Future Colliders: 
ELC-Anzatz

Vladimir SHILTSEV (Fermilab)

Physics Limits of Ultimate Beams - III

January 22, 2021 – Snowmass’21 AF1
Content:

• **Part I**
  – Ultimate colliders: *Scope and Approach*
  – Ultimate colliders: *ELC-Ansatz, Units*
  – Ultimate colliders: *Limits of E*
  – Ultimate colliders: *Limits of L*
  – Ultimate colliders: *Limits of C*
  – Other considerations: *T*

• **Part II**
  – Circular *pp / ee / μμ*
  – Linear and Plasma *ee / γγ / μμ*
  – Exotic (crystal) *μμ / μμ*

• **Conclusions / Q&A / Discussion**
In search of uniform approach to discuss far future/ultimate machines

• BASED ON EXISTING TECHNOLOGIES
  – Circular ee
  – Linear ee/γγ
  – Circular pp
  – Circular μμ

• BASED ON EMERGING TECHNOLOGIES
  – ERL ee/γγ
  – Plasma ee/γγ
  – Linear μμ / Plasma μμ

• EXOTIC SCHEMES
  – Crystal linear μμ/ττ
  – Crystal linear ττ
  – Crystal circular pp
“ELC – Ansatz”

- We will evaluate possible (ultimate) future colliders on base of
  - Feasibility of **Energy**
  - Feasibility of **Luminosity**
  - Feasibility of **Cost**

- For each machine type / technology we will start with what is the state-of-the-art now and attempt to make "1-2-several" orders of magnitude steps in **Energy**
  - see how it affects **Luminosity**
  - see how it affects **Cost**

- Leave it to others to judge where the lower limit on **L** and upper limit on **C** are… other limits may appear
“ELC – Ansatz” : Choice of Units

- Units of Energy will be **TeV**
  - most often cme $= 2 \times E_{\text{beam}}$, sometimes - per beam

- Units of Luminosity will be **$ab^{-1}/yr$**
  - e.g., $1e35$ over $1e7$ sec/yr… HL-LHC will have $0.3$ $ab^{-1}/yr$
  - factor of $\sim 2$ uncertainty in peak lumi / machine availability

- Units of power(total facility) will be **TWh/yr**
  - Eg CERN/LHC $\sim 200$MW and $1.1-1.3$ TWh/yr

- Units of Cost will be **LHCU**
  - cost of the LHC construction, approx. $10B$ - see below
  - for other machines the cost will be estimated using $\alpha\beta\gamma$ model with uncertainty $O(2)$ - see below
  - the $\alpha\beta\gamma$ model needs to be extended for novel approaches
Limits on *Energy* (1)

- Linear vs Circular

\[ \Delta U_{SR} = \frac{90 \text{keV} \cdot E_e^4 (\text{GeV})}{R^m} \]

\[ \Delta U_{SR} < E_e \]

\[ E_e < 500 \text{ GeV} \cdot \left( \frac{R}{10 \text{ km}} \right)^{1/3} \]

- Circular does not make sense beyond these energies

\[ \text{for muons: } \times \left( \frac{m_{\mu}}{m_e} \right)^{1/3} \]

\[ E_{\mu} < 600 \text{ TeV} \cdot \left( \frac{R}{10 \text{ km}} \right)^{1/3} \]

\[ \text{for protons: } \times \left( \frac{m_p}{m_e} \right)^{1/3} \]

\[ E_p < 10 \text{ PeV} \cdot \left( \frac{R}{10 \text{ km}} \right)^{1/3} \]
Limits on Energy (2)

- Particles don’t survive acceleration
  - Unstable particles
    - for muons: \( G \gtrsim 3 \text{ MeV m}^{-1} \)
    - for \( \tau \)-leptons: \( G \gtrsim 0.3 \text{ TeV m}^{-1} \)
  - Lossy transport from cell to cell (loss in plasma material, c-t-c efficiency)

\[
\frac{dN}{dt} = -N/\sqrt{\tau_0}
\]

\[
\frac{N}{N_0} \approx \left( \frac{m_\mu c^2}{E} \right)^\kappa, \quad \kappa = \left( \frac{m_\mu c}{\tau_0 G} \right)
\]
Limits on *Energy* (3)

- Corollary limits

  - **Space/area available**
    
    - Circumference 100 km, $B < 16 \text{T}$, $E < 50 \text{TeV}$
    - Circumference 40,000 km, $B = 1 \text{T}$, $E < 1.3 \text{PeV}$
    - Length 50 km, $G < 0.1 \text{GV/m}$, $E < 5 \text{TeV}$
    - Length 10 km, $G < 1 \text{TV/m}$, $E < 10 \text{PeV}$

- **Power available**

- **Money available**
Limits on Luminosity (1)

- General Equation
  - rewrite with norm.emm.
  - HEP demand
    - limits, eg, beam power
Limits on *Luminosity* (2)

- Another example

\[ L \sim \frac{p^2}{\int n_b E^2 \cdot \Phi_{5p}} \]

\[ \sigma_{5p} \text{ is limited (e.g. 1Å)} \]

\[ \sigma = \frac{r_p N_p}{n \pi \varepsilon} \]

\[ \Delta Q_{sc} = \frac{r_p N_p}{n \varepsilon \alpha \beta \delta^2} \]

\[ V_{\perp} \sim N_p \cdot \text{impedance} \]

\[ PU \sim \frac{z}{f_0 n_b} \sin \phi \]

- beam-beam limit
- space-charge limit
- beam loading
- event pile-up
Limits on \textit{Luminosity} (3)

- particle production
- beamstrahlung
- synchrotron radiation
- SR/meter
- IR rad damage
- $\nu$-radiation dose
- instabilities
- jitter/emittance growth
All Colliders: Past, Existing, under Discussion

On base of Shiltsev & Zimmermann, Rev. Mod. Phys. (2021)

- $E^4/3$
- $E$

Peak luminosity $L$ ($10^{30}$ cm$^{-2}$ s$^{-1}$)

Center of mass energy $E_{cm}$ (GeV)
Paradigm Shift looming for > 0.1-1 PeV…

\[
L \propto s
\]

\[
ab^{-1}/yr
\]

other limits?

L, power limited

Center of mass energy, TeV

Luminosity, cm\(^{-2}\) s\(^{-1}\)

... to be discussed below…
Limits on Cost (1)

- Cost is set by technology
  - Accelerator technology
  - Civil construction technology
  - Power production, delivery and distribution technology
2014 Cost analysis:

17 “Data Points” - Costs of Big Accelerators:

- **Actually built:**
  - RHIC, MI, SNS, LHC
- **Under construction:**
  - XFEL, FAIR, ESS
- **Not built but costed:**
  - SSC, VLHC, NLC
  - ILC, TESLA, CLIC, Project-X, Beta-Beam, SPL, ν-Factory

Wide range:
- 4 orders in Energy, >1 order in Power, >2 orders in Length
- Almost 2 orders in cost
  - (normalized to US TPC)
The $\alpha\beta\gamma$ cost model:

$$\text{Cost}(\text{TPC}) = \alpha L^{1/2} + \beta E^{1/2} + \gamma P^{1/2}$$

a) Is for a “green field” facility!

b) US-Accounting!

c) There is hidden correlation btw $E$ and technology progress

d) Pay attention to units $(10 \text{ km} \text{ for } L, 1 \text{ TeV} \text{ for } E, 100 \text{ MW} \text{ for } P)$

- $\alpha \approx 2\text{B }$/sqrt(L/10 km)
- $\beta \approx 10\text{B }$/sqrt(E/TeV) for SC/NC RF
- $\beta \approx 2\text{B }$/sqrt(E/TeV) for SC magnets
- $\beta \approx 1\text{B }$/sqrt(E/TeV) for NC magnets
- $\gamma \approx 2\text{B }$/sqrt(P/100 MW)
Total Cost vs $\alpha\beta\gamma$-Model (Log-Log)

The $\alpha\beta\gamma$-model is good to +-30%
Illustrations

Comment:

$\sqrt{\text{functions}}$ are quite accurate over wide range because such dependence well approximates the "initial cost"– effect:

![Illustration](image_url)

---

*Fig. 9.5. Variation of costs of power plant versus its capacity.*
Take LHC as an Example:

- $\alpha\beta\gamma$ – Model:
  - 40 km of tunnels
  - 14 TeV c.o.m SC magnets
  - $\sim$150 MW of site power

TOTAL PROJECT COST: \textbf{14B$ \pm 4.5B$}

- ITF T.Roser talk @ PLUB-II (USD 2021):
  - existing injector complex \textbf{4.6 B$}
  - new accelerator systems \textbf{4.06 B$}
  - new infrastructure and civil \textbf{2.75 B$}
  - explicit labor \textbf{\sim1.4 B$}

Total: \textbf{12.8B$}
“αβγ – Model” : Caveats

- (once again) note three warning signs:
  - “…+- 30% …green field… US accounting…”
  - rounded powers and coefficients, e.g., $\sqrt{x} \approx x^{0.4…0.6}$

- Analysis was done in 2013:
  - inflation 7yrs x 3% = 21%
  - many more projects have been costed since then, others updated

- “Not-yet-built” machine costs estim’d by proponents

- “One person study” – limited research on the subject

- That’s why:
  - a) ITF work is very important
  - b) I’ll use LHCU for rough estimates/analysis here
Costs of future technologies are not well known:
- plasma, lasers, crystals, “magic cheap” magnets, tunnels, HTS, etc

Costs of civil construction and power systems are driven by larger economy (not by us)... “stable”

Having injector (~1/3 of cost) $\rightarrow$ factor of 2 in energy reach

Follows from the model:
- Cost is weak function of luminosity (see next slide)
  - Also, LHC $10B$, HL-LHC $1B$ with $x5$ increase in luminosity
  - It’s OK to start high $E$, low $L$...CESR, Tevatron increased $L > 100x$, LHC >10x
- Cost is moderate function of length/circumference
- Cost is strong function of Energy and technology

Of course, the model does not tell us “what’s affordable”
- but at least allows approximately sort proposals in categories
  - E.g., “Less than LHCU”, “1-2 LHCU”, “More than 3 LHCU”, etc
Example: Muon Colliders
cost(color) vs $E$ vs Lumi
Some projects discussed @ EPPSU’19 and Snowmass’21
Colliders: Probability to Be Built

\[ \sim \frac{1}{\text{Cost}^2} \quad \text{(is it true?)} \]

\[
\frac{1}{(1 \times \text{LHC})^2} = 100\% \\
\frac{1}{(2 \times \text{LHC})^2} = 25\% \\
\frac{1}{(3 \times \text{LHC})^2} = 10\% 
\]

...Tao HAN: \textit{calibration is function of community size, budget and time}

...Mike KORATZINOS: \textit{lower cost?} \\
\[ \Rightarrow \frac{1}{(1+\text{Cost}^2)} \ldots \]
Energy Ranges and Reference Points: *leptons vs hadrons*

About 1:7

\[ M_{\text{NewPhysics}} = \sqrt{s}/2 \]

\[ \mu\mu @ 14 \text{ TeV} = pp @ 100 \text{ TeV} \]

Equivalent reach in *pp* after rescaling for *pdf*’s
Reference Points and (Far) Future

Circular pp

- LHC
- FCC VLHC
- Sea UELN
- GLT

Circular ee

- LEP FCCee

Circular μμ

- MC-1.5
- MC-6
- MC-14

Linear ee/μμ

- SLC ILC
- ILC-1 CLIC

Plasma ee/μμ

- LPWA/BPWA ALIC-3
- ALIC-30

Crystal μμ/ττ

- XC-0.1
- XC-1
- XC-10
Construction Time… \(~\text{SQRT(Cost)}\)?

Peak spending rate depends on $$/\text{yr}$$ limit and on \# of available experts ... currently, btw 0.2 to 0.5 B$/yr (total World’s HEP \(~4B$$)
The latest: **EIC at BNL**

Reference Funding Profile

- Reprioritized and New Funding
- FY2021 Budget of $30M supports schedule for CD-1
- Funding Profile Set prior to CD-2
- $100M Investment by New York State for Conventional Construction

~50% of NP budget
“Social Cost” and Social Limits

• There is probably a limit on cost of ultimate accelerators
  – how much society (national, regional, global) wants/can afford spending on HEP…slowly varies in time (grows?)
  – current estimate for global big collider ~ 2-3 LHCU ?

• Since recently – awareness of the “carbon footprint”/”ecology” limit for large facilities
  – current estimate for global big collider ~ 1(2) TWH/yr ?
  – disturbed environment (land use, radiation, pollutants, etc)

• Scarcity of materials
  – Helium, Nb, W, etc ?
Commissioning Time: \( T \sim \text{Complexity} \)


The evolution of the performance of continuously improving facilities where every next step brings *x-fold* improvement on top of previous improvement can be further simplified in an approximate formulas:

\[
C \cdot P = T, \tag{3}
\]

where the factor \( P = \ln (\text{luminosity}) \) is the “*performance*” gain over time interval \( T \), and \( C \) is a machine-dependent coefficient equal to average time needed to increase the performance (in the case of colliders — luminosity) by \( e = 2.71 \ldots \) times, or boost the “*performance*” \( P \) by 1 unit. Both, \( T \) and \( C \) have the dimension of time,

A. Get the energy (“one particle”):
\( C \sim O(1) \text{ yr} \) – for known technologies, longer for new, \( \sim \# \text{ of elements} \)

B. Get the (ultimate) luminosity:
depends on luminosity risk – eg for \( P=4.5 \) (risk \( \sim 100 \text{ in L} \)) and \( C \sim 2 \to T=4.5 \times 2=9 \text{ yrs} \)
Content:

- **Part I**
  - Ultimate colliders: *Scope and Approach*
  - Ultimate colliders: *ELC-Ansatz, Units*
  - Ultimate colliders: *Limits of E*
  - Ultimate colliders: *Limits of L*
  - Ultimate colliders: *Limits of C*
  - Other considerations: $T$

- **Part II**
  - Circular $pp / ee / \mu\mu$
  - Linear and Plasma $ee / \gamma\gamma / \mu\mu$
  - Exotic (crystal) $\mu\mu / \mu\mu$

- **Conclusions / Q&A / Discussion**
Circular $pp$ Colliders

- Can use Tevatron and LHC as reference point
- Parameter sets exist for SCC, FCC-hh, SppC, VLHC, Eloisatron
- Major advantages:
  - known technology and physics
  - good power efficiency $ab^{-1}/$TWh
- Major limitations:
  - Size (magnetic field $B$)
  - Power
  - Beam-beam, burn-off, instabilities
  - Synchrotron radiation
  - Cost
**pp Luminosity**

- **Beam-beam limit #1**

- **Synchrotron radiation**:  
  - Total power  
  - or SR/meter

W.Baretta (1996)
Other limits

- Pile up
- IR radiation damage
- Resistive wall instability
- TMCI
- e-cloud
- Turn around time vs Luminosity evolution

\[ PV = \frac{Z, \sigma_{in}}{\sigma_{in}} \approx \frac{PV}{\sigma_{in}} = f_{V, n_2, \sigma_{in}} \approx f_{V, n_2, \sigma_{in}} \log E \]

\[ Rad = Z, E \]

\[ \rightarrow Z \sim \frac{Rad}{E} \]

\[ N_{RW} \equiv \tau_{RW} f_0 = \frac{\sqrt{2\pi(E_p/e)a^3}}{I_B Z_0 \langle \beta \rangle} \sqrt{\frac{(1 - \Delta v)\sigma_W}{cR^3}}. \]

\[ N_{\text{thr}} = 1.24 \times 10^{10} \times \sqrt{\frac{\sigma_s}{0.1 \, \text{m}}} \frac{E}{3 \text{TeV}} \frac{v_s}{0.005} \times \left( \frac{a}{0.9 \, \text{cm}} \right)^3 \frac{520 \, \text{km}}{C} \times \frac{250 \, \text{m}}{\langle \beta \rangle} \]
Qualitative Cost Dependencies - 100 TeV pp

Total Project Cost (arb units) vs Dipole field $B$ (arb units)

- **red** line: base cost parameters set
- **black** line: tunnel 5 times cheaper
- **blue** line: magnets 5 times cheaper

*for illustration purposes only*
**Circular pp**

**10 ab^{-1}/yr**
- **10 ab^{-1}/yr**
- **B=16T, 100 km, P=4 TWh, Cost=2.3 LHCU**
- **B=3.2T, 1900 km, P=3-4 TWh, Cost=4-5 LHCU**
- **B=20T, 1600 km, P=10+ TWh, Cost> 30 LHCU**

**1 ab^{-1}/yr**
- **1 ab^{-1}/yr**
- **B=8 T, 27 km, P=1.1 TWh, Cost=1 LHCU**
- **B=10T, 233 km, P=1.5 TWh, Cost=3.3 LHCU**
- **B=1T, 40000 km, P> 30 TWh, Cost> 30 LHCU**

**0.1 ab^{-1}/yr**
- **0.1 ab^{-1}/yr**
- **B=4.5 T, 6 km, P=0.3 TWh, Cost=0.5 LHCU**
- **B=10T, 300 km, P=2-3 TWh, Cost=3.5 LHCU**
- **B=3.2T, 1900 km, P=3-4 TWh, Cost=4-5 LHCU**

**0.01 ab^{-1}/yr**
- **0.01 ab^{-1}/yr**
- **B=4.5 T, 6 km, P=0.3 TWh, Cost=0.5 LHCU**
- **B=10T, 300 km, P=2-3 TWh, Cost=3.5 LHCU**

**1 fb^{-1}/yr**
- **1 fb^{-1}/yr**
- **B=4.5 T, 6 km, P=0.3 TWh, Cost=0.5 LHCU**
- **B=10T, 300 km, P=2-3 TWh, Cost=3.5 LHCU**

**pp Colliders: Lumi and Cost vs Energy**

**7 TeV**
- **7 TeV**
- **B=4.5 T, 6 km, P=0.3 TWh, Cost=0.5 LHCU**
- **B=10T, 300 km, P=2-3 TWh, Cost=3.5 LHCU**

**70 TeV**
- **70 TeV**
- **B=16T, 100 km, P=4 TWh, Cost=2.3 LHCU**
- **B=3.2T, 1900 km, P=3-4 TWh, Cost=4-5 LHCU**

**700 TeV**
- **700 TeV**
- **B=10T, 300 km, P=2-3 TWh, Cost=3.5 LHCU**
- **B=3.2T, 1900 km, P=3-4 TWh, Cost=4-5 LHCU**

**7 PeV**
- **7 PeV**
- **B=10T, 300 km, P=2-3 TWh, Cost=3.5 LHCU**
- **B=3.2T, 1900 km, P=3-4 TWh, Cost=4-5 LHCU**

**70 PeV**
- **70 PeV**
- **B=10T, 300 km, P=2-3 TWh, Cost=3.5 LHCU**
- **B=3.2T, 1900 km, P=3-4 TWh, Cost=4-5 LHCU**

**Fermilab**
“Globaltron” and other ideas

- Cost saving magnets
  - superferric
  - permanent

- Better/cheaper conductors
  - Iron-based SC, graphene?

- Can they give factor of ~5 in \$/\text{Tm)?
  - (doubts so far)… leave it to AF7

V. Kashikhin, FNAL beams-doc-8948 (2020)
Circular $e^+e^-$ Colliders

- Let’s skip them… dead end… SR power

$$L = \frac{3}{16\pi r_e^2 (m_e c^2)} \frac{\xi_y P_T}{\beta_y^*} \gamma^{-3}$$

- E.g. $>0.5$ TeV ring will be
  - Big ($>200$-$300$ km?)
  - Low luminosity $O(10 \text{ fb-1/yr})$
  - A lot of RF $\rightarrow$ expensive $>1.5$-$2$ LHCu
Circular Muon Colliders

- Parameter sets exist for 1.5, 3, 6, 10, 14 TeV
- Major advantages:
  - factor of x7 in $E_{\text{reach}}$
  - best power efficiency $ab^{-1}/\text{TWh}$
  - Traditional core technologies
- Major limitations:
  - Muon production
  - Muon cooling
  - Neutrino radiation

MC Luminosity

- $L \sim B$ field

- Assuming:
  - Enough muons can be produced
  - $L \sim Power \times Energy$
  - At some energy, neutrino radiation sets the limit
  - Ultimate lumi depends on suppression factor $\Phi$

\[ L = \gamma n_b \cdot \frac{N_0^2}{\nu \epsilon p x} \]
\[ \sim \text{freq} \cdot 300 \cdot B \]
\[ \dot{N} = N_0 \cdot n_b \cdot \text{freq} \sim 2 \cdot 10^9 \cdot 5 \sim 10^{12} / s \]
\[ L \sim B \cdot \text{P} \cdot \frac{N_0}{\nu} \]
\[ D(\text{dose}) \sim \text{freq} \cdot N_0 \cdot n_b \cdot \frac{\delta^3}{\Phi} \quad D \sim \frac{P_0^2}{\Phi} \]
\[ \text{Power} \sim \frac{D \Phi}{\delta^2} \leq P_0 \]
\[ L \sim \frac{B}{\text{freq}} \cdot \frac{(D \Phi)^2}{\epsilon} \]
Neutrino Radiation Mitigation $\Phi \sim 100$ possible

Move collider ring components, e.g. vertical bending with 1% of main field

Opening angle ± 1 mradian

O(100) larger than decay cone $\Rightarrow$ gain O(100) in radiation

In straights, additional improvement in horizontal

Need to study impact on beam and operation, e.g. dispersion control
**MC: Lumi and Cost vs Energy**

**Circular \( \mu \mu \)**

- **10 ab\(^{-1}\)/yr**
  - B=10 T, \( \Phi=1 \)
  - 27(14) km
  - P=1.5 TWh
  - Cost=1-1.3 LHCU

- **1 ab\(^{-1}\)/yr**
  - B=10 T, \( \Phi=10 \)
  - 27 km
  - P=2 TWh
  - Cost=1.6 LHCU

- **0.1 ab\(^{-1}\)/yr**
  - B=10 T
  - 6 km, \( \Phi=1 \)
  - P=1.3 TWh
  - Cost=0.7 LHCU

- **0.01 ab\(^{-1}\)/yr**
  - B=10 T, \( \Phi=100 \)
  - 500 km
  - P=6 TWh
  - Cost=4.2 LHCU

- **1 fb\(^{-1}\)/yr**
  - B=10 T, \( \Phi=100 \)
  - 1000 km
  - P=9 TWh
  - Cost=7 LHCU

**Fermilab**
New muon production schemes

- **LEMMA** and **Gamma-Factory**
  - small emittance
  - Intensity N/emm?
  - beam-beam

- Both require (expensive?) aux. $e^+$ or $p^+$ machines

---

**Linear lepton Colliders**

- Mostly $e^+e^-/e^-e^-$/$\gamma\gamma$
  - Muons possible, but $\mu$-sources are expensive (and limited prod’n $dN/dt$)
  - Protons possible, but lose factor of 7 in effective cme energy reach in $hh$
  - $NC$ $RF$, $SC$ $RF$, plasma, wakefields

- Major advantages:
  - No SR losses
  - RF acceleration well developed

- Major limitations:
  - $L$ scales with power, jitter/size and beamstrahlung
  - plasma/wakefield acceleration is not fully matured yet (many unknowns - energy staging, $e^+$, power, cost, etc)

\[
E_b = eGL
\]

\[
\mathcal{L} = \left(\frac{1}{\sigma_y^*}\right) \left(\frac{N_e}{\sigma_x^*}\right) \frac{H_D}{4\pi}
\]

\[
\eta \equiv \frac{P_b}{P_{\text{wall}}}
\]

\[
N_e n_b f_r = \eta P_{\text{wall}} / \left(e E_{\text{c.m.e.}}\right)
\]

\[
L \propto \frac{\eta_{\text{linac}} P_{\text{wall}} N_\gamma}{E \sigma_y}
\]

\[
N_\gamma \approx \frac{2\alpha_r e N}{\sigma_x}
\]

(luminosity spectrum)
**ee/γγ or μμ Linear Collider Luminosity**

- Other considerations:
  - Positron production and acceleration in plasma
    - Can be solved by switching to ee/γγ
  - Beamstrahlung
    - Can be solved by ultrashort bunches or switching to ee/γγ or μμ
      (see M. Peskin @ PLUB-II and Swapan C. talk today)
  - Instabilities in RF structures or plasma
  - Jitter/emittance control
    - Problems grow with more elements and smaller beams at IP → limit at 1A
      (T. Raubenheimer, PRSTAB 2000)

Linear RF and Plasma: *Lumi* and *Cost* vs *Energy*

**Linear ee/γγ**

- **10 ab⁻¹/yr**
  - 18 km
  - P = 3 TWh
  - Cost = 2.4-4 LHCU

- **1 ab⁻¹/yr**
  - LHC-1
  - CLIC-3
  - PWFA-10
  - LWFA-30
  - DLA-30

- **0.1 ab⁻¹/yr**
  - 40 km
  - P = 1.5 TWh
  - Cost = 1.5 LHCU
  - 50 km
  - P = 1 TWh
  - Cost = 2.5 LHCU

- **0.01 ab⁻¹/yr**
  - 50 km
  - P = 1 TWh
  - Cost = 2.5 LHCU

- **1 fb⁻¹/yr**
  - 21-42 km
  - P = 6-100 P Wh
  - Cost = 5-7 LHCU
  - 50-80 km
  - P = 2.5 TWh
  - Cost = 5-7 LHCU
  - ~500 km
  - P = 2.5 TWh
  - Cost = 13-18 LHCU
Exotic Colliders

- Acceleration in structured media, eg CNTs or crystals (only muons!!!)
- Major advantages:
  - solid density $\to$ 1-10 TV/m gradients
  - continuous focusing and acceleration (no cells, one long channel, particles get strongly cooled *betatron radiation*)
  - small size promises low cost
- Major limitations:
  - “blue sky”, $O(10)$ papers, plans for proof-of-principle experiment *E336* @FACET-II (S.Corde, T.Tajima, et al)
  - how to drive Xtals? lasers, beams?
  - Cost is unknown, power is unknown, luminosity - (how low?)

$E \text{[GV/m]} \approx 100\sqrt{n_0 [10^{18} \text{ cm}^{-3}]}$

Exotic Coliders: Line of Thinking

Size is limited <10 km → calls for the highest gradients → crystals → muons

$L = f \frac{N_1 N_2}{A}$

$L \approx 1 \text{ Å}^2 = 10^{-16} \text{ cm}^{-2}$

$P = fn_{ch} \cdot NE$

Luminosity calls for more particles in the smallest beam size

This is the smallest beam size at IP

The power is limited <10MW

$\rightarrow N$ is small at high $E \rightarrow$ low $L$
XC Luminosity

- Considerations:
  - Muons/bunch < Xtal electrons excited
  - Employ many channels
  - Limit beam power $O(10\text{MW})$
  - Combine $n\text{\_channels}$ to gain $L$ via crystal funnel (? Is it possible)

\[ L = \frac{f N^2}{A} \]

\[
\begin{align*}
100\lambda_p & \times \lambda_p^2 \\
N_0 & \sim 10^3 \\
n_{ch} & \sim 100
\end{align*}
\]

\[ P = f n_{ch} N E \]

\[ f = 10^6 \text{ Hz} \]

\[ L \left[ \text{sm}^{-2} \text{ s}^{-1} \right] \approx 4 \times 10^{33-35} \]

\[ \frac{P^2 \left[ \text{MW} \right]}{E^2 \left[ \text{TeV} \right] f n_{ch} \left[ 10^8 \text{ Hz} \right]} \]
Xtal Collider

\[ n \sim 10^{22} \text{ cm}^{-3}, \quad 10 \text{ TeV/m} \rightarrow 1 \text{ PeV} = 1000 \text{ TeV} \]

\[ n_\mu \sim 1000, \quad n_B \sim 100, \quad f_{\text{rep}} \sim 10^6, \quad L \sim 10^{30-32} \]

\[ n \sim 10^{22} \text{ cm}^{-3}, \quad 10 \text{ TeV/m} \rightarrow 1 \text{ PeV} = 1000 \text{ TeV} \]
Xtal Colliders: *Lumi* and *Cost vs Energy*

**Linear Xtal $\mu\mu$**

- **10 ab$^{-1}$/yr**
  - $G=1$ TV/m
  - 0.1-1 km
  - $P=\leq 1$ TWh
  - Cost $=\leq 1$ LHCU?

- **1 ab$^{-1}$/yr**
  - $G=1$-10 TV/m
  - 0.1-1 km
  - $P=\leq 1$ TWh
  - Cost $=\leq 1$ LHCU?

- **0.1 ab$^{-1}$/yr**
  - $G=1$-10 TV/m
  - 1-10 km
  - $P=\leq 2$ TWh
  - Cost $=\leq 10$ LHCU?

- **0.01 ab$^{-1}$/yr**
  - $G=1$-10 TV/m
  - 1-10 km
  - $P=\leq 2$ TWh
  - Cost $=\leq 10$ LHCU?

- **1 fb$^{-1}$/yr**
  - $G=1$-10 TV/m
  - 1-10 km
  - $P=\leq 2$ TWh
  - Cost $=\leq 10$ LHCU?
Main Conclusions:

- **For ultimate high energy colliders:**
  - Major thrust is *Energy*
  - Major concern/limit is *Cost*
  - Main focus is *Luminosity* and *Power*

- **Cost:**
  - Critically dependent on core acceleration technology
  - Existing injectors and infrastructure greatly help

- **High Energy means low Luminosity:**
  - Don’t expect more than 0.1-1 ab⁻¹/yr at 30TeV-1 PeV
  - Assume *Power* limited to 1-3 TWh/yr
Main Conclusions (2):

• **For considered collider types:**

  • *Circular* $pp$ – limit is close or below 100 TeV (14 TeV cm)
  • *Circular* $ee$ – limit is $\sim$0.4-0.5 TeV
  • *Circular* $\mu\mu$ – limit is between 30 and 100 TeV
  • *Linear RF* $ee/\gamma\gamma$ – limit is between 3 and 10 TeV
  • *Plasma* $ee/\gamma\gamma$
  • *Exotic crystal* $\mu\mu$ – promise of 0.1-1 PeV, low Luminosity

• **Muons are particles of the future**

Helpful/cited references (next slide)
(Some) References:

1. $\alpha\beta\gamma$ model – V.Shiltsev, *JINST* 9 T07002 (2014).
2. RMP Colliders – V.Shiltsev, F.Zimmermann, *Rev.Mod.Phys.* (2021); see also arxiv
6. F.Zimmermann – *NIMA* 909 (2018): 33-37; see also ARIES Workshops summary
9. Granada ALEGRO – Input to *EPPSU*, #007a (Granada, 2019)
Thank You for Your Attention!