

OPTIMAL CONTROL OF A LEVITATED PARTICLE AT THE SQL

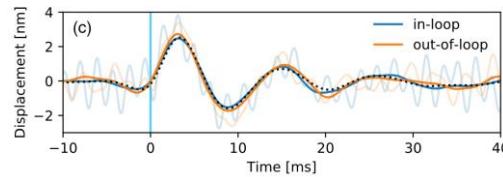
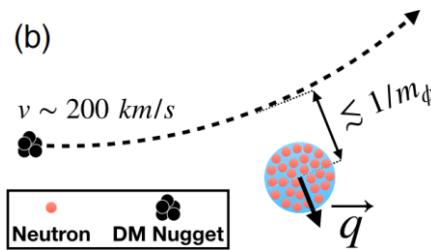
arXiv:2012.15188

Optomechanics for dark matter detection – 2nd edition
April 9th 2021

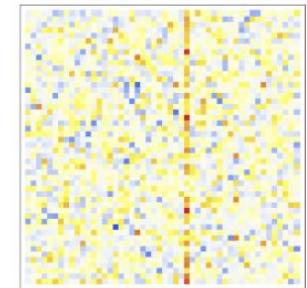
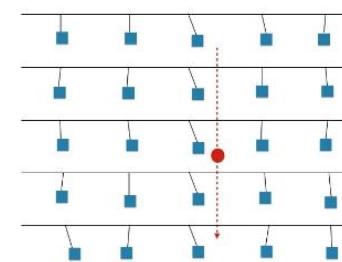
Lorenzo Magrini

*Vienna Center for Quantum Science and Technology
Faculty of Physics, University of Vienna*

Optically monitoring levitated particles for detection of (small) impulses

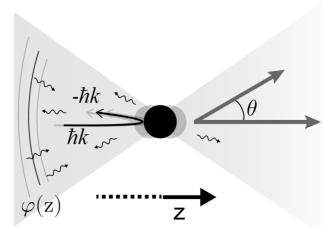


F. Monteiro *et al.*, PRL **125** (2020)

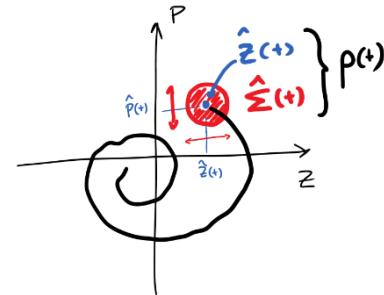


D. Carney *et al.*, PRD **102** (2020)

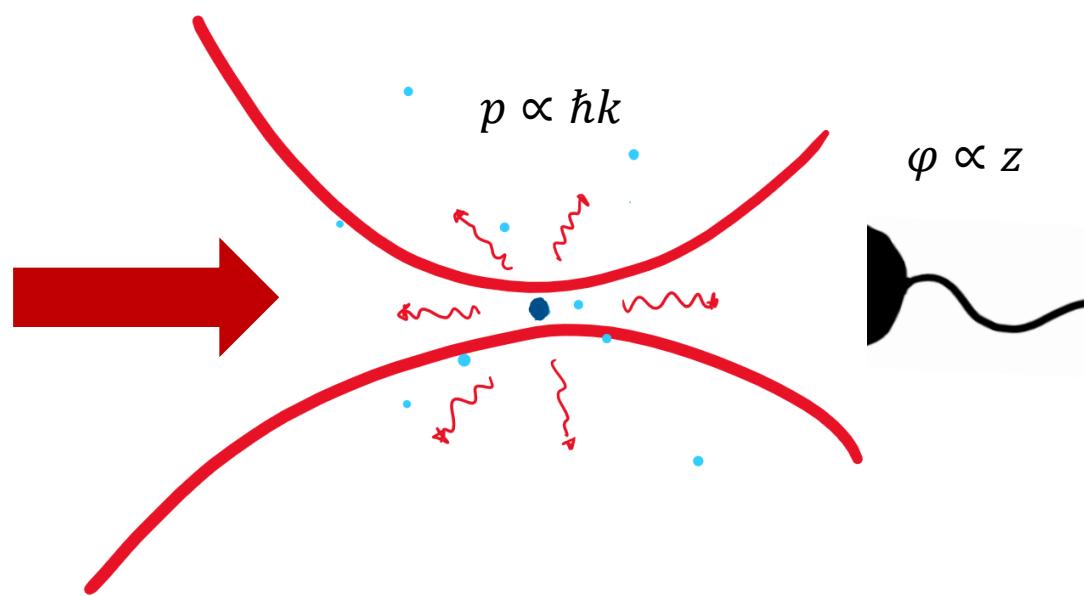
1. Position measurement at the quantum limit



2. Real time quantum filtering (and optimal control)

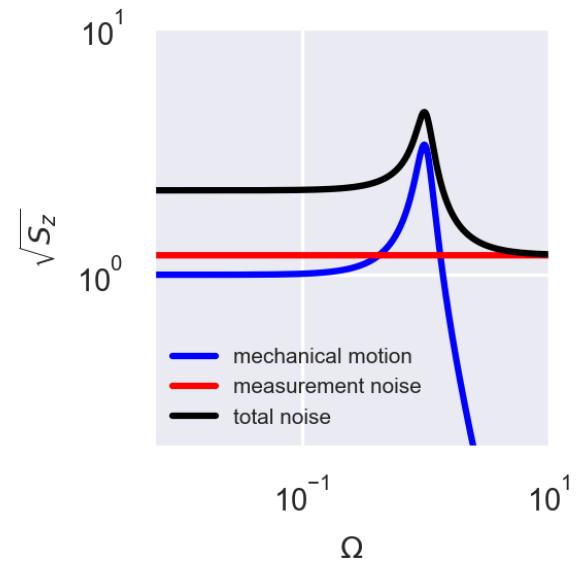
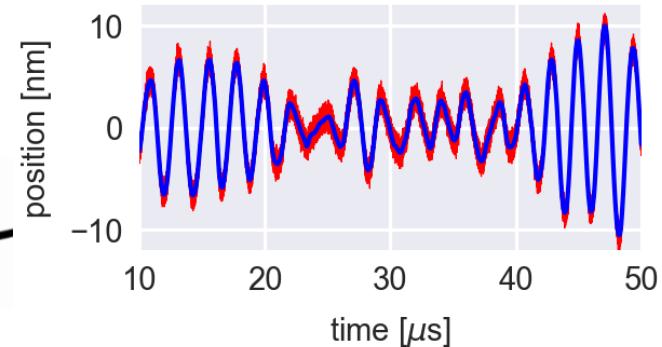


Quantum measurements and the standard quantum limit - ideal

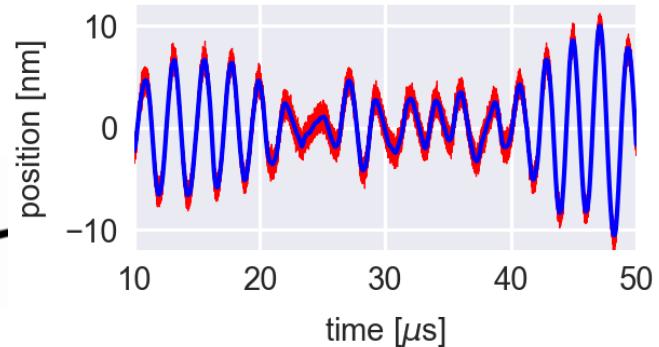
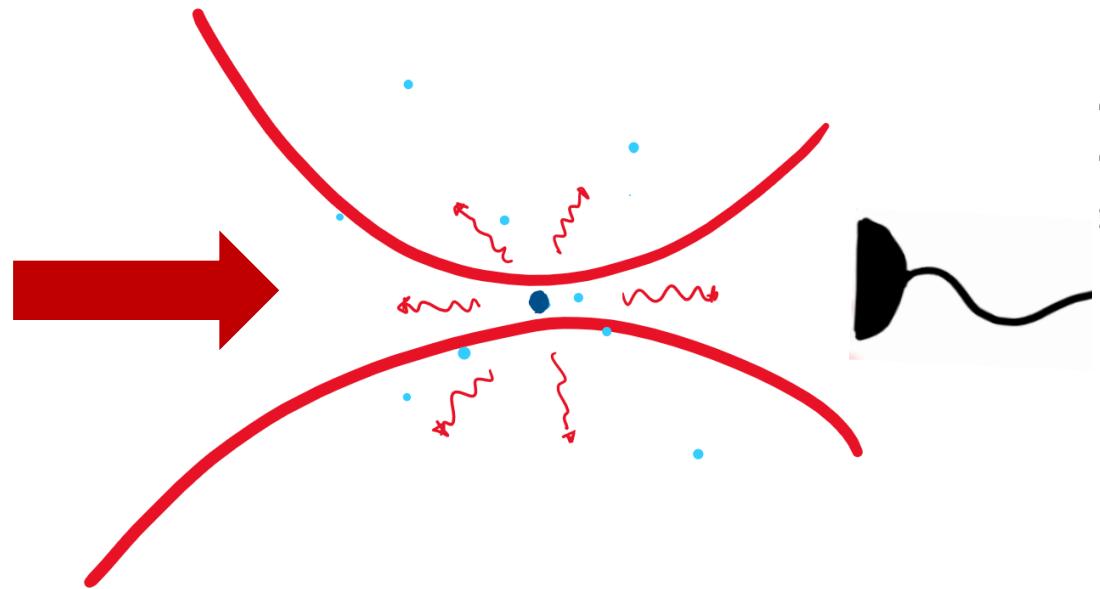


$$S_\zeta = S_z^{zpf}(\Omega) + S_F^{ba} |\chi(\Omega)|^2 + S_z^I$$

$$\sqrt{S_F^{ba} S_z^I} = \hbar$$



Quantum measurements and the standard quantum limit - real



$$S_\zeta = S_z^{zpf}(\Omega) + (S_F^{ba} + S_F^{th}) |\chi(\Omega)|^2 + S_z^{imp}$$

S_F^{tot}

$$\sqrt{S_F^{tot} S_z^{imp}} = \frac{\hbar}{\sqrt{\eta}}$$

imp
z

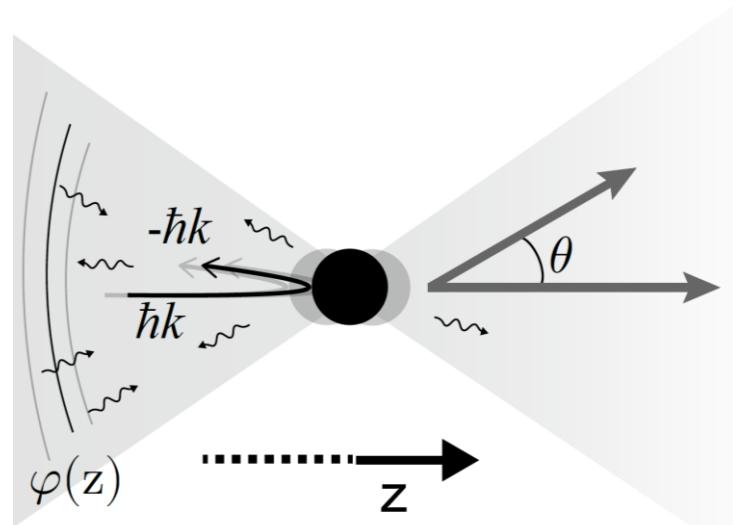
distributions of information and back-action are not uniform

$$\Delta\varphi \sim 2 \frac{2\pi}{\lambda} z$$

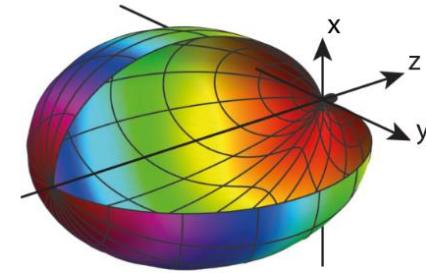
$$\Delta p \sim 2\hbar k$$

$$\Delta\varphi \sim \text{constant}$$

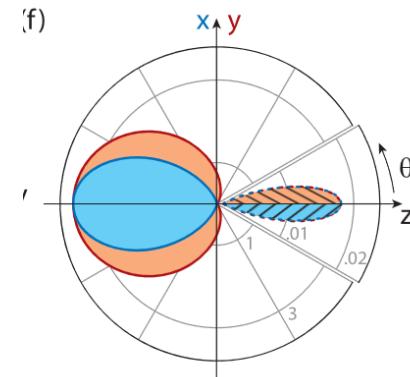
$$\Delta p \sim 0$$



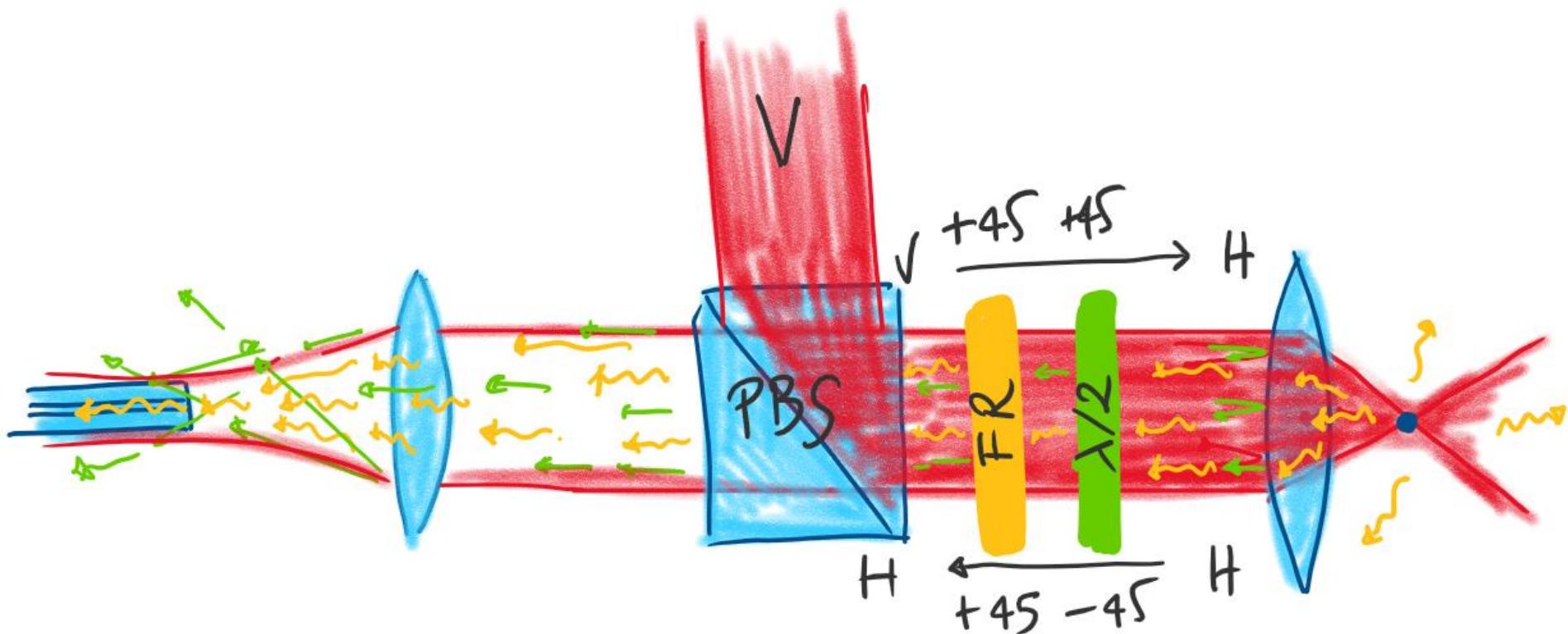
(e)



(f)



phase sensitive confocal detection of the dipole



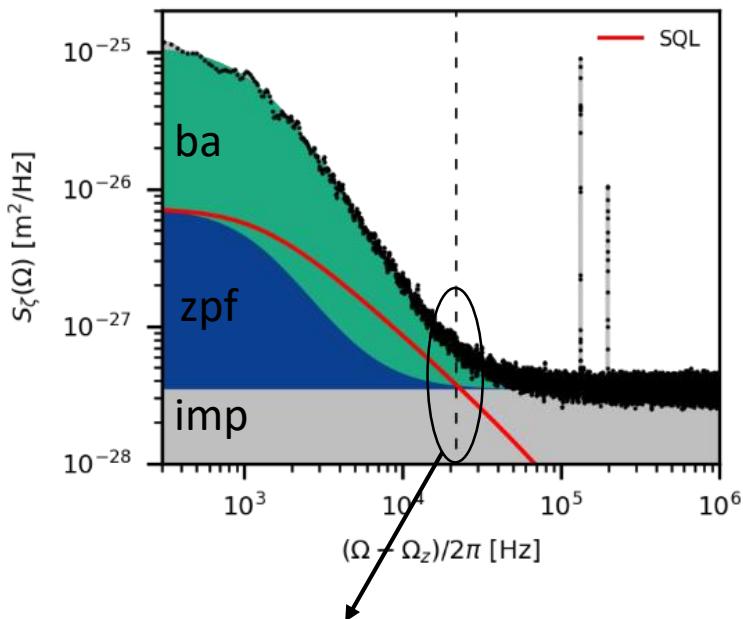
Efficiency and selectivity of the confocal microscope

$$\eta \sim 0.34$$

$$\eta^* \sim 0.15$$

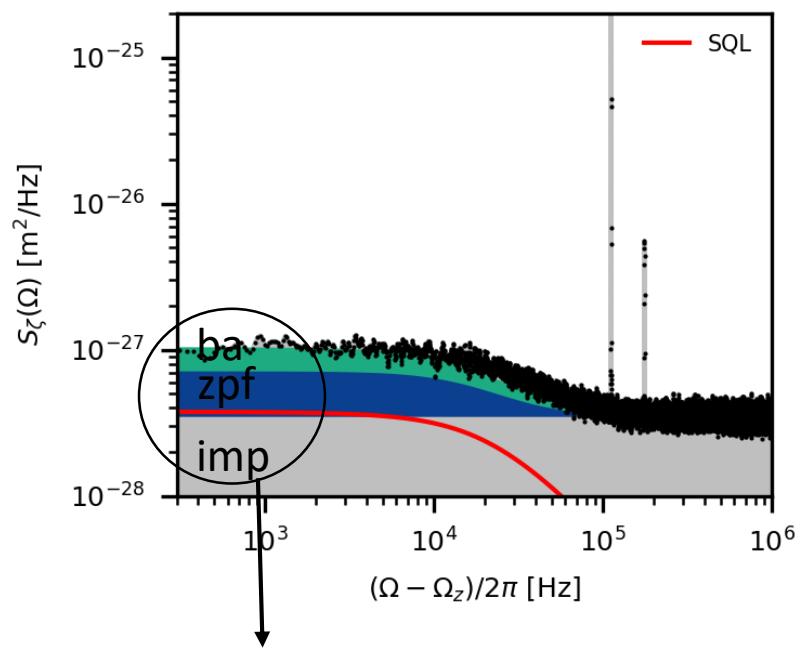
$$\eta_{background} \sim 0.001$$

Measurement close to the SQL



$$S_{\zeta\zeta}(\Omega) = 1.7S_{SQL}(\Omega) > S_{SQL}(\Omega)$$

$$\frac{1}{2} \text{imp} + \frac{1}{2} \text{ba}$$



$$S_{\zeta\zeta}(\Omega_z) = 2.7S_{SQL}(\Omega_z) > 2S_{SQL}(\Omega_z)$$

$$\frac{1}{2} \text{zpf} + \frac{1}{4} \text{imp} + \frac{1}{4} \text{ba}$$

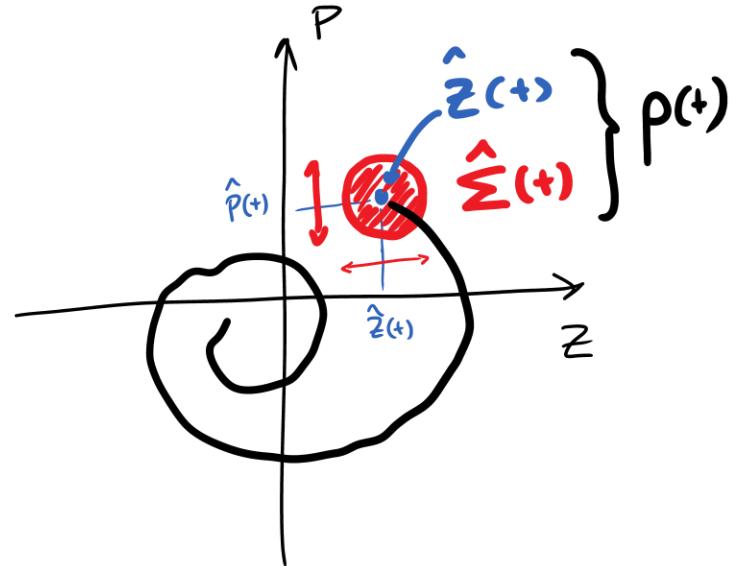
Open quantum system under continuous measurement

$\rho(t)$ state conditioned on the measurement output $\zeta(t)$

If the state is Gaussian, it is completely determined by the first and second moments of the state space operators $\mathbf{z} = [z, p]$:

$$\hat{\mathbf{z}}(t) = \text{tr}(\mathbf{z}\rho(t))$$

$$\hat{\Sigma}(t) = \text{Re}[\text{tr}(\mathbf{z}\mathbf{z}^T\rho(t))] - \hat{\mathbf{z}}(t)\hat{\mathbf{z}}(t)^T$$

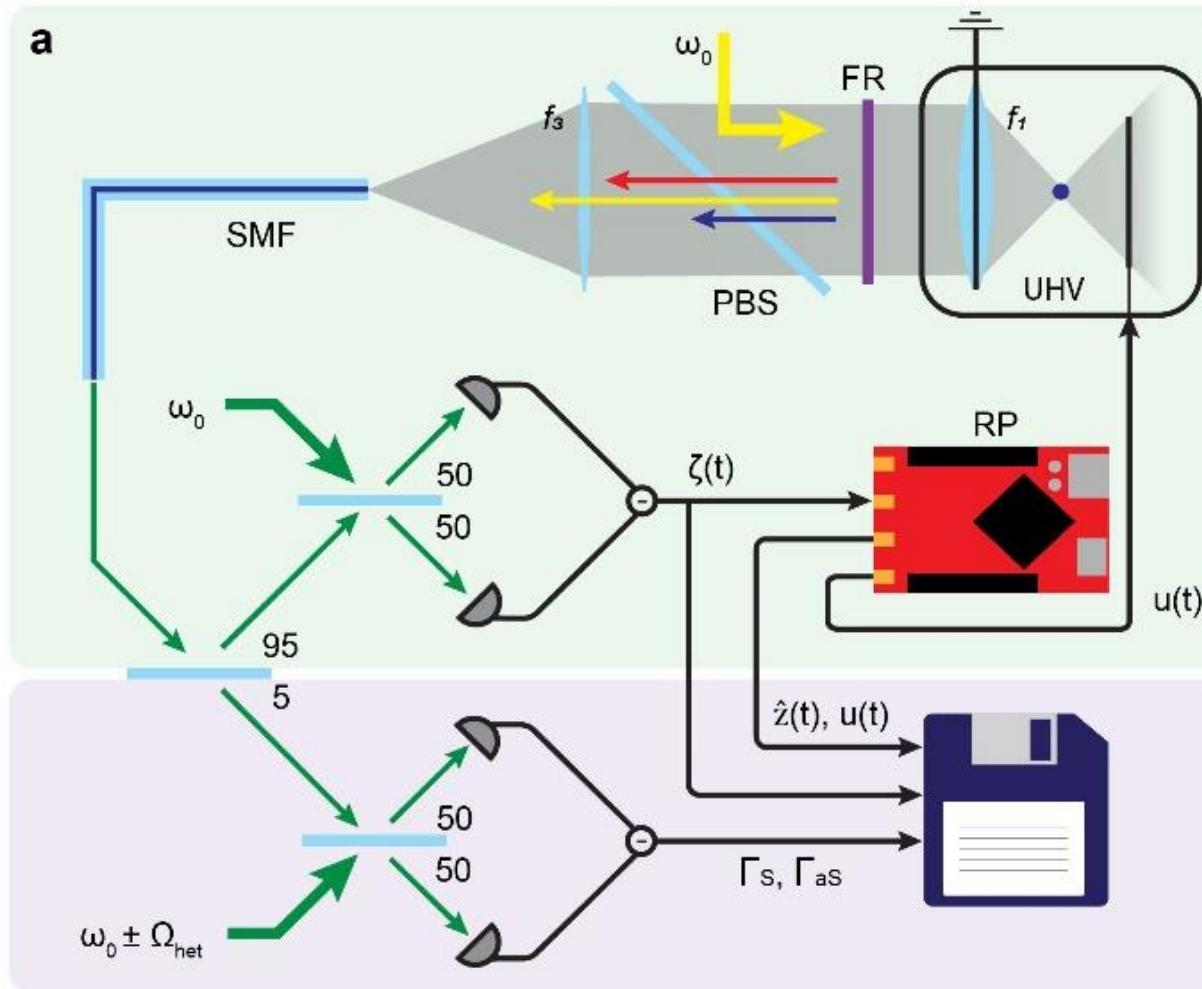


And the dynamical equations of $\hat{\mathbf{z}}$ and $\hat{\Sigma}$ are equivalent to **the classical Kalman filter!**

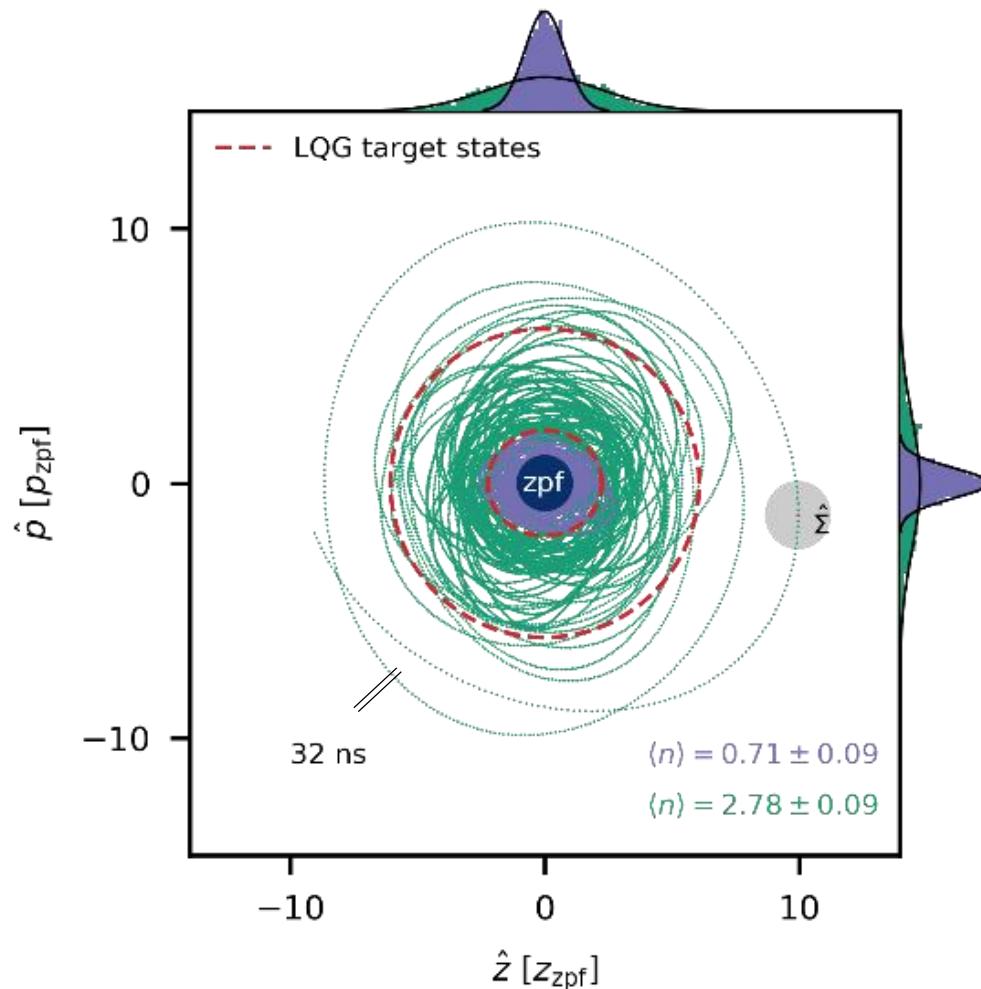
Belavkin, Radio Eng. Electron. Phys. (USSR) 25, 1445 211 (1980)

Kalman, R. E. ASME J. Basic Eng. (1960).

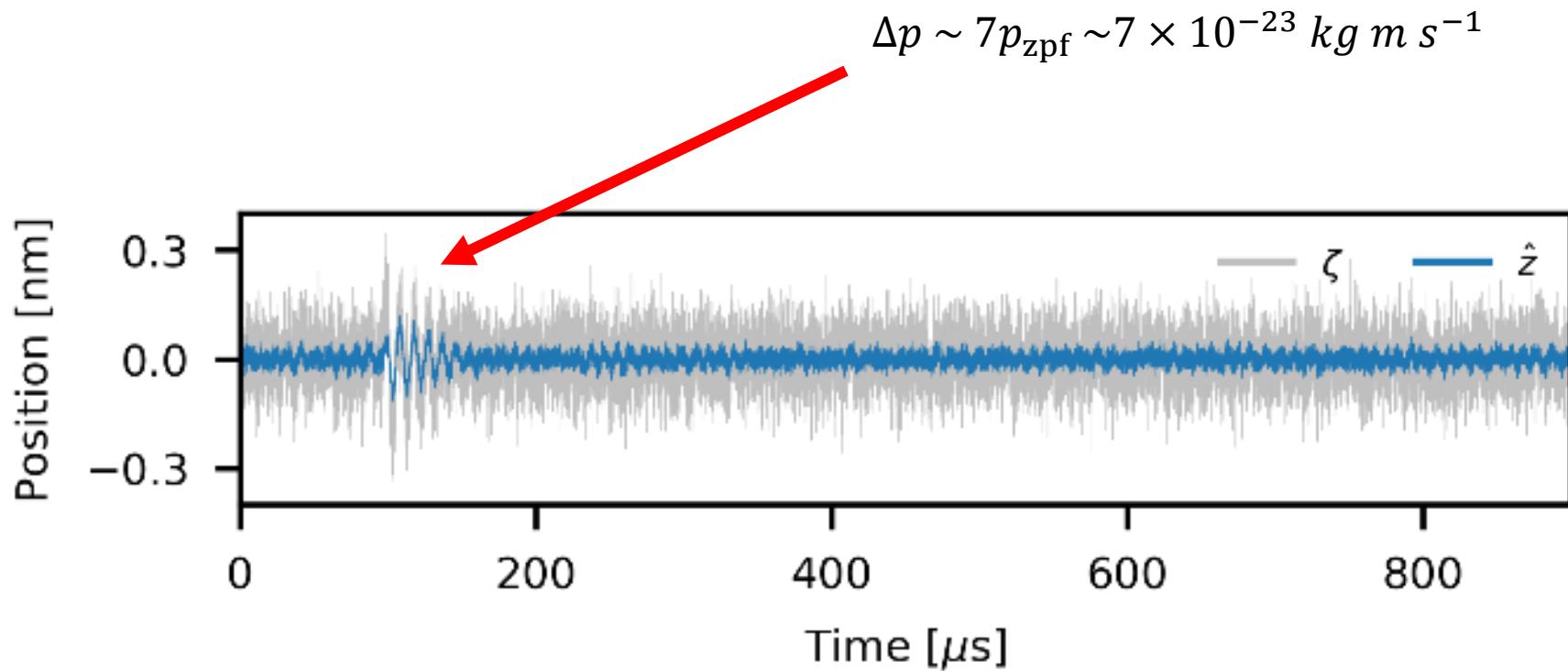
The setup



The quantum trajectory



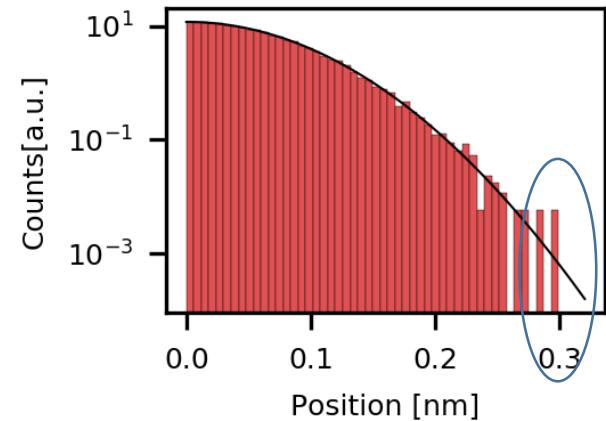
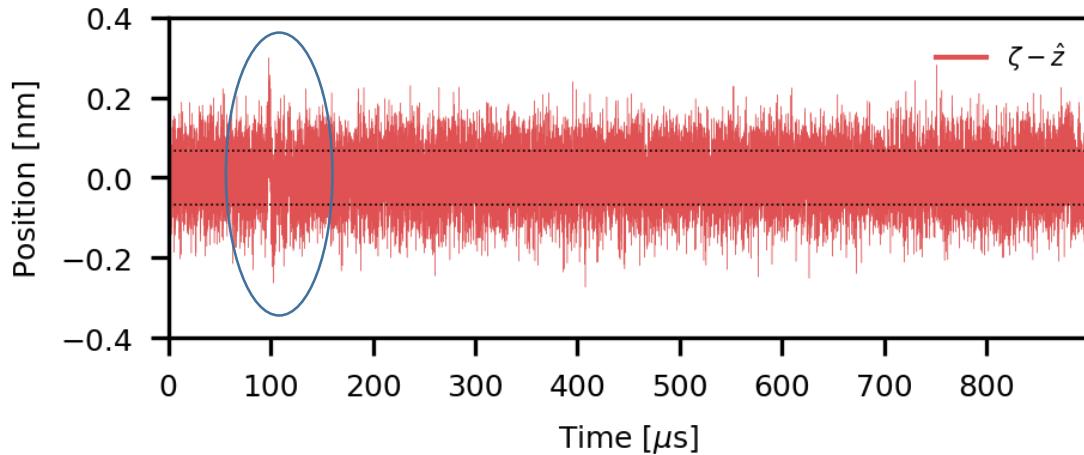
Real time estimation and control



Model verification

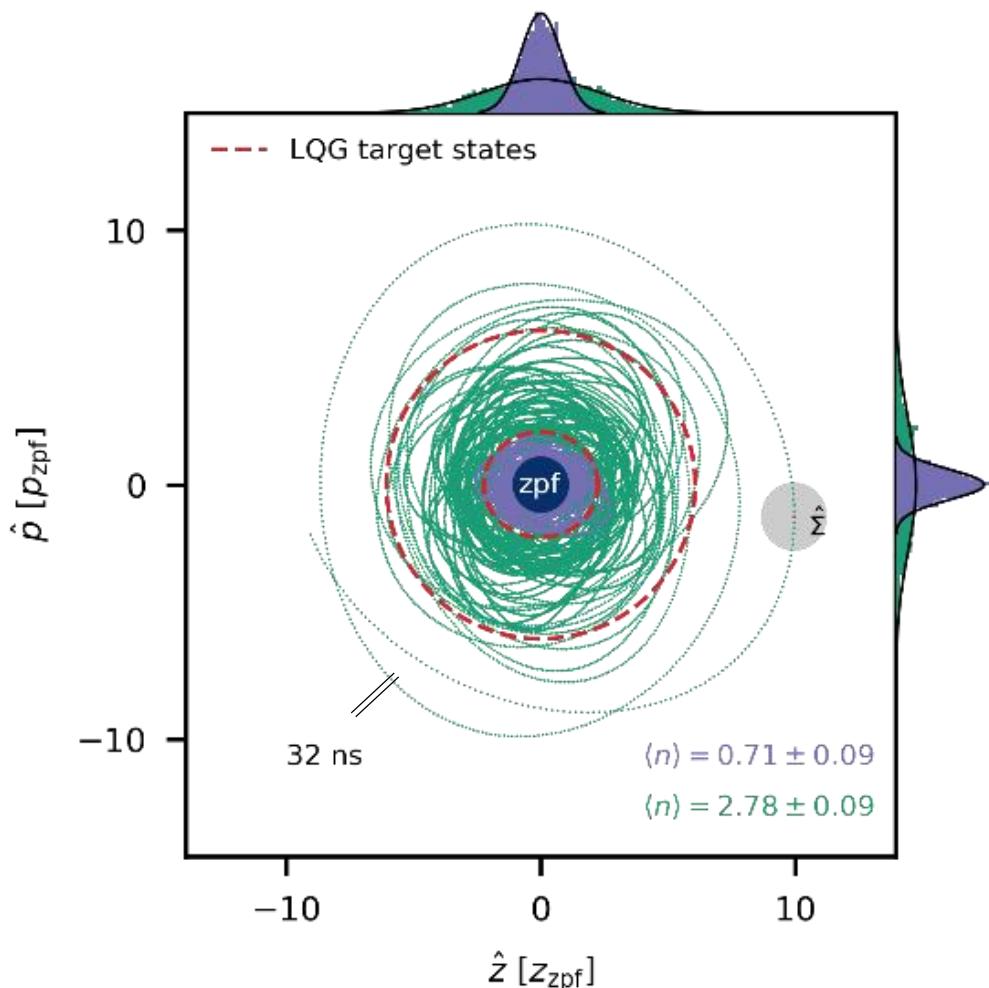
Analysis of the innovation sequence:

$$\epsilon = \zeta - \hat{z}$$



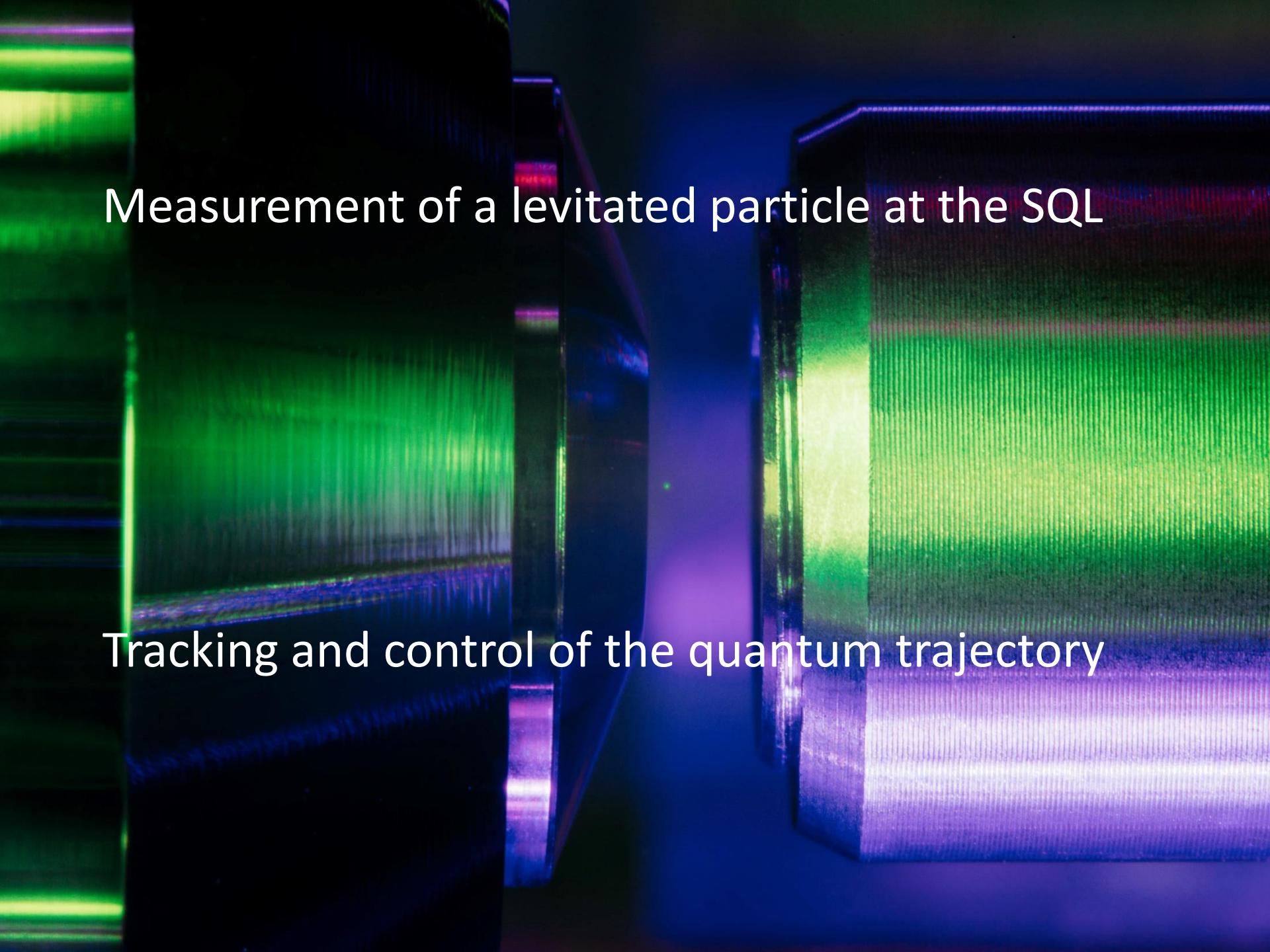
The distribution of the innovation sequence does not depend on the average state covariance (temperature), only on the covariance of the conditional state (measurement quality)

The quantum trajectory



Sensitivity to impulses does not require the state to be cold, its enough that the conditional state is!

$$\Delta p_{\min} = \sqrt{p_{\text{zpf}}^2 + \hat{\Sigma}_{pp}}$$



Measurement of a levitated particle at the SQL

Tracking and control of the quantum trajectory

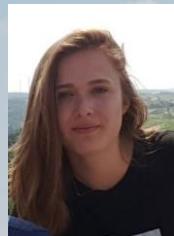
KUGI GROUP AT THE TUW



SUNGKUN HONG



SEBASTIAN HOFER



CONSTANZE BACH



PHILIPP ROSENZWEIG



ANDREAS DEUTSCHMANN



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April 9th 2021



Continuous monitoring of a quantum systems

Generalized measurement

$$P(z) = \text{tr}[M_z \rho M_z^\dagger]$$

$$\rho \rightarrow \frac{M_z \rho M_z^\dagger}{P(z)}$$

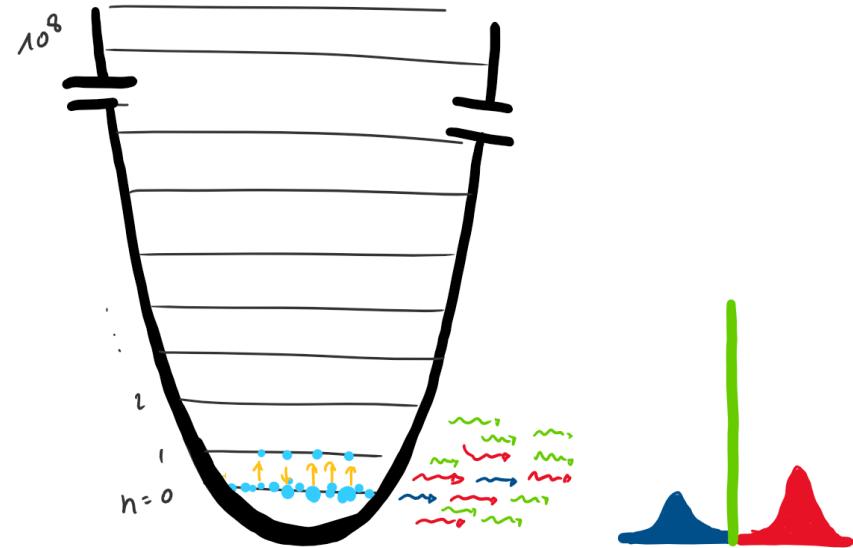
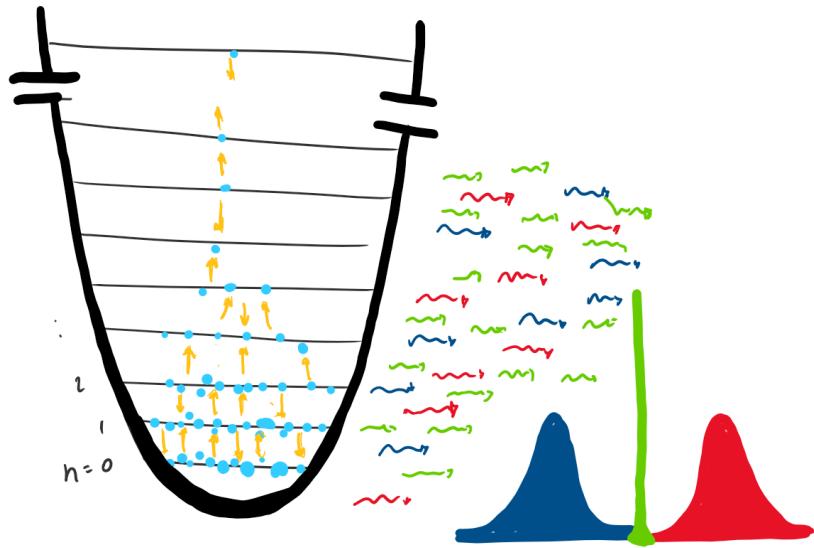
Sequence of generalized measurements (+ unitary dynamics $M_z \rightarrow M_z U_0(\text{dt})$)

$$\rho \rightarrow \frac{M_{z_0} \rho M_{z_0}^\dagger}{P(z_0)} \rightarrow \frac{M_{z_1} M_{z_0} \rho M_{z_0}^\dagger M_{z_1}^\dagger}{P(z_0|z_1)} \rightarrow \frac{M_{z_2} M_{z_1} M_{z_0} \rho M_{z_0}^\dagger M_{z_1}^\dagger M_{z_2}^\dagger}{P(z_0|z_1|z_2)} \rightarrow \dots$$

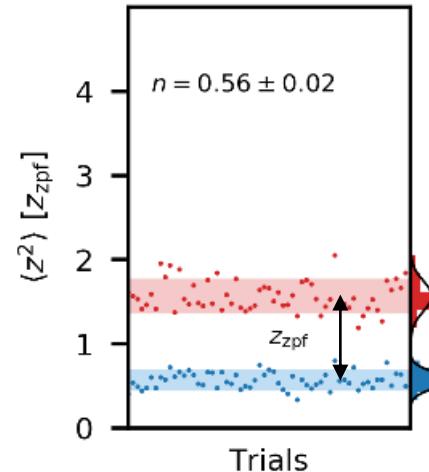
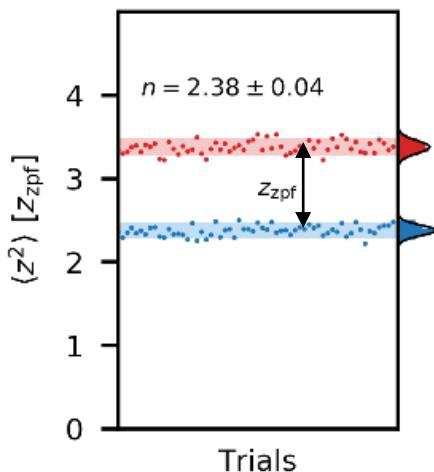
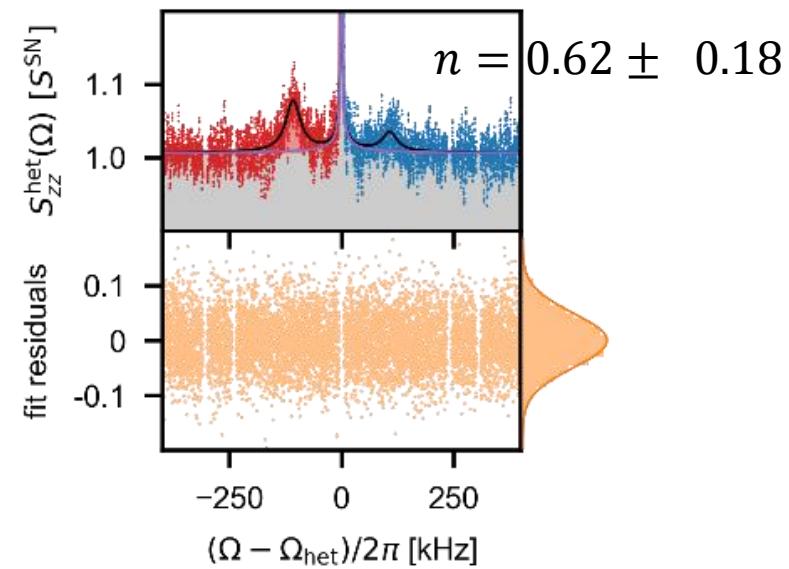
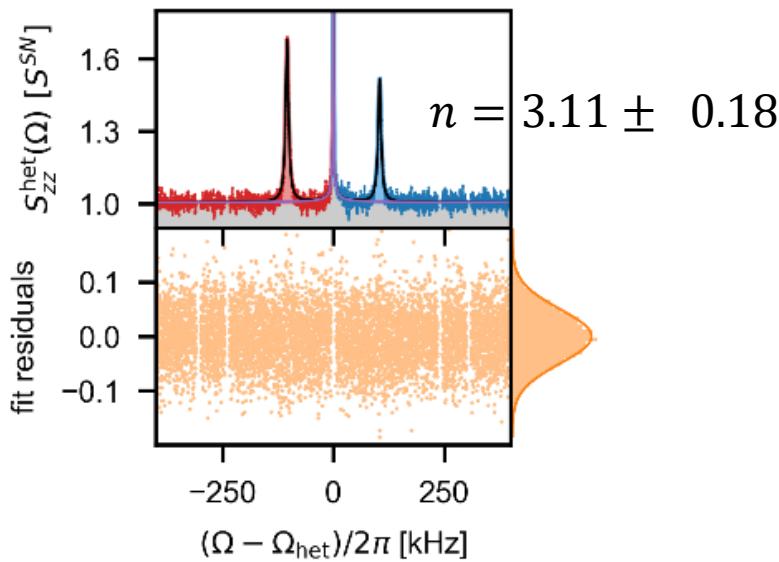
the sequence can be generalized to a stochastic differential equation driven by the signal

$$\{z_t\} = z_0, z_1, z_2 \dots$$

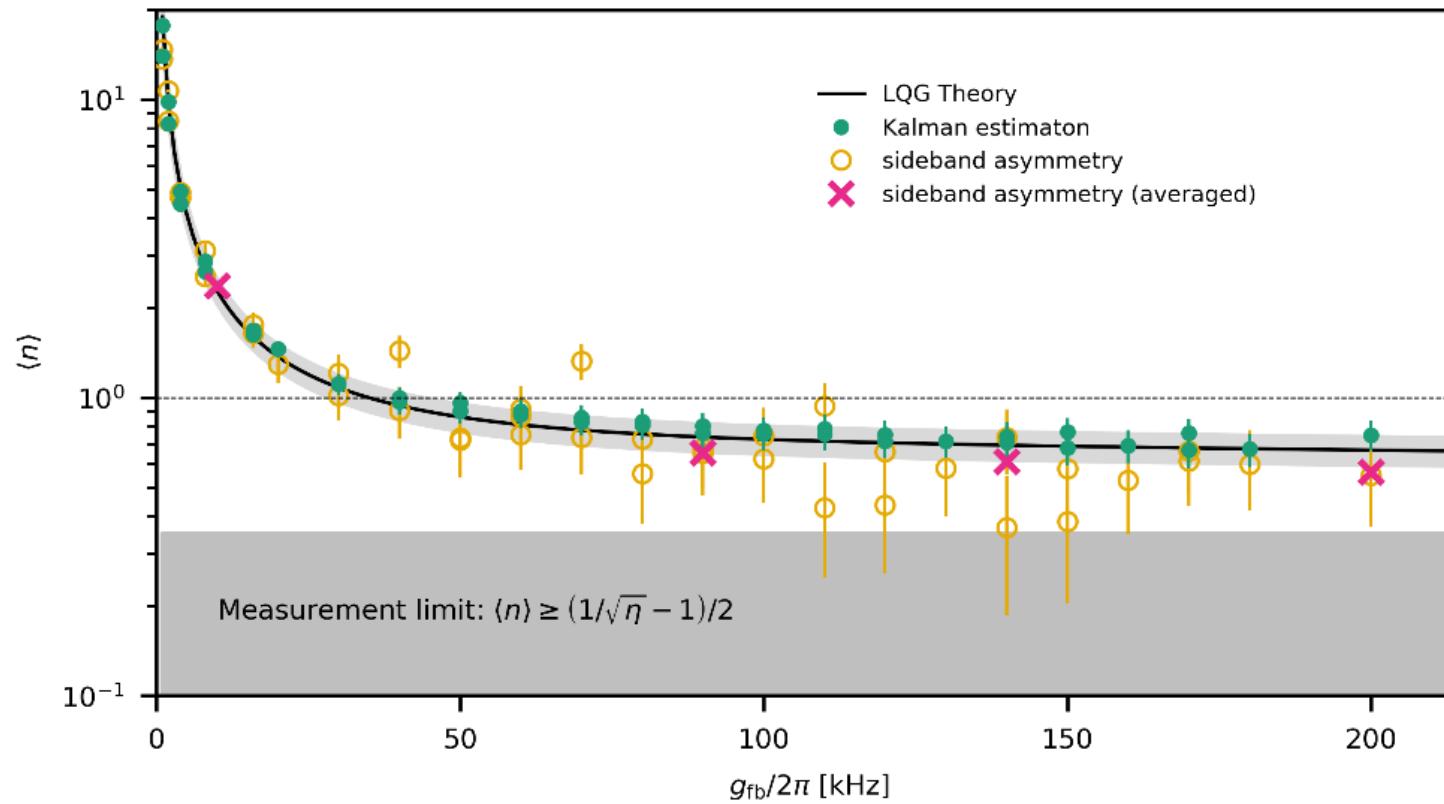
scattering of photons



heterodyne noise analysis 2

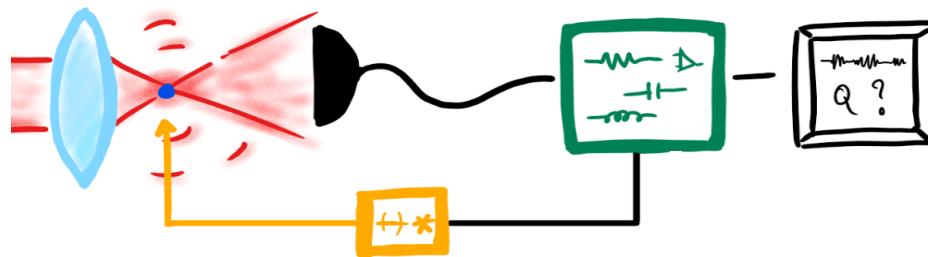


Ground state cooling



SYSTEM

OBSERVER



$$\mathbf{z} = [z, p]$$

SYSTEM MODEL INPUT PROCESS NOISE

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{G}\mathbf{w}(t)$$

$$\dot{\hat{\mathbf{z}}}(t) = \mathbf{A}\hat{\mathbf{z}}(t) + \mathbf{B}\mathbf{u} + \mathbf{L}(\zeta(t) - \mathbf{C}\hat{\mathbf{z}}(t))$$

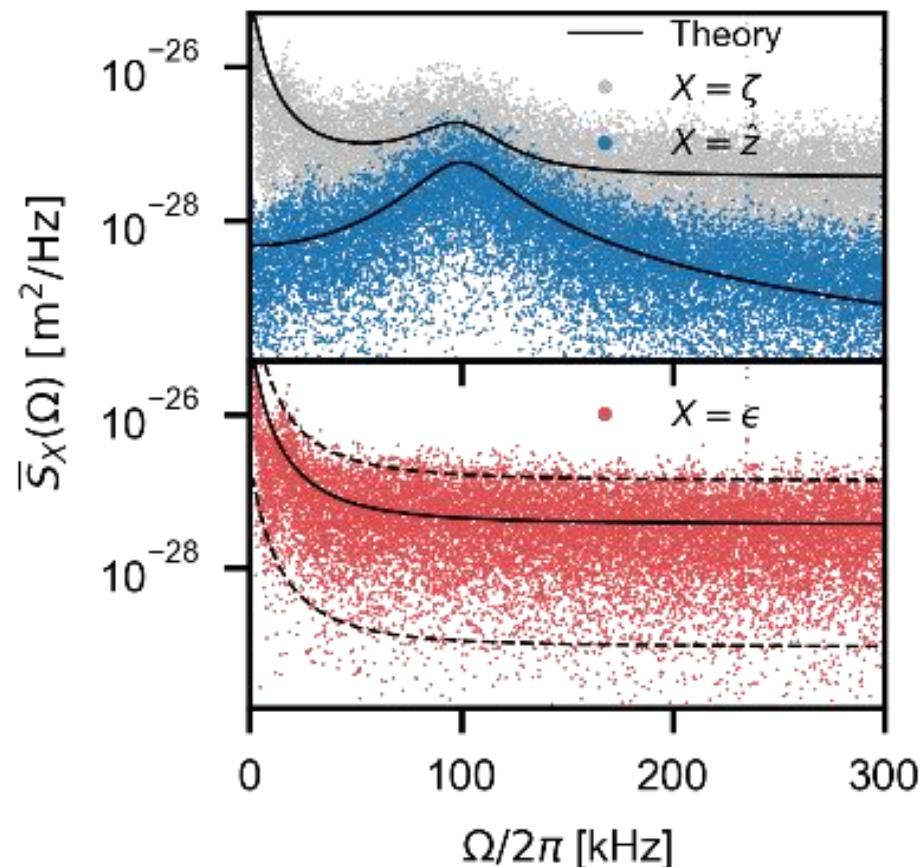
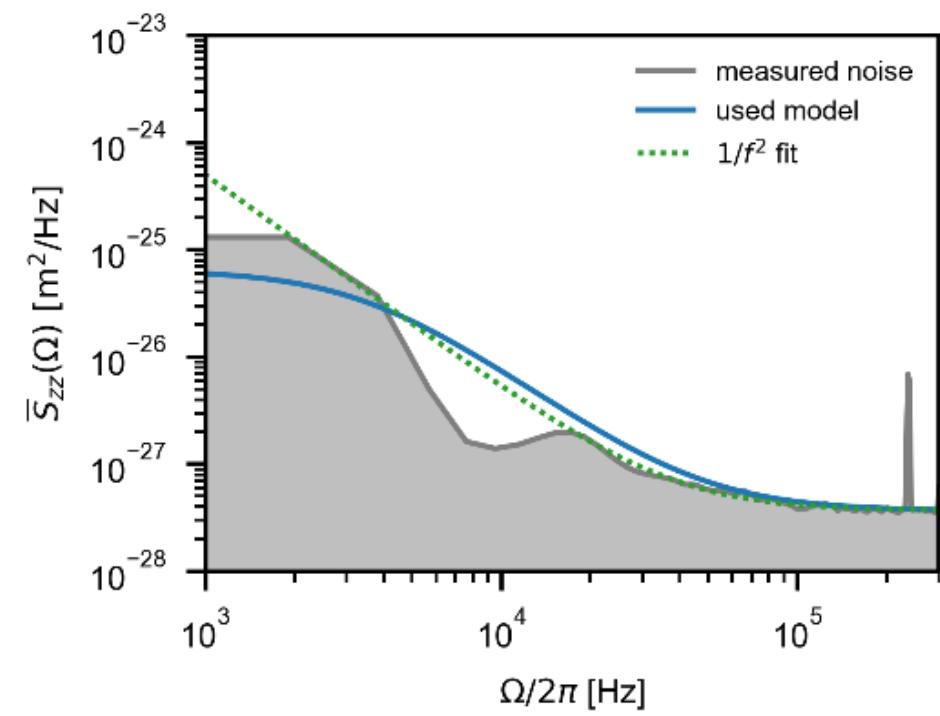
$$\zeta(t) = \mathbf{C}\mathbf{z}(t) + \mathbf{v}(t) \quad \text{MEASUREMENT NOISE}$$

$$\zeta(t) = \mathbf{C}\mathbf{z}(t)$$

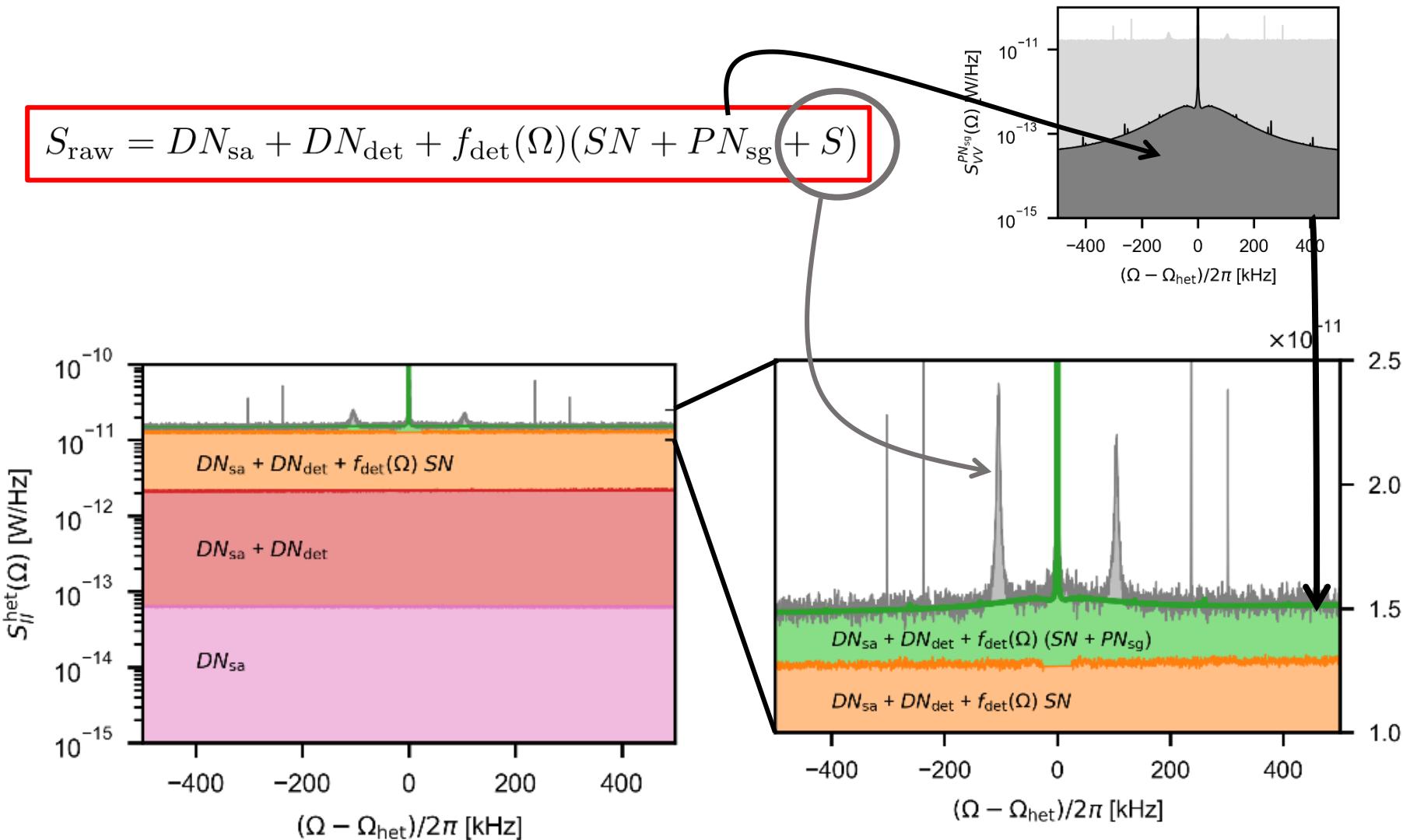
DETECTION MODEL

$$\mathbf{L} = \mathbf{L}(\mathbf{w}(t), \mathbf{v}(t))$$

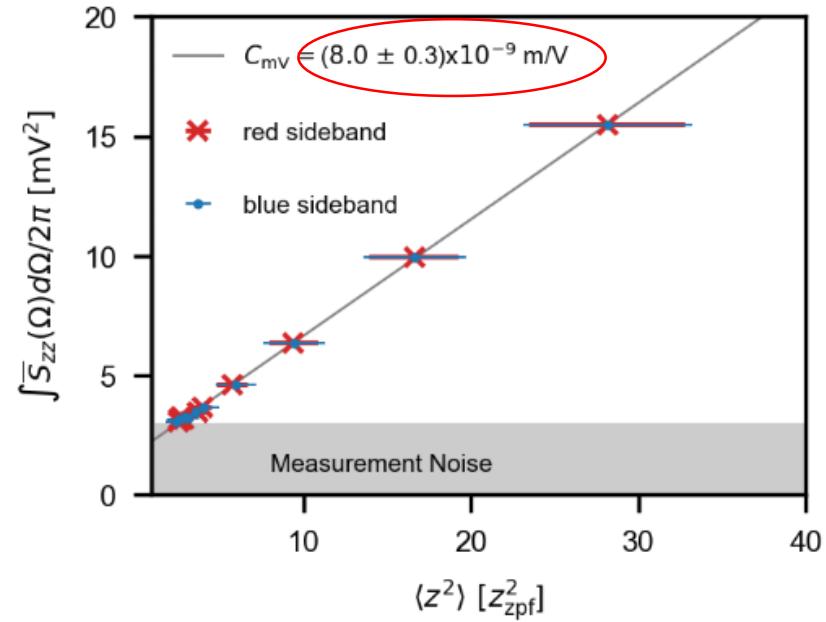
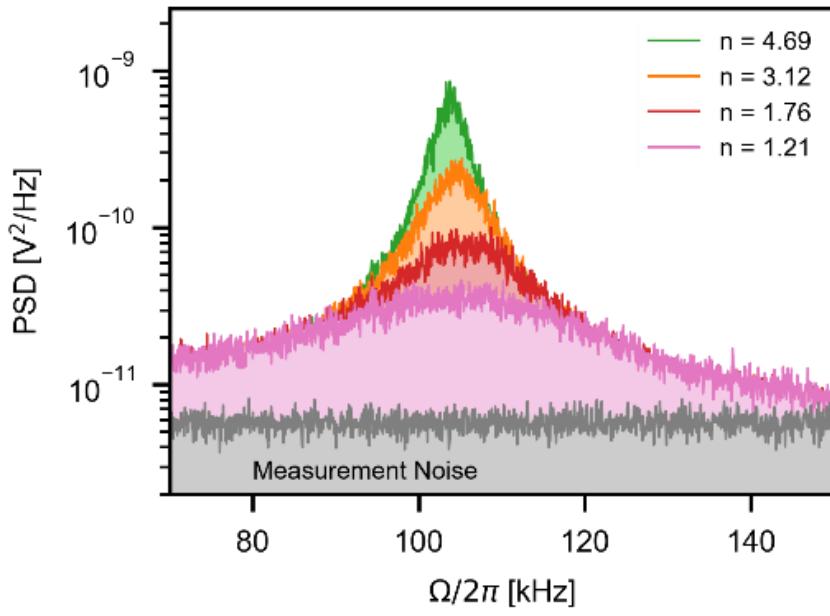
pink noise modelling



heterodyne noise analysis 1



position calibration



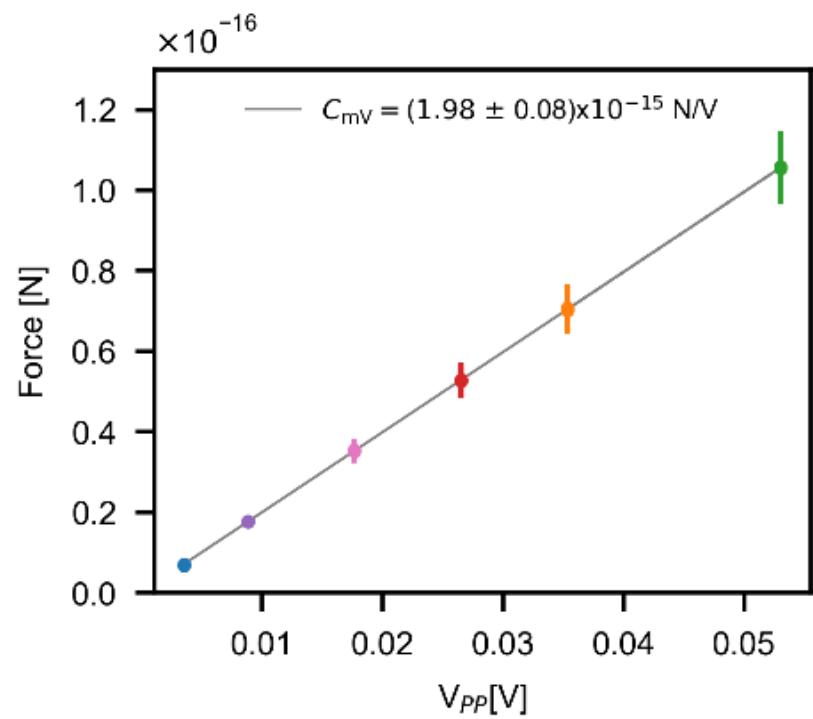
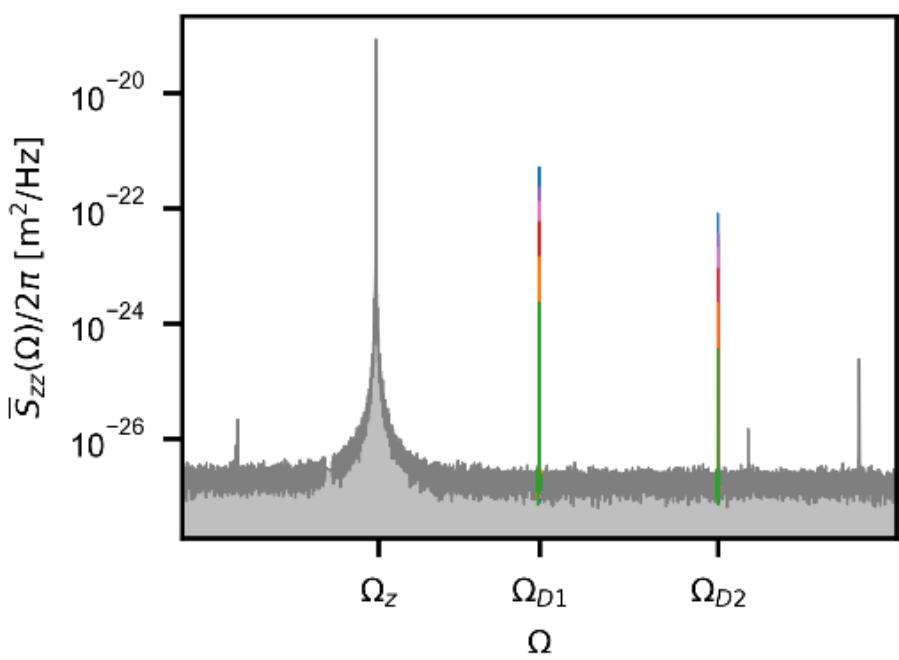
From geometrical considerations we know:

...and expect:

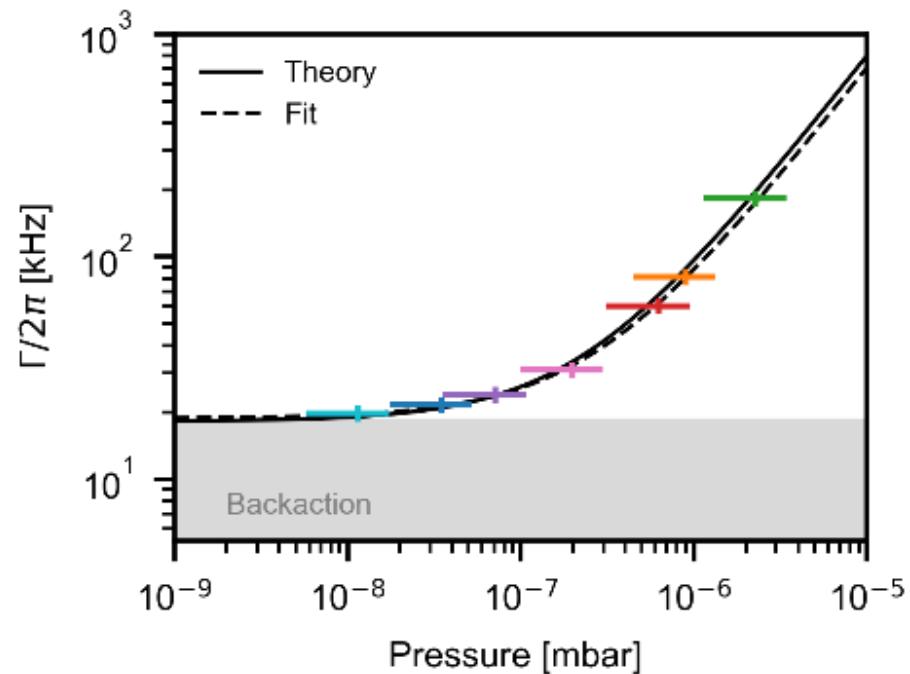
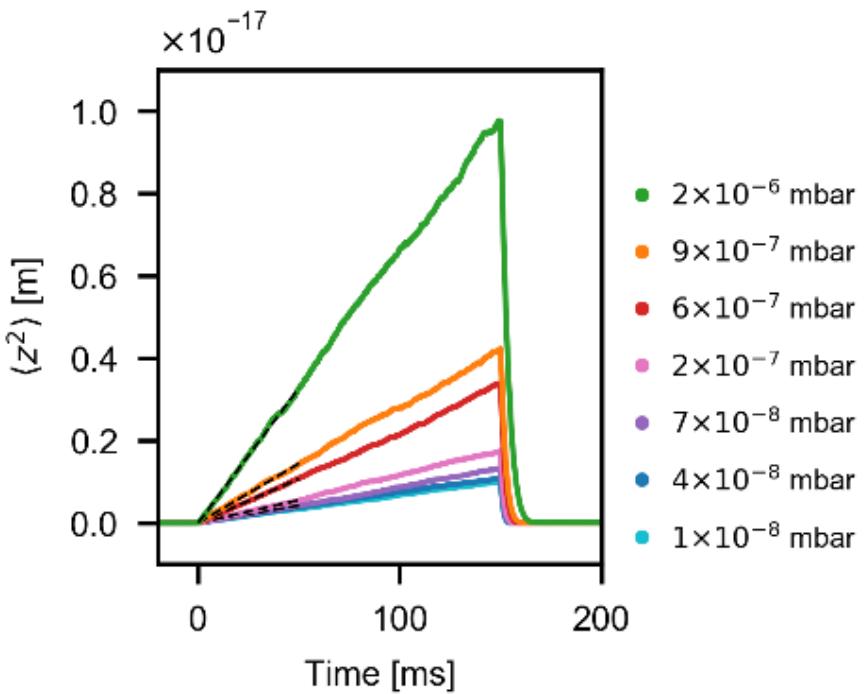
$$\chi = \frac{\partial \varphi}{\partial z} = \sqrt{\frac{\eta}{\eta^*}} \sqrt{\left(A^2 + \frac{2}{5}\right)} k \quad [\text{rad/m}]$$

$$C_{\text{mV}} = \frac{h\nu}{(-e)\eta_q g_t \sqrt{P_S P_{\text{LO}}} \chi} = 7.9 \times 10^{-9} \text{ m/V}$$

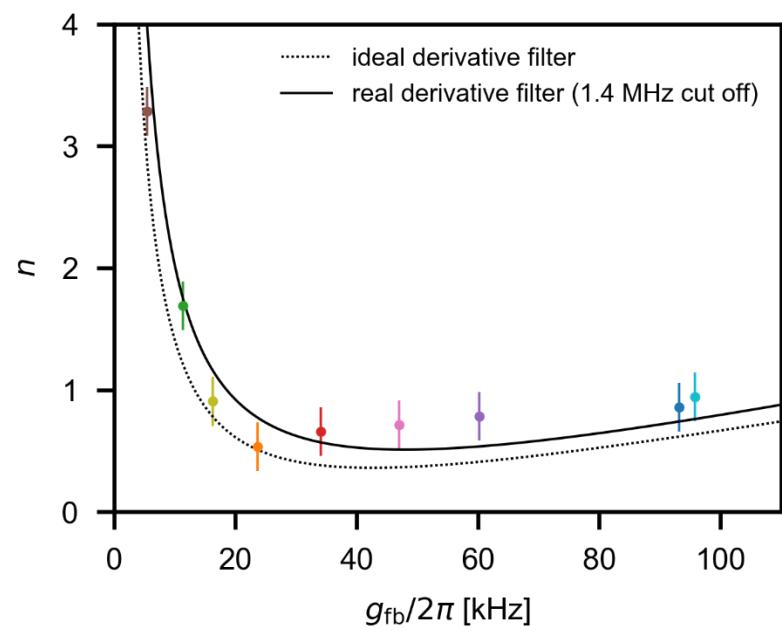
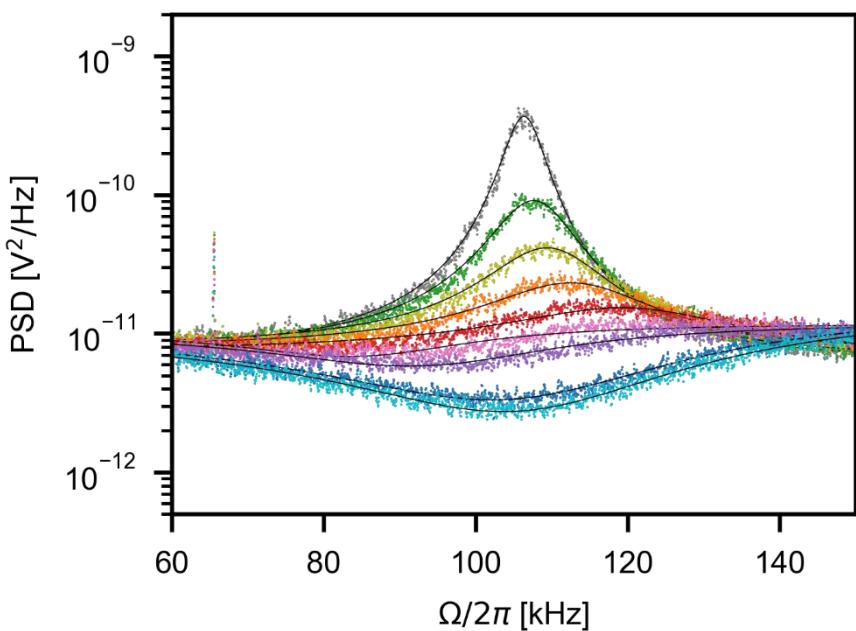
force calibration



back-action and thermal force



derivative feedback



Optimal estimation and control

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Optimal estimation and control