New 6D scheme &Space charge effects

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Friday MAP

9/9/11

- Introduction
- Long Space-Charge Theory
- The Challenge
- Solution
  1. Philosophy
  2. New taper after merge
  3. New trans cooling lattice
  4. Emittances
- Can we do even beter ?
- Conclusion

Introduction

- With high charges and low emittances expect coherent effect
- Study of transverse space charge, in 6D and Final, look ok

- But Longitudinal space charge in final 6D appears serious

Theory of Longitudinal Space Charge

- For a Gaussian bunch, the space charge defocusing is greatest in the core
- De-focusing means increased longitudinal beta
- higher beta means higher equilibrium long emittance: Cooling is weaker

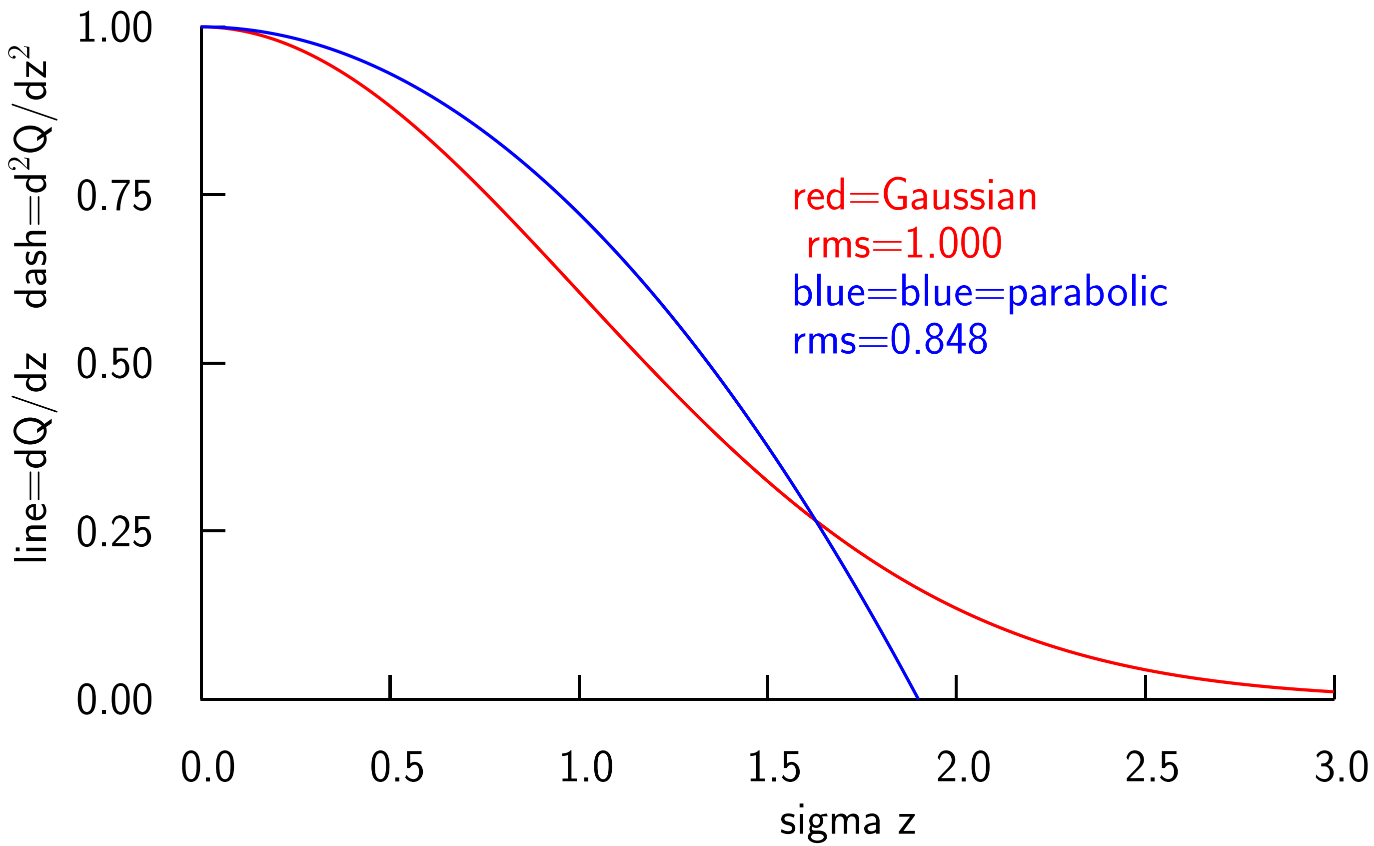
- In the tails, longitudinal focusing is even increased
- Long focusing is increased: Long beta is less
- Equilibrium long emittance is less: Cooling is stronger

As a result, with strong space charge, the tails get pulled in until the bunches approximate ellipsoids with near uniform charge density, and negligible tails.

The projected charge densities, in z, is now parabolic:

$$\frac{dQ}{dx} = \rho_o \left(1 - \frac{z^2}{z_m^2}\right)$$

Compare Gaussian with Parabolic



The resulting distributions have:

- rms widths of ≈ 85% of the Gaussians
- maximum z $z_m = 1.9 \times \sigma_z$

Theory from Allen[?] for parabolic

$$\mathcal{E}_{sc} = \left(\frac{\rho_o\ g\ a^2}{z_m^2\ 2\epsilon_o}\right)\ z \quad \text{where} \quad \rho_o = \frac{3Q}{4\pi\ a^2\ z_m}$$

z and z_m are defined in the bunch center of mass frame

$$\text{giving} \quad \mathcal{E}_{sc} = \left(\frac{3\ Q\ g}{8\ \pi\ \epsilon_o\ (z_m)^3}\right)\ z_c \qquad = \left(\frac{3\ Q\ g}{\gamma^2\ \epsilon_o\ 8\ \pi\ z_{m\ lab}^3}\right)\ z_{lab}$$

$$\mathcal{E}'_{sc} = \frac{d\mathcal{E}_{sc}}{dz_{lab}} \approx \left(\frac{3\ Q\ g}{\gamma^2\ \epsilon_o\ 8\ \pi\ 1.9^3\ \sigma_z^3}\right) = 0.017\ \frac{Q\ g}{\epsilon_o\ \gamma^2\ \sigma_z^3} \tag{1}$$

In the absence of a pipe, g is a constant, dependent just on z_m/a, and the gradient of the electric field $\mathcal{E}'_{sc}$ is also constant.

With a pipe, $\mathcal{E}'_{sc}$ is not constant, but g can defied so that $\mathcal{E}'_{sc}$ gives the average gradient of $\mathcal{E}_{sc}$. g now depends on both z_m/a and b/a where b is the beam pipe radius.

C. Allen, N. Brown, M. Reiser; "Image effects for beams in axisymmetric systems"; Particle Accelerators; vol. 45; p 149 (1994)

Numerically calculated values of $g(z_m/a, b/a)$ are given in reference ?? and can be approximated by:

$$g \approx \frac{1}{\left(1/F_z^n + 1/F_b^n\right)^{1/n}}$$

where

$$F_z = 0.66 + 2.2 \ln\left(1 + 0.4 \left(\frac{z_m}{a} - 1\right)\right)$$

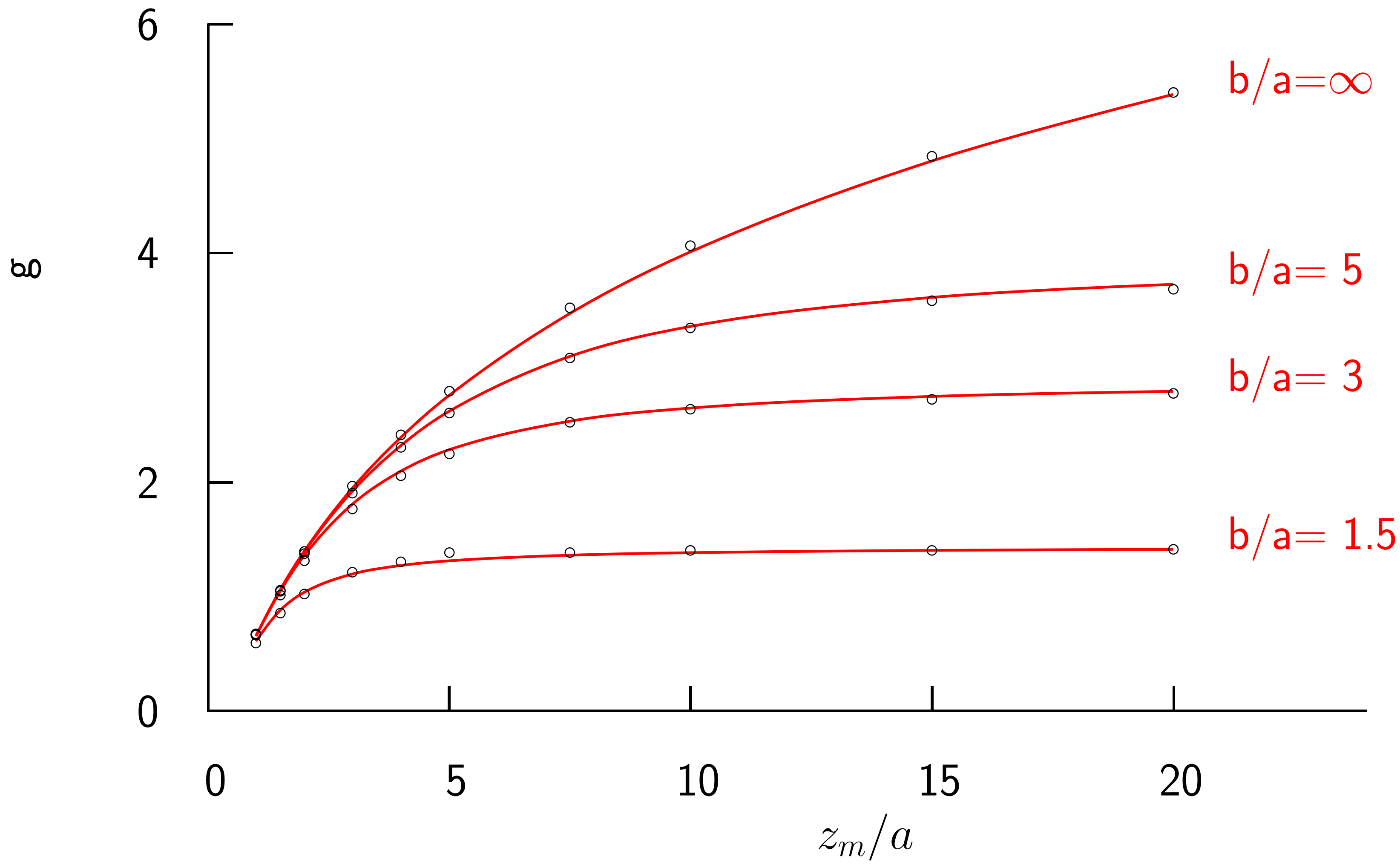
$$F_b = 0.6 + 2.1 \ln\left(\frac{b}{a}\right)$$

$$n = 1.6 + 1.5 \ln\left(\frac{b}{a}\right)$$

Using our approximation of the ICOOL Gaussian bunches to the space charge limited ellipsoidal forms:

$$\frac{z_m}{a} = \frac{\gamma \sigma_z}{\sigma_x} \qquad \frac{b}{a} = \frac{b}{1.9\ \sigma_x}$$

Numerically calculated values and the above fit are



With the assumption that the bunch is very long compared with its radial width, then, from S.Y.Lee p341 eq. 3.346[?]:

$$\mathcal{E}_{\rm sc} \ = \ \frac{e\ g_o}{4\pi\ \epsilon_o\ \gamma^2}\ \frac{d\lambda}{dz}$$

where , $\lambda = dN_/dz$ is the line beam density, $\epsilon_o = 8.8\ 10^{-12}$, and $e = 1.6\ 10^{-19}$

For a Gaussian bunch: $$\lambda \ = \ \frac{N_\mu}{\sqrt{2\pi}\ \sigma_z}\ \exp\left(-\frac{z^2}{2\ \sigma_z^2}\right)$$

$$\mathcal{E}_{\rm sc} \ = \ \frac{e\ g_o}{4\pi\ \epsilon_o\ \gamma^2}\ \frac{N_\mu\ z}{\sqrt{2\pi}\ \sigma_z^3}\ \exp\left(-\frac{z^2}{2\ \sigma_z^2}\right)$$

At z=0, where $\mathcal{E}'_{sc}$ is maximal:

$$\mathcal{E}'_{sc} \ = \ \frac{d\mathcal{E}_{\rm sc}}{dz} \ = \ \frac{e\ g_o}{4\pi\ \epsilon_o\ \gamma^2}\ \frac{N_\mu}{\sqrt{2\pi}\ \sigma_z^3} \ \approx \ 0.032\ \frac{Q\ g_o}{\epsilon_o\ \gamma^2\ \sigma_z^3} \tag{2}$$

which is about 2 times eqn. 1, reflecting the greater central line density for the Gaussian distribution.

The geometry factor g_o is here derived from a radial integration and is given as $g_o = [1 + 2\ \ln(b/a)]$ for a bunch with constant radial charge density to radius a. It is somewhat greater than the Allen determination, for long bunches, for an average gradient, of $g \ = \ F_b = [0.6 + 2.1\ \ln(b/a)]$

Example distributions

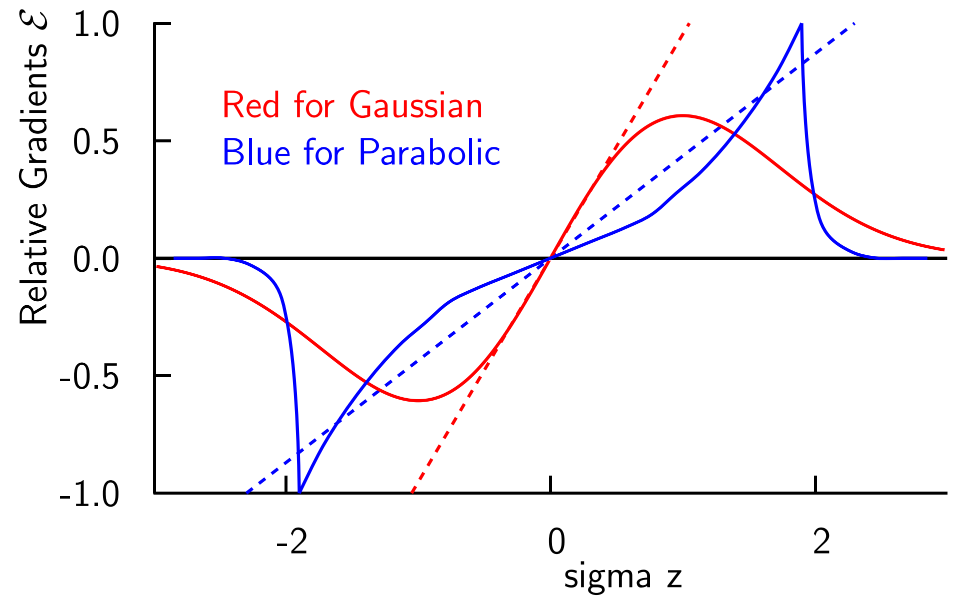
For

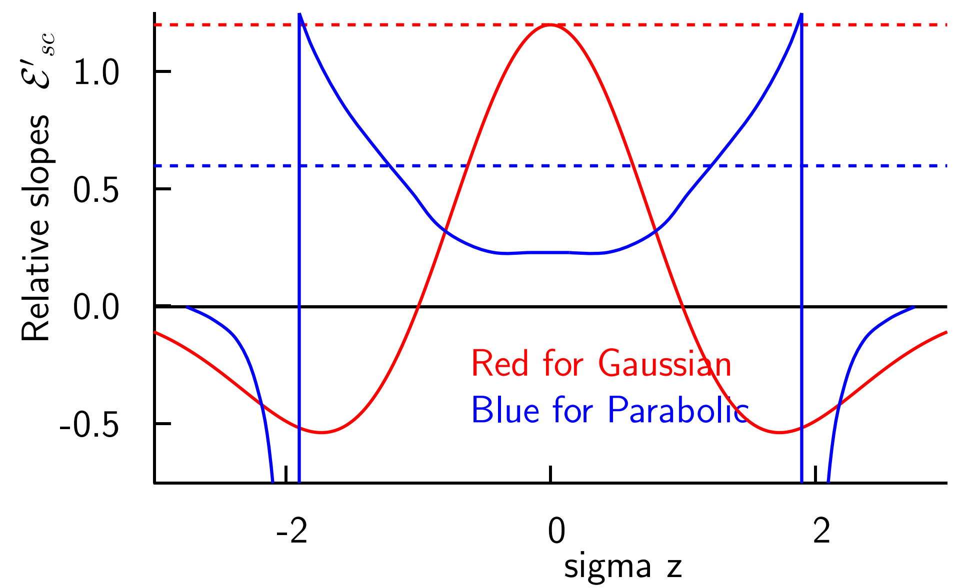
$b \;=\; 2\,a$

$z_m \;=\; 10\,a$

The central slope, and thus field gradient, for the Gaussian is more than 2 $\times$ the central slope for the parabolic,

But the average slopes of the two are similar





rf Requirement

To reach the same performance as simulated without space charge, the needed ${\mathcal{E}'}_{\rm rf}$ must equal the simulated contribution ${\mathcal{E}'}_{\rm sim}$ plus the magnitude of the space charge defocus ${\mathcal{E}'}_{\rm sc}$

$$\frac{{\mathcal{E}'}_{\rm rf}}{{\mathcal{E}'}_{\rm sim}} = 1 + \xi \quad \text{where} \qquad \xi = \frac{{\mathcal{E}'}_{\rm sc}}{{\mathcal{E}'}_{\rm sim}}$$

Assuming the bunch length small compared with the rf wavelength:

$${\mathcal{E}'}_{\rm sim} = \frac{d}{dz}\left(\mathcal{E}_o \ \sin\left(\frac{\omega z}{c}\right)\right) = \frac{\omega \ \mathcal{E}_{rf} \ \eta}{c}\cos(\phi)$$

where ϕ is the rf phase with respect to the zero crossing, ω is the rf frequency, and $\mathcal{E}_{rf}$ is the simulated rf gradient, and η is the fraction of the lattice filled with rf.

To be conservative we will use the SY Lee numbers from eq. 2

$${\mathcal{E}'}_{\rm sc} = 0.032 \, \frac{Q \ g}{\epsilon_o \ \gamma^2 \ \sigma_z^3}$$

So

$$\xi \approx 0.032 \, \frac{Q \ g \ c}{\epsilon_o \ \gamma^2 \ \sigma_z^3 \ (\omega \ \mathcal{E} \ \eta \ \cos(\phi))_{sim}} \qquad (3)$$

Space charge Gradients at the end of old 6D cooling

In the following table $\sigma_{x,y}$ is an average over the cell length and is dominated by its value away from the local focus at the absorber

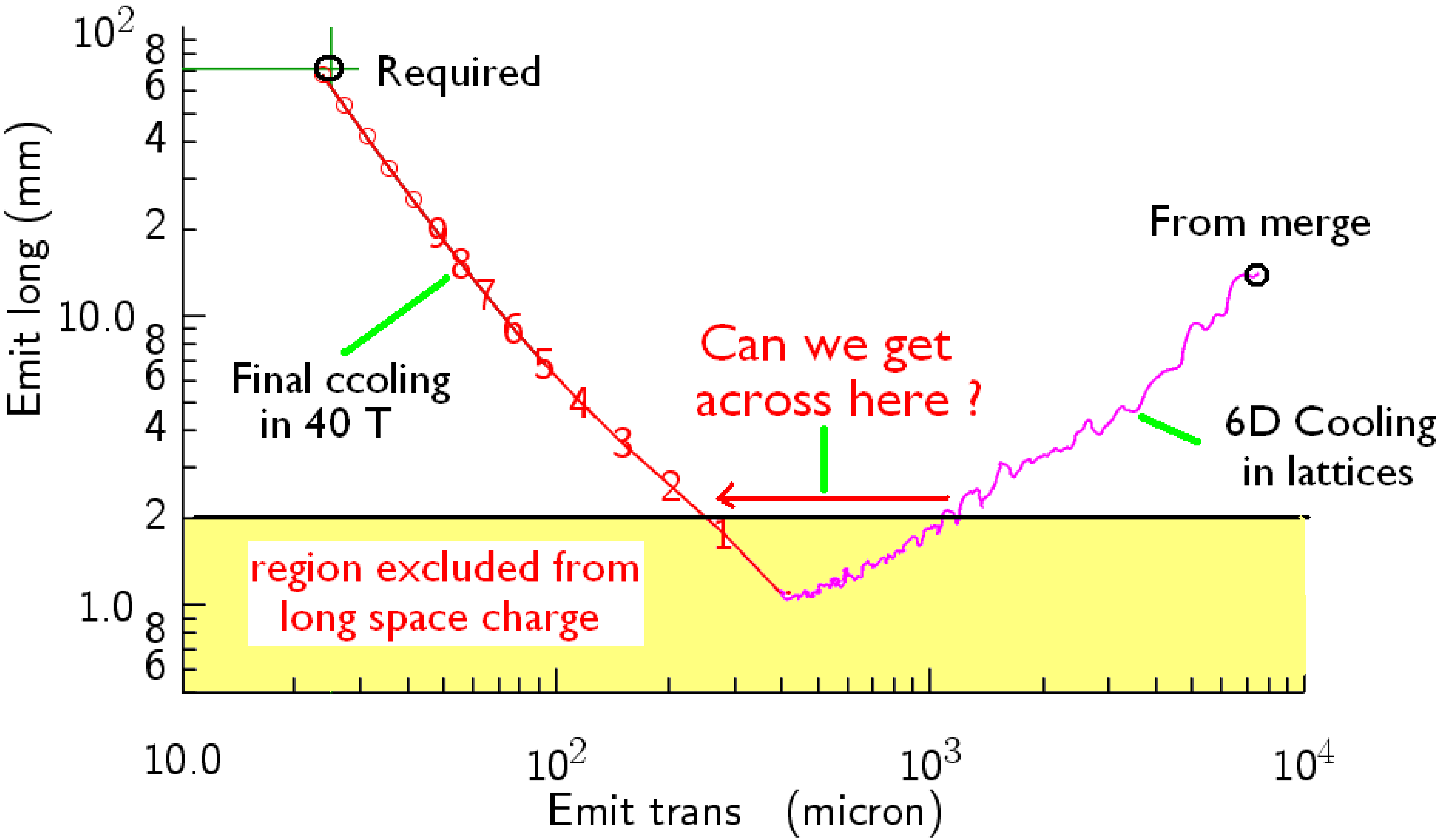
ξ is the required increase in rf gradient to conteract the space charge defocusing

	N_μ 10^{12}	mom MV/m	$\epsilon_\parallel$ mm	$\sigma_{x,y}$ mm	σ_z mm	freq MHz	$\mathcal{E}_{\rm rf}$ MV/m	η	b/a	g	$\mathcal{E}'$ MV/m^2	$\mathcal{E}'_{rf}$ MV/m^2	ξ
Current	4.81	207	1.1	12	16.6	805	20.05	0.5	3	1.75	261	155	1.68
Higher $\epsilon_\parallel$	4.81	207	2.0	12	27	805	20.05	0.5	3	2.25	70	155	0.45

Using 1.3 GHz rf with gradients increased by $\sqrt{1300/805}$ increases the focus strength by a factor of 2, but not the factor of $(1+\xi) = 2.68$ required,

- With $\xi \propto 1/\sigma_z^3$, increasing σ_z is the best bet
- Lowering the energy helps somewhat,
- T0 increase σ_z, increase the longitudinal emittance
- With the longitudinal emittance increased to 2.0 mm (from 1.1 mm) the required increase in rf is reduced to 45% which is probably doable

The new cooling challenge



Philosophy

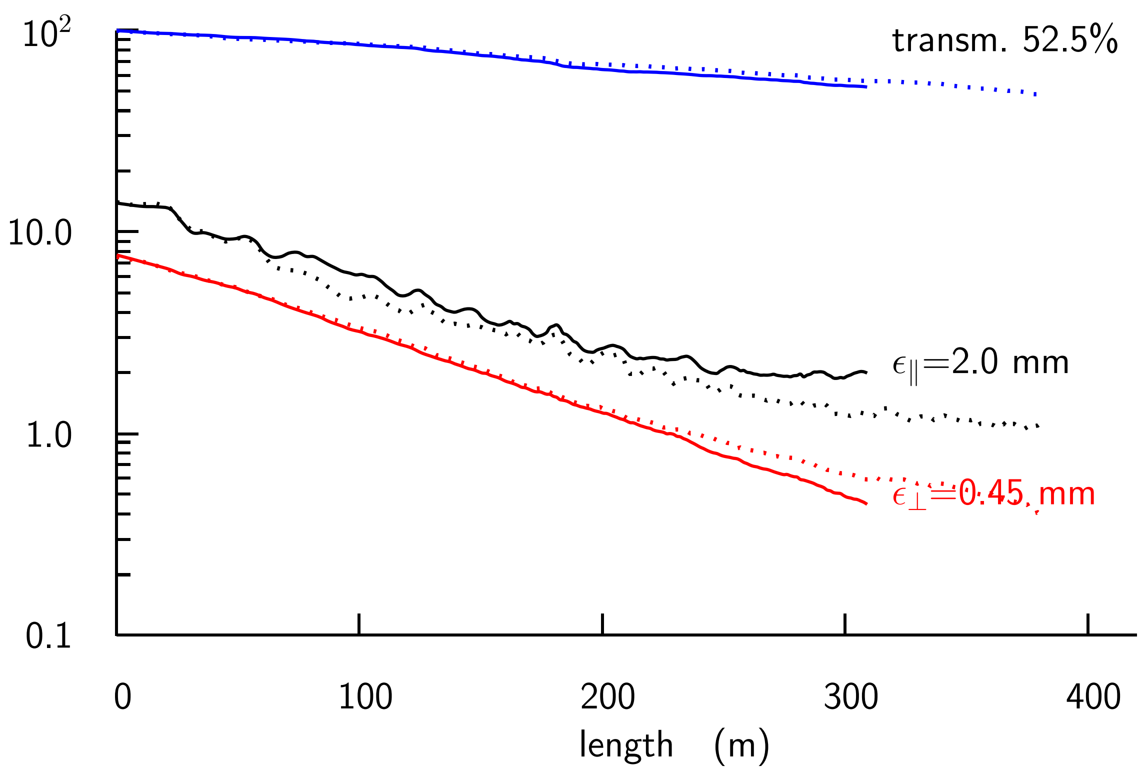
- Keep $\epsilon_{\parallel} \geq 2$ mm (vs. 1 mm) to reduce space charge
- Use only hydrogen, not LiH that is less efficient
- In tapered cooling after merge, up to last stage with hydrogen, weaken emittance exchange to keep $\epsilon_{\parallel}$ above 2 mm

- Then design special stages for cooling only in transverse direction, but with enough exchange to keep long emit constant
  - Use non-flip (Fernow) cells to lower required fields and current densities
  - Lower momentum to further lower required fields and current densities
  - Now reduce cell length, restoring fields and densities, lowering betas, and giving more absorbers per meter
  - These will later be used in new tapered sequence, but are just examples now

New 6D cooling design for after merge

- New tapered sequence (tapr15a) based on old design (tapr12f)
  - Reduce emittance exchange in all Guggenheim stages
  - End after #34 (instead of #37, eliminating LiH stages

file	45	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37
Old ϵ %	3	2.6	2.2	1.9	1.6	1.3	1.0	.88	.76	.65	.65	.65	.65	.65	.6	.6	.4
Modified	4.5	2.2	2.3	1.5	1.3	1.3	1.0	.9	.7	.6	.5	.45	.35	.25			

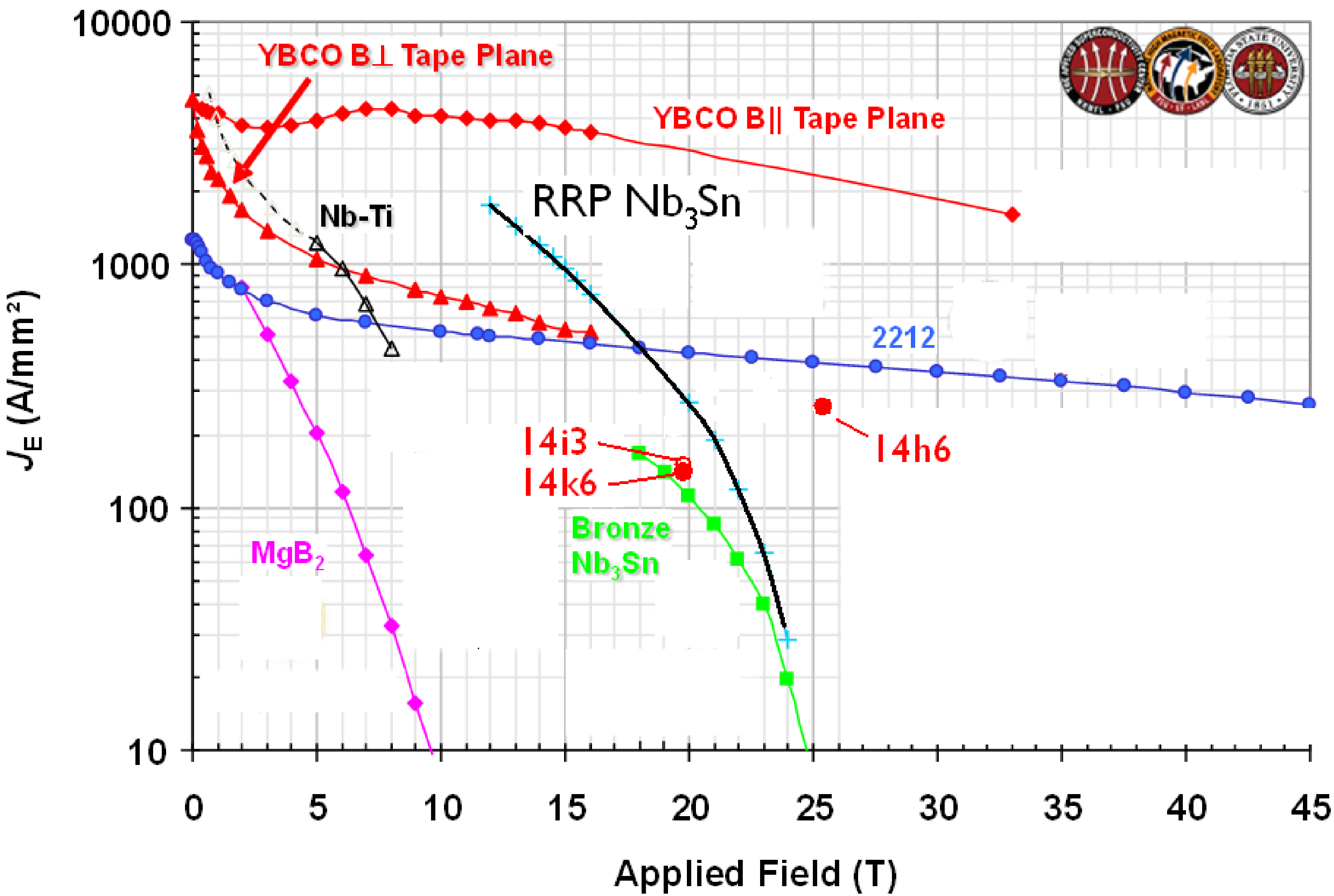


$\rightarrow$ final $\epsilon_\perp = 450(\mu m)$
cf 400 old

$\rightarrow$ final $\epsilon_\parallel = 2\ (mm)$
cf 1 old

Choice of current densities

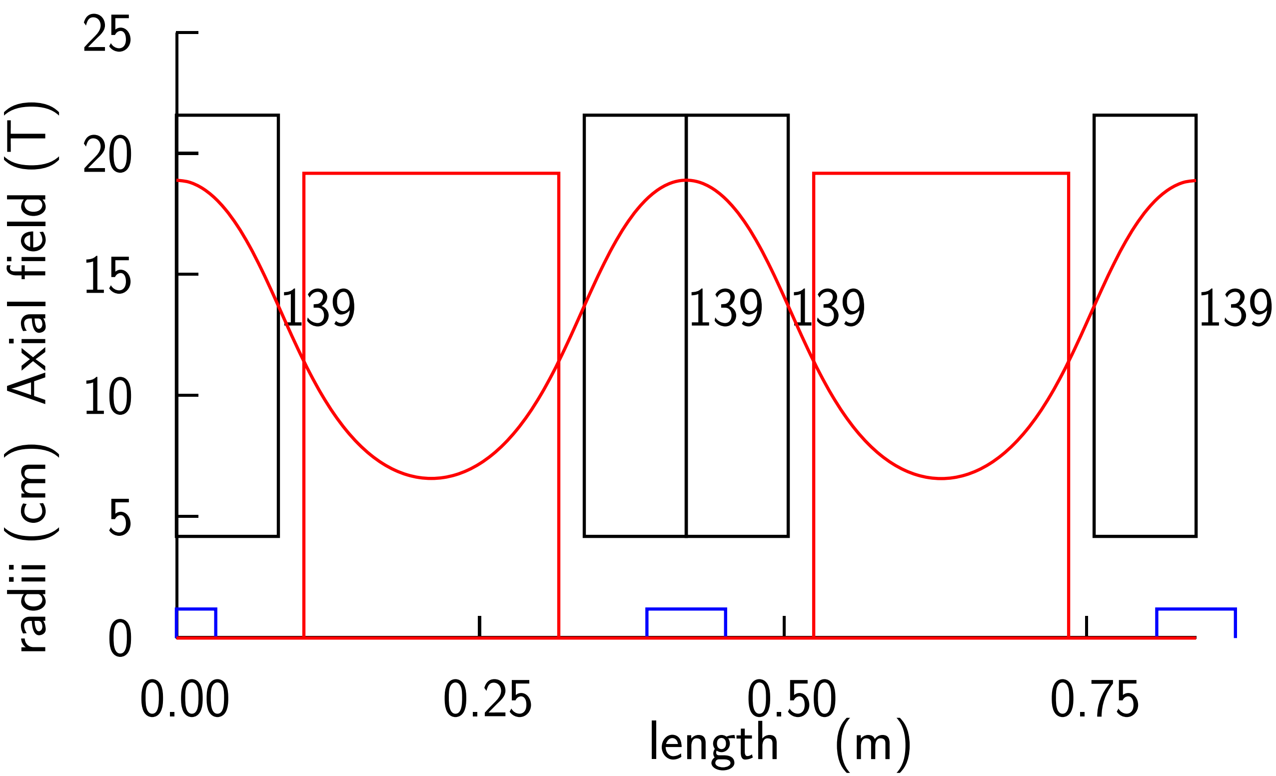
Stay below 50% of current densities of RRP Nb_3Sn

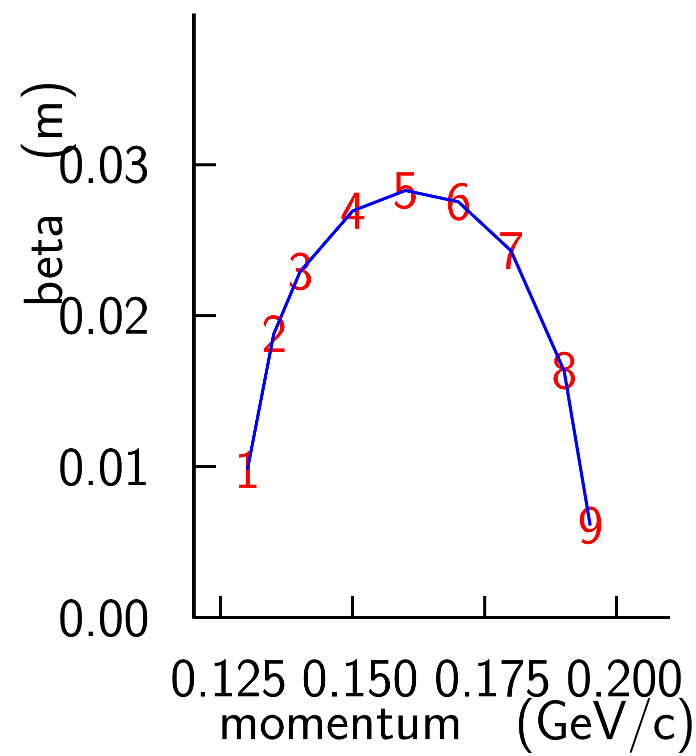


Example of cooling with constant $\epsilon_{\parallel}$

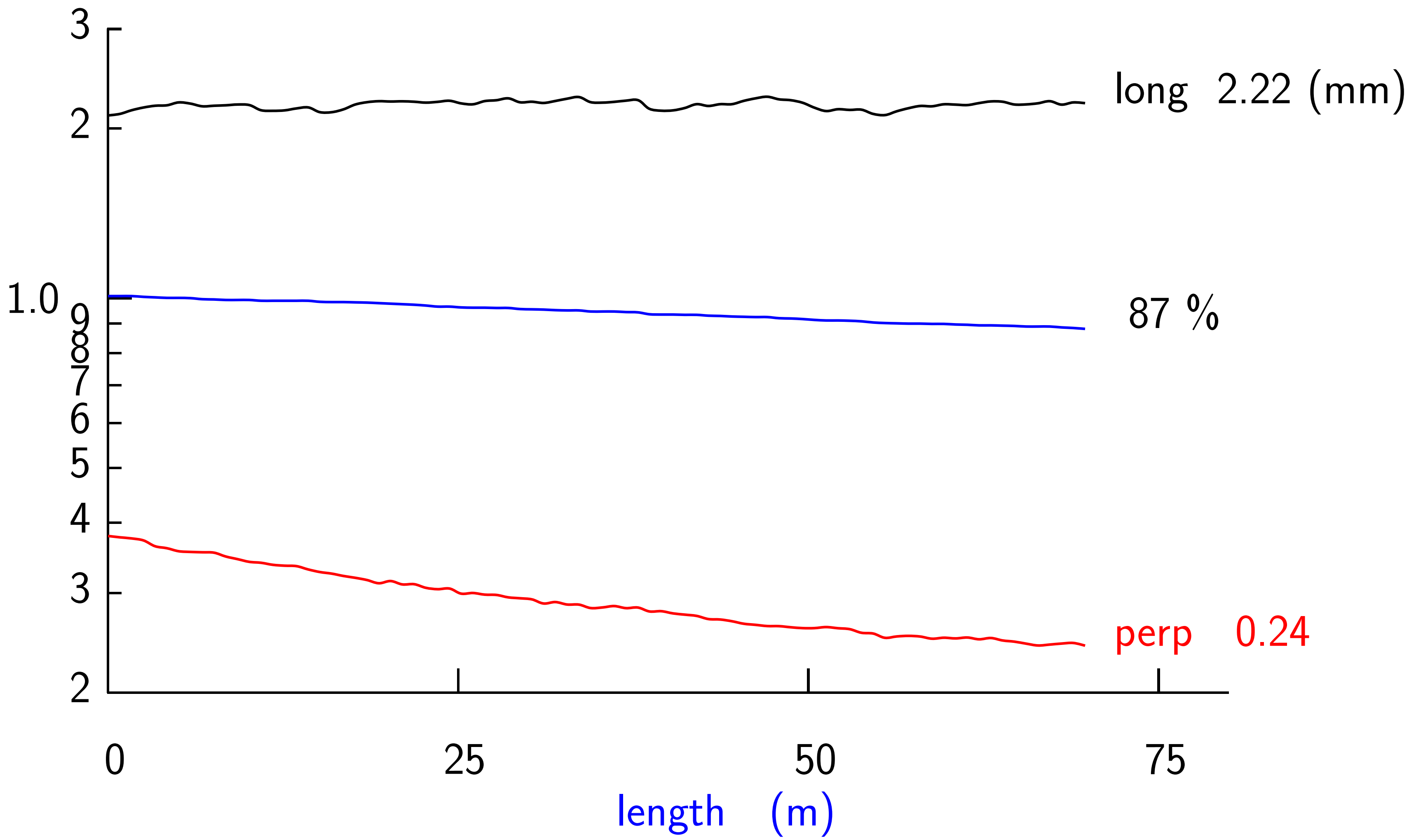
	file beta	file cool	cell cm	H_2 cm	j A/mm^2	B_o T	B_{max} T	$\beta_\perp$ cm	p_o MeV/c	$\Delta p/p$ %	r_1 cm	r_2 cm	L cm
1	nf103	tapr14e3	42	$\pm$2.7	139	18.5	19.7	3.6	160	$\pm$21	4.2	21.6	16.8

- 42 (vs. 68.75) cm cell, momentum 160 (vs. 200) MeV/c, and good $\Delta p/p$

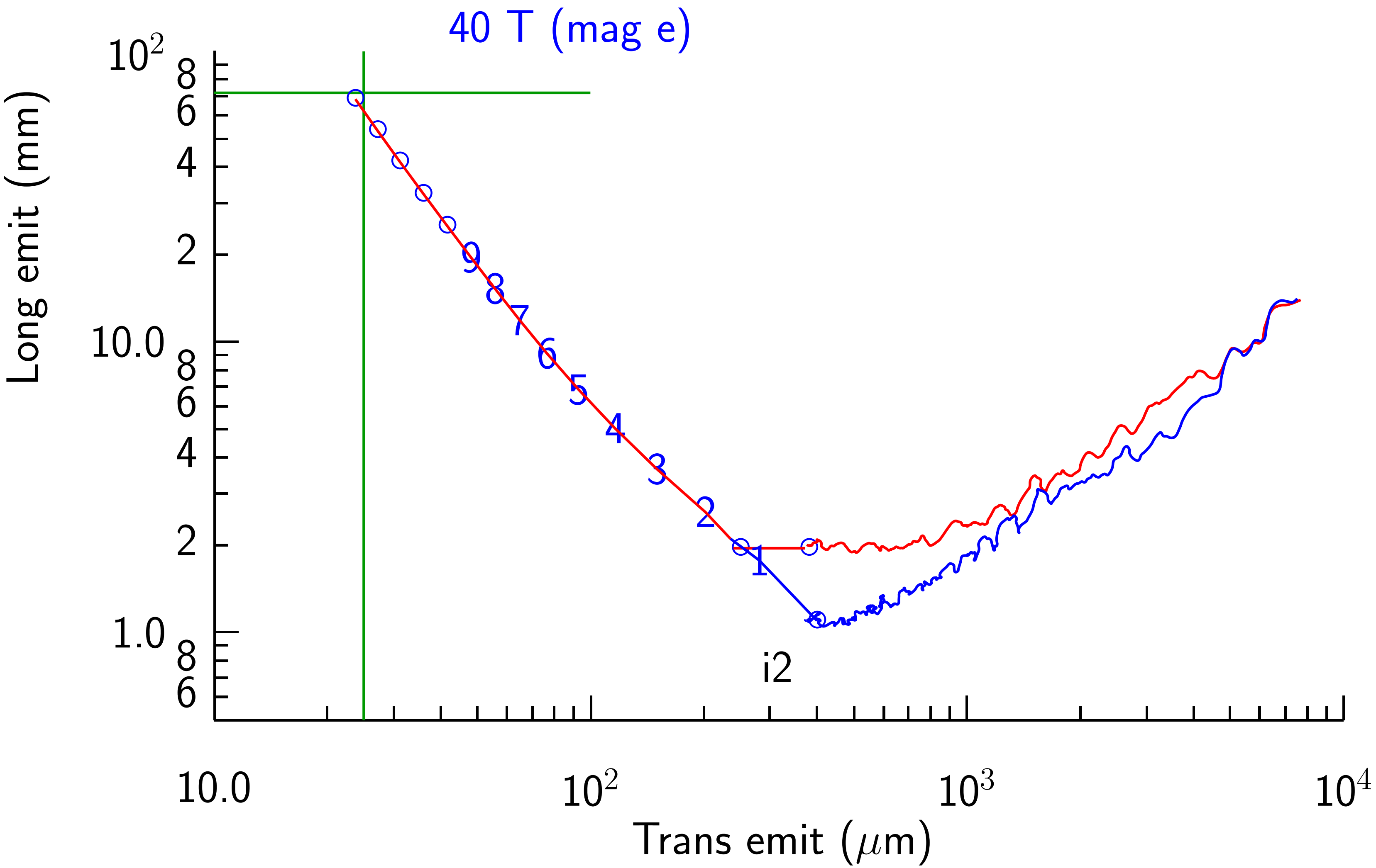




ICOOL Simulation



Emittance plot



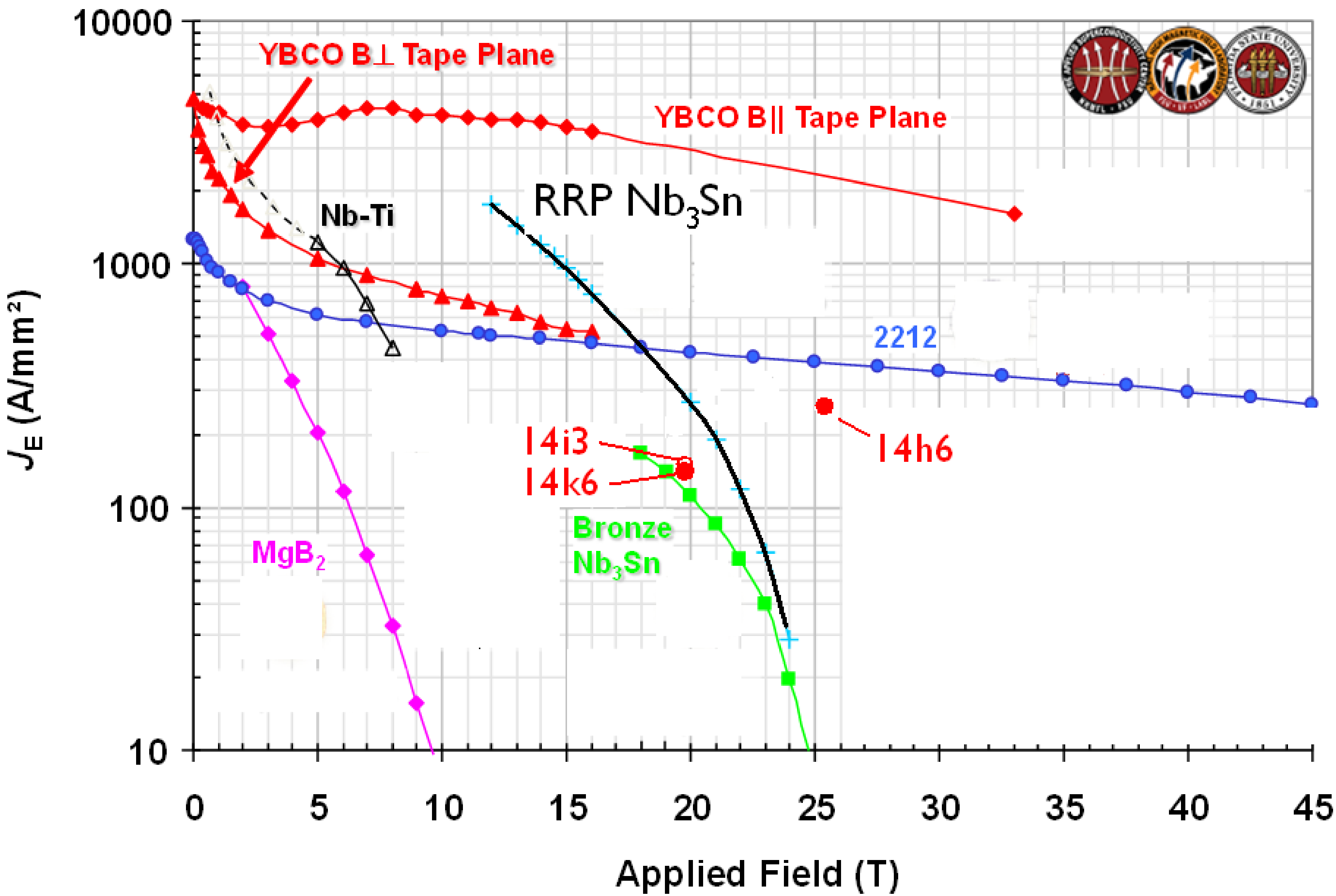
This was easy !

Can we do better ?

	file beta	file cool	cell cm	H_2 cm	j A/mm^2	B_o T	B_{max} T	$\beta_\perp$ cm	p_o MeV/c	$\Delta p/p$ %	r_1 cm	r_2 cm	L cm
1	nf103	tapr14e3	42	± 2.7	160	18.5	19.7	3.6	170	± 21	4.2	21.6	16.8
2	nf104	tapr14k6	42	± 2.0	148	19.5	19.8	1.86	150	± 17	2.4	17.4	16.8
3	nf104	tapr14h3	35	± 1.4	276	24.5	25.5	1.38	150	± 16	2.75	12.75	14

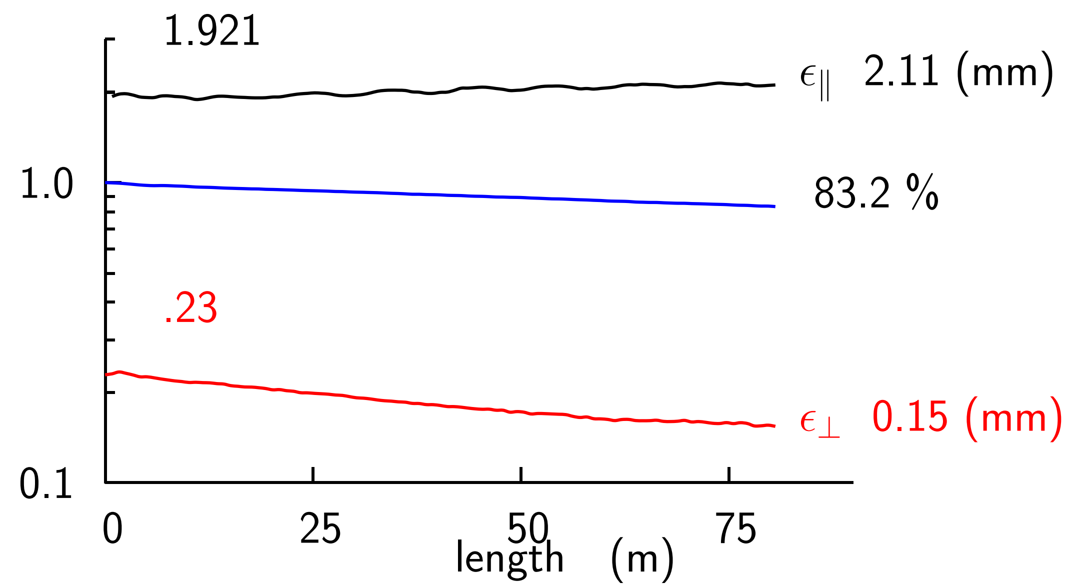
1. As discussed above
2. Modify to lower beta at expense of less $\Delta p/p$, and lower momentum
3. Use of YBCO now allows smaller cell

Include use of YBCO ?

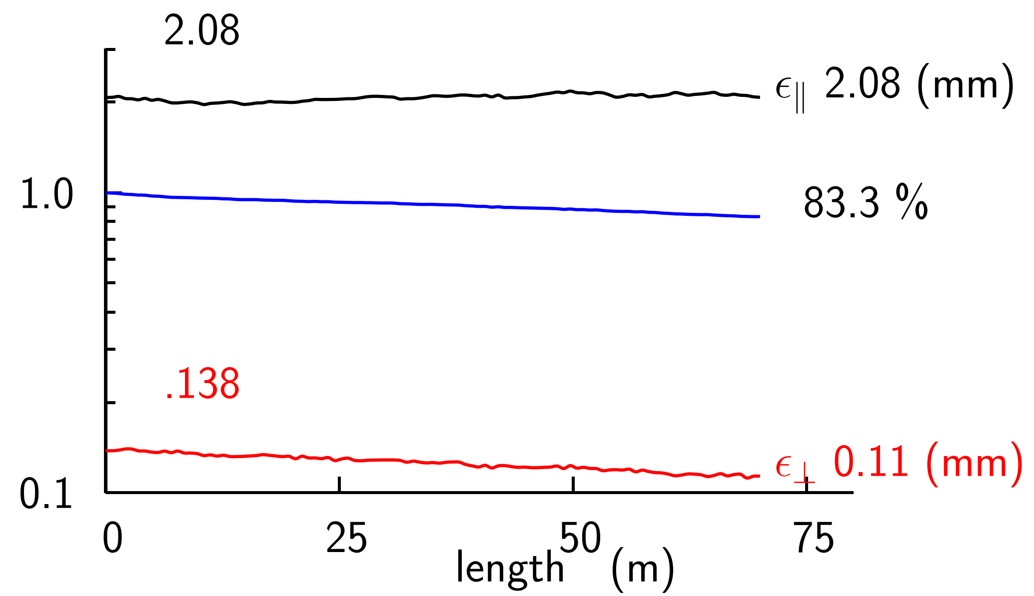


ICOOL Simulations

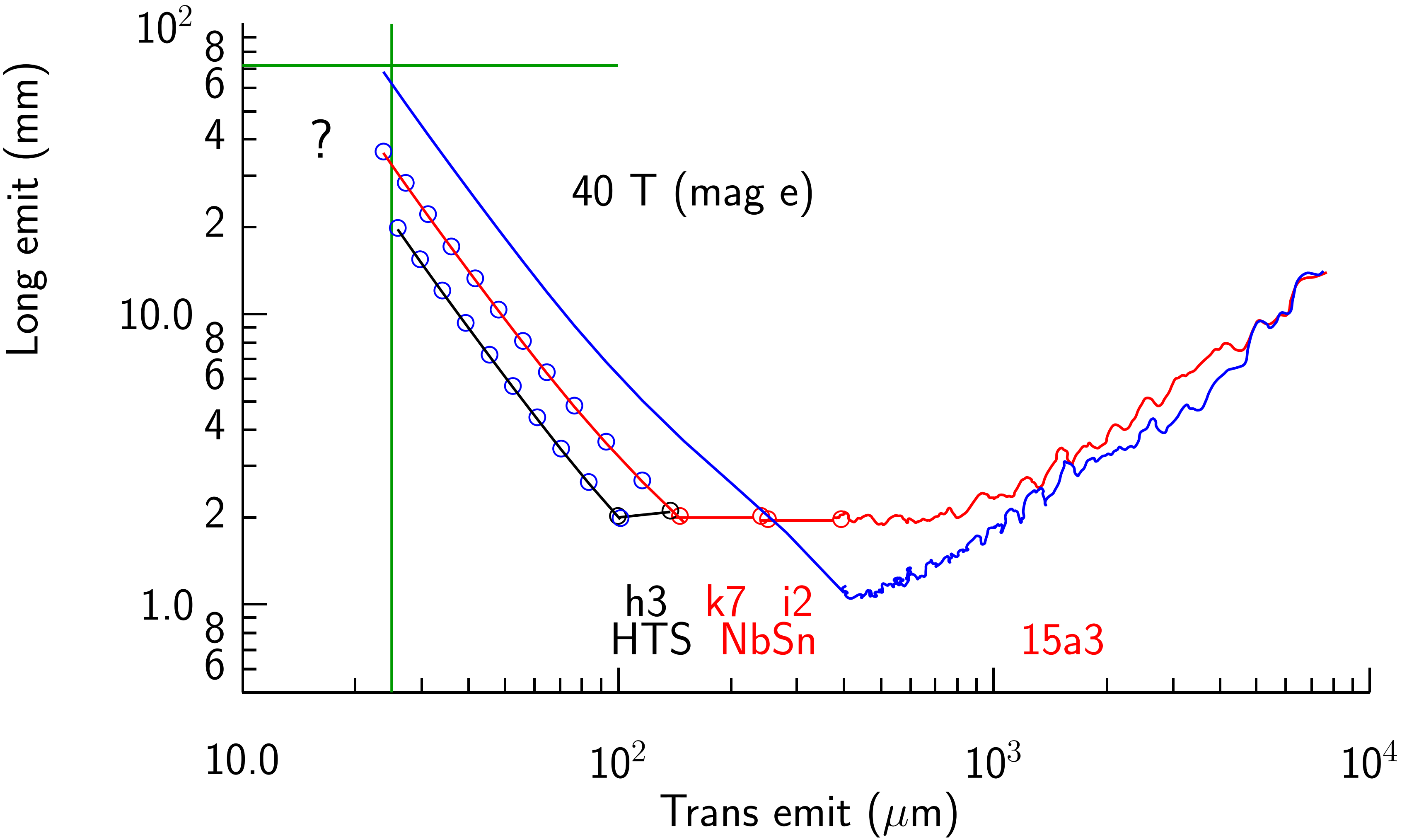
Best using Nb_3Sn



Using YBCO



Emittance plot



This is exciting   Can we get to lower transverse emittances ?

Conclusion

- Cooling in the presence of space charge will make bunches more 'parabolic'
- Allen et al have full simulations of the parabolic case

- Using these formulae the current last 6D cooling will not work
- But redesign is straightforward:
  - Reduce emittance exchange in tapered lattice,
  - Adding a now lattice with non-flip fields

- Adding more stages of non-flip lattices can further lower trans emittance $\rightarrow$ possibility of lowering final emittances

- This is all preliminary

- Need to study final emittance exchange below 25 μm