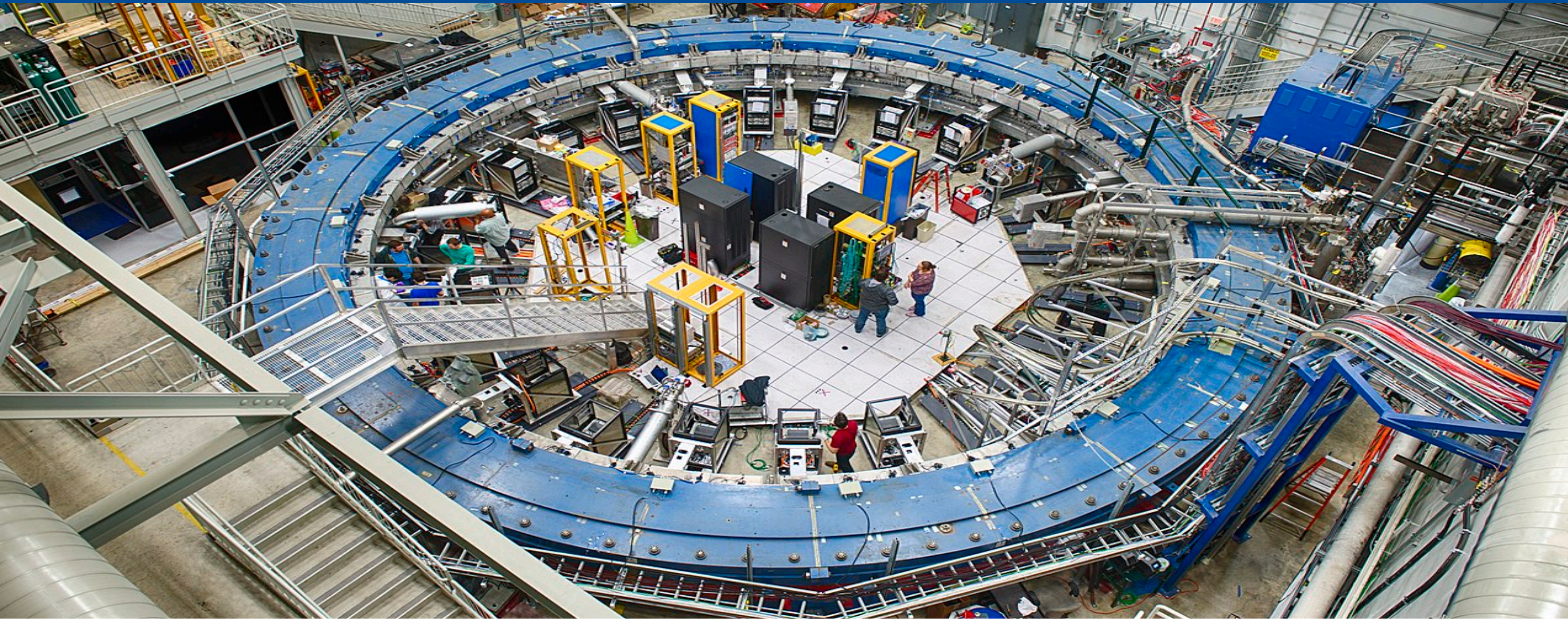


Beam dynamics characterization and uncertainties in the Muon g-2 Experiment at Fermilab

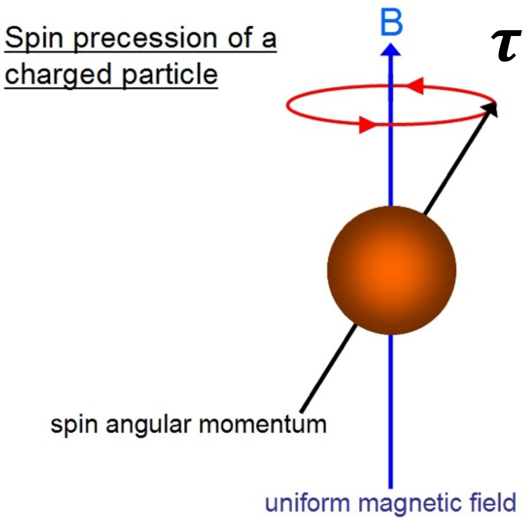
David A. Tarazona
ANL HEP seminar
February 15, 2021

MICHIGAN STATE UNIVERSITY

Contents

- Muon g-2 Overview
- Muon Campus
- Storage Ring
- Contributions to *g-2*
- Research Plans

Muon g-2 Overview: a_μ



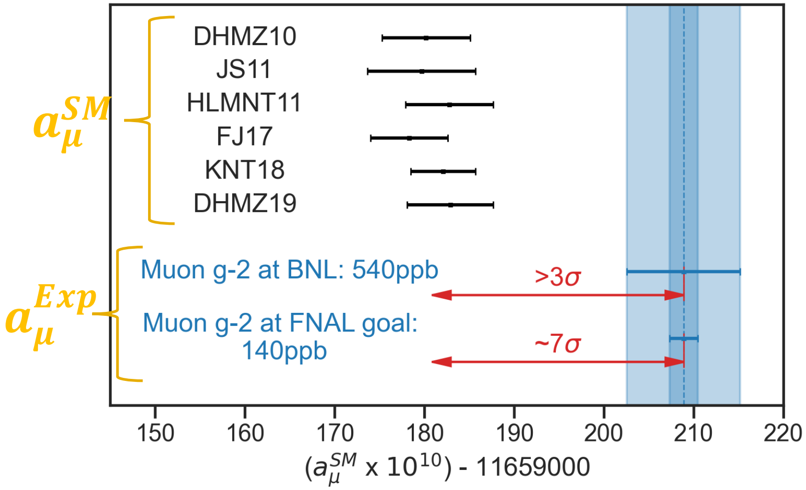
$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \boldsymbol{B}$$

$$\boldsymbol{\mu} = g \frac{e}{2m_\mu} \boldsymbol{S}$$

- Dirac equation (relativistic QM) predicts

$$g = 2$$

- Experiments and Standard Model indicate:



$$g \neq 2$$

$$\Downarrow$$

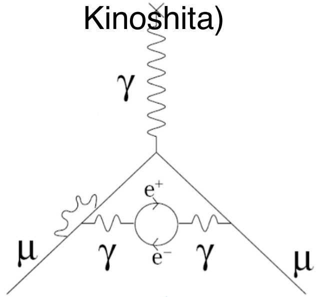
$$g \equiv 2(1 + a_\mu)$$

Motivation

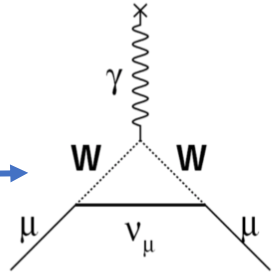
$$a_\mu^{SM} \neq a_\mu^{Exp}$$

Muon g-2 Overview: a_μ from Standard Model

12672 QED diagrams
(up to 10th-order,
Kinoshita)



Dirac equation

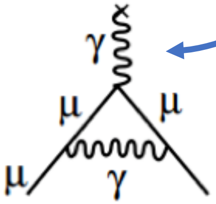


Electroweak contributions

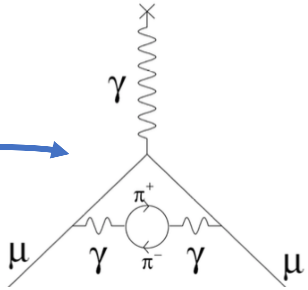
Standard Model

$$g_\mu = 2.00233183622(756)\cdots$$

a_μ^{SM}



1st-order QED (Schwinger)

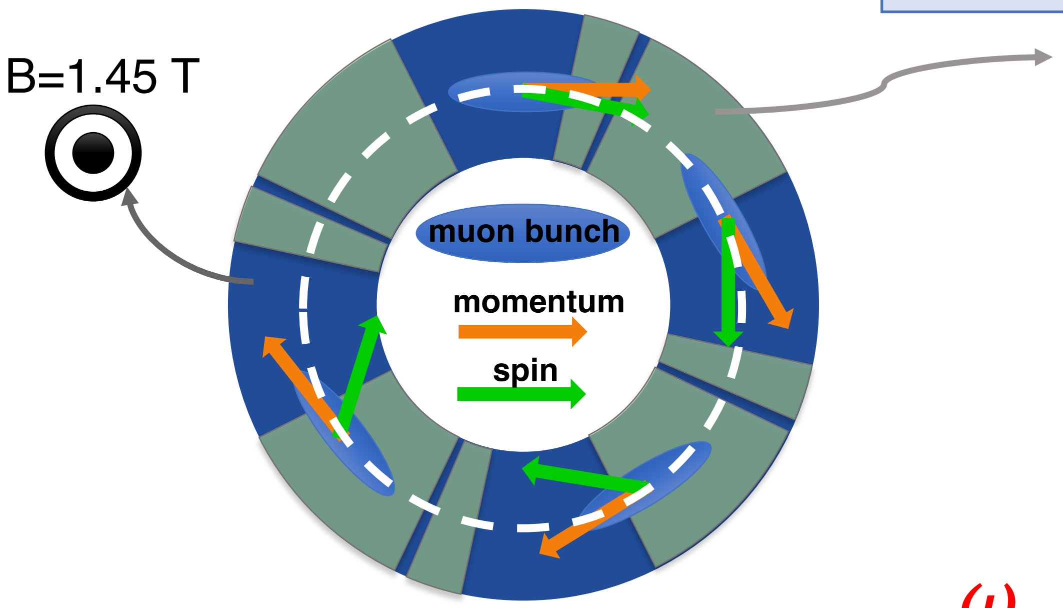


Hadronic contributions

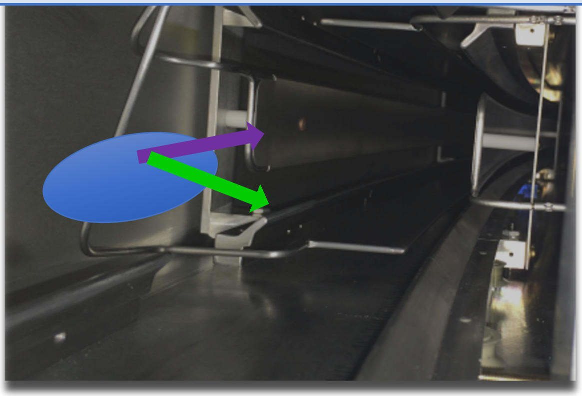
*KNT *Phys.Rev.D* 101 (2020) 014029

Muon g-2 Overview: a_μ from Fermilab

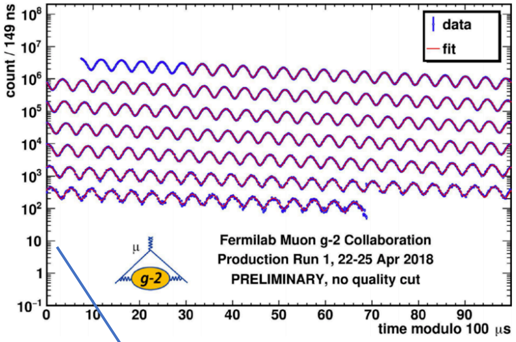
$$\omega_a = \omega_S - \omega_C$$



Electrostatic Quadrupole System (EQS)



$$\omega_a = -\frac{e}{m} a_\mu \langle B \rangle$$



ω_a: Spin precession frequency relative to the momentum direction of muons in the of the storage ring

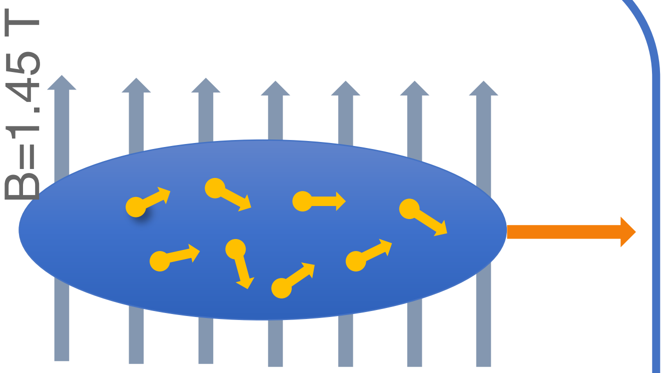
$$N(t, E_{th}) = N_0(E_{th}) \exp^{-t/\gamma\tau_\mu}[1 + A(E_{\text{th}}) \cos(\omega_a t + \varphi_0(E_{th}))]$$

Muon *g*-2 Overview: Beam Requirements

For the general case in lab frame ($p_0 = mc/\sqrt{a_\mu}$=3.094GeV/c):

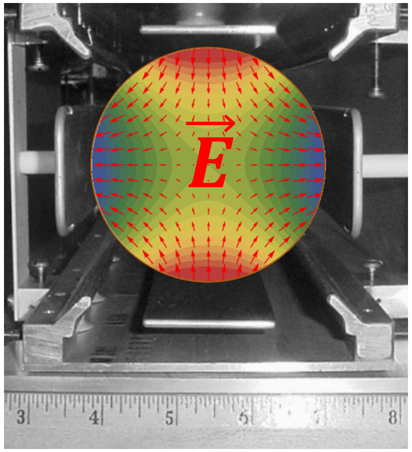
$$\omega_a = \langle (\vec{\omega}_s - \vec{\omega}_C)_y \rangle = -\frac{e}{m} a_\mu \langle B \rangle + \frac{e}{m} a_\mu \left\langle \left(\left(\frac{\gamma}{\gamma+1}\right) (\vec{\beta} \cdot \vec{B})\vec{\beta} + \left(1 - \frac{1}{(1+\delta)^2}\right) \frac{\vec{\beta}\times\vec{E}}{c} \right)_y \right\rangle$$

Pitch Correction "C_P"



- **Muon's** vertical motion aligns with B-field

E-field Correction "C_E"



Injected muon bunch's Momentum Acceptance:

$$\delta_{max} = \frac{dp}{p_0}$$

$$\Downarrow$$

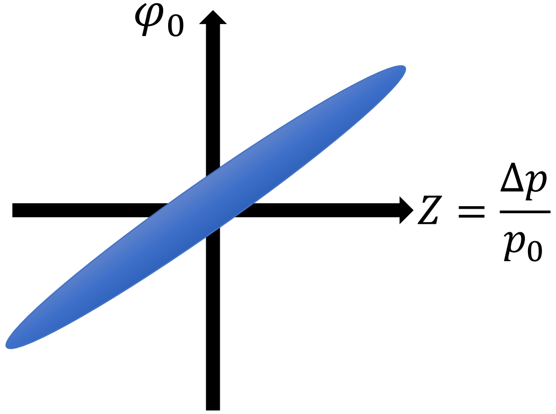
$$\pm 0.5\%$$

Muon *g*-2 Overview: Beam Requirements

$$N(t, E_{th}) = N_0(E_{th})\exp^{-t/\gamma\tau_\mu}[1 + A(E_{\mathrm{th}})\cos(\omega_a t + \varphi_0(E_{th}))]$$

$$\Delta\omega_a \approx \frac{\Delta\varphi_0}{\Delta t}$$

Lost Muons Correction "C_{LM}"



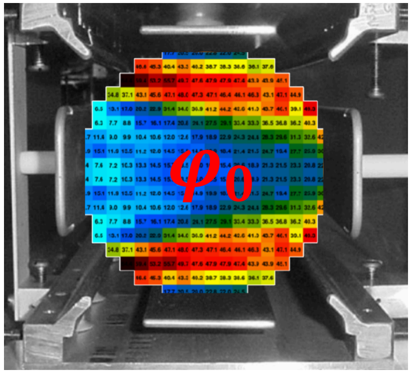
$$\frac{d\varphi_0(t)}{dt} \approx \left.\frac{d\varphi_0}{dZ}\right|_{t=t_0} \frac{dZ(t)}{dt}$$

Lost muons' Z

≠

Stored muons' Z

Phase Acceptance Correction "C_{PA}"

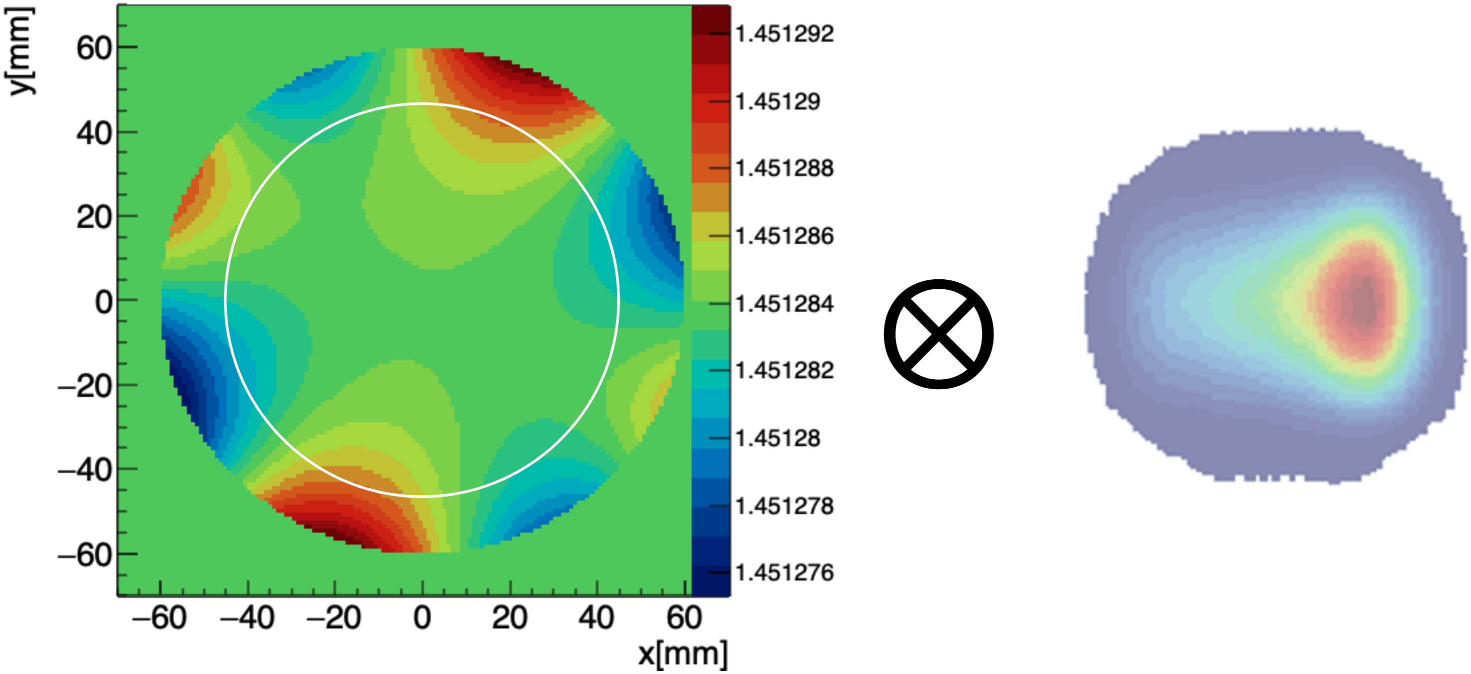


Unstable beam → $\frac{\Delta\varphi_0}{\Delta t} \neq 0$

φ_0 from positron detection effects

Muon *g*-2 Overview: Beam Requirements

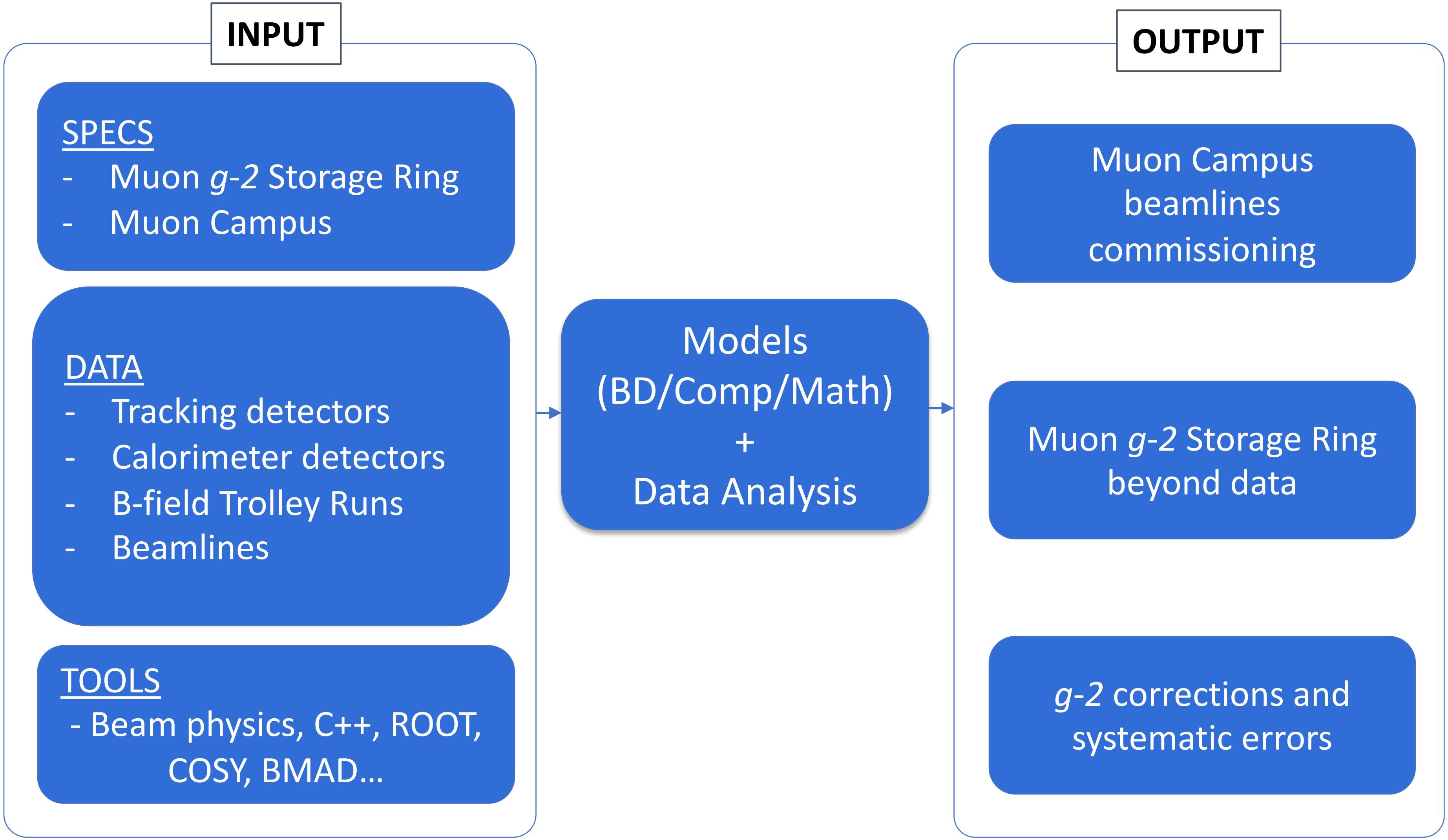
$\langle \boldsymbol{B} \rangle$



- $\langle B \rangle$ is the magnetic field experienced by the muon beam around the storage ring.

- Requirement -> Stored beam azimuthal characterization.

Research Work: Workflow



INPUT

SPECS
- Muon *g-2* Storage Ring
- Muon Campus

DATA
- Tracking detectors
- Calorimeter detectors
- B-field Trolley Runs
- Beamlines

TOOLS
- Beam physics, C++, ROOT, COSY, BMAD…

Models (BD/Comp/Math) + Data Analysis

OUTPUT

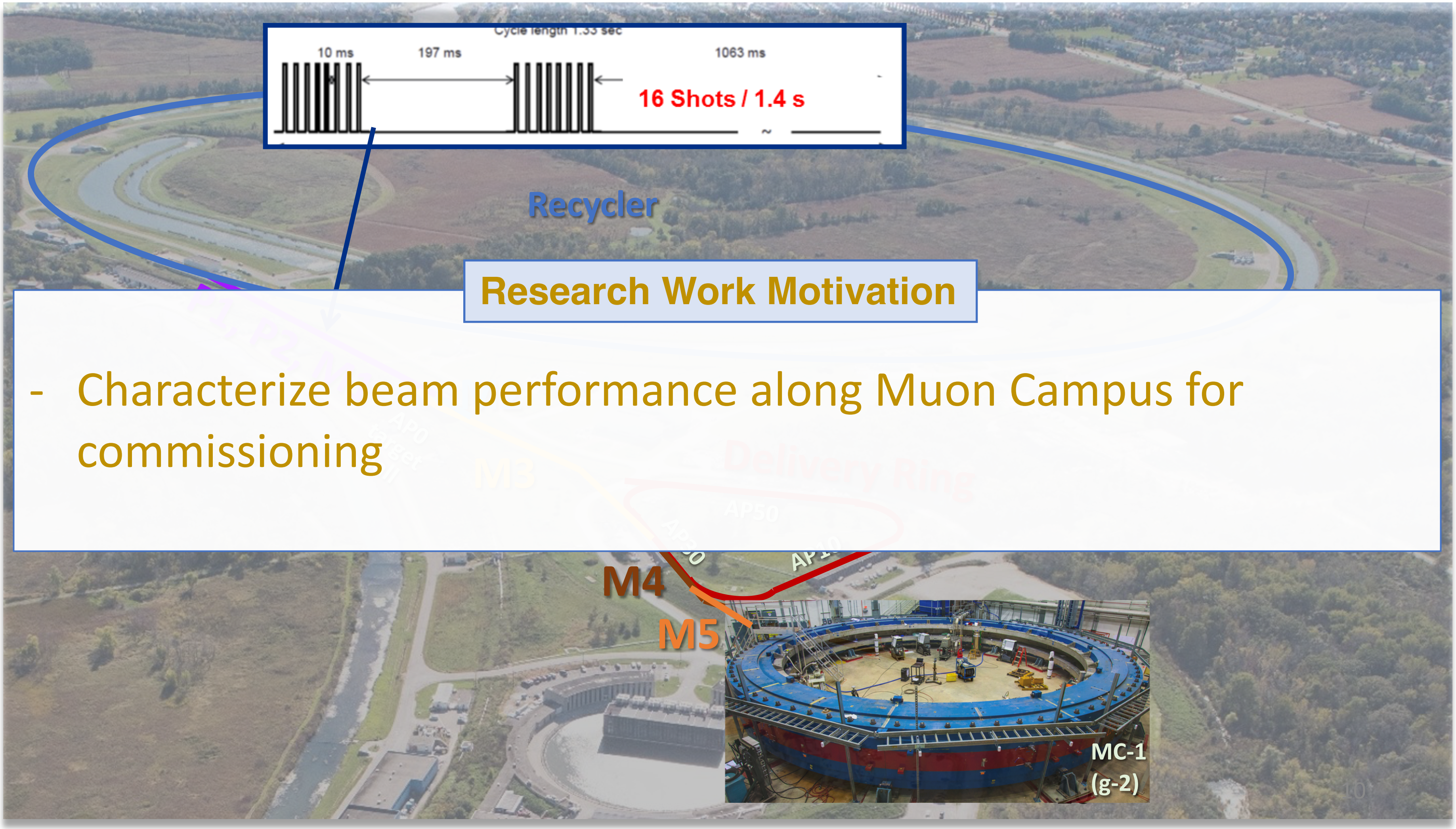
Muon Campus beamlines commissioning

Muon *g-2* Storage Ring beyond data

g-2 corrections and systematic errors

Muon Campus

- >748k highly polarized muons/fill expected at entrance of the Storage Ring (TDR)
- Batches of 10^{12} protons hit “pion-production” target ($\pi^+ \rightarrow \mu^+ + \nu_\mu$)



Research Work Motivation

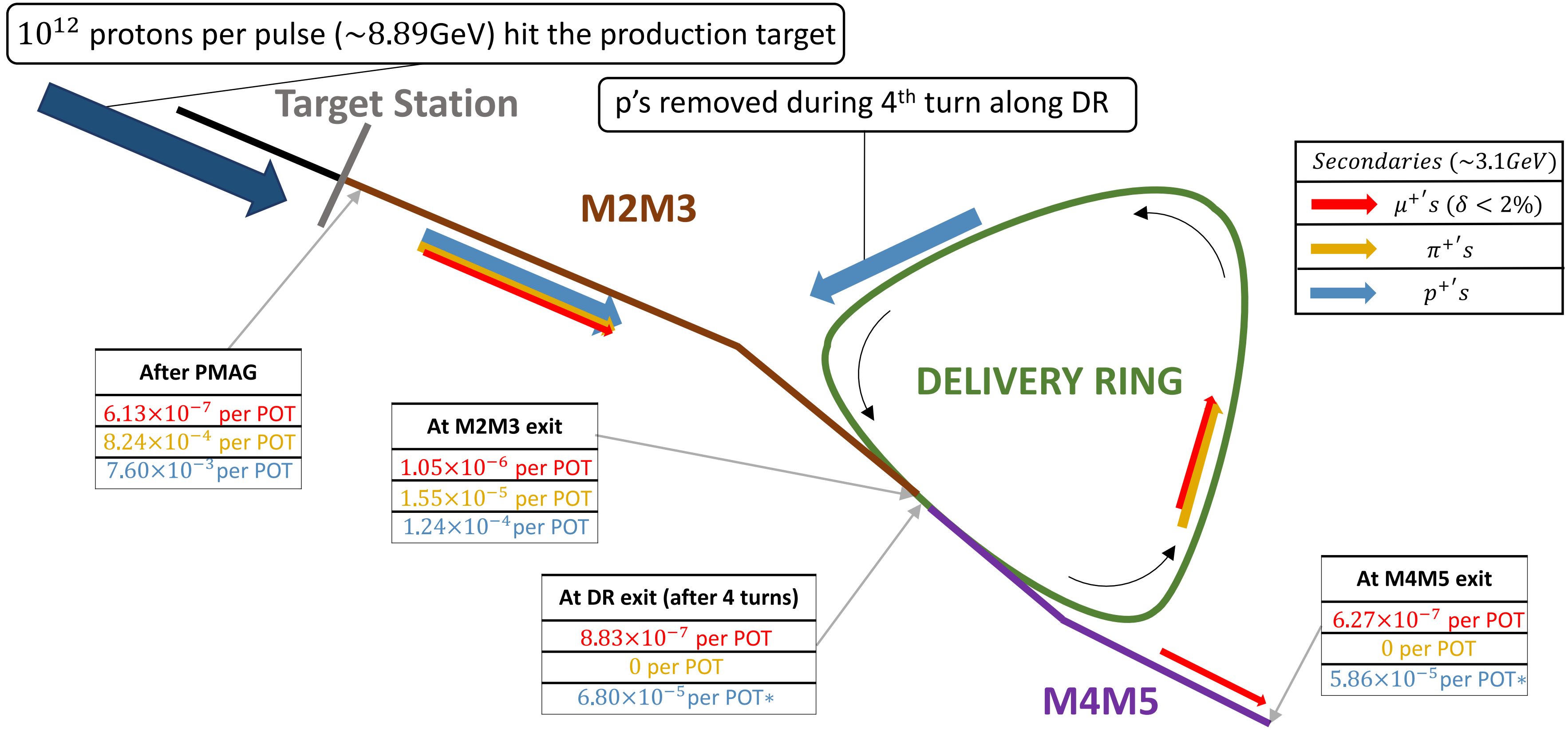
- Characterize beam performance along Muon Campus for commissioning

Muon Campus studies

Main purposes of the *COSY*-based Muon Campus model:

- Characterize beam performance
- Validate numerical models within *g*-2 Collaboration
- Quantify impact of nonlinearities on muon beam production

Muon Campus: Beam Performance



10^{12} protons per pulse ($\sim$8.89GeV) hit the production target

p's removed during 4th turn along DR

$Secondaries\ (\sim 3.1GeV)$	
🟥	$\mu^{+'}s\ (\delta < 2\%)$
🟧	$\pi^{+'}s$
🟦	$p^{+'}s$

After PMAG
6.13×10^{-7} per POT
8.24×10^{-4} per POT
7.60×10^{-3}per POT

At M2M3 exit
1.05×10^{-6} per POT
1.55×10^{-5} per POT
1.24×10^{-4}per POT

At DR exit (after 4 turns)
8.83×10^{-7} per POT
0 per POT
6.80×10^{-5}per POT$*$

At M4M5 exit
6.27×10^{-7} per POT
0 per POT
5.86×10^{-5}per POT$*$

* Without proton removal system

COSY end-to-end simulation considers:

- Fringe fields
- Spin Dynamics
- High-order effects
- Apertures of the beam line elements
- Primary decay modes ($\pi^+ \rightarrow \mu^+ + \nu_\mu$ and $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$)
- Horizontal and vertical misalignments (randomly Gaussian-distributed [$\sigma_x = \sigma_y = 0.25\text{mm}$])

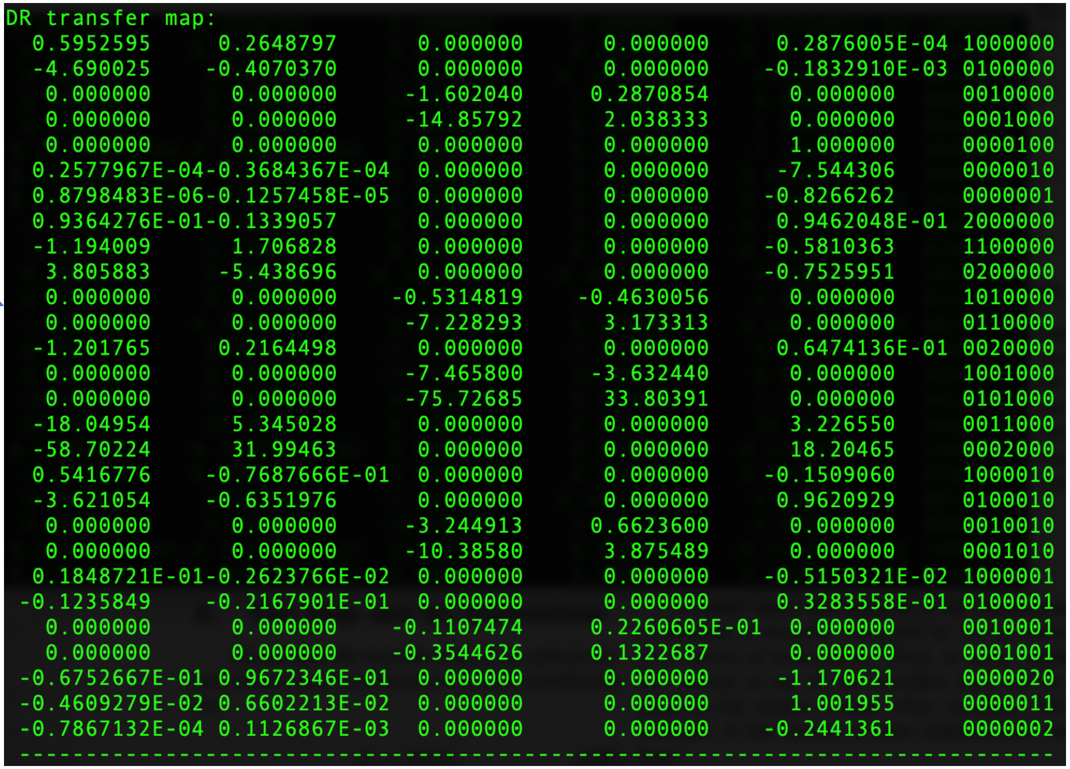
Muon Campus: *COSY*-INFINITY

$$z_2 = \sum_{ijklm=0}^{\infty,\infty,\infty,\infty,\infty} a^z_{ijklm} x_1^i {x'}_1^j y_1^k {y'}_1^l {\delta_p^m}_1 = \sum_{ijklm=0}^{1,1,1,1,1} a^z_{ijklm} x_1^i {x'}_1^j y_1^k {y'}_1^l {\delta_p^m}_1 + \text{nonlinearities}$$

$z \rightarrow x, x', y, y', \delta_p = \Delta p / p_0$ muon coordinates rel. to ideal muon.

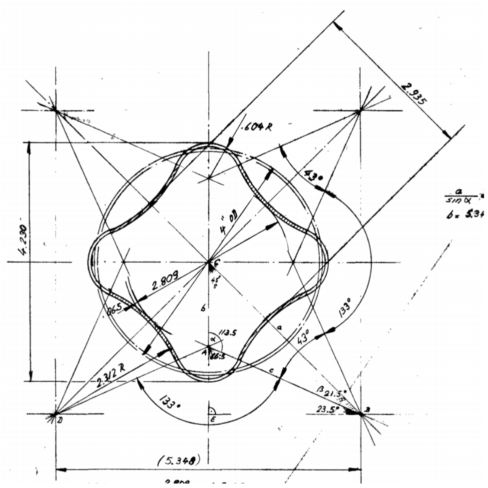
a^z_{ijklm}

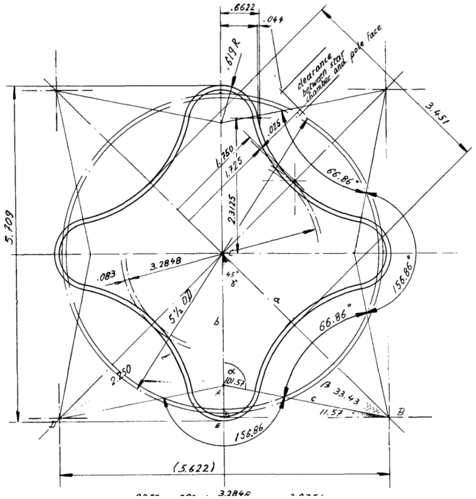
$\mathcal{M}(\vec{z}_1) = \vec{z}_2$



```
DR transfer map:
  0.5952595     0.2648797      0.000000      0.000000     0.2876005E-04 1000000
  -4.690025    -0.4070370      0.000000      0.000000    -0.1832910E-03 0100000
   0.000000      0.000000     -1.602040     0.2870854      0.000000     0010000
   0.000000      0.000000     -14.85792      2.038333      0.000000     0001000
   0.000000      0.000000      0.000000      0.000000      1.000000     0000100
  0.2577967E-04-0.3684367E-04  0.000000      0.000000     -7.544306     0000010
  0.8798483E-06-0.1257458E-05  0.000000      0.000000    -0.8266262     0000001
  0.9364276E-01-0.1339057      0.000000      0.000000     0.9462048E-01 2000000
  -1.194009      1.706828      0.000000      0.000000    -0.5810363     1100000
   3.805883     -5.438696      0.000000      0.000000    -0.7525951     0200000
   0.000000      0.000000    -0.5314819    -0.4630056      0.000000     1010000
   0.000000      0.000000     -7.228293      3.173313      0.000000     0110000
  -1.201765     0.2164498      0.000000      0.000000     0.6474136E-01 0020000
   0.000000      0.000000     -7.465800     -3.632440      0.000000     1001000
   0.000000      0.000000     -75.72685      33.80391      0.000000     0101000
  -18.04954      5.345028      0.000000      0.000000      3.226550     0011000
  -58.70224      31.99463      0.000000      0.000000      18.20465     0002000
  0.5416776    -0.7687666E-01  0.000000      0.000000    -0.1509060     1000010
  -3.621054    -0.6351976      0.000000      0.000000     0.9620929     0100010
   0.000000      0.000000     -3.244913     0.6623600      0.000000     0010010
   0.000000      0.000000     -10.38580      3.875489      0.000000     0001010
  0.1848721E-01-0.2623766E-02  0.000000      0.000000    -0.5150321E-02 1000001
 -0.1235849    -0.2167901E-01  0.000000      0.000000     0.3283558E-01 0100001
   0.000000      0.000000    -0.1107474     0.2260605E-01  0.000000     0010001
   0.000000      0.000000    -0.3544626     0.1322687      0.000000     0001001
 -0.6752667E-01 0.9672346E-01  0.000000      0.000000     -1.170621     0000020
 -0.4609279E-02 0.6602213E-02  0.000000      0.000000      1.001955     0000011
 -0.7867132E-04 0.1126867E-03  0.000000      0.000000    -0.2441361     0000002
 -------------------------------------------------------------------------------
```

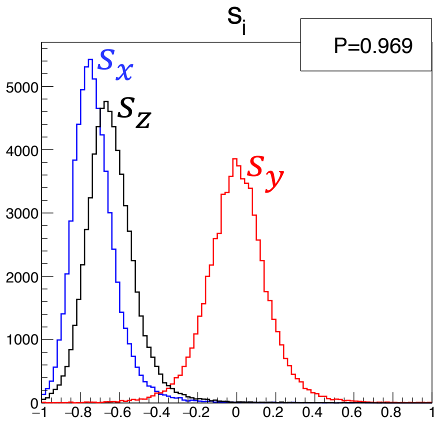
Muon Campus: Beam collimation and pion-to-muon decay

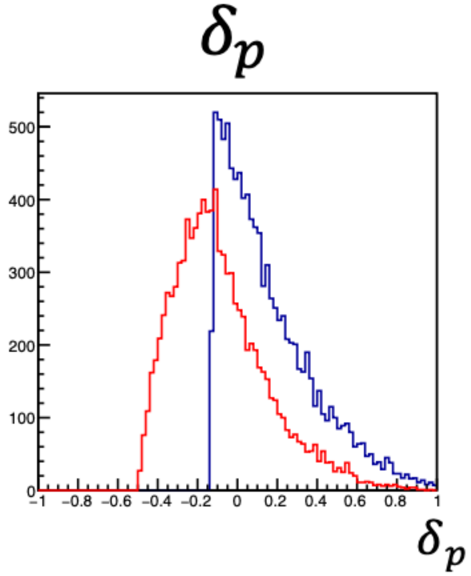




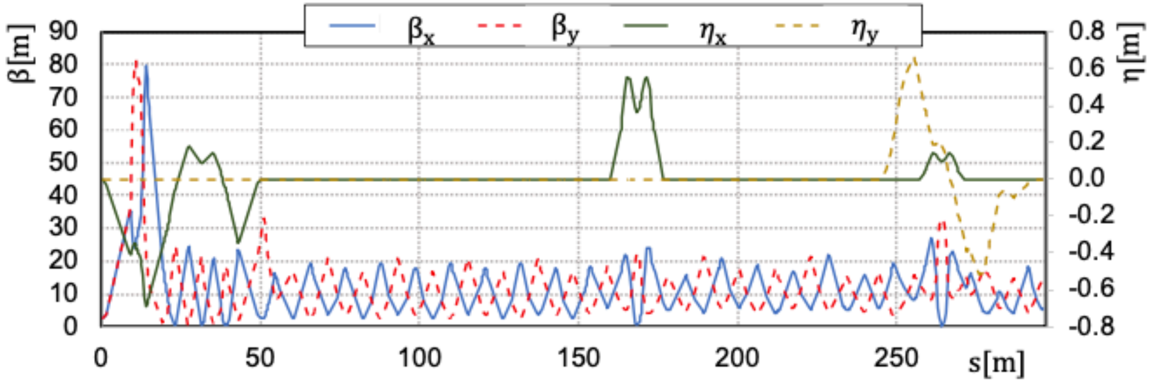
- Collimation across Muon Campus beamline elements.

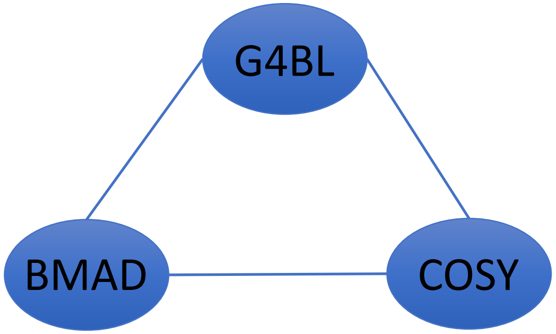
- Muon kinematics and spin dynamics from weak decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$





Muon Campus: Models validation

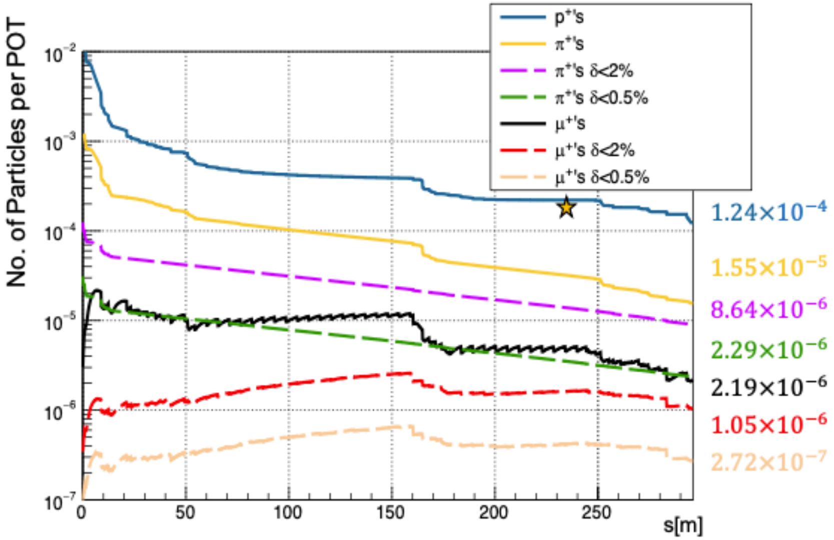




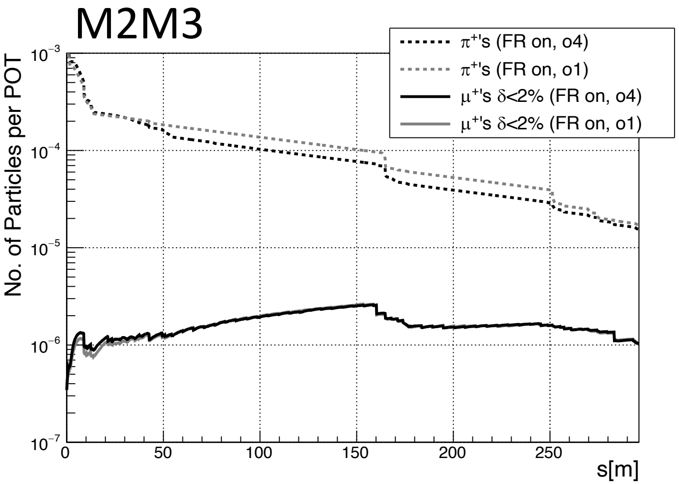
G4BL

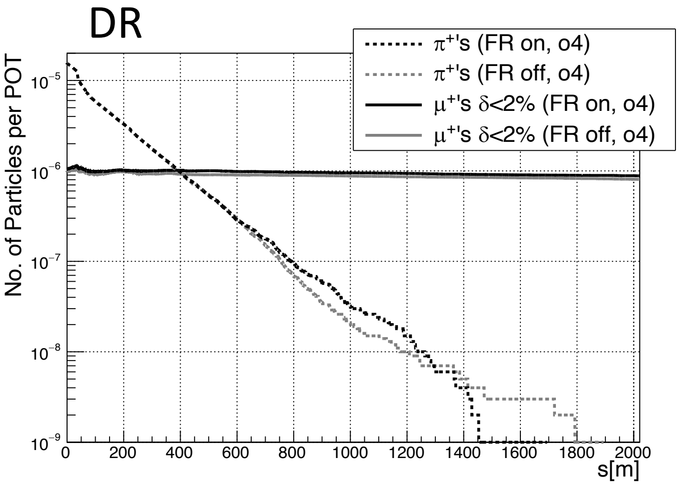
BMAD

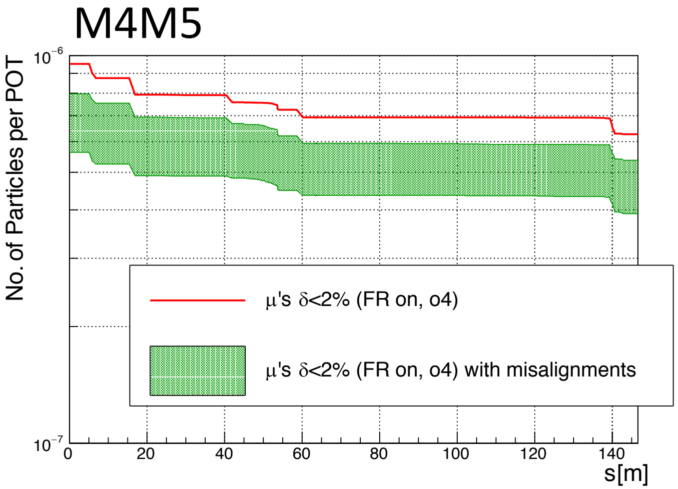
COSY



Muon Campus: Nonlinearities





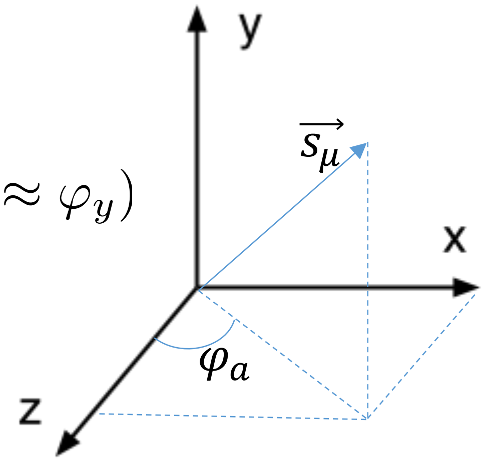


- Fringe fields and high-order guide fields increment muons/pions population by 9.4%.

Muon Campus: Spin-orbit correlation studies, DR

The following beamline higher-order systematic effect will be present in the E989 measurement*:

$(\varphi_a \approx \varphi_y)$



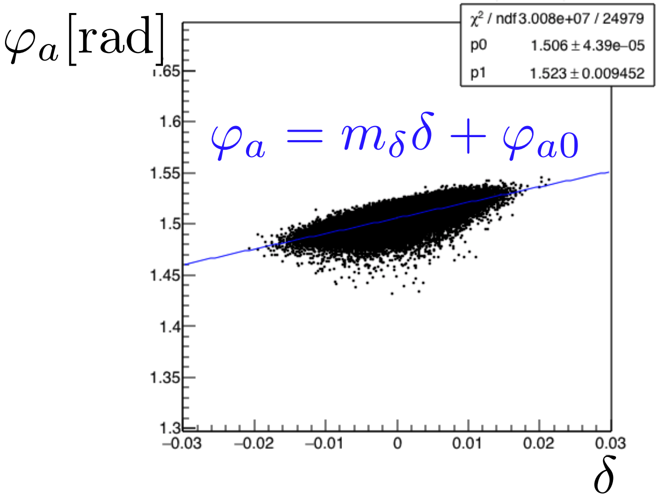
$$\langle \Delta a \rangle = \frac{m}{qB} \langle \Delta \omega_a \rangle$$

$$= \frac{m}{qB} \left(\frac{d\langle \phi_a \rangle}{d\gamma} \frac{d\langle \gamma \rangle}{dt} \right)$$

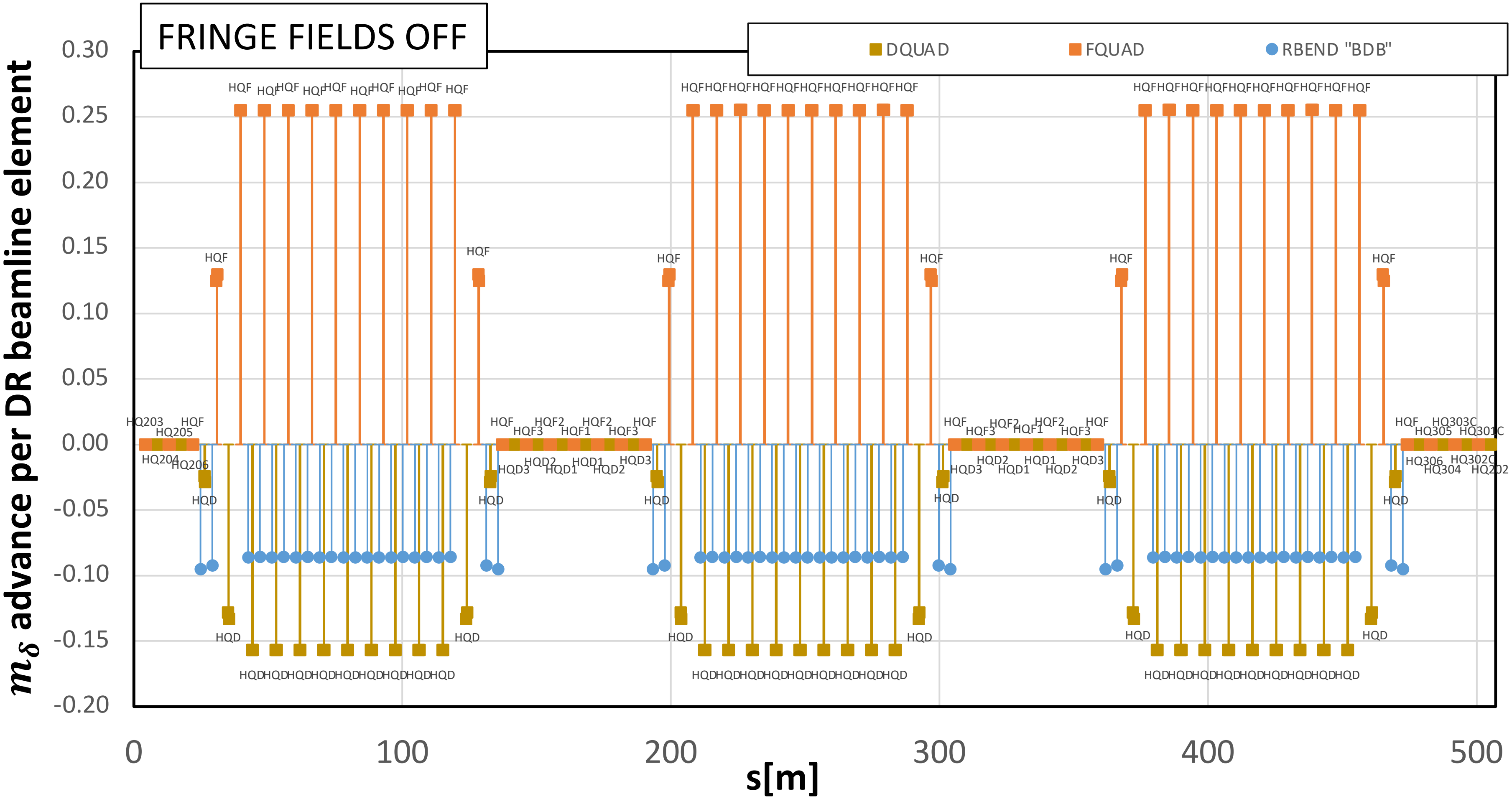
$$\gamma = \gamma_0 (1 + \beta_0^2 \delta) \quad \text{for} \quad \delta \to 0$$

$$\frac{d\langle \phi_a \rangle}{d\gamma} = \frac{1}{\gamma_0 \beta_0^2} \frac{\boldsymbol{d\langle \phi_a \rangle}}{\boldsymbol{d\delta}} \quad \text{for} \quad \delta \to 0$$

$$\boldsymbol{m_\delta = \left. \frac{d\varphi_a}{d\delta} \right|_{\delta \to 0}}$$

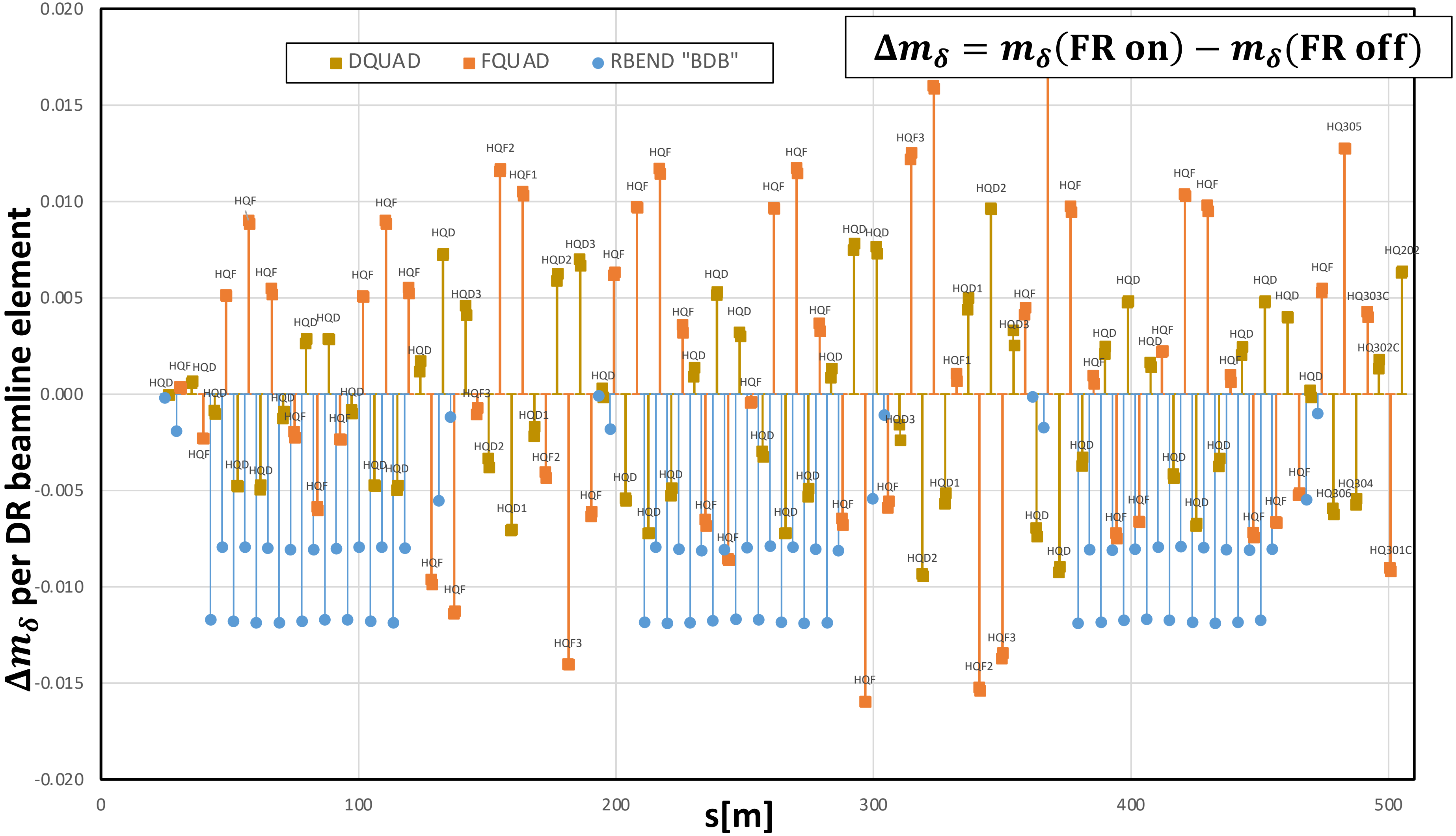


Muon Campus: Spin-orbit correlation studies, DR



- The advance of the spin-orbit correlation m_δ across the focusing quadrupoles "HQF" prevails among the other elements within the DR.

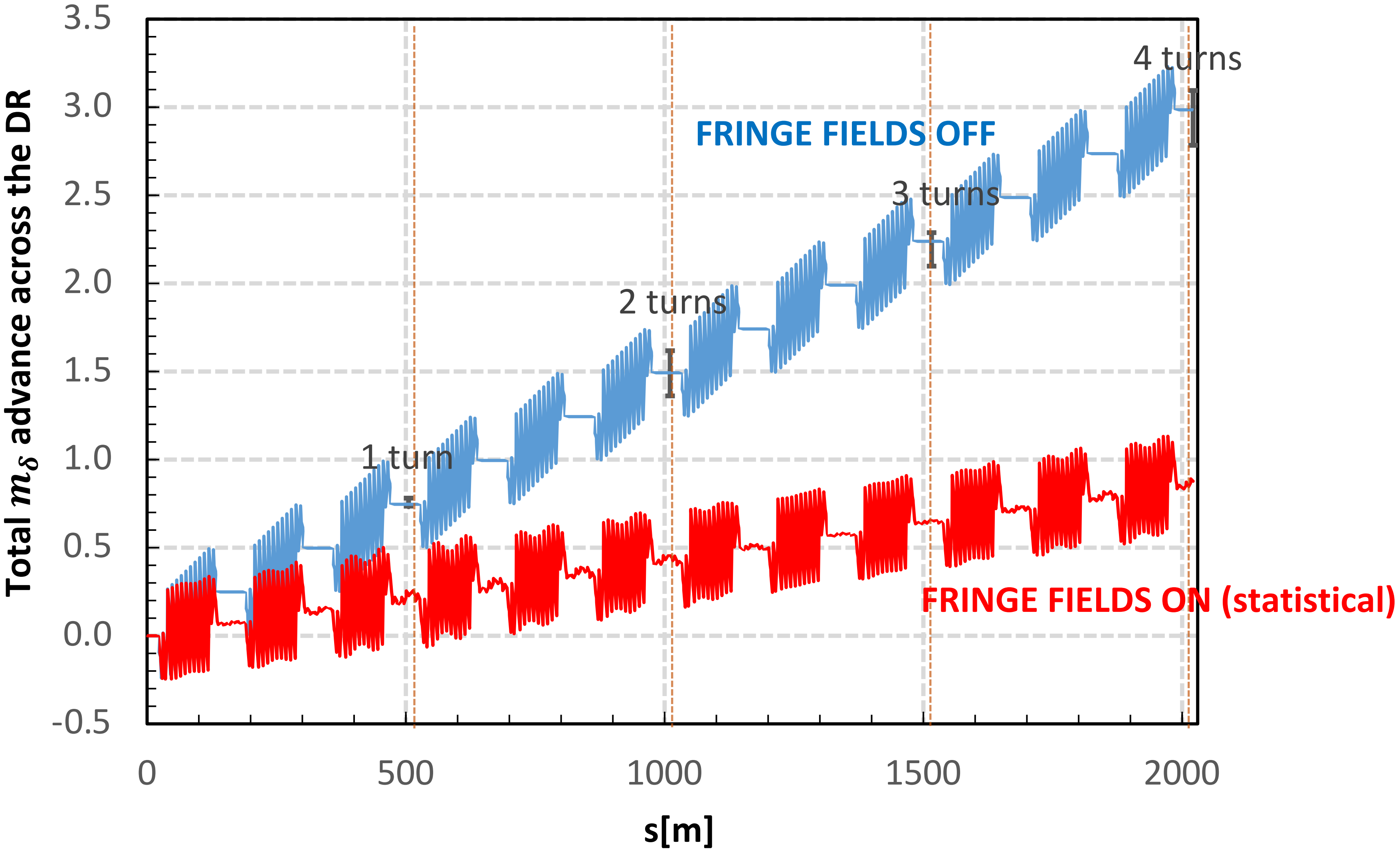
Muon Campus: Spin-orbit correlation studies, DR



$$\Delta m_\delta = m_\delta(\mathrm{FR\ on}) - m_\delta(\mathrm{FR\ off})$$

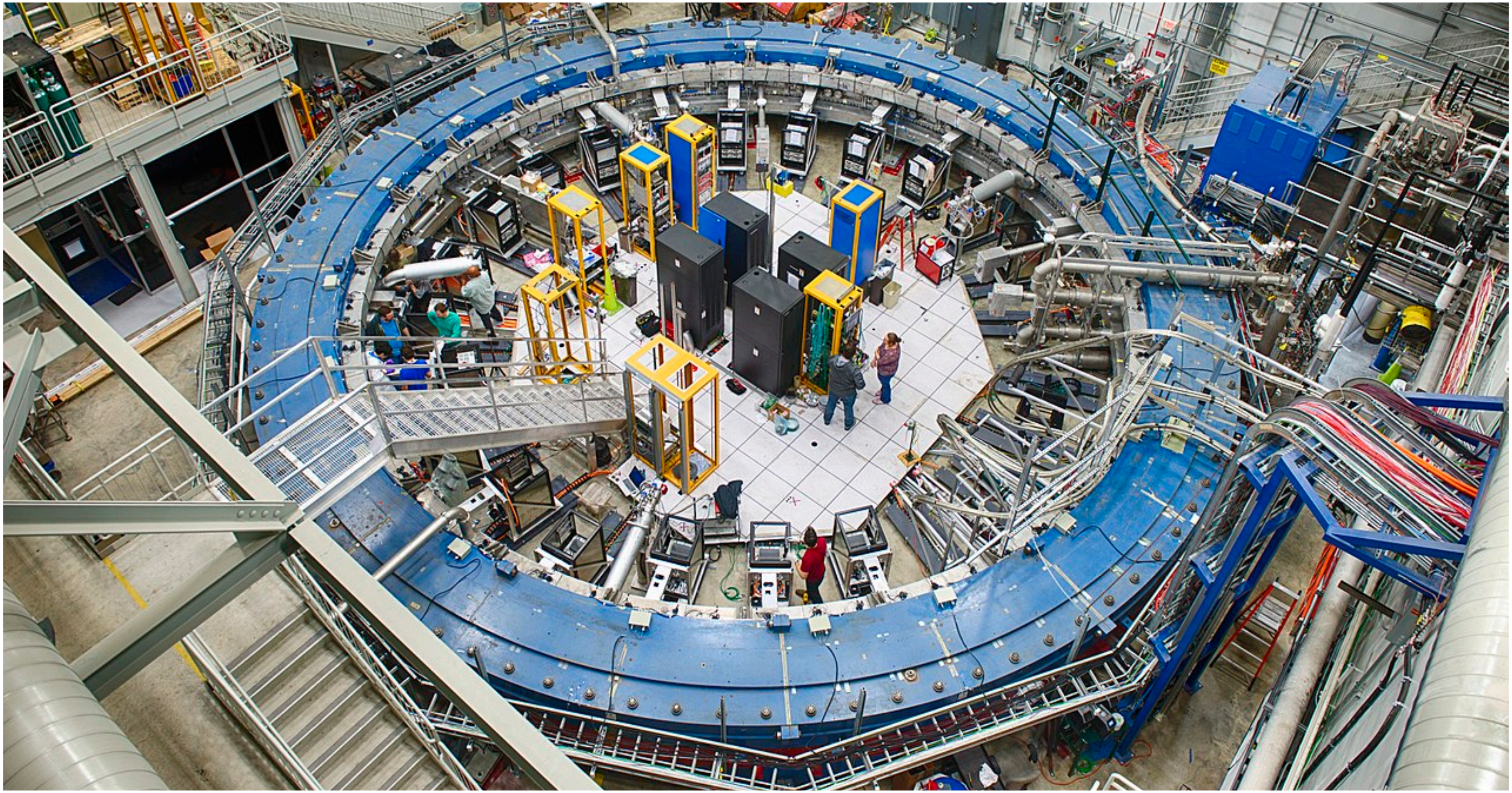
- Rectangular magnets are the main contributors to the reduction of m_δ due to fringe fields.

Muon Campus:

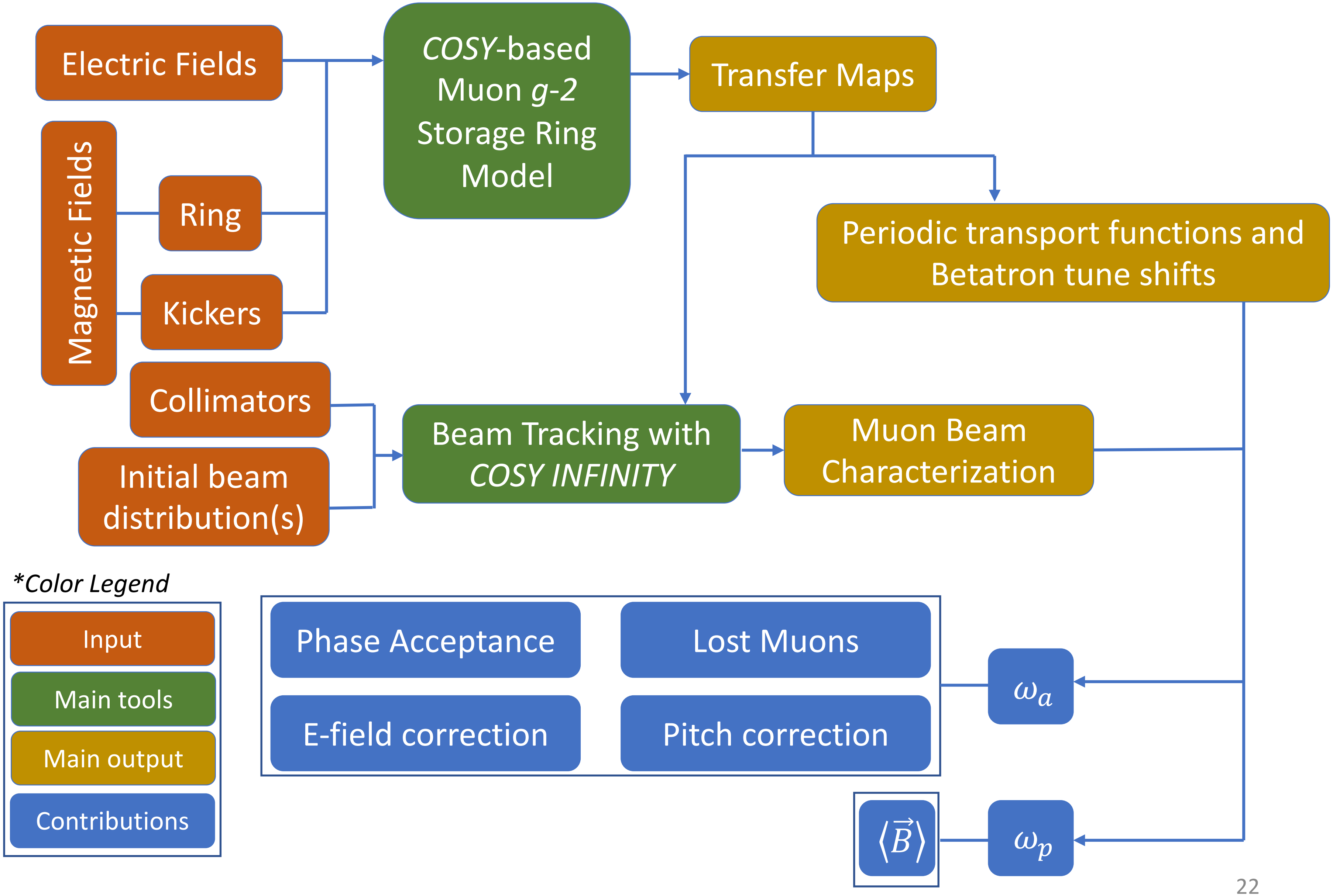


- Potential to control spin-orbit correlations via DR rectangular magnets.

Storage Ring Model: Features and implementation



Features and implementation



Electric Fields

Magnetic Fields

Ring

Kickers

Collimators

Initial beam distribution(s)

COSY-based Muon *g-2* Storage Ring Model

Transfer Maps

Periodic transport functions and Betatron tune shifts

Beam Tracking with *COSY INFINITY*

Muon Beam Characterization

**Color Legend*

- 🟧 Input
- 🟩 Main tools
- 🟨 Main output
- 🟦 Contributions

Phase Acceptance

Lost Muons

E-field correction

Pitch correction

ω_a

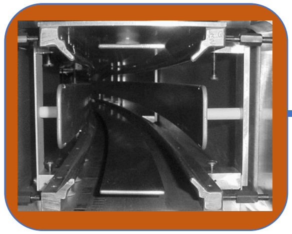
ω_p

$\langle \vec{B} \rangle$

Features and implementation

Input

- Electric fields:
  - Main field:



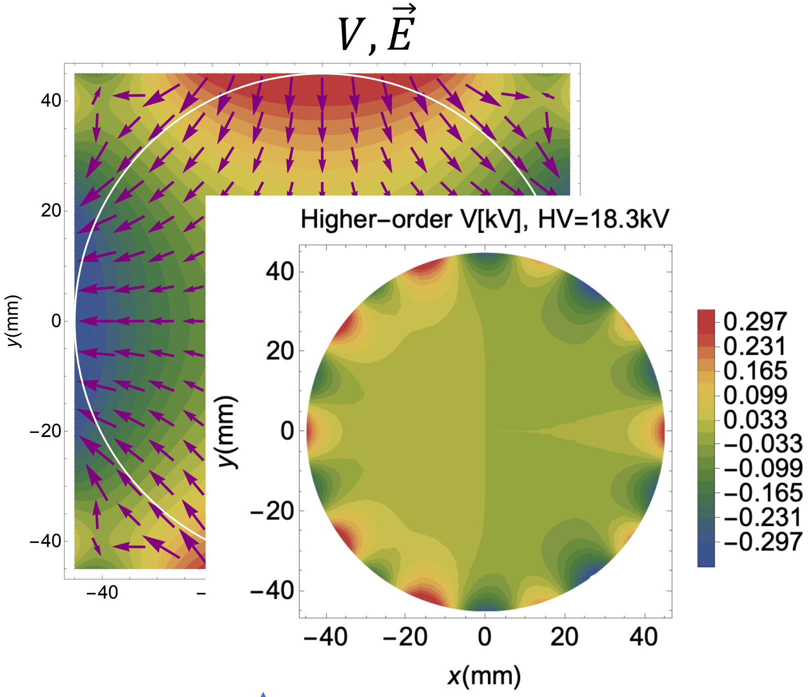
- Conformal mapping
- Fringe fields calc.

$$V = V(x,y,s) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} a_{k,l}(s)\frac{x^k y^l}{k!l!}$$

$$\Delta V(x,y,s) = 0$$

$$\begin{aligned} a_{k,l+2} = & -a''_{k,l} - kha''_{k-1,l} + kh'a'_{k-1,l} - a_{k+2,l} - (3k+1)ha_{k+1,l} \\ & - 3kha_{k-1,l+2} - k(3k-1)h^2 a_{k,l} - 3k(k-1)h^2 a_{k-2,l+2} \\ & - k(k-1)^2 h^3 a_{k-1,l} - k(k-1)(k-2)h^3 a_{k-3,l+2}, \end{aligned}$$

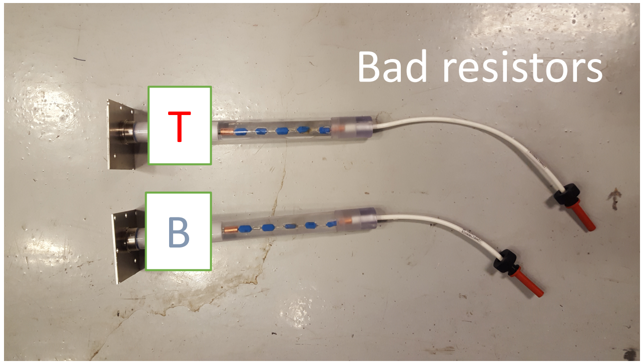
k	$l=0$	$l=2$	$l=4$	$l=6$	$l=8$	$l=10$
0	0	1.520E+07	-3.605E+06	-2.210E+12	2.569E+13	-2.712E+22
1	0	-6.410E+06	3.549E+06	8.389E+12	-8.251E+13	2.860E+23
2	-1.520E+07	3.605E+06	2.210E+12	-2.569E+13	2.712E+22	-2.059E+24
3	0	-2.028E+06	-7.457E+12	7.167E+13	-2.745E+23	1.246E+25
4	0	-2.210E+12	1.887E+13	-2.712E+22	1.821E+24	7.217E+30
5	0	4.660E+12	-4.219E+13	2.402E+23	-9.955E+24	-1.370E+32
6	2.210E+12	-7.863E+12	2.712E+22	-1.377E+24	-7.217E+30	1.575E+33
7	0	1.135E+13	-1.830E+23	6.442E+24	1.218E+32	-1.419E+34
8	0	-2.712E+22	8.170E+23	7.217E+30	-1.241E+33	-1.425E+41
9	0	1.029E+23	-2.978E+24	-1.005E+32	9.880E+33	3.907E+42
10	2.712E+22	-2.895E+23	-7.217E+30	8.544E+32	1.425E+41	-6.221E+43
11	0	6.634E+23	7.306E+31	-5.689E+33	-3.366E+42	7.515E+44
12	0	7.217E+30	-4.691E+32	-1.425E+41	4.649E+43	-7.654E+45
13	0	-3.957E+31	2.382E+33	2.705E+42	-4.884E+44	6.947E+46
14	-7.217E+30	1.558E+32	1.425E+41	-3.056E+43	4.336E+45	-5.815E+47
15	0	-4.887E+32	-1.924E+42	2.647E+44	-3.440E+46	4.593E+48
16	0	-1.425E+41	1.613E+43	-1.951E+45	2.525E+47	-3.478E+49
17	0	1.022E+42	-1.051E+44	1.295E+46	-1.756E+48	2.556E+50
18	1.425E+41	-5.173E+42	5.933E+44	-8.016E+46	1.176E+49	-1.840E+51

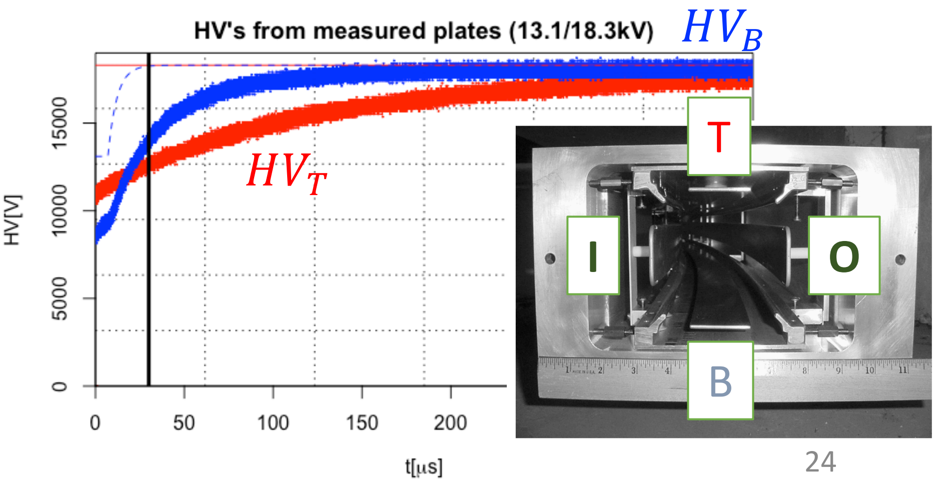


Features and implementation

Input

- Electric fields:





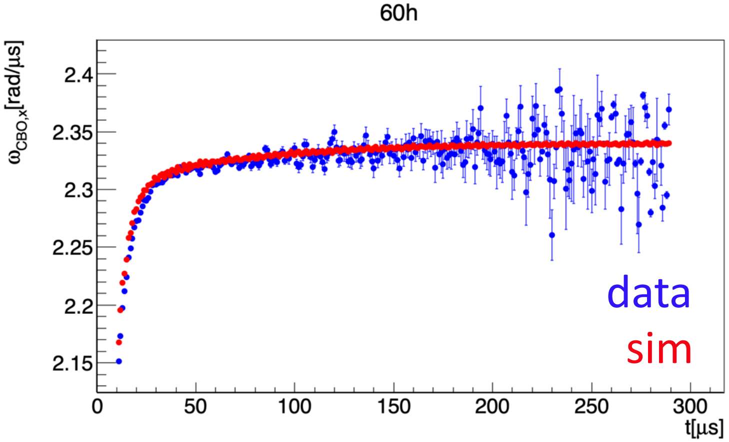
- Two resistors connected to Q1L were damaged during Run-1
- Consequently, HV at Q1L behaved in several failure modes
- HV_T and HV_B were not measured during Run-1…

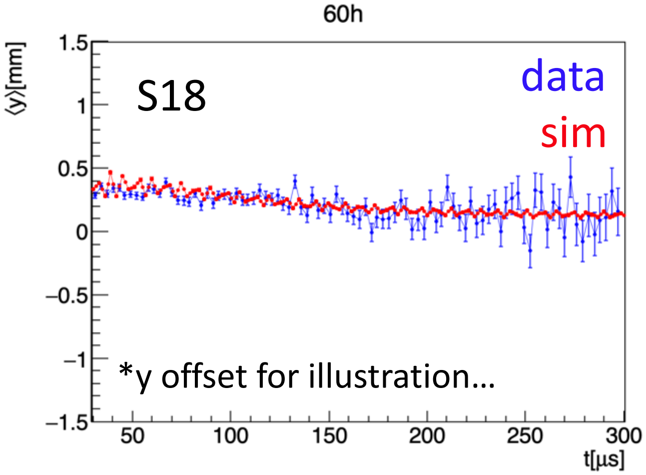
Features and implementation

Input

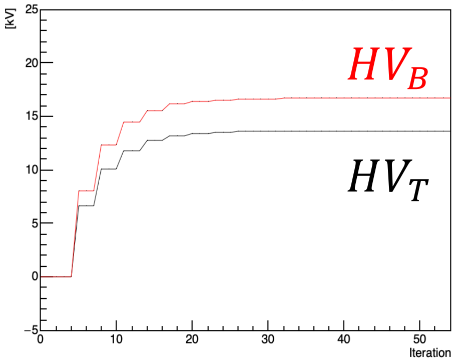
- Electric fields (First *g-2* Run):

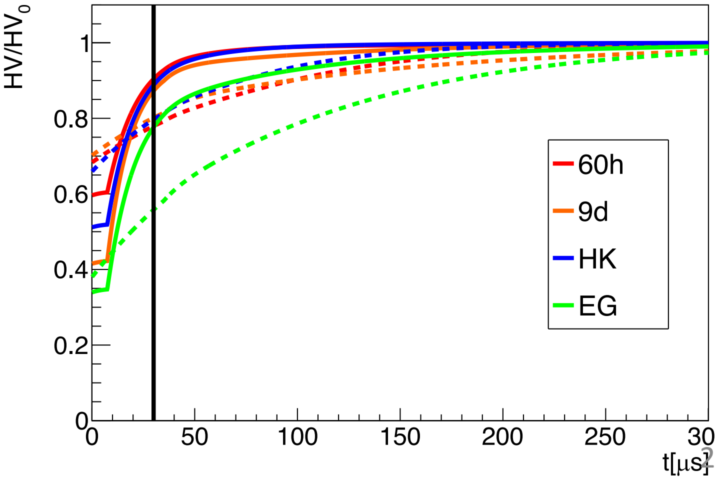
$$f_{obj}(HV_T, HV_B) = \left(1 - \frac{y_{fp}^{\text{sim}}(HV_T, HV_B; t)}{y_0^{\text{data}}(t)}\right)^2 + \left(1 - \frac{\omega_{CBO,x}^{\text{sim}}(HV_T, HV_B; t)}{\omega_{CBO,x}^{\text{data}}(t)}\right)^2$$





*y offset for illustration…



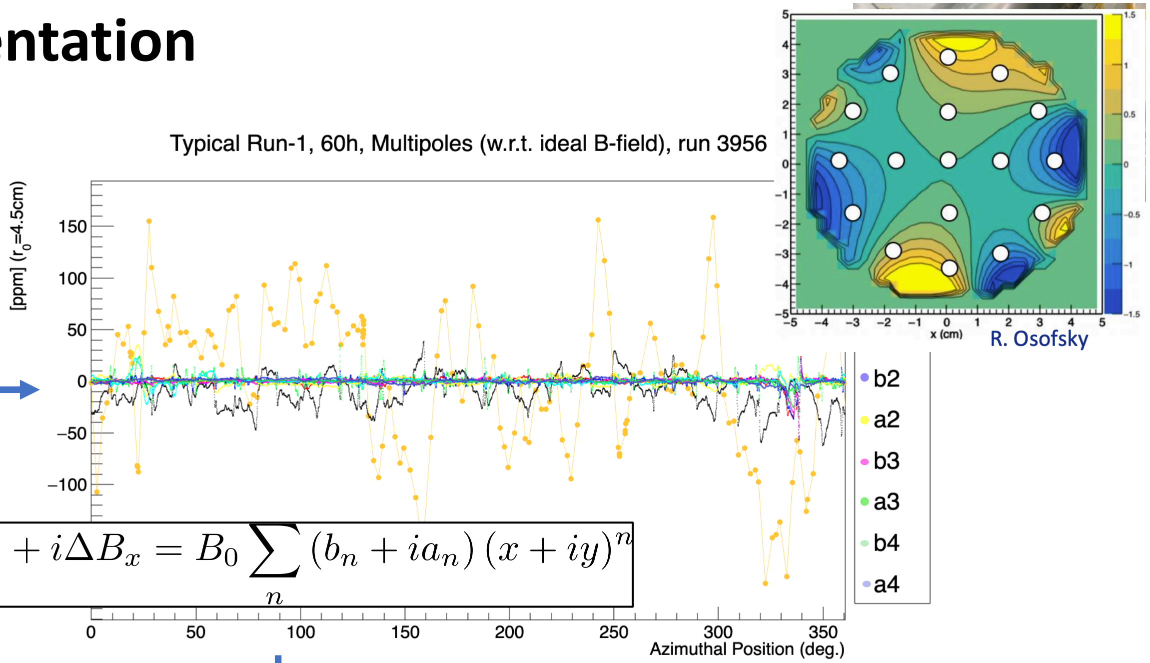


Features and implementation

Input

- Magnetic fields:
  - Ring:

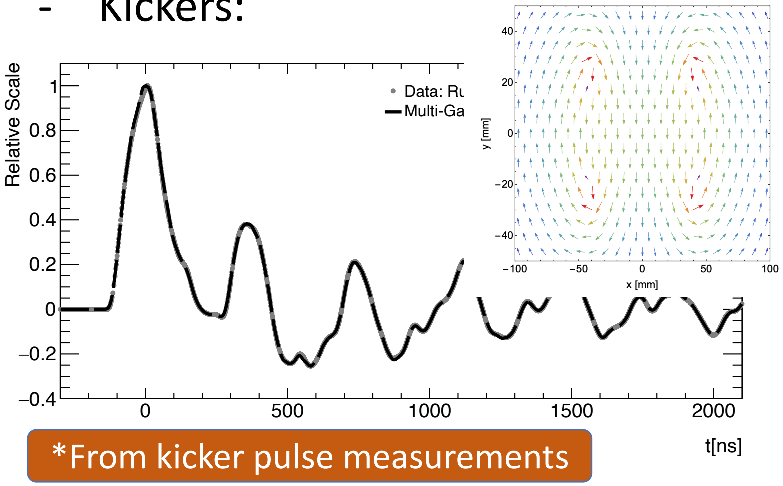
Trolley Data



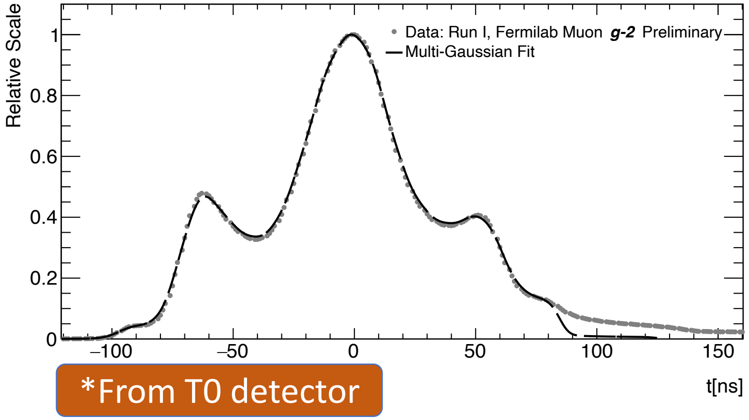
$$\Delta B_y + i\Delta B_x = B_0 \sum_n \left(b_n + i a_n\right)\left(x + iy\right)^n$$

$$\mathcal{M}_{E,B}(L) = \left(\mathcal{M}_E(L_i/2)\cdot\mathcal{M}_L(-l)\cdot\mathcal{M}_B(\phi,l)\cdot\mathcal{M}_E(L_i/2) + \mathcal{C}_x(L_i,\phi) + \mathcal{C}_y(L_i,\phi)\right)^n .$$

  - Kickers:



***From kicker pulse measurements**

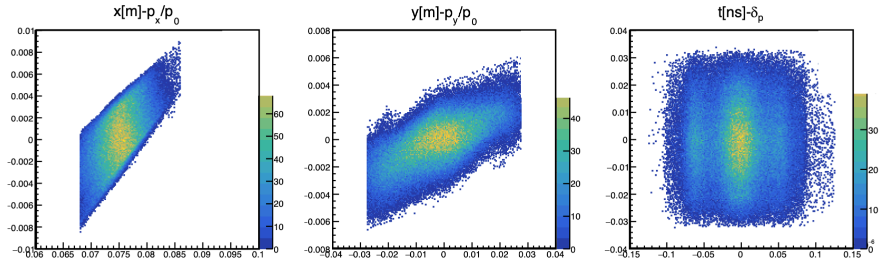


***From T0 detector**

Features and implementation

Input

- Initial beam distribution
  - At $t \approx 0\mu s$:



End of M5 → Inflector (D. Rubin) → COSY kickers → ⋯

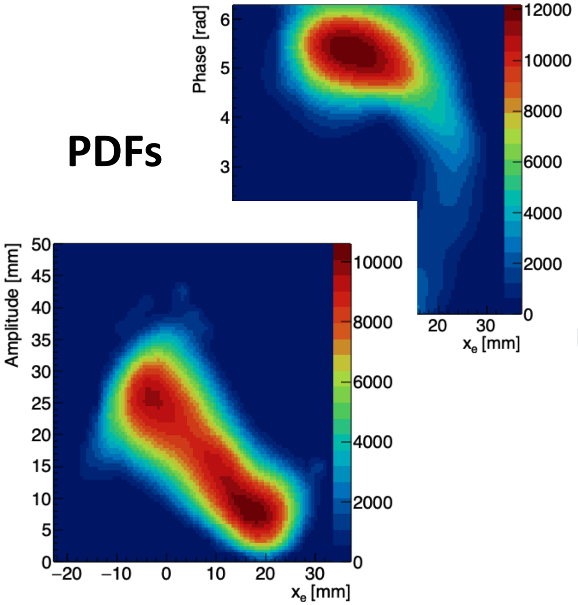
  - At $t \approx 4\mu s$:

Trackers $x - y - t$

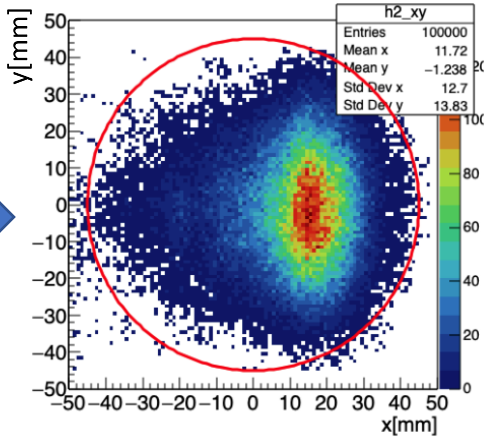
FR's angular frequencies

→ χ^2 minimization + … →

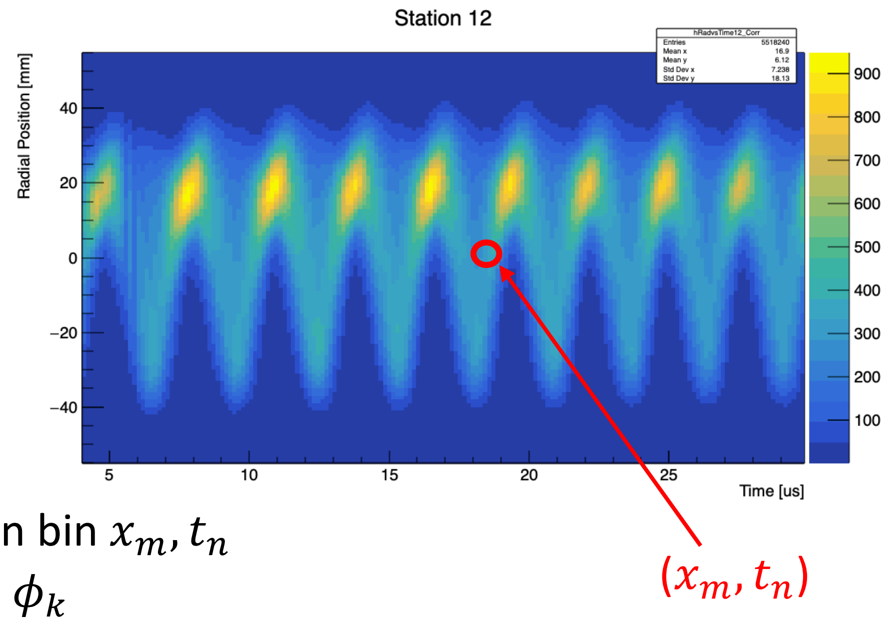
PDFs



→ COSY (α, β, γ)'s and D_x →



Method, radial motion



Mom., Ampl., Phase Bin

$$N_{mn} = \sum_i \sum_j \sum_k \beta_{ijkmn} f_{ijk}$$

Data Bin

How much of each f_{ijk} gets into data bin

- N_{mn} is number of measured tracks within bin x_m, t_n
- f_{ijk} is number of muons with x_e^i, A_j, and ϕ_k
- β_{ijkmn} is the PDF of single muon with x_e^i, A_j, and ϕ_k integrated over bin x_m, t_n:

$$\beta_{ijkmn} = \epsilon(x_m) \int_{x_m=r_-}^{r_+} \int_{t_m=t_-}^{t_+} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}\left[x_m-(x_e^i+x_{off}+A_j \cos(\omega t_n+\phi_k))\right]^2} dx_m dt_n$$

Acceptance

x_{off} (found empirically) includes closed-orbit, tracker reco offset & scraping offset

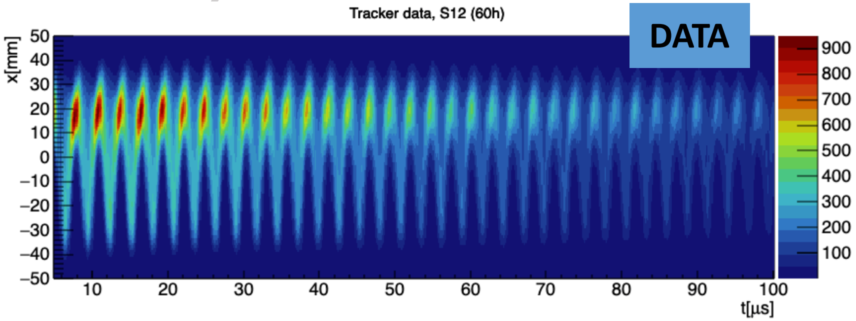
- The method's objective is to find optimal f_{ijk}'s such that

$$X^2 = \sum_m \sum_n \frac{\left(\sum_{ijk} \beta_{ijkmn} f_{ijk} - N_{mn}\right)^2}{\sigma^2_{N_{mn}}}$$

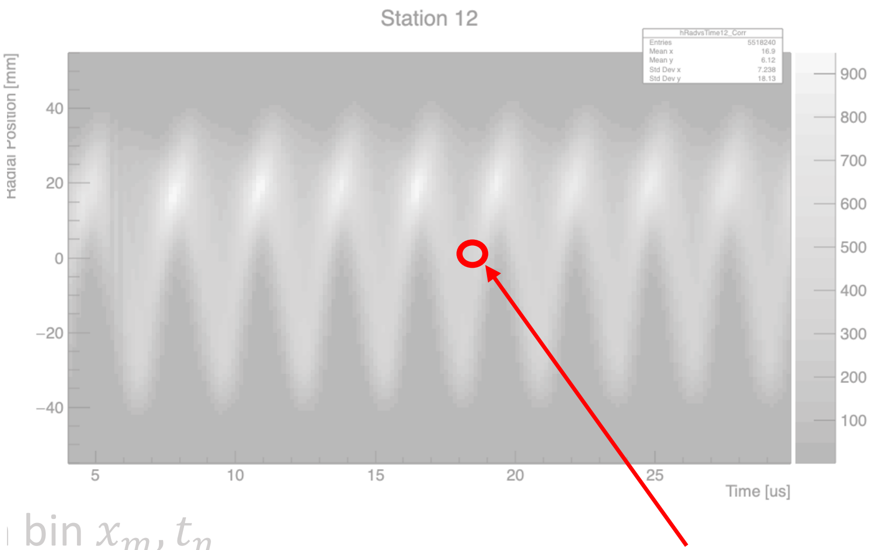
is minimized via NNLS Solver. All f_{ijk}'s together compose beam's PDF $f_x(x_e, A_x, \phi_x)$.

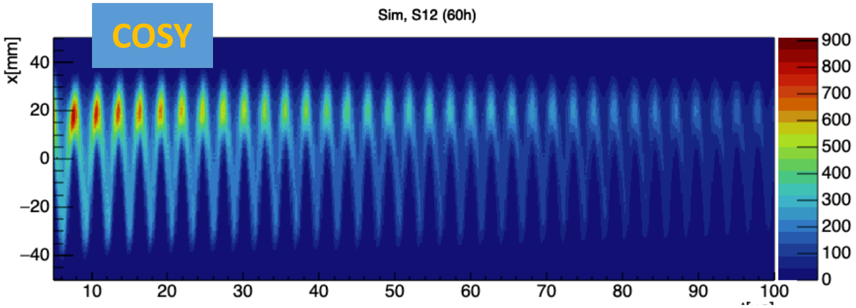
(nonnegative least squares)

Method, radial motion



Da





bin x_m, t_n

(x_m, t_n)

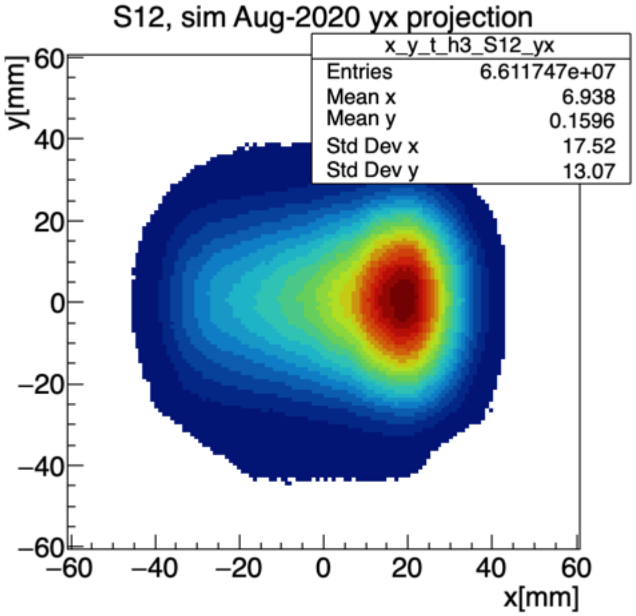
ϕ_k

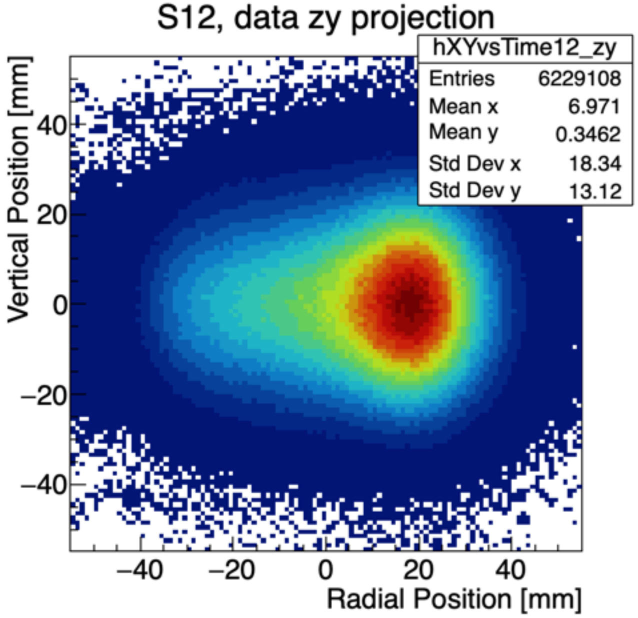
A_j, and ϕ_k integrated over bin x_m, t_n:

$\beta_{ijkmn} = \epsilon(x_m) \int$

Acceptance

- The method's c





acker

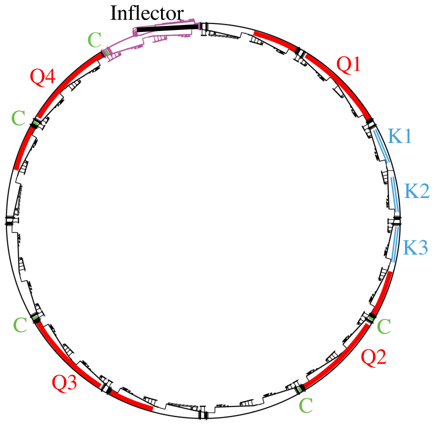
ffset

is minimized

x).

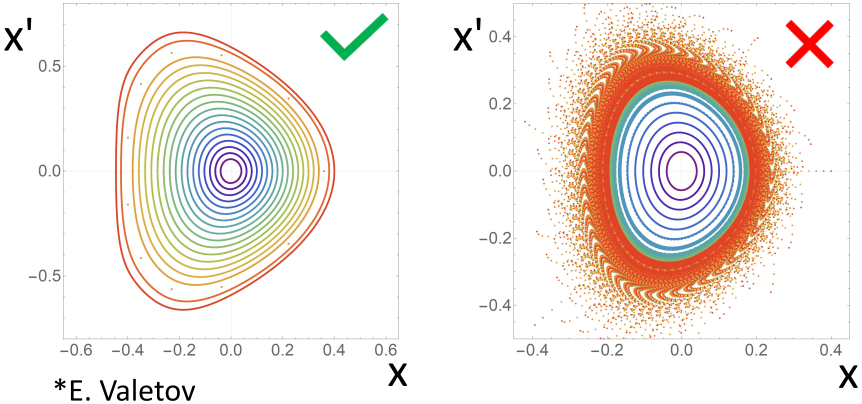
(nonnegative least squares)

Features and implementation



Main tools

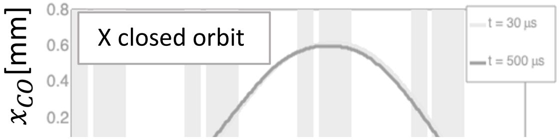
- Muon *g-2* Storage Ring *COSY* Model:
  - Built on data input and ring specs.
  - Conventional scraping.
  - Collimation.
  - ***COSY INFINITY*** tools (e.g., optimizers, Differential Algebra, Normal Form) available.
  - Suitable for both tracking and beam-physics framework.
- Beam Tracking with *COSY*
  - Use of nonlinear transfer maps:
    - Fast.
    - Reproduceable.
    - Spin transfer maps.
  - Symplectic tracking.
  - Parallel computing.

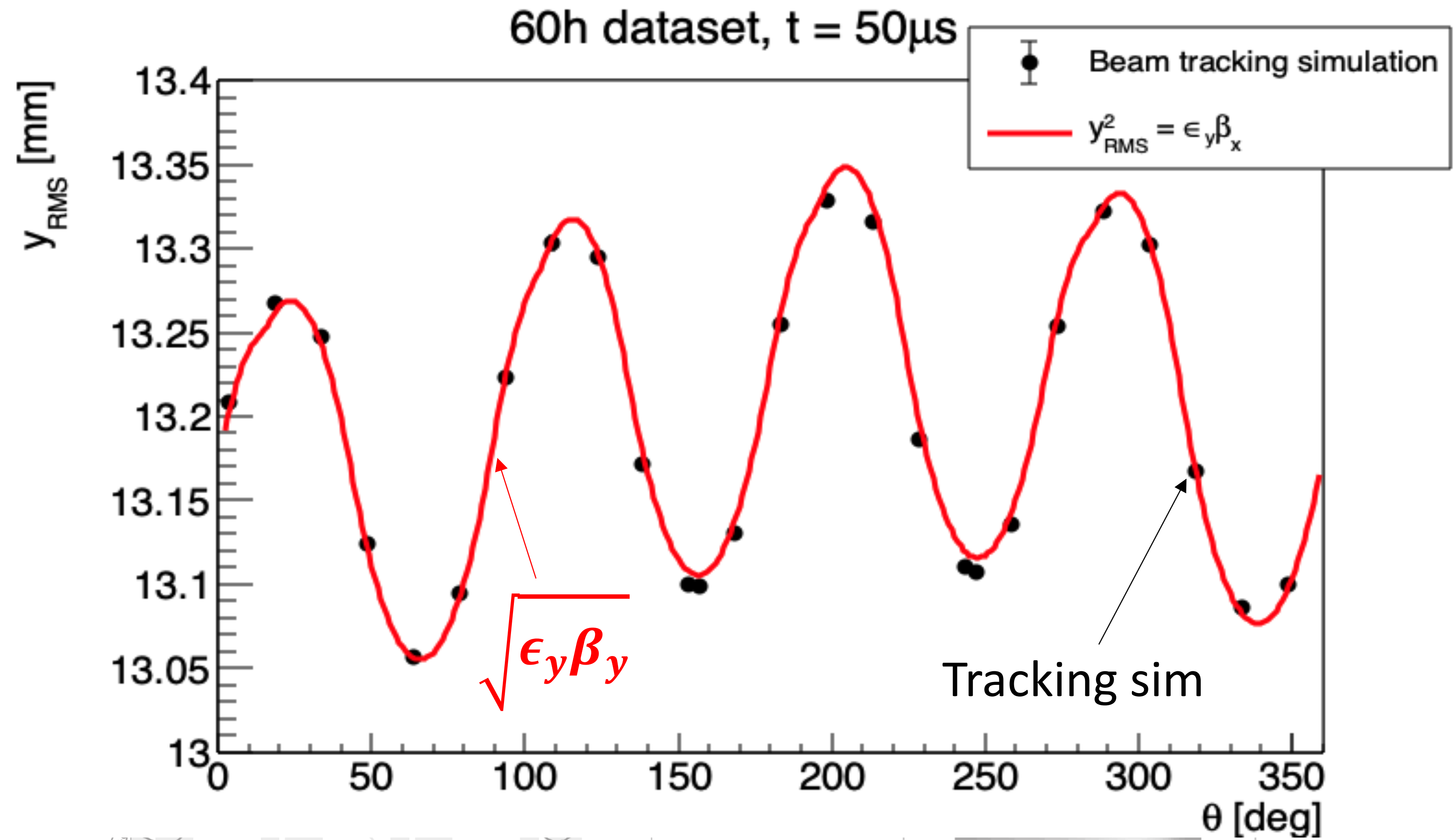


*E. Valetov

GM2-doc-24739

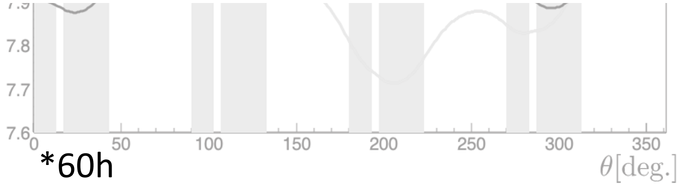
Features and implementation



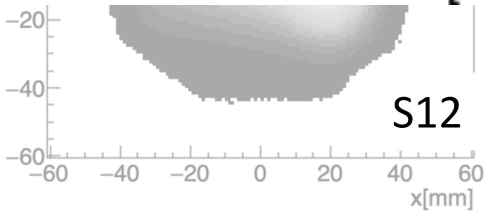


$\sqrt{\epsilon_y \beta_y}$

Tracking sim



*60h

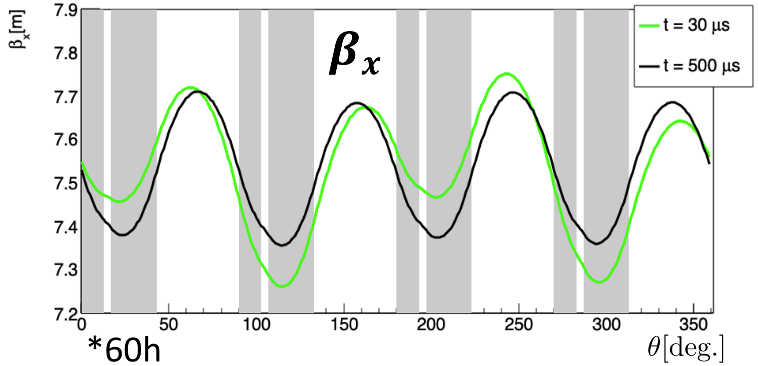


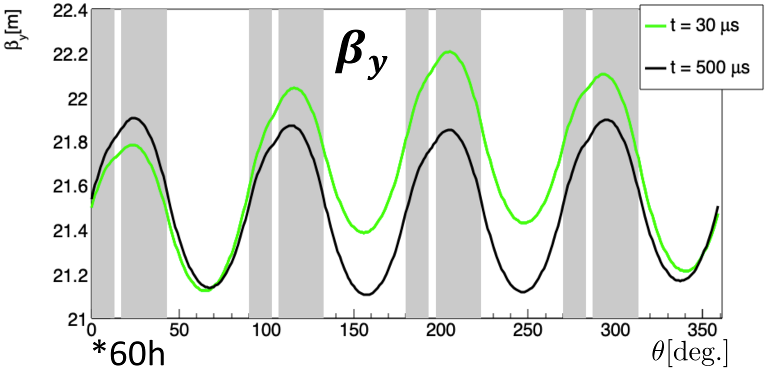
S12

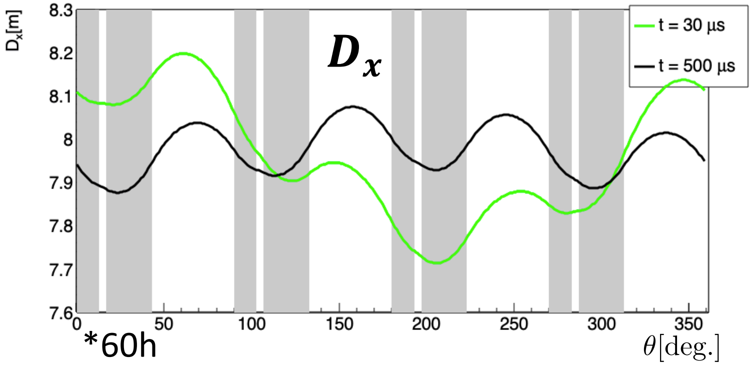
COSY: Features and implementation

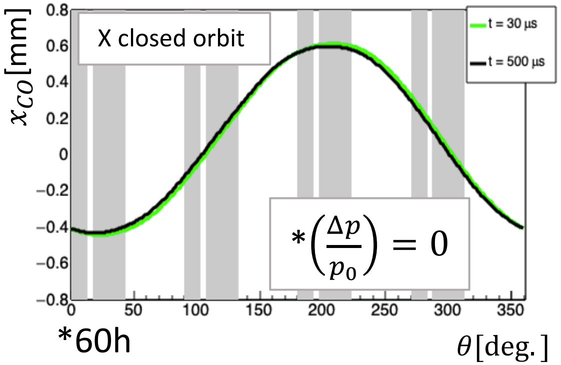
Main output

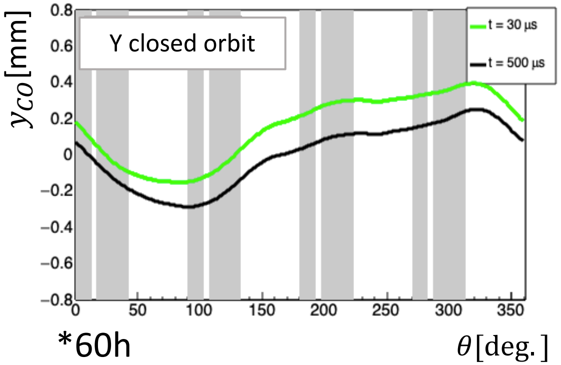
Beam azimuthal description



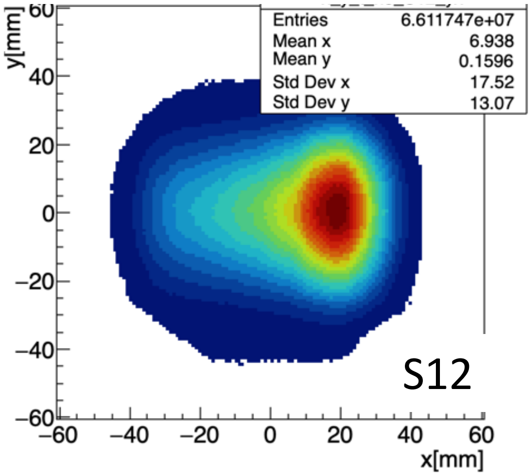








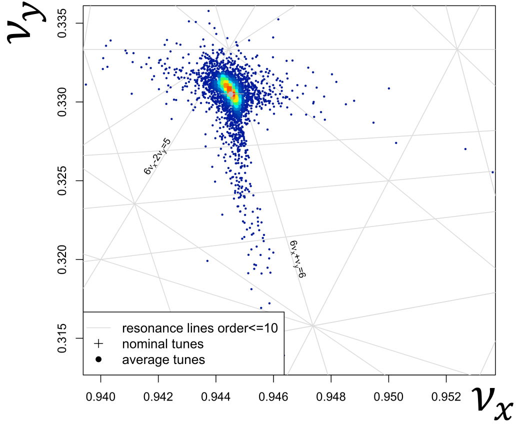
Beam $x-y-t-\theta$ distributions



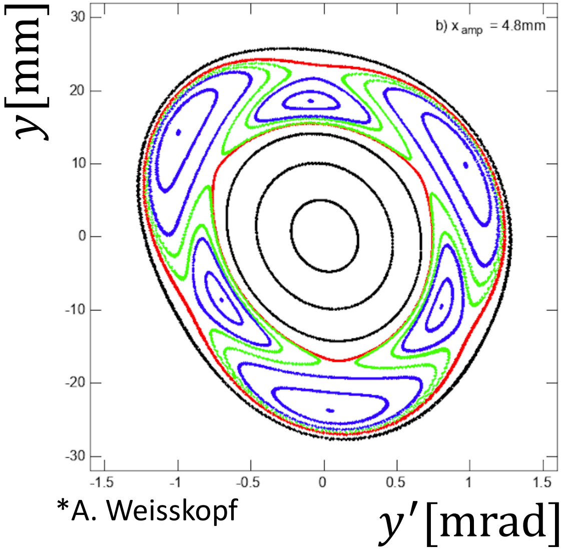
Features and implementation

Main output

Tune footprint

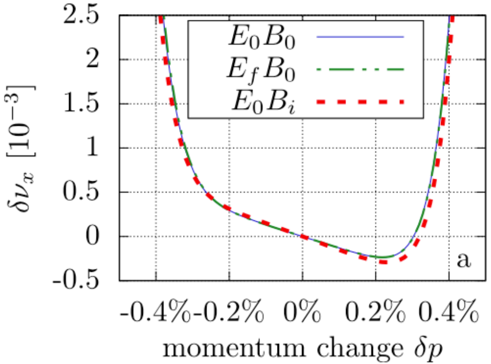


Nonlinear amplitude modulation



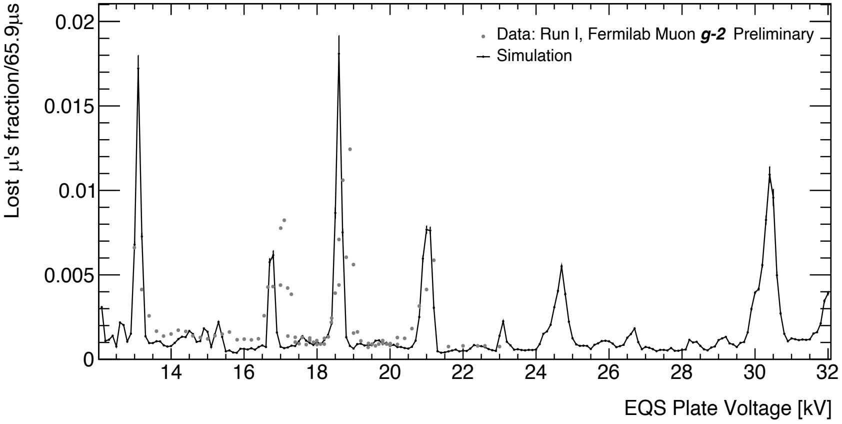
*A. Weisskopf

Tune shifts

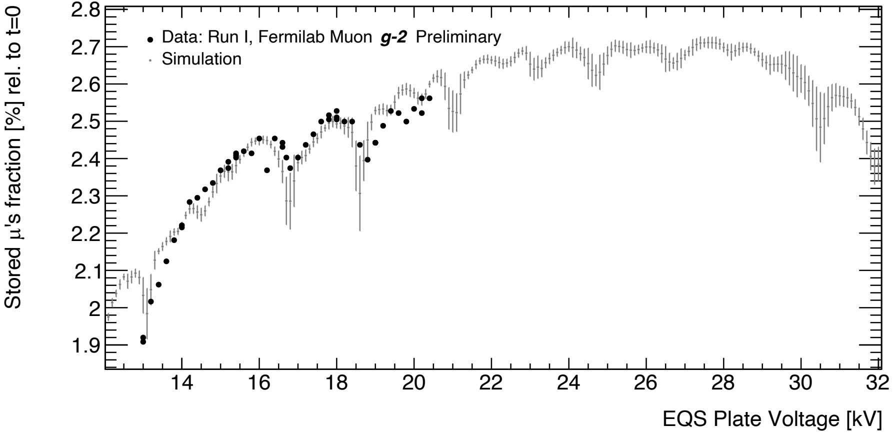


*A. Weisskopf

Betatron resonances



Storing efficiency



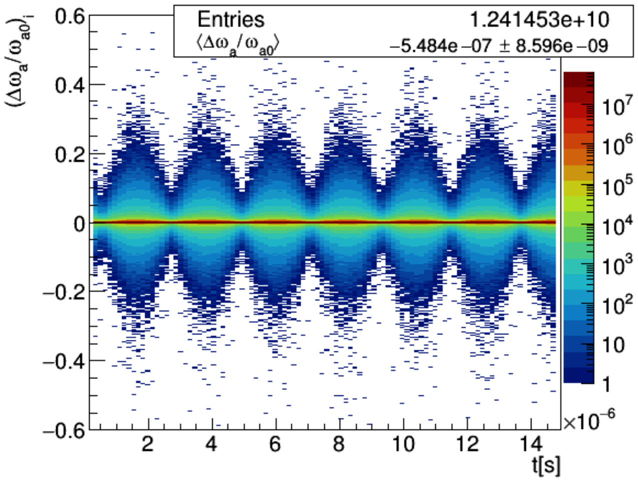
Contributions to ω_a corrections

Contributions to ω_a corrections

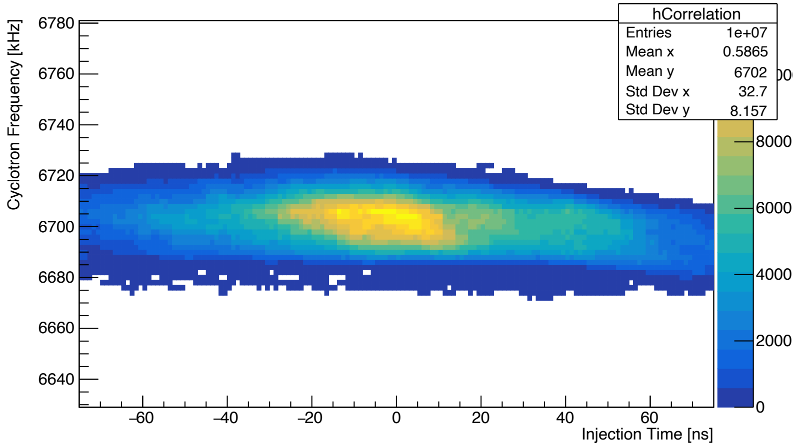
$$\omega_a = \omega_{a0}(1 + \langle \Delta\omega_a^E \rangle + \langle \Delta\omega_a^B \rangle) \approx \omega_{a0}(1 + \boldsymbol{C_E} + \boldsymbol{C_P})$$

$$C_E = -\frac{n_0\beta_0^2}{1-n_0} 2\langle \delta^2 \rangle \quad \text{and} \quad C_P = -\frac{n_0}{2\rho_0^2}\langle y^2 \rangle$$

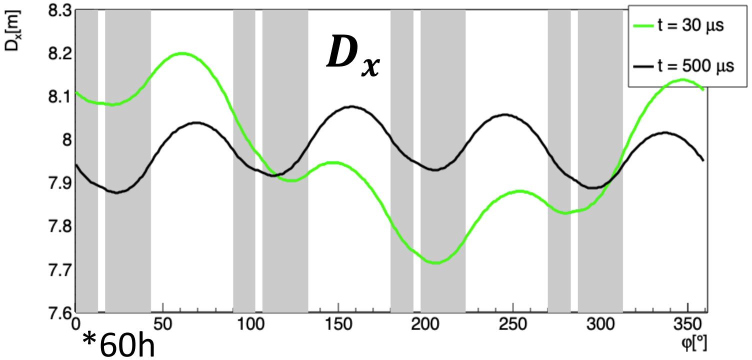
E-field correction + Pitch correction



- Pitch correction linear approximations + momentum spread asymmetry + EQS continuous plates approximation + EQS up to 20th high-order multipole + EQS fringe fields add $\boldsymbol{\delta(C_E + C_P) < 10\text{ppb}}$.



- Momentum-time correlation modelling for FR analysis.



*60h

- Distorted radial dispersion during Run-1 adds $\boldsymbol{\delta C_E \sim 5\text{ppb}}$.

- Using *COSY* β_y's, $y_{RMS}(\theta)$ variations introduce $\boldsymbol{\delta C_P \sim 1\text{ppb}}$ (T. Barret and J. Mott).

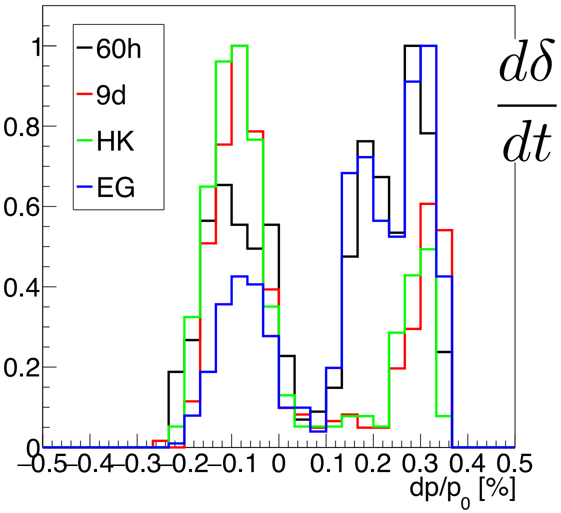
Contributions to ω_a corrections

Lost Muons

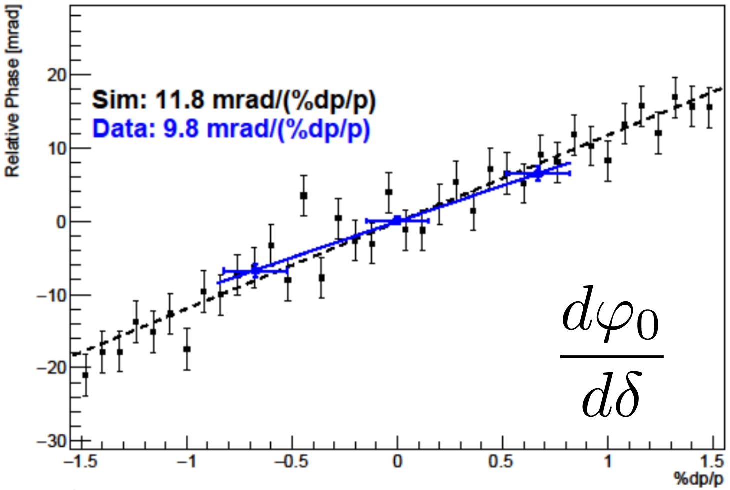
$$\Delta\omega_a \approx \frac{\Delta\varphi_0}{\Delta t}$$

	C_{ml} [ppb]
60h	-14.13 ± 4.30
9d	2.18 ± 2.86
HK	4.16 ± 4.47
EG	-24.57 ± 2.47

- Off by <12 ppb from data-driven corrections (E989 Note 248, H. Binney).

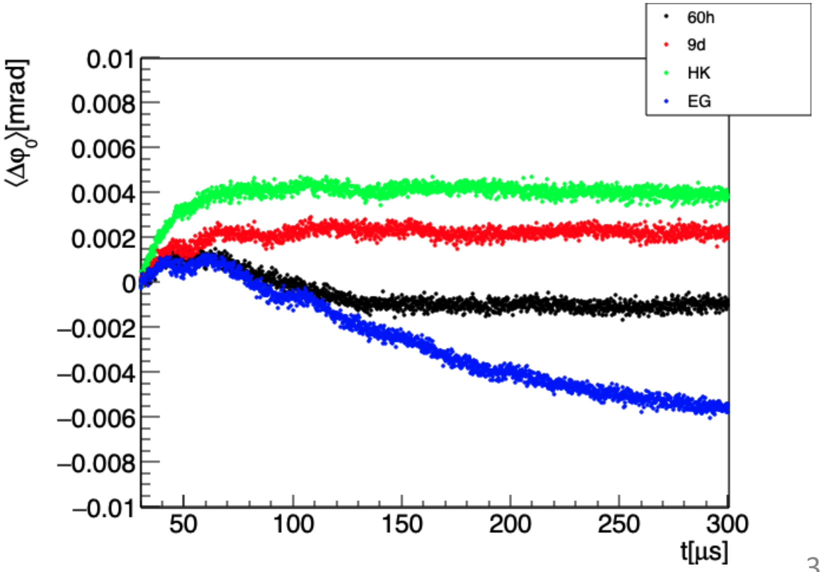


$$\frac{d\delta}{dt}$$



$$\frac{d\varphi_0}{d\delta}$$

*E. Valetov



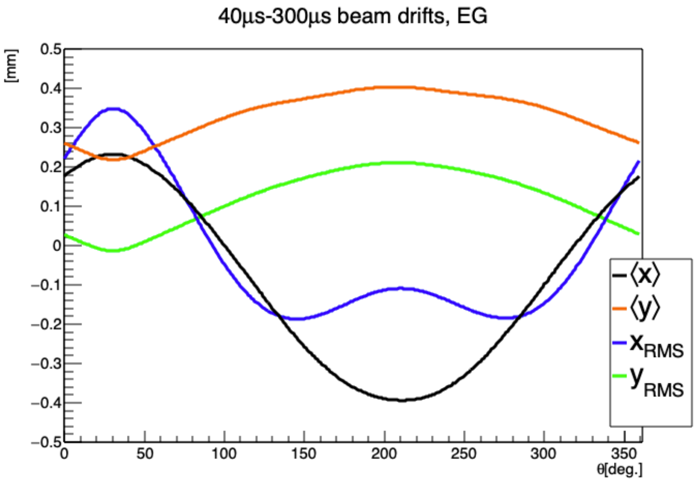
Contributions to ω_a corrections

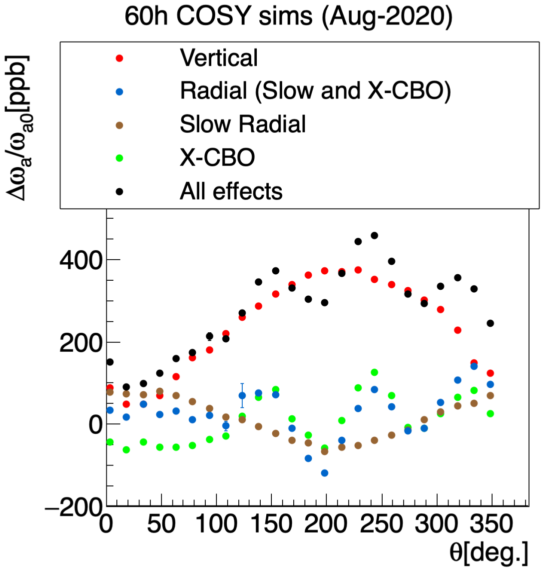
$$\Delta R \propto \Delta\omega_a^{PA} \approx \frac{\Delta\varphi^{PA}}{\Delta t}$$

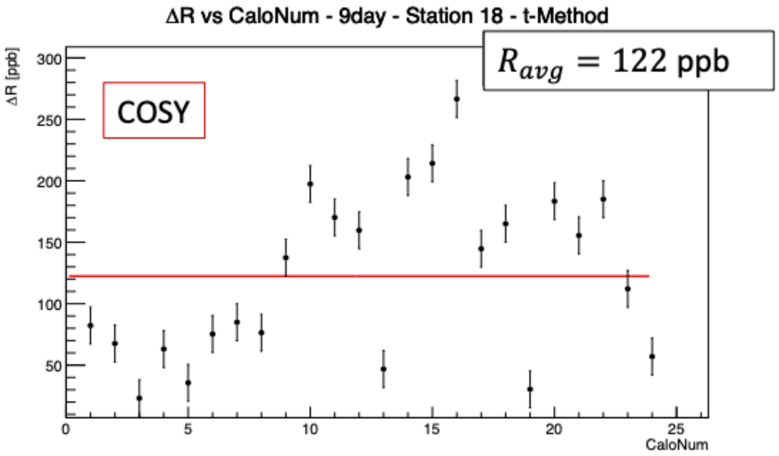
Phase Acceptance

$$< \varphi_0(t) >= \text{atan}\left(\frac{\sum_{xy}\sin\left(\varphi_a(x,y)\right)\cdot M_B(x,y,t)\cdot\epsilon_C(x,y)\cdot A(x,y)}{\sum_{xy}\cos\left(\varphi_a(x,y)\right)\cdot M_B(x,y,t)\cdot\epsilon_C(x,y)\cdot A(x,y)}\right)$$

- M_T's around ring projected from tracker data with Run-1 beam functions.

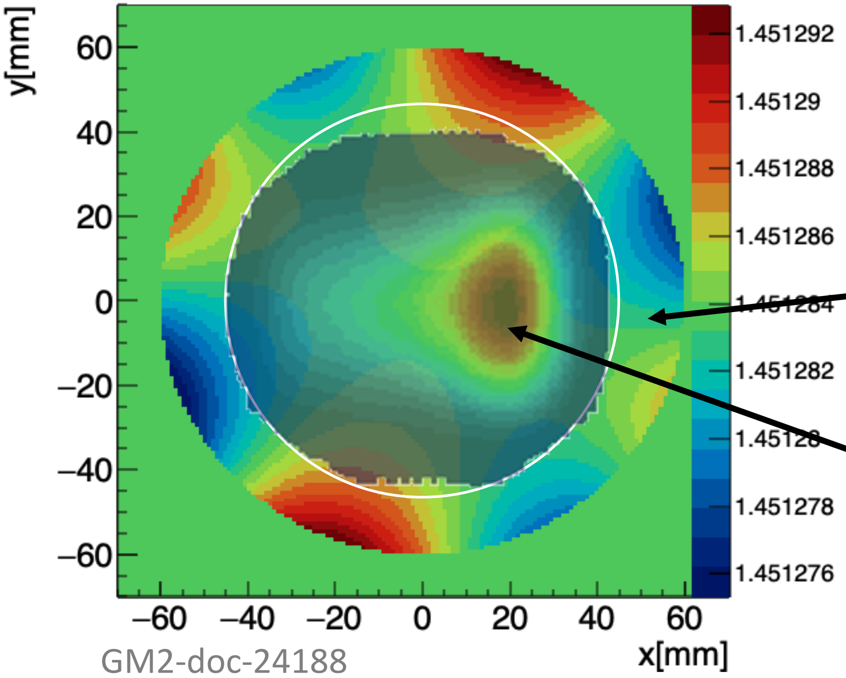

40µs-300µs beam drifts, EG


60h COSY sims (Aug-2020)


ΔR vs CaloNum - 9day - Station 18 - t-Method

Contributions to $\langle \vec{\boldsymbol{B}} \rangle$

$\langle B \rangle$: Combining B and the stored beam



GM2-doc-24188

$B_i(x, y)$ from Field Team, and implemented in model.

$M_T(x, y, \theta)$ and optical lattice functions from data-driven simulations

- $\langle \vec{B} \rangle$ is sensitive to radial closed orbit distortion by ~ 13 ppb relative to ideal case in Run 1.
- Effect of beam width variations around the ring is negligible (< 3 ppb).

- Vertical closed orbit distortions from Quad station misalignments under study.

Research Plans

***g-2*:**

- Repurpose models to account for further sources of error (e.g. electric station misalignments) and support further measurements.

- Quantify beam-dynamics driven uncertainties for remaining data-production runs (momentum-time correlation, phase acceptance).

Mu2e:

- Participate in Cosmic Ray Veto (CRV) installation and commissioning.
- Commissioning of slow resonant extraction (measurements and modelling).
- Commissioning of M4 beamline for Mu2e.
- Optimize focus to Mu2e.

Thank You