Muon g-2 and EDM theory motivation

Dominik Stöckinger, TU Dresden

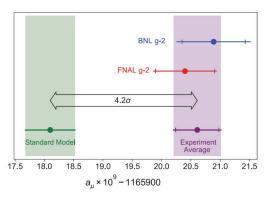
Workshop: Potential Fermilab Muon Campus & Storage Ring Experiments, 24th May 2021

- Overview and SM theory
- 2g-2 and BSM important general remarks
- 3 Comparisons a_{μ} and other dipole observables
- General lessons and conclusions

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Finally: Fermilab Run 1 versus Theory Initiative SM value

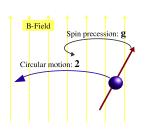


Limits on EDMs:
$$|d_e|<8.7\times10^{-29}e\,{\rm cm}$$

$$|d_\mu|<1.5\times10^{-19}e\,{\rm cm}$$
 Limit on $\mu\to e\gamma$:
$$BR(\mu\to e\gamma)<4.2\times10^{-13}$$

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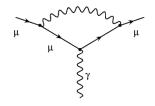
The landscape of dipole observables



$$H_{\mathrm{eff}} = -2(1+a_{\mu})\frac{e}{2m_{\mu}}\vec{B}\cdot\vec{S} - 2d_{\mu}\vec{E}\cdot\vec{S}$$

EDM is T, P, CP-violating!

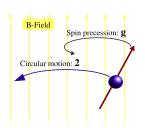
$$a_{\mu} = -2m_{\mu} \operatorname{Re}(c), \quad d_{\mu} = -e \operatorname{Im}(c), \quad BR_{(\mu \to e\gamma)} \approx \sim |c^{\mu e}|^2$$



$$\sim ar{u}(p') \Big[\gamma_{\mu} F_1 + rac{i}{2m} \sigma_{\mu
u} q^{
u} (\operatorname{Re}(c) - i \gamma_5 \operatorname{Im}(c)) \Big] u(p)$$

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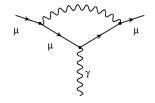
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Note: $\mu o e$ conversion and e.g. neutron-EDM are not only given by dipole operators but might be "dipole-dominated"

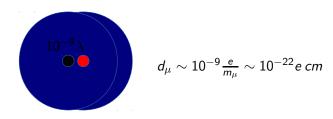


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Units and first expectations: a_{μ} , d_{μ} and $\mu \rightarrow e \gamma$ are dipole observables

$$a_{\mu} = -2m_{\mu} \operatorname{Re}(c), \quad d_{\mu} = -e \operatorname{Im}(c), \quad BR_{(\mu \to e\gamma)} \approx \sim |c^{\mu e}|^2$$

- ullet suppose $a_{\mu}\sim 10^{-9}$, and also ${
 m Im}(c)/{
 m Re}(c)= an\phi\sim 1$
- expect



Compton wavelength λ

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SM contributions

• All sectors of the SM contribute to a_{μ} : $a_{\mu}^{\text{QED}} \sim 10^{-3}$ $a_{\mu}^{\text{HVP}} \sim 10^{-3}$











• Electron/muon EDM in SM needs quarks of all generations ($\leadsto \approx 0$)!

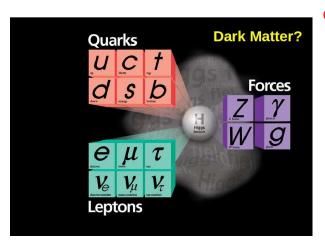




[Pospelov, Ritz '13] 4-, 5-loop diagrams $d_e \sim 10^{-44} e \ cm$ (10⁻³⁴ × Compton!)

• Similarly: CLFV essentially zero in SM (even with neutrino masses) $(\mu \to e\gamma, \ \mu \to e$ conversion etc)

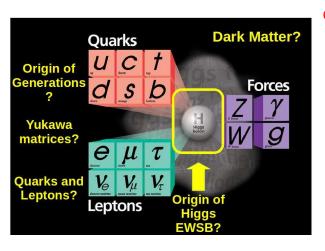
Open questions require Beyond the Standard Model (BSM) physics



Open questions!

- experimental clues needed! $\rightsquigarrow g-2!$
- relevant and deep questions may be related to g-2
- Origin of baryon-antibaryon asymmetry?
 → FDMs!

Open questions require Beyond the Standard Model (BSM) physics



Open questions!

- experimental clues needed! $\rightsquigarrow g-2!$ not easy to explain!
- relevant and deep questions may be related to g-2
- Origin of baryon-antibaryon asymmetry?
 FDMs!

Two important general points on a_{μ}

SM prediction too low by
$$\approx (25 \pm 6) \times 10^{-10}$$

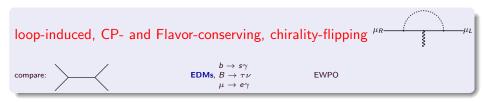
$$\begin{array}{c} \text{discrepancy} \approx 2 \times \textit{a}_{\mu}^{\text{SM,weak}} \\ \text{but: expect } \textit{a}_{\mu}^{\text{NP}} \sim \textit{a}_{\mu}^{\text{SM,weak}} \times \left(\frac{\textit{M}_{W}}{\textit{M}_{\text{NP}}}\right)^{2} \times \text{couplings} \end{array}$$

 Many experiments needed to investigate CPV and flavor structure of potential new physics

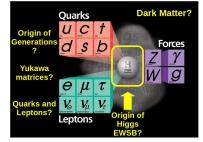
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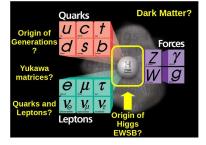
Window to muon mass generation mechanism?

Dark Matter? Hard to see in detectors

but could couple to muon → large effects possible!

many examples, but within simple models; need at least three new fields

generally: dark matter direct detection constraints important!



Window to muon mass generation mechanism? allows significant chiral enhancements,

but such models are constrained by collider, flavour etc

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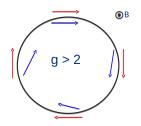
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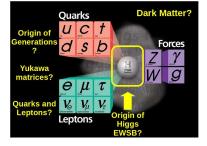
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Window to the muon mass generation mechanism (Higgs/Yukawa sectors)

(continuous spin rotation requires rest mass!)





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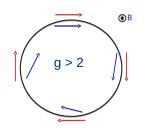
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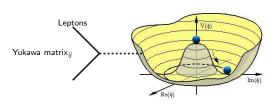
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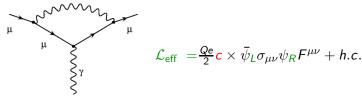
(continuous spin rotation requires rest mass!)





(changed by new physics?)

Connection to chirality flip, and structure of BSM



But:

EW gauge invariant a_{μ} -operator:

$$\bar{L}\sigma_{\mu\nu}\mu_R F^{\mu\nu}\langle H \rangle$$

$$a_{\mu} \sim m_{\mu} \times \underbrace{(\text{some VEV}) \times (\mu_{L \leftrightarrow R}\text{-flipping param.})}_{\text{potential enhancement! Often } \propto m_{\mu}} \times \underbrace{(\text{other couplings})}_{\text{typical}} \times \frac{(\text{other couplings})}{M_{\text{typical}}^2}$$

 $m_{\mu}(SM) \sim (SM \text{ Higgs-VEV}) \times (\text{muon Yukawa coupling})$

Typical behaviour: \sim chirality flip (\rightsquigarrow Higgs!) and masses

• EWSM:
$$\alpha \frac{m_{\mu}^2}{M_W^2}$$

$$\begin{array}{c|c} \times \langle H \rangle & \nu_L \\ \hline \mu_R & \mu_L & W & \mu_L \end{array}$$

• LQ:

$$g_Lg_R \frac{m_\mu m_t}{M_{LQ}^2}$$



• 2HDM:

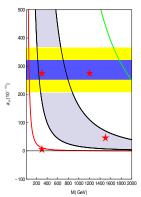
$$lpha^2 an^2 eta rac{m_\mu^2}{M_H^2}$$



• SUSY:

$$\alpha \frac{m_{\mu}^2 \tan \beta}{M_{\text{SUSY}}^2} \frac{\mu}{M_{\text{SUSY}}}$$

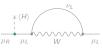




Details: see backup slides [Athron,Balazs,Jacob,Kotlarski, DS,Stöckinger-Kim, 2104.03691]

Typical behaviour: \sim chirality flip (\rightsquigarrow Higgs!) and masses

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• LQ: $g_Lg_R \frac{m_\mu m_t}{M_{LQ}^2}$

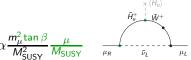


Couplings \sim new flavor structure, formally $\propto m_{tt} m_t$

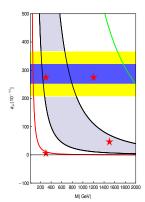
• 2HDM: $\alpha^2 \tan^2 \beta \frac{m_{\mu}^2}{M_H^2}$



Couplings \sim new flavor structure or "minimal flavor violating"



New flavor-independent and new flavor-dependent contributions



Details: see backup slides [Athron,Balazs,Jacob,Kotlarski, DS,Stöckinger-Kim, 2104.03691]

SUSY:

Relations and estimates

also: [Giudice, Paradisi, Passera 2012] [Crivellin, Hoferichter, Schmidt-Wellenburg 2018]

Can unify description of MDM, EDM, CLFV: generalize $\mathcal{L}_{\text{eff}} \sim c^{ij}$ to leptons i, j with coefficients c^{ij} , $c^{ij} \propto \text{VEV} \times \text{chir.-flip}$:

$$egin{aligned} a_{\mu} &= -2m_{\mu} \mathrm{Re}(c^{\mu\mu}) \ a_{\ell} &= -2m_{\ell} \mathrm{Re}(c^{\ell\ell}) \ d_{\ell} &= -e \, \mathrm{Im}(c^{\ell\ell}) \end{aligned}$$
 $BR(\mu
ightarrow e \gamma) = rac{e^2 m_{\mu}^3}{\pi \Gamma_{\mu}} (|c^{\mu e}|^2 + |c^{e\mu}|^2)$

Relations and estimates

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$$\begin{split} d_{\mu} &\approx \left(\frac{\Delta a_{\mu}}{3\times 10^{-9}}\right) 2\times 10^{-22} \mathrm{e\,cm} \times \tan\phi_{\mu},\\ d_{e} &\approx \left(\frac{\Delta a_{e}}{7\times 10^{-14}}\right) 10^{-24} \mathrm{e\,cm} \times \tan\phi_{e},\\ BR(\mu \to e\gamma) &\approx \left(\frac{\Delta a_{\mu}}{3\times 10^{-9}}\right)^{2} 2\times 10^{-13} \left(\frac{\theta_{\mu e}}{10^{-5}}\right)^{2}, \end{split}$$

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- SM: $\tan \phi \ll 1$, $\theta_{\mu e} \ll 1$.
- Current EDM, MEG limits: $\tan \phi_{\mu} \lesssim 1000$, $(\tan \phi_{e} \ll 1 \text{ or } \Delta a_{e} \ll 10^{-14})$, $\theta_{\mu e} \lesssim 10^{-5}$
- New physics strongly restricted in $\theta_{\mu e}$ and $\tan \phi_e$ but not in $\tan \phi_{\mu} \rightsquigarrow \text{improve}!$
- Naive scaling $c^{\ell\ell} = m_{\ell} \times \text{const.}$:

$$\Delta a_e:\Delta a_\mu=m_e^2:m_\mu^2,\quad d_e:d_\mu=m_e:m_\mu\ \stackrel{\mathsf{Exp.}}{\Rightarrow}\ |d_\mu^\mathsf{naive\ sc.}|\lesssim 10^{-27}$$

- New physics possibilities: new flavor structures (LQ, sleptons, 2HDM-Yukawas), new flavor-independent parameters (complex Higgsino mass, gaugino masses)
- Note: neutron EDM and $\mu \rightarrow e$ conversion sensitive to non-dipole operators! Note 2: naive scaling is different from writing $a_{\mu} = C_{\text{BSM}} \frac{m_{\mu}^{c}}{M^{2}}$ \leftrightarrow $c^{\mu\mu}_{\text{A}} \sim m_{\mu} \times c^{\text{dimensionless BSM-couplings}}_{\text{BSM}} = 0$

Summary of main points

discrepancy
$$\approx 2 \times a_{\mu}^{\rm SM,weak}$$
 but: expect $a_{\mu}^{\rm NP} \sim a_{\mu}^{\rm SM,weak} \times \left(\frac{M_W}{M_{\rm NP}}\right)^2 \times {\rm couplings}$ a_{μ} is loop-induced, CP- and flavor-conserving and chirality-flipping

Which models can still accommodate large deviation?

Many (but not all) models!
→ Connection to dark matter? Window to muon mass generation?

but always: experimental constraints!

Questions for a_{μ} versus d_{μ} (and a_{e} and $\mu \rightarrow e(\gamma)$):

- Is there new physics in a_{μ} ? Dark matter/new Yukawas/Higgs?
- Is there more flavor structure beyond SM Yukawas? (LQ, SUSY, 2HDM, ...)
- Is CPV connected to flavor/generations? (SUSY Higgsino/gaugino masses vs new Yukawas)
- Are there $\mathcal{O}(1)$ sources of CPV (in the muon sector)? $d_{\mu}^{\Delta a_{\mu},\mathcal{O}(1)} \sim 10^{-22} \, \mathrm{e\,cm}$
- ullet Does naive scaling hold (exactly/approximately) for a_ℓ or for d_ℓ ? $d_\mu^{
 m naive\ sc.}\lesssim 10^{-27}{
 m e\,cm}$

Looking forward to experimental programme. very promising future! $_{\sim \infty}$

There are many more examples. . .

SUSY: MSSM, MRSSM

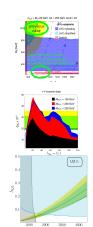
- MSugra...many other generic scenarios
- Bino-dark matter+some coannihil.+mass splittings
- Wino-LSP+specific mass patterns

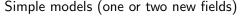
Two-Higgs doublet model

• Type I, II, Y, Type X(lepton-specific), flavour-aligned

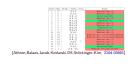
Lepto-quarks, vector-like leptons

ullet scenarios with muon-specific couplings to μ_L and μ_R





- Mostly excluded
- light N.P. (ALPs, Dark Photon, Light $L_{\mu}-L_{\tau}$)



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Example BSM idea

- fundamental new QFT symmetry
- predicts Higgs potential/mass
- dark matter candidate
- ullet chirality flip enhancement $\leadsto g-2$
- viable (LHC)?

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Example BSM idea Minimal SUSY Standard Model

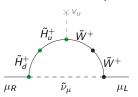
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Superpartners and SUSY Higgs sector $\leadsto an \beta = rac{v_u}{v_d},$ Higgsino mass μ

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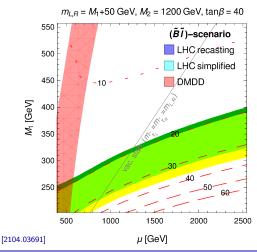
MSSM can explain g-2 and dark matter



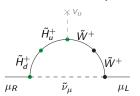
$$a_{\mu}^{
m SUSY} pprox 25 imes 10^{-10} \; rac{ aneta}{50} \; rac{\mu}{M_{
m SUSY}} \left(rac{500 {
m GeV}}{M_{
m SUSY}}
ight)^2$$

- "Dark matter mass" versus μ
- explains g-2 in large region (expands for $\tan \beta \neq 40$)
- DM explained by stau/slepton-coannihilation
- this automatically evades (current) LHC limits





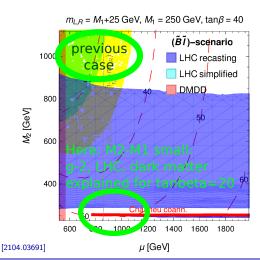
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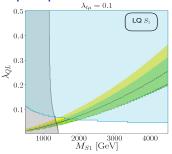
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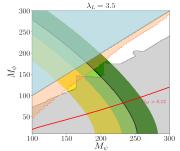
- Strong LHC limits on M₂
- DM also explained by Wino-coannihilation
- again evades (current) LHC limits





Leptoquarks and Model L with 2 fields $_{0.5}$ $_{-0.1}$ $_{-0.1}$

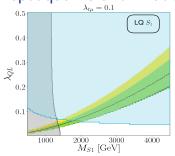


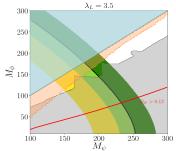


[Athron, Balazs, Jacob, Kotlarski, DS, Stöckinger-Kim, 2104.03691]

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Leptoquarks and Model L with 2 fields

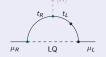




[Athron, Balazs, Jacob, Kotlarski, DS, Stöckinger-Kim, 2104.03691]

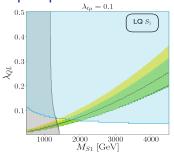
$$a_{\mu}$$
 from LQ (or VLL) $\mathcal{L}_{S_1} = -\left(\lambda_{QL}Q_3 \cdot L_2S_1 + \lambda_{t\mu}t\mu S_1^*\right)$

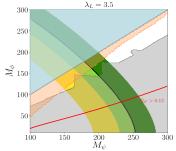
Specific LQ that works:



- Chiral enhancement $\sim y_{\mathsf{top}}, y_{\mathsf{VLL}}$ versus y_{μ}
- LHC: lower mass limits
- Flavour constraints →
 assume only couplings to muons
- Viable window above LHC (without m_{μ} -finetuning)

Leptoquarks and Model L with 2 fields





[Athron, Balazs, Jacob, Kotlarski, DS, Stöckinger-Kim, 2104.03691]

au from 2-field model L

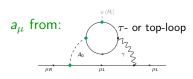
- No chiral enhancement, need very large couplings
- LHC: lower mass limits
- Dark matter candidate, but incompatible with large a_{μ} General result: a_{μ} and DM require at least three new fields!



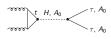
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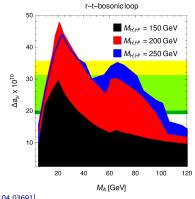
BSM with smaller masses, hidden from colliders?

Aligned 2-Higgs doublet model, rich new Higgs/Yukawa sectors



LHC constraints:





[2104.03691]

- can explain g-2
- need large new Yukawa couplings
- under pressure, testable at LHC, lepton colliders, B-physics

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Two important general points

$$\begin{array}{c} {\rm discrepancy} \approx 2 \times \textit{a}_{\mu}^{\rm SM, weak} \\ {\rm but: \ expect \ } \textit{a}_{\mu}^{\rm NP} \sim \textit{a}_{\mu}^{\rm SM, weak} \times \left(\frac{\textit{M}_{W}}{\textit{M}_{\rm NP}}\right)^{2} \times {\rm couplings} \end{array}$$



Questions: Which models can(not) explain it?

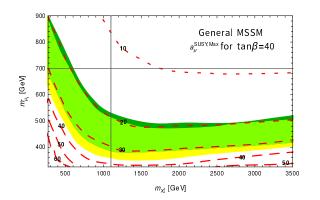
Why is a single number so interesting?

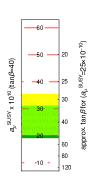
"Why are you happy about a discrepancy?"

 \Rightarrow we might make significant progress!

Full MSSM overview in 7 plots

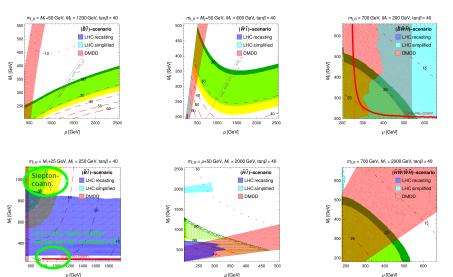
[Peter Athron, Csaba Balasz, Douglas Jacob, Wojciech Kotlarski, DS, Hyejung Stöckinger-Kim, 2104.03691]





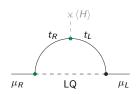
Full MSSM overview in 7 plots

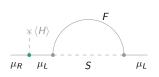
[Peter Athron, Csaba Balasz, Douglas Jacob, Wojciech Kotlarski, DS, Hyejung Stöckinger-Kim, 2104.03691]



Summary: Bino-LSP: a_{μ} and DM. Wino-/Higgsino-LSP: a_{μ} . Both cha<slepton: pproxdisfavoured.

One-field, two-field models (renormalizable, spin 0, 1/2)





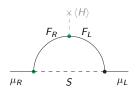
- many models: excluded
- very special models: chiral enhancement specific leptoquarks, specific 2HDM versions
- however, no dark matter



- even more models: excluded
- no chirality flip
- few models: either a^{BNL}_{II} or dark matter



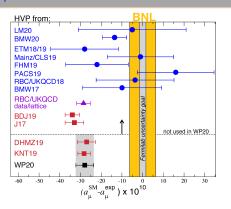
Three-field models



- many models: viable, large chirality enhancements
- \bullet can explain ${\it a}_{\mu}^{\rm BNL}$ and LHC and dark matter

Details on hadronic vacuum polarization

Status of Hadronic Vacuum Polarisation contributions



Lattice QCD + QED

- impressive progress, but...
- large spread between results
- tensions when looking at 'Euclidean
- time window' comparisons
- · large systematic uncertainties (e.g. from non-trivial extrapolation to continuum limit, finite size)

Dispersive/lattice hybrid

('window' method)

For WP20: Dispersive data-driven from DHMZ and KNT

TI White Paper 2020 value:

$$a_{\mu}^{HVP} = 6845 (40) \times 10^{-11}$$

- TI WP2020 prediction uses dispersive data-driven evaluations with minimal model dependence
- a,,HVP value and error obtained by merging procedure accounts for tensions in input data and differences in data treatment & combination (going beyond usual χ^2_{min} inflation) Thomas Teubner

