Measurement of magnetic permeability of steel laminations of Booster gradient magnets

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A few words about myself

- Fermi National Accelerator Laboratory
  - PARTI 2011 summer student
- Moscow Institute of Physics and Technology
  - 2011, BS, “Subterahertz phonon generation by long Josephson junctions”, Institute of Radio-engineering and Electronics of RAS
- California Institute of Technology
  - SURF 2010 summer student, “Characterization of ceria thin films”
Problem

- Calculation of impedance of Booster gradient magnets
- Unknown magnetic permeability of the steel in the desired frequency range
Idea of measurement

Electromagnetic wave propagation in strip lines depends upon properties of materials, including magnetic permeability.

Microstrip line

Network analyzer

Strip line
Basic element of transmission line:  

Telegrapher’s equations: 

\[
\begin{align*}
\frac{\partial U}{\partial x} &= -IR - L \frac{\partial I}{\partial t} \\
\frac{\partial I}{\partial x} &= -C \frac{\partial U}{\partial t}
\end{align*}
\]

Harmonic solutions: 

\[
U = A \exp(i\omega t - ikx) + B \exp(i\omega + ikx)
\]

\[
I = \frac{A\gamma}{R + i\omega L} \exp(i\omega t - ikx) - \frac{B\gamma}{R + i\omega L} \exp(i\omega t + ikx)
\]

\[
k^2 = -i\omega C(R + i\omega L)
\]

Wave impedance: 

\[
\rho = \frac{U}{I} = \sqrt{\frac{R + i\omega L}{i\omega C}}
\]
Microstrip line parameters

The simplest formulae (valid if $W \gg H$) for parameters per unit length:

\[
C = \varepsilon \varepsilon_0 \frac{W}{H} \quad L = \mu_0 \frac{H}{W} \quad R = \frac{(1+i)}{W} \left( \text{sqrt} \left( \frac{\omega \mu}{2\sigma} \right)_{\text{strip}} + \text{sqrt} \left( \frac{\omega \mu}{2\sigma} \right)_{\text{ground}} \right)
\]

More complicated formulae exist, which take into account edge effects.

\[
\varepsilon = \varepsilon' - i \varepsilon'' \quad \mu(\omega) = \mu'(\omega) - i \mu''(\omega)
\]

Loss tangent: \( \tan \delta = \frac{\varepsilon''}{\varepsilon'} \)

If resistive losses are negligible (for example, in the case of copper), then

\[
\rho \approx \sqrt{\frac{L}{C}} \approx \rho_0 \left(1 + i \frac{\delta}{2}\right) \quad kl \approx l \omega \sqrt{LC} \approx \omega \tau \left(1 - i \frac{\delta}{2}\right)
\]
**S-parameters**

**Definition:**

\[
\begin{pmatrix}
 b_1 \\
 b_1
\end{pmatrix} =
\begin{pmatrix}
 S_{11} & S_{12} \\
 S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
 a_1 \\
 a_2
\end{pmatrix}
\]

S-parameters are measured by network analyzer

**Our case (symmetric):**

\[
\begin{aligned}
 S_{11} &= \frac{i(\kappa^2 + 1) \tan kl}{2\kappa + i(\kappa^2 + 1) \tan kl} \\
 S_{21} &= \frac{2\kappa}{2\kappa \cos kl + i(\kappa^2 + 1) \sin kl}
\end{aligned}
\]

\[
\kappa = \frac{\rho}{Z_0}
\]
Experimental setup

Copper microstrip line

Steel microstrip line

Copper strip line

Network analyzer
Copper microstrip line

\[ W = 12\text{mm} \quad H = 1.4\text{mm} \]

\[ \rho_0 = 17.4\Omega \]

\[ \tau = 1.91 \times 10^{-9} \text{sec/\text{rad}} \]

\[ \delta = 0.02 \]
Tapered copper microstrip line

\[ W = 12\, \text{mm} \quad H = 1.4\, \text{mm} \]

\[ \rho_0 = 17.3\, \Omega \]

\[ \tau = 1.84 \cdot 10^{-9} \, \text{sec/rad} \]

\[ \delta = 0.02 \]
Strip transmission line

\[ W = 12\text{mm} \quad H = 1.4\text{mm} \]

\[ \rho_0 = 10.1\Omega \]

\[ \tau = 2 \cdot 10^{-9}\text{sec/rad} \]

\[ \delta = 0.02 \]
Weakly-linked resonator

\[ W = 12mm \]
\[ H = 0.8mm \]

\[ \tau = 1.88 \cdot 10^{-9} \text{ sec/ rad} \]

instead of

\[ \tau = 1.85 \cdot 10^{-9} \text{ sec/ rad} \]
Possible causes of additional phase shift

- Improper model:
  - Radiation (excluded by strip line measurements)
  - Transition from coaxial to microstrip (must depend upon the width of a microstrip, excluded)

- Network analyzer calibration errors
  - No standard thru connector in calibration kit => different phase shifts of S11 and S21. If this is the case, then the sum of S21 and thru connector phase shifts should be equal to the S11 phase shift, which is found to be true: $11 + 28 ≈ 40$. 
How to take into account resistive losses:

\[
\tau_s = \tau_c \sqrt{1 + \frac{R}{i\omega L}}
\]

\[
\rho_s = \rho_c \sqrt{1 + \frac{R}{i\omega L}}
\]

\[
L = \frac{\rho_c \cdot \tau_c}{l}
\]

\[
R = \frac{(1+i)}{W} \sqrt{\frac{2\sigma_s}{\omega \mu \mu_0}} LR \left( \frac{1}{\pi} + \frac{1}{\pi^2} \ln \frac{4\pi W}{T} \right)^* 
\]

\[
LR = 0.94 + 0.132 \frac{W}{H} - 0.0062 \left( \frac{W}{H} \right)^2
\]

\[
\sigma_s = 2.3 \cdot 10^6 \frac{S}{m}
\]

Copper microstrip was made up from the copper strip of approximately the same geometry as the steel strip and was pressed with glass by magnet poles

$$\rho_0 = 9.7 \Omega$$

$$\tau = 2.01 \cdot 10^{-9} \text{sec/ rad}$$

$$\delta = 0.027$$
Steel microstrip in the magnetic field
Steel microstrip analysis with LL-FMR

Landau-Lifshitz ferromagnetic resonance model*: 

\[
\mu = 1 + \frac{\mu_s}{1 + i \frac{f}{f_a} - \left( \frac{f}{f_r} \right)^2}
\]

*C. P. Neo, Y. Yang, J. Ding, “Calculation of complex permeability of magnetic composite materials using ferromagnetic resonance model”, Journal of Applied Physics, April 2010
Magnetic permeability

- Real part of steel magnetic permeability
  - 0T real $\mu$ LL-FMR
  - 0T real $\mu$ direct
  - 1T real $\mu$ LL-FMR
  - 1T real $\mu$ direct
  - 2T real $\mu$ LL-FMR
  - 2T real $\mu$ direct

- Imaginary part of magnetic permeability
  - 0T imag $\mu$ LL-FMR
  - 0T imag $\mu$ direct
  - 1T imag $\mu$ LL-FMR
  - 1T imag $\mu$ direct
  - 2T imag $\mu$ LL-FMR
  - 2T imag $\mu$ direct
Results

- Technique for determining necessary parameters is developed
- Experimental investigation of the problem is carried out
- Estimation of magnetic permeability is obtained
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