On radiation damping in planar channeling in the presence of multiple scattering

V.N. Baier *, V.M. Katkov

Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russian Federation

Abstract

The effect of radiative damping of transverse oscillations of positrons in the planar channel of a crystal is considered under realistic conditions when multiple scattering of the positrons on the atomic electrons is taken into account. © 1997 Published by Elsevier Science B.V.

1. The influence of the radiation reaction on the particle motion in a system with a strong transverse focusing is now under active study (see Refs. [1–3] and references therein). One possible realization of such a system is the planar channeling of positrons, where positrons are moving in the superstrong microscopic focusing field formed by crystalline planes. The existence of the phenomenon of radiation damping of transverse oscillation of positrons was found nearly two decades ago [4].

2. In Ref. [4] the motion of a particle in an oscillatory transverse potential well $U(x) = kx^2/2$ was considered under the influence of the force of radiative braking as given in Ref. [5]. The following set of equations was derived,

$$\ddot{x} + a\dot{x} + \frac{kx}{m\gamma} = 0, \quad \dot{\gamma} + \frac{ka}{mc^2}\gamma^2 x^2 = 0, \quad (1)$$

* E-mail: baier@inp.nsk.su.

where x is the transverse coordinate, $\gamma = \varepsilon/mc^2$ is the Lorentz factor of the positron, $a = 2ke^2/3m^2c^3$; below we will put $c = \hbar = 1$.

Averaging over particle oscillations in a channel (over "fast" variables) one obtains

$$\dot{\varepsilon}_\perp + a\varepsilon_\perp + \frac{a}{4m}\varepsilon_\perp^2\gamma = 0, \quad \dot{\gamma} + \frac{a}{m}\gamma^2\varepsilon_\perp = 0, \quad (2)$$

where $\varepsilon_\perp$ is the transverse energy of the particle. This set contains "slow" variables only. It coincides with the set obtained in Ref. [3] by two different methods and given in terms of γ and $J_x = \varepsilon_\perp/\omega$, where $\omega = \sqrt{k/m\gamma}$ is the oscillation frequency.

The solution of this set (see Ref. [3]) is

$$\varepsilon_\perp(t) = \frac{\varepsilon_\perp(0)e^{-at}}{R^{1/5}(t)}, \quad \gamma(t) = \frac{\gamma(0)}{R^{4/5}(t)},$$

$$R(t) = 1 + \tfrac{5}{8}\varrho_0(1 - e^{-at}), \quad (3)$$

where $\varrho_0 = 2\varepsilon_\perp(0)\gamma(0)/m$. If one introduces the so-called multipolarity parameter $\varrho = 2\gamma^2\overline{v_\perp^2}$, where $\overline{v_\perp^2}$ is

Table 1
Parameters of the potential for the (110) plane and characteristics of damping and antidamping processes

Crystal	T (K)	d_{pl} (10^{-8} cm)	U_0 (eV)	l_a (cm)	A	β_0	η	r_c
Cd	293	1.26	23	2.3	1.7	9	6.8	1.1
Si	293	1.92	23	5.5	2.6	10.6	7.3	1.5
Ge	293	2.00	40	3.4	3.2	11.1	7.5	1.6
W	293	2.24	130	0.33	4.1	11.8	7.7	1.7

the mean square of the transverse velocity (when $\varrho \ll 1$ the radiation is dipole, when $\varrho \gg 1$ the radiation is of magnetic bremsstrahlung nature, the case $\varrho \sim 1$ is intermediate and many harmonics of radiation are essential), then $\varrho = 2\varepsilon_\perp(t)\gamma(t)/m$ and ϱ_0 is the initial value of this parameter.

When $at \ll 1$, one has

$$\varepsilon_\perp(t) = \frac{\varepsilon_\perp(0)}{R_1^{1/5}(t)}, \quad \gamma(t) = \frac{\gamma(0)}{R_1^{4/5}(t)},$$

$$R_1(t) = 1 + \tfrac{5}{8}\varrho_0 at, \tag{4}$$

so that the transverse energy and total energy initially damp with power laws in the case when $\varrho_0 \gg 1$. However, the exponential damping factor in the transverse energy becomes more important for longer time (distance). When $at \gg 1$, one has

$$\varepsilon_\perp(t) = \frac{\varepsilon_\perp(0)e^{-at}}{R_2^{1/5}}, \quad \gamma(t) = \frac{\gamma(0)}{R_2^{4/5}},$$

$$R_2 = 1 + \tfrac{5}{8}\varrho_0. \tag{5}$$

The exponential damping factor in Eqs. (3)–(5) can be written in the form $\exp(-l/l_a)$, where l is the length of the way of the particle in the crystal. The characteristic damping length l_a for (110) plane in different crystals is given in Table 1 along with the distance between the planes d_{pl} and the depth of the potential well U_0. Note that the length l_a is quite close to the standard radiation length L_{rad} in corresponding media and only for diamond l_a is a few times shorter than L_{rad}.

The form of the transverse potential for planar channeling of electrons is very different from the oscillator potential and atomic nuclei are situated in the middle of the channel, so that the multiple scattering of electrons is amplified compared with an amorphous

medium. The transverse potential for positrons is more similar to the oscillator one and the positrons in the channel are moving mostly far from the atomic nuclei, so that multiple scattering of the positrons is diminished compared with an amorphous medium. So, only the case of channeling of particles with positive charge is interesting from the point of view of damping of the transverse oscillations of a particle moving in a planar channel. Although in real crystals the planar potential is quite different from the oscillator one and the spread of frequencies of motion of a positron in a channel is of the order of one [6], nevertheless damping in the oscillator potential is a very useful model the importance of which is connected with the existence of an exact analytical solution.

3. The only interesting situation is the case when positrons are moving far from the crystalline planes, when the main scattering process is scattering on atomic electrons. Connected with this scattering increment of the transverse energy is (see Ref. [6], Section 10)

$$\frac{\Delta\varepsilon_\perp}{\Delta l} = \frac{\varepsilon}{2}\frac{\Delta\vartheta_s^2}{\Delta l}, \tag{6}$$

where ϑ_s^2 is the mean value of the square of the x-component of the angle of the multiple scattering of a positron. The interaction of a positron with atomic electrons can be split into two parts. The first contribution to the square of the angle of the multiple scattering gives scattering on electrons situated inside planes forming the planar channel in which the positron is moving. The maximal momentum transfer follows from the condition that a positron should not go out of the channel and minimal momentum transfer is defined by maximal remoteness of the positron from the atomic plane,

$$q_{max}^2 = 2\varepsilon U_0 = \varrho_p m^2, \quad q_{min} = \frac{1}{d_{pl}},$$

$$\frac{q_{max}^2}{q_{min}^2} = \varrho_p \frac{d_{pl}^2}{\lambda_C^2} \sim \varrho_p \times 10^5, \tag{7}$$

where U_0 is the depth of the transverse potential well in which channeling occurs, d_{pl} is the distance between neighboring planes, $\lambda_C = 1/m = \hbar/mc$ is the electron Compton wavelength, $\varrho_p = 2\varepsilon U_0/m^2$. In the region $q \geqslant q_{min}$ one can consider scattering as a scattering on

free electrons and its cross section is defined, within logarithmic accuracy, by the local density of atomic electrons $n(x)$ depending on the positron coordinate x ($1/q < d_{pl}$). So, we have for the square of the angle of multiple scattering the following expression,

$$\frac{\Delta\vartheta_s^2}{\Delta l} == \frac{4\alpha^2 n(x)}{\varepsilon^2} \int_{q_{min}}^{q_{max}} \frac{q_x^2 d^2 q}{q^4} = \frac{2\pi\alpha^2 n(x)}{\varepsilon^2} \ln \frac{q_{max}^2}{q_{min}^2},$$

(8)

where $\alpha = e^2 = 1/137$, $q^2 = q_x^2 + q_y^2$. Note that the ratio q_{max}^2/q_{min}^2 and consequently the square of the angle of multiple scattering does not depend on the particle's mass. Now we compare the square of the angle of multiple scattering of relativistic protons moving in a planar channel with the expression for ionization losses (stopping power) in a corresponding amorphous medium:

$$\left(\frac{\delta\varepsilon}{\delta l}\right)_{am} = \frac{4\pi\alpha^2 n_{am}}{m} \left[\ln\left(\frac{2\gamma_M m}{\omega_0}\right) - \frac{1}{2}\right],$$

(9)

where $\gamma_M = \varepsilon/M$, M is the proton mass, and $\omega_0^2 = 4\pi n_{am}/m$ is the plasma frequency; n_{am} is the mean density of electrons in an amorphous medium. Taking into account that

$$\frac{\omega_0}{m} = \sqrt{\frac{4\pi\alpha n_{am}}{m^3}} \sim \sqrt{Z} \times 10^{-5},$$

$$\frac{2\gamma_M}{\sqrt{Z}} = \frac{2\varepsilon U_0}{M U_0 \sqrt{Z}} \sim \varrho_p,$$

(10)

and comparing Eqs. (7), (8) with Eqs. (9), (10) we find that

$$\frac{\Delta\vartheta_s^2}{\Delta l} \simeq \frac{m}{2\varepsilon^2} \left(\frac{\delta\varepsilon}{\delta l}\right)_{am} \frac{n(x)}{n_{am}}.$$

(11)

We use the expression (11) for calculation of the dechanneling length for protons with energy 10 GeV $\leqslant \varepsilon \leqslant$ 200 GeV moving in the (110) or (111) channels in Si giving a result which agrees satisfactorily with the experimental data [7].

The contribution of electrons situated outside planes which are forming the channel under consideration (contribution of long distances, $x \geqslant d_{pl}$) to the mean square of the angle of multiple scattering has the form

$$\frac{\Delta\vartheta_s^2}{\Delta l} = \frac{4\alpha^2 n_{am}}{\varepsilon^2} \int_{\omega_0}^{1/d_{pl}} \frac{q_x^2 d^2 q}{q^4} = \frac{2\pi\alpha^2 n_{am}}{\varepsilon^2} \ln \frac{1}{\omega_0^2 d_{pl}^2},$$

$$\omega_0^2 d_{pl}^2 \simeq \frac{4\pi\alpha Z d_{pl}^2}{d_{pl}^3 m} = 4\pi\alpha Z \frac{\lambda_C}{d_{pl}} \simeq Z \times 10^{-4}.$$

(12)

The total contribution of both short and long distances to the increment of the transverse energy is

$$\frac{\Delta\varepsilon_\perp}{\Delta l} = \frac{\pi\alpha^2}{\varepsilon} \left(n_{am} \ln \frac{10^4}{Z} + n(x) \ln(\varrho_p \times 10^5)\right).$$

(13)

The ratio r_c of the contribution of long distances to the contribution of short distances is given in Table 1.

For the oscillatory potential $U(x) = kx^2/2$ (which describes either averaged characteristics of a one-parametric potential, or the motion of the particle near the bottom in any potential well), one has from the Poisson equation

$$n(x) = \frac{k}{4\pi\alpha} = \frac{2U_0}{\pi\alpha d_{pl}^2} = \text{const}.$$

To proceed with the inclusion of multiple scattering in Eq. (2), we use Eqs. (12) and (13) and the last equation, which are self-consistent for the oscillatory potential. Besides, when one uses Eqs. (12) and (13), it is necessary to substitute $q_{max}^2 \to 2\varepsilon\varepsilon_\perp$ (or $\varrho_p \to \varrho$), since in the case when the increment of the transverse energy $\Delta\varepsilon_\perp \gg \varepsilon_\perp$ in one interaction (time of scattering is much shorter than the period of motion) one cannot describe the motion in terms of a classical trajectory.

Furthermore, the analysis below shows that within the adopted accuracy (logarithmic accuracy) one can put in the equations of motion that $\varrho = 1$ in the argument of the logarithm. As a result we obtain

$$\dot{\varepsilon}_\perp + a\varepsilon_\perp + \frac{a}{4m}\varepsilon_\perp^2\gamma - \frac{b}{\gamma} = 0, \quad \dot{\gamma} + \frac{a}{m}\gamma^2\varepsilon_\perp = 0,$$

(14)

where

$$b = \frac{k\alpha}{4m}\left(\ln 10^5 + A \ln \frac{10^4}{Z}\right), \quad A = n_{am}\frac{\alpha\pi d_{pl}^2}{2U_0}.$$

Values of A (the expression for A is valid for crystal structures fcc$_{(d)}$, bcc) for some crystals are given in Table 1.

It is convenient to rewrite the set of equations (14) in terms of functions $\gamma(t)$ and $r(t) = \varepsilon_\perp(t)\gamma(t)$ for which the set of equations is simplified considerably:

$$\dot{r} + ar + \frac{5ar^2}{4m} - b = 0, \quad \dot{\gamma} + \frac{a}{m}\gamma r = 0, \qquad (15)$$

It is seen that the first equation of this set contains the function $r(t)$ only and this is a differential equation with separable variables, the solution of which is straightforward. With the function found, $r(t)$, one can integrate the second equation of the set of equations (15). The solution of the set of equations can be presented in the form

$$\varrho(t) = \frac{2r(t)}{m} = \frac{4}{5}\frac{G(t)}{F(t)},$$

$$\varepsilon_\perp(t) = \frac{4\varepsilon_\perp(0)}{5\varrho_0}\frac{G(t)}{F^{1/5}(t)(2\eta)^{4/5}}\exp\left[\tfrac{2}{5}(\eta-1)at\right],$$

$$\gamma(t) = \gamma(0)\left(\frac{2\eta}{F(t)}\right)^{4/5}\exp\left[-\tfrac{2}{5}(\eta-1)at\right],$$

$$(16)$$

where

$$F(t) = \beta_1 - \beta_2 e^{-a\eta t},$$

$$\beta_1 = 1 + \eta + \tfrac{5}{4}\varrho_0, \quad \beta_2 = 1 - \eta + \tfrac{5}{4}\varrho_0,$$

$$G(t) = \beta_1(\eta-1) + \beta_2(\eta+1)e^{-a\eta t},$$

$$\eta = \sqrt{1 + 5b/ma}. \qquad (17)$$

The value of the parameters for some usable crystals is given in Table 1. When the multiple scattering is turned off ($\eta = 1$), one returns to the solution (3).

Now we will proceed with a qualitative analysis of the set of equations (15) which we rewrite in the form

$$\frac{d\varrho}{dT} + \varrho + \frac{5\varrho^2}{8} - 2\beta = 0, \quad \frac{d\gamma}{dT} + \frac{\gamma\varrho}{2} = 0, \qquad (18)$$

where $T = at$; we introduce

$$\beta = \frac{3}{8}\left(\ln(\varrho 10^5) + A\ln\frac{10^4}{Z}\right),$$

$$\beta_0 \equiv \frac{b}{ma} = \frac{3}{8}\left(\ln(10^5) + A\ln\frac{10^4}{Z}\right). \qquad (19)$$

The parameters β_0 for some usable crystals are shown in Table 1. So, in this analysis we take into account that β is logarithmically dependent on energy. When

$\varrho_0 \gg 1$ and $T \ll 1$ one can neglect the terms $\varrho - 2\beta$ in the first equation (18) since the value of ϱ is still large at $T \ll 1$. Then we have the set of equations

$$\frac{d\varrho}{dT} + \frac{5\varrho^2}{8} = 0, \quad \frac{d\gamma}{dT} + \frac{\gamma\varrho}{2} = 0, \qquad (20)$$

the solution of which is

$$\varepsilon_\perp(T) = \frac{\varepsilon_\perp(0)}{R_1^{1/5}(T)}, \quad \gamma(t) = \frac{\gamma(0)}{R_1^{4/5}(T)},$$

$$\varrho = \frac{\varrho_0}{R_1(T)}, \quad R_1(T) = 1 + \tfrac{5}{8}\varrho_0 T. \qquad (21)$$

It coincides with Eq. (4) where multiple scattering is neglected.

When $T \sim 1/\sqrt{\beta}$ from Eq. (21) one has $\varrho \sim \sqrt{\beta}$. In this situation all terms in the first equation in Eq. (18) is of order one and multiple scattering is turned on. In this case with a good accuracy we have $\beta \simeq \beta_0$. With further increase of the T function ϱ tends to its stationary value,

$$\frac{d\varrho_s}{dT} = 0, \quad \varrho_s + \frac{5\varrho_s^2}{8} - 2\beta = 0,$$

$$\varrho_s \simeq \tfrac{4}{5}(\sqrt{1 + 5\beta_0} - 1). \qquad (22)$$

Note, that for the crystals we considered (diamond, Si, Ge, W) for the (110) plane one has $\varrho_s \simeq 5$ and one can neglect the dependence of β on ϱ ($\beta \simeq \beta_0$). In this region of T the total energy decreases exponentially, $\gamma(T) \propto \exp(-\varrho_s T/2)$ and the transverse energy increases exponentially since the value $\varrho = 2\gamma\varepsilon_\perp/m$ remains constant.

In the opposite case the $\varrho_0 \ll 1$ value of $\varrho + \tfrac{5}{2}\varrho^2$ remains small compared with 2β until $T \sim 1/\sqrt{\beta}$. In this case one can neglect damping effects (terms $\propto \varrho$, ϱ^2) and the function ϱ varies only due to multiple scattering:

$$\frac{d\varrho_s}{dT} - 2\beta_0\left(1 + \frac{\ln\varrho}{C}\right) = 0, \qquad (23)$$

where $C = 25$ for diamond and $C = 31$ for W. Solving this equation by the method of successive approximations we find after the first step

$$\varrho(T) \simeq 2\beta_0 T\left(1 + \frac{\ln(2\beta_0 T + \varrho_0) - 1}{C}\right) + \varrho_0. \qquad (24)$$

At $\varrho_0 \ll T \leqslant 1/\sqrt{\beta}$, $\varrho(T) \simeq 2\beta_0 T$ is independent of the initial value ϱ_0 and (with logarithmic accuracy) is defined by the value β_0. With increasing T the function ϱ attains a value of the order of $\sqrt{\beta}$ and one has to take into account all the terms in the set of equations (18) and we have the situation discussed above: $\varrho \rightarrow \varrho_s$, the total energy decreases exponentially, while the transverse energy increases exponentially.

So, we have shown that with logarithmic accuracy (this is just the accuracy with which we could calculate β) one can neglect the dependence of $\beta(\varrho)$ and put $\beta = \beta_0$.

The solutions (16) of the set of equations (15) are illustrated also in Figs. 1 and 2. The ratio $\varepsilon_\perp(T)/\varepsilon_\perp(0)$ versus $T = at$ is given in Fig. 1. Curve 1 in (a), (b) and (c) present the case $\eta = 1$ when the multiple scattering is turned off. These curves show damping of the transverse energy and coincide, naturally, with results of Ref. [3]. However, the multiple scattering changes the situation drastically. For low ϱ_0 the transverse energy is increasing from the very beginning, while for $\varrho_0 = 100$ the transverse energy first is decreasing but starting from $T \sim 1$ it ceases to decrease and begins to increase. So, under this condition decrease of the transverse energy is possible in a very short interval of $T = at$ and only when ϱ_0 is high enough. The ratio $\gamma(T)/\gamma(0)$ versus $T = at$ is given in Fig. 2. Here one can see that in the absence of multiple scattering the total energy first decreases (for high ϱ_0) and then tends to some constant, while the multiple scattering causes an unlimited decrease of the total energy. So, the behavior of the curves in the figures illustrates numerically the results of the above qualitative analysis.

4. Using the results obtained we can analyze the behavior of a positron beam entering the oriented crystal at different energies. At low energy the condition $\varrho_0 \ll 1$ is fulfilled for all the particles of the beam ($\varrho_0 \leqslant \varrho_p \ll 1$). In this case the positrons quit the channel during a time which is much shorter than the radiation damping time ($\sim 1/a$) and in this case one can neglect radiation damping. Indeed, the characteristic dechanneling time t_d is (see Eqs. (23), (24))

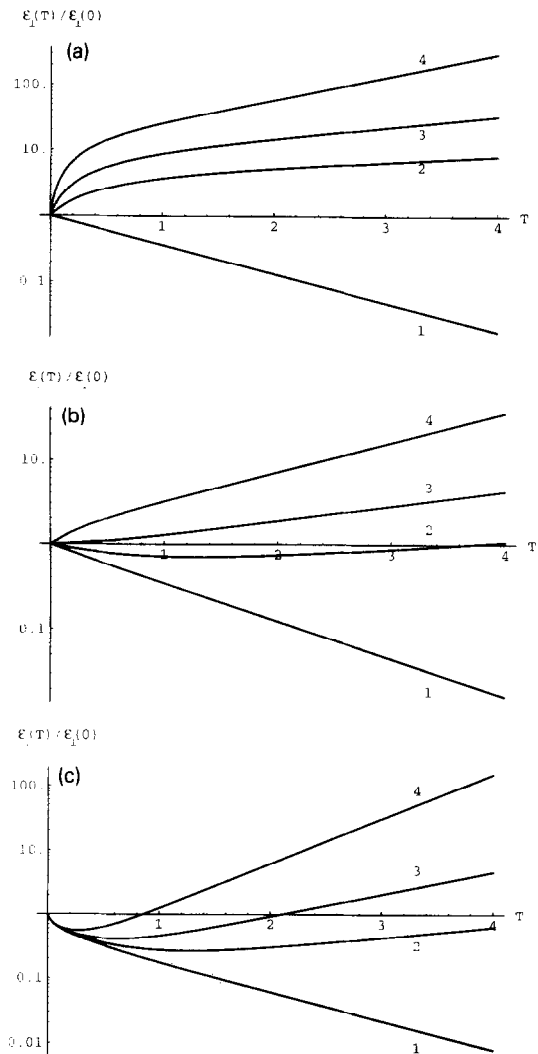$$\varepsilon_\perp = \frac{\beta_0 am^2}{\varepsilon} t_d = U_0,$$

Fig. 1. Ratio $\varepsilon_\perp(T)/\varepsilon_\perp(0)$ versus $T = at$ for different values of parameter ϱ_0. (a) For $\varrho_0 = 0.1$ curve 1 is for $\eta = 1$ when the multiple scattering is turned off; curves 2, 3, and 4 are for $\eta = 1.5, 2, 3$ respectively. (b) Same for $\varrho_0 = 1$. (c) Same for $\varrho_0 = 100$ but curves 1, 2, 3, 4 are for $\eta = 1, 2, 3, 5$ respectively.

$$t_d = \frac{\varepsilon U_0}{\beta_0 am^2} = \frac{\varrho_p}{2\beta_0 a} \ll \frac{1}{a}. \tag{25}$$

At high energy when $\varrho_p \gg 1$ the initial distribution of the positrons over ϱ_0 for an oscillator potential has the form (see Eq. (9.24), of Ref. [6])

$$dN(\varrho_0) = \frac{d\varrho_0}{\sqrt{\varrho_0}} \frac{1}{2\sqrt{\varrho_p}}. \tag{26}$$
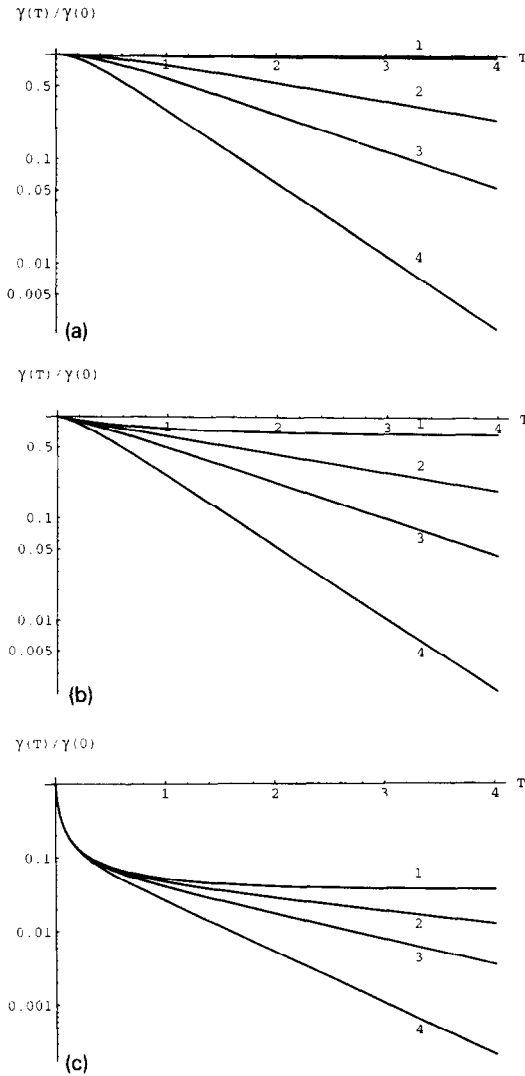
Fig. 2. Ratio $\gamma(T)/\gamma(0)$ versus $T = at$ for different values of parameter ϱ_0. (a) For $\varrho_0 = 0.1$ curve 1 is for $\eta = 1$ when the multiple scattering is turned off; curves 2, 3, 4 are for $\eta = 2, 3,$ 5 respectively. (b) Same for $\varrho_0 = 1$. (c) Same for $\varrho_0 = 100$.

Let us consider a fraction of positrons with $\varrho_0 \leqslant 1$, which is small,

$$N(\varrho_0 \leqslant 1) = \int_0^1 dN(\varrho_0) = \frac{1}{\sqrt{\varrho_p}}. \tag{27}$$

These are particles moving well inside a channel with energy $\varepsilon_\perp \leqslant U_0/\varrho_p \ll U_0$. As one can see from the above analysis, during a time $t \sim 1/a\sqrt{\beta_0}$ all the particles of this group acquire a value $\varrho \simeq \varrho_s$, while the

energy of the particles diminishes slightly. Later on the transverse energy increases,

$$\varepsilon_\perp \sim U_0 \frac{\varrho_s}{\varrho_p} \exp(\varrho_s at/2) \tag{28}$$

and during a time $t \sim (2/\varrho_s a) \ln \varrho_p$ positrons of this group go out of the channel. So, the total dechanneling time of this fraction is

$$t_d \sim \left(\frac{1}{\sqrt{\beta_0}} + \frac{c_1 \ln \varrho_p}{2(\sqrt{1 + 5\beta_0} - 1)} \right) \frac{1}{a}, \tag{29}$$

where $c_1 = 5$. An analogous consideration for the main fraction of positrons ($\varrho_0 \sim \varrho_p$) gives $c_1 = 1$.

The analysis above was performed in the frame of classical electrodynamics. As is known [6], quantum effects are governed by the parameter $\chi_c = \varepsilon U_0/dm^3$. Since quantum effects in radiation are turned on rather early, the value $\chi_c \geqslant 0.1$ can be considered as a boundary of the quantum region. From this estimate one sees that quantum effects become significant starting from the energy $\varepsilon \sim 60$ GeV in tungsten and $\varepsilon \sim 600$ GeV in silicon. We neglect also the diffusion of the transverse energy in the scattering process. Both these effects could be considered in a consistent way using the distribution function of channeled particles only.

5. The idea to use particle channeling in oriented crystals in the accelerator technique is a very attractive one. However, it appears that even in ideal conditions there is damping of the transverse oscillations without loss of the total energy only in the case $\varrho_0 \ll 1$; otherwise the particle loses its total energy along with damping of transverse oscillations. Inclusion of the interaction of the channeling particle with atomic electrons deteriorates the situation drastically. Damping on a rather limited scale is possible only in the case $\varrho_0 \gg 1$ in a very narrow interval of time. Otherwise we have antidamping: the transverse energy is increasing while the total energy is decreasing. Furthermore, we do not take into account scattering of the positrons on fluctuations of the planar potential, which can contribute essentially when the positrons are moving close to the atomic planes, connected with this scattering radiation.

References

[1] Z. Huang, P. Chen and R.D. Ruth, Phys. Rev. Lett. 74 (1995) 1759.

[2] Z. Huang, P. Chen and R.D. Ruth, Radiation damping in focusing-dominated systems, in: Proc. of the 16th IEEE Particle Accelerator Conf. and Int. Conf. on High Energy Accelerators, Texas, 1996, p. 3326.

[3] Z. Huang, P. Chen and R.D. Ruth, Nucl. Instr. and Meth. B 119 (1996) 192.

[4] A.G.Bonch-Osmolovskii and M.I. Podgoretskii, Sov. J. Nucl. Phys. 29 (1979) 216.

[5] L.D. Landau and E.M. Lifshitz, Classical theory of fields, 4th English Ed. (Pergamon Elmsford, New York, 1975).

[6] V.N. Baier, V.M. Katkov and V.M. Strakhovenko, Electromagnetic processes at high energies in oriented single crystals (World Scientific, Singapore, 1997).

[7] V.M. Biryukov, Yu.A. Chesnokov, N.A. Galyaev et al., Nucl. Instr. and Meth. B 86 (1994) 245.