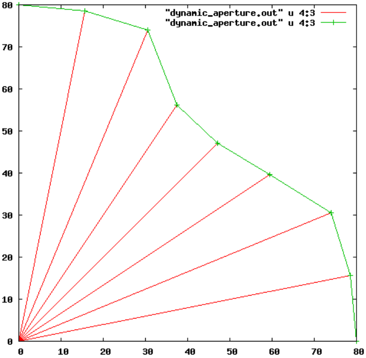


Figure: Dynamic aperture under TM mode (identical with or without crab cavity).

TM mode: emittance

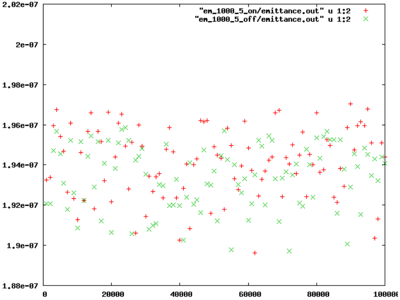


Figure: Emittance along x-axis up to 10^5 turns with crab cavity on (red) and off (green).

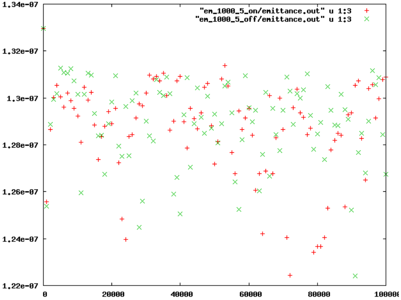


Figure: Emittance along y-axis up to 10^5 turns with crab cavity on (red) and off (green).

Conclusion

Interpolation

- Smooth
- Matches tabulated data
- Close to Mathematica quadratic interpolation.

Simulation

TM mode (at 26 GeV):

- small footprint change
- dynamic aperture not affected
- some emittance change, but bounded in the same vicinity

The effect of the crab cavities on the beam is small seen from this simulation.

Future work

Simulation of the TEM mode cavity at various energies of SPS and LHC.

Acknowledgement

I would like to thank:

- Dr. Tanaji Sen and Dr. Hyung Jin Kim for advising the research
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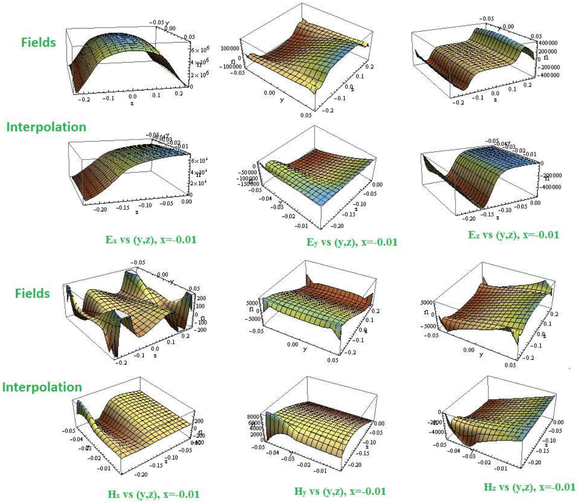
- Fermilab and Lee Teng Internship Program
- Visa office at Fermilab and Stony Brook University for making this happen

Interpolation results in comparison with Mathematica interpolation (cont'd)



Interpolation results in comparison with Mathematica interpolation (cont'd)