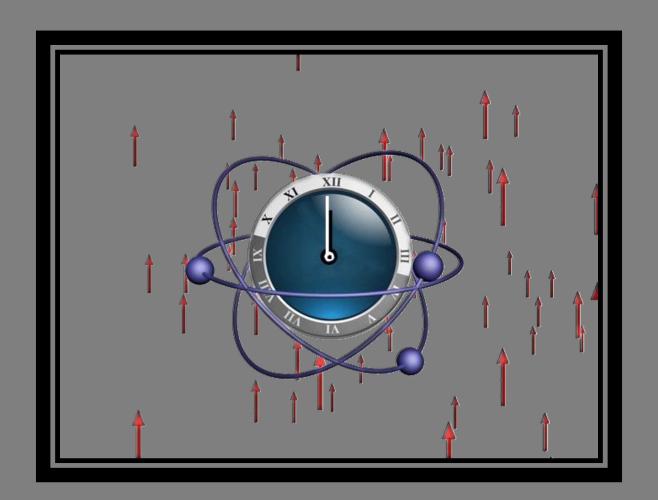
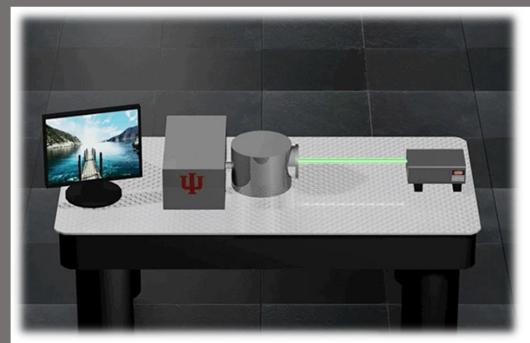
Lorentz and CPT Violation in Muon g-2 Experiments Arnaldo J. Vargas Loyola University New Orleans

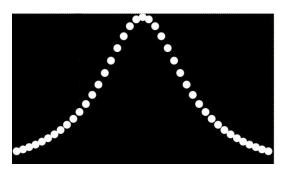


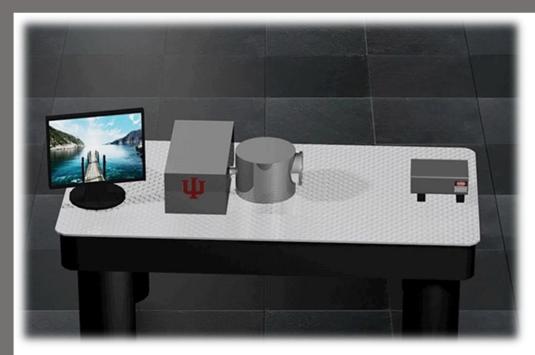




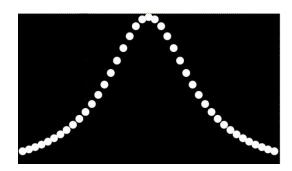


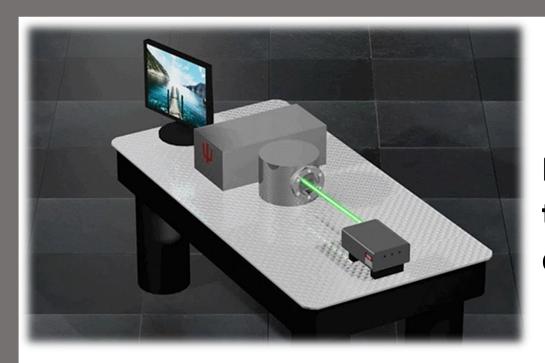
Original inertial reference frame



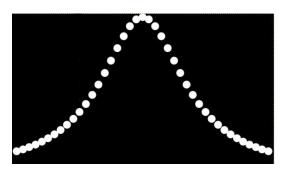


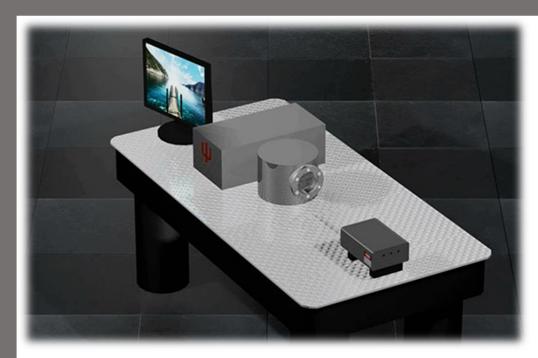
Original inertial reference frame



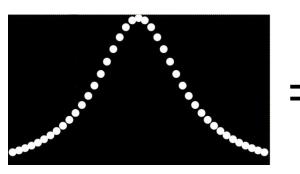


Original inertial reference frame

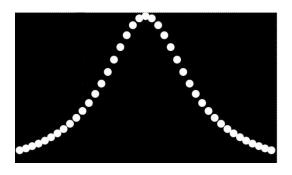


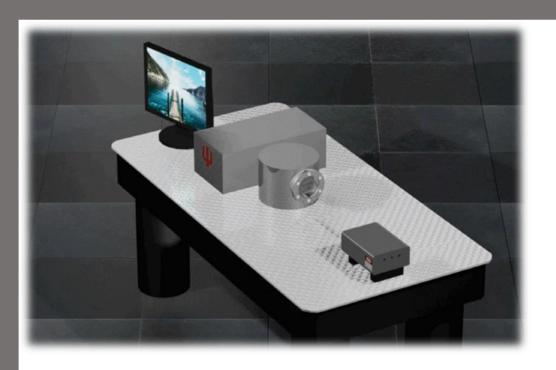


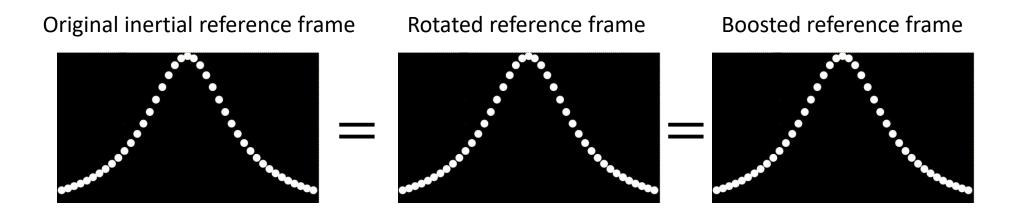
Original inertial reference frame



Rotated reference frame







The SME Lagrangian

Colladay and Kostelecký, PRD **55**, 6760 (1997) Colladay and Kostelecký, PRD **58**, 116002 (1998) Kostelecký, PRD **69**, 105009 (2004)

$$\mathcal{L}_{SME} = \mathcal{L}_{SM} + \mathcal{L}_{GR} + \mathcal{L}_{LV} \leftarrow \text{Lorentz violation}$$
Conventional physics

Facilitates the systematic test of Lorentz and CPT symmetry

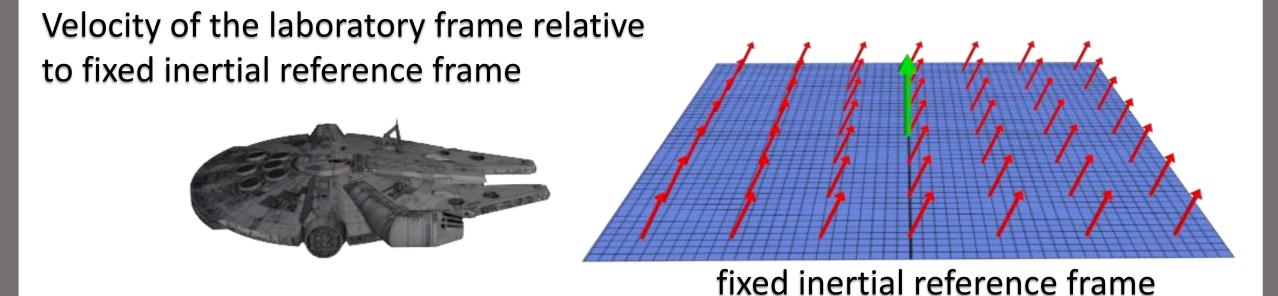
- ❖ Models for Lorentz violation applicable to diverse physical scenarios
- Compare and classify tests of Lorentz symmetry
- Predicts signals for Lorentz and CPT violation

Lorentz violation can be represented as constant uniform background fields that permeate Minkowski spacetime

$$\mathcal{L}\supset k^{\mu}\mathcal{O}_{\mu}(x)$$

 $m{k}^{\mu}$ Lorentz-violating background field \mathcal{O}^{μ} field operator

Lorentz violation happens when the experimental results depend on the spacetime orientation of the laboratory frame



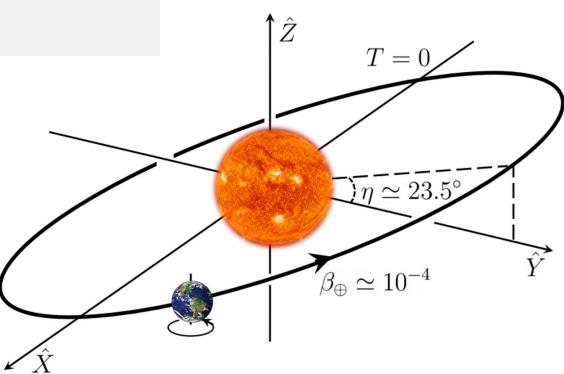
The components k^{μ} of the background field are called the coefficients for Lorentz violation or SME coefficients

The coefficients for Lorentz-violation transform under coordinate transformations, therefore it is important to report all the results in the same reference frame

 $\mathcal{L}\supset k^{\mu}\mathcal{O}_{\mu}(x)$

The reference frame used is the Sun-centered frame (SCF)

Kostelecký and Russell, arXiv:0801.0287v7



The Standard-Model Extension (SME) Lagrangian

$$\mathcal{L}_{SME} = \mathcal{L}_{SM} + \mathcal{L}_{GR} + \mathcal{L}_{LV}$$

$$\mathcal{L}_{LV} \supset \mathcal{O}^{\mu_1 \dots \mu_n} k_{\mu_1 \dots \mu_n}$$

The components $k_{\mu_1...\mu_n}$ are called coefficients for Lorentz violation

The field operator $\mathcal{O}^{\mu_1...\mu_n}$ represents standard field operators

Example of a Lorentz-violating term

$$\overline{\psi}_f \gamma_\alpha \psi_f \mathcal{V}_f^\alpha$$

 $\psi_f(x)$ is a fermionic field and f labels the flavor of the field

Combination	Result
å ^{UR(5)}	$< 6.5 \times 10^{-27} \text{ GeV}^{-1}$
-	$> -3.5 \times 10^{-27} \text{ GeV}^{-1}$
$\tilde{a}^{\text{UR}(5)} = m \tilde{g}^{\text{UR}(6)}$	(-1 to 1) × 10 ⁻³⁴ GeV ⁻¹
$\tilde{a}^{\text{UR}(5)} \pm m \tilde{g}^{\text{UR}(6)}$	$(-4 \text{ to } 4) \times 10^{-25} \text{ GeV}^{-1}$
å ^{UR(5)}	$(-0.001 \text{ to } 2.8) \times 10^{-17} \text{ GeV}^{-1}$
¿UR(6)	$(-8.5 \text{ to } 0.0025) \times 10^{-20} \text{ GeV}^{-2}$
₫ ^{UR(6)}	$(-5.4 \text{ to } 5.4) \times 10^{-14} \text{ GeV}^{-2}$

			V) '
		$0.99c_{X=Y} + 0.10c_{XZ}$	$(3.8 \pm 5.6) \times 10^{\circ}$
Combination	Result	$0.94c_{XY} - 0.35c_{YZ}$	$(-0.4 \pm 2.8) \times 10^{\circ}$
$ ilde{b}_X$	$(-0.9 \pm 1.4) \times 10^{-31} \text{ GeV}$	$0.35c_{XY} + 0.94c_{YZ}$	$(3.2 \pm 7.0) \times 10^{\circ}$
$ ilde{b}_Y$	$(-0.9 \pm 1.4) \times 10^{-31} \text{ GeV}$	$0.18c_{TX} - 0.98c_{T(Y+Z)}$	$(0.95 \pm 18) \times 10^{\circ}$
$ ilde{b}_Z$	$(-0.3 \pm 4.4) \times 10^{-30} \text{ GeV}$	$0.98c_{TX} + 0.18c_{T(Y+Z)}$	(5.6 ± 7.7) × 10°
	\tilde{d}_Q) $(0.9 \pm 2.2) \times 10^{-27} \text{ GeV}$	$c_{T(Y-Z)}$	(-11±19) × 10
	\tilde{l}_Q) $(-0.8 \pm 2.0) \times 10^{-27} \text{ GeV}$	c_{TT}	$(-8.8 \pm 5.1) \times 10^{-1}$
$+\tan \eta (\tilde{d}_{YZ} - \tilde{H}_{XT})$ \tilde{b}_{X}	$(2.8 \pm 6.1) \times 10^{-29} \text{ GeV}$	"	$(-14 \pm 28) \times 10$
$ ilde{b}_{Y}$	$(6.8 \pm 6.1) \times 10^{-29} \text{ GeV}$	\overline{c}_{TT}	$(-0.014 \pm 0.028) \times 10$
$ ilde{b}_X$	$(0.1 \pm 2.4) \times 10^{-31} \text{ GeV}$	"	$(4.6 \pm 4.6) \times 10$
\tilde{b}_Y	$(-1.7 \pm 2.5) \times 10^{-31} \text{ GeV}$	c_{TT}	$(-4 \text{ to } 2) \times 10^{\circ}$
$ ilde{b}_{Z}$	$(-29 \pm 39) \times 10^{-31} \text{ GeV}$	$c_{(TX)}$	$(-30 \text{ to } 1) \times 10^{\circ}$
$ ilde{b}_{\perp}$	$< 3.1 \times 10^{-29} \text{ GeV}$	$c_{(TY)}$	$(-80 \text{ to } 6) \times 10^{\circ}$
$ ilde{b}_Z $	$< 7.1 \times 10^{-28} \text{ GeV}$		$(-11 \text{ to } 1.3) \times 10^{\circ}$
r_e	< 3.2 × 10 ⁻²⁴	$c_{(TZ)}$ $3c_{(TX)} + 0.51c_{(TY)} + 0.22c_{(TZ)}$	
$ ec{b} $	•		$(-2.9 \pm 6.3) \times 10^{\circ}$
$r_{\omega_a^-, \text{diurnal}}$	$< 1.6 \times 10^{-21}$	$c_{XX} - c_{YY}$	
$ \tilde{b}_J , \ (J=X,Y)$	$< 10^{-27} \text{ GeV}$	$\frac{1}{2}c_{(XY)}$	$(2.1 \pm 0.9) \times 10^{\circ}$
$ \tilde{b}_J^* , \ (J=X,Y,Z)$	$< 10^{-22} \text{ GeV}$	$\frac{1}{2}C(XZ)$	$(-1.5 \pm 0.9) \times 10^{\circ}$
		$\frac{1}{2}c_{(YZ)}$	$(-0.5 \pm 1.2) \times 10^{\circ}$
		- (/	

Combination

¿UR(4)

 $\begin{aligned} c_{XX} + c_{YY} - 2c_{ZZ} \\ \lambda^{ZZ} \end{aligned}$

 $0.10c_{X}$

Result

 $(-1.3 \text{ to } 0.2) \times 10^{-15}$ > -1.2×10^{-16} > -6×10^{-20}

 $(3.1 \pm 1.9) \times 10^{-11}$ $(2.6 \pm 5.3) \times 10^{-11}$ $> -5 \times 10^{-21}$ $(-9.0 \pm 11) \times 10^{-17}$

 $(-106 \pm 147) \times 10^{-16}$

 $(13.3 \pm 9.8) \times 10^{-16}$

 $< 1.5 \times 10^{-15}$ > -5×10^{-13}

Combination	Result
$\frac{1}{2}C(YZ)$	$(2.1 \pm 4.6) \times 10^{-16}$
$\frac{1}{2}C(XZ)$	$(-1.6 \pm 6.3) \times 10^{-16}$
$\frac{1}{2}c_{(XY)}$	$(7.6 \pm 3.5) \times 10^{-16}$
$c_{XX} = c_{YY}$	$(1.15 \pm 0.64) \times 10^{-15}$
$ c_{XX} + c_{YY} - 2c_{ZZ} - 0.25(\tilde{\kappa}_{e-})^{ZZ} $	< 10 ⁻¹²
$ \frac{1}{2}c(xy) $	$< 8 \times 10^{-15}$
$ c_{XX} - c_{YY} $	$< 1.6 \times 10^{-14}$
$ c_{XX} + c_{YY} - 2c_{ZZ} $	< 10 ⁻⁵
$ c_{TJ} + c_{JT} $, $(J = X, Y, Z)$	< 10 ⁻²
c_{XX}	$(-3 \text{ to } 5) \times 10^{-15}$
cyr	$(-0.7 \text{ to } 2.5) \times 10^{-15}$
c _{ZZ}	$(-1.6 \text{ to } 2.5) \times 10^{-15}$
c(YZ)	$(-2.5 \text{ to } 1.8) \times 10^{-15}$
C _{0.X}	$(-7 \text{ to } 4) \times 10^{-15}$
COY	$(-0.5 \text{ to } 1.5) \times 10^{-15}$
c _{oz}	$(-4 \text{ to } 2) \times 10^{-17}$
$[0.05c_{XX} + 0.55c_{YY} + 0.41c_{ZZ}]$	$< 1.3 \times 10^{-15}$
$+0.16c_{(XY)} = 0.14c_{(XZ)} = 0.47c_{(YZ)}$	
$+0.22c_{(0X)} + 0.74c_{(0Y)} - 0.64c_{(0Z)} + c_{00}$	
$[0.58c_{XX} + 0.04c_{YY} + 0.38c_{ZZ}]$	$< 2.5 \times 10^{-15}$
$-0.14c_{(XY)} - 0.47c_{(XZ)} + 0.12c_{(YZ)}$	
$+0.76c_{(0X)} - 0.19c_{(0Y)} - 0.62c_{(0Z)} + c_{00}$	l
$\tilde{d}_{XY} = \tilde{H}_{ZT} + \tan \eta \tilde{H}_{YT}$	$(0.1 \pm 1.8) \times 10^{-27} \text{ GeV}$
\tilde{H}_{ZT}	$(-4.1 \pm 2.4) \times 10^{-27} \text{ GeV}$
$\dot{N}_{VT} = \dot{d}_{ZX}$	$(-4.9 \pm 8.9) \times 10^{-27} \text{ GeV}$
$+\tilde{H}_X(r + \tan q)(\tilde{g}_T - 2\tilde{d}_+ + \tilde{d}_Q)$	$(1.1 \pm 9.2) \times 10^{-27} \text{ GeV}$
dxx	$< 2 \times 10^{-14}$
$ d_{YY} , d_{ZZ} $	$< 3 \times 10^{-15}$
Jd _{use}	$< 2 \times 10^{-15}$
	$< 2 \times 10^{-14}$
([axal)	$< 7 \times 10^{-15}$
Max	$< 5 \times 10^{-14}$
	< 5 × 10 ⁻¹⁵
$ d_{TZ} $	< 8 × 10 ⁻¹⁷
$ \tilde{d}_J , (J = \lambda X)$	< 10 ⁻²² GeV
$ d_x $	< 10 ⁻¹⁹ GeV
gxr , gxz	10 ⁻¹⁷ GeV
$ g_{XY} , g_{XZ} $ $ g_{YZ} , g_{YX} $	10-12 GeV
$ \tilde{g}xx $, $ \tilde{g}xy $	
$ \tilde{g}_{D,J} , (J = X, Y, Z)$	107 No
$ \tilde{g}_{D,J} , (J = X, Y, Z)$	10-22 GeV
19 <i>D</i> , <i>J</i> 1, (0 = 2, 7)	And Get

Muon Sector

Combination	Result
b_Z^μ	$-(1.0 \pm 1.1) \times 10^{-23} \text{ GeV}$
$\sqrt{(b_X^{\mu^*})^2 + (b_Y^{\mu^*})^2}$	$< 1.4 \times 10^{-24} \text{ GeV}$
$\sqrt{(b_X^{\mu^*})^2 + (b_Y^{\mu^*})^2}$	$< 2.6 \times 10^{-24} \text{ GeV}$
$\sqrt{(\hat{b}_{X}^{\mu})^{2}+(\hat{b}_{Y}^{\mu})^{2}}$	$< 2 \times 10^{-23} \text{ GeV}$
$b_Z^\mu = 1.19(m_\mu d_{Z0}^\mu + H_{XY}^\mu)$	$(-1.4 \pm 1.0) \times 10^{-22} \text{ GeV}$
b_Z^μ	$(-2.3 \pm 1.4) \times 10^{-22} \text{ GeV}$
$m_{\mu}d_{Z0}^{\mu} + H_{XY}^{\mu}$	$(1.8 \pm 6.0) \times 10^{-23} \text{ GeV}$
$ c^{\mu} $	< 10-11
$\tilde{a}_{\mu}^{UR(5)} = m_{\mu} \tilde{g}_{\mu}^{UR(6)}$	$(-1 \text{ to } 1) \times 10^{-34} \text{ GeV}^{-1}$
č _μ ^{UR(6)}	$(-8.5 \text{ to } 0.0025) \times 10^{-20} \text{ GeV}^{-2}$
$ c^{\tau} $	< 10 ⁻⁸
$\dot{a}_{\tau}^{UR(5)} = m_{\tau} \dot{g}_{\tau}^{UR(6)}$	$(-2 \text{ to } 2) \times 10^{-33} \text{ GeV}^{-1}$

Minimal Lorentz-violating terms contain field operator of mass dimensions 3 or 4

Minimal Lorentz-violating term

Nonminimal Lorentz-violating term

Mass dimension 3

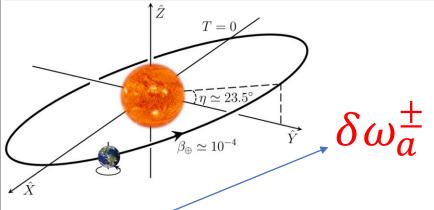
$$\overline{\psi}_f \gamma_{\alpha} \psi_f \mathcal{V}_f^{\alpha}$$

Mass dimension 5

$$\overline{\psi}_f \gamma_\alpha \partial_\beta \partial_\nu \psi_f V_f^{\alpha\beta\nu}$$

Low-energy experiments are expected to be more sensitive to minimal terms than to nonminimal terms in a Lorentz-violation effective field theory.

The minimal SME only considers Lorentz-violating operators of mass dimension 3 and 4.



Components of magnetic field in SCF

$$\delta \omega_a^{\pm} \simeq 2 \widecheck{b}^{\pm} \cdot \widehat{B} + \frac{B_I}{B} (\widecheck{T}^{\pm})^{IJ} (\beta_{LF})_J$$

Laboratory's velocity in SCF

Lorentz-violating anomalous precession frequency shift

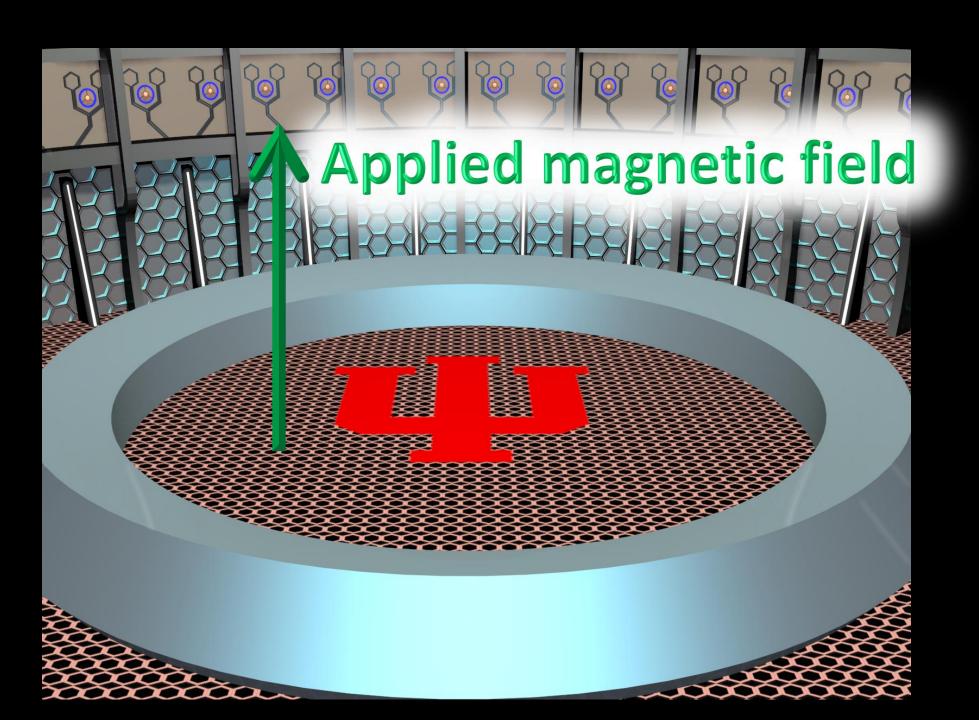
- + for positive muon
- for negative muon

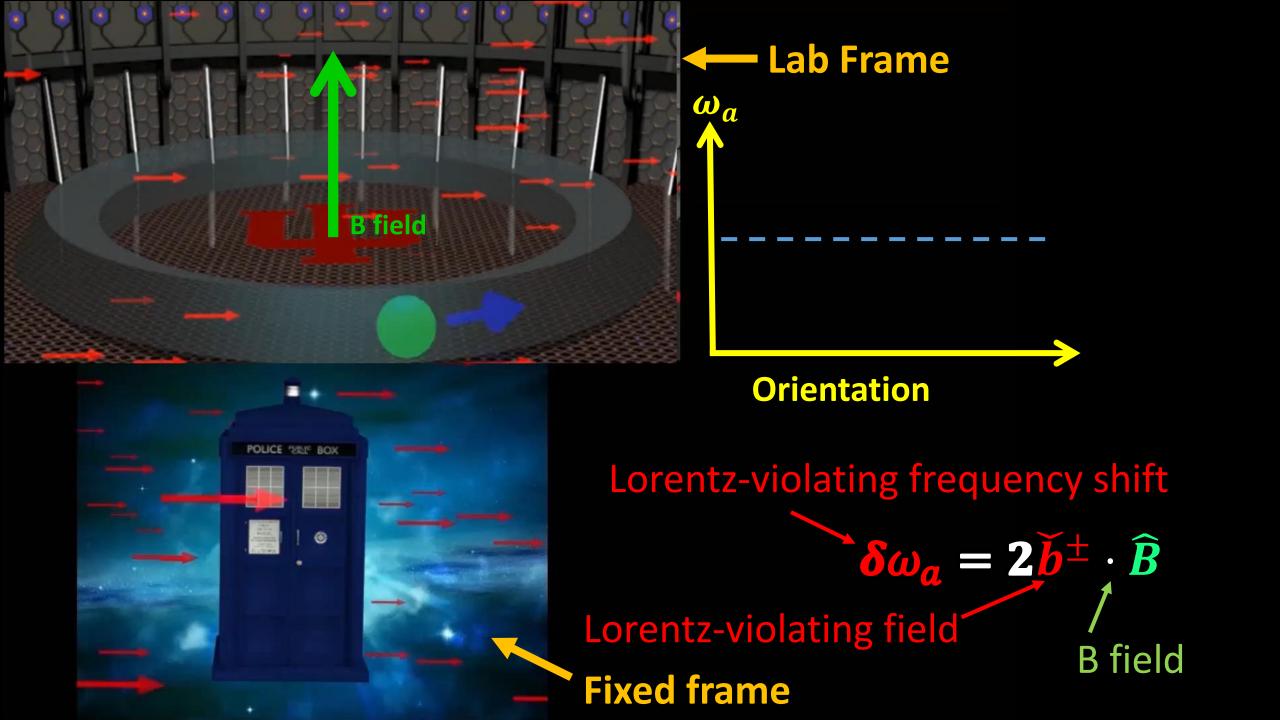
Direction of the magnetic field in SCF

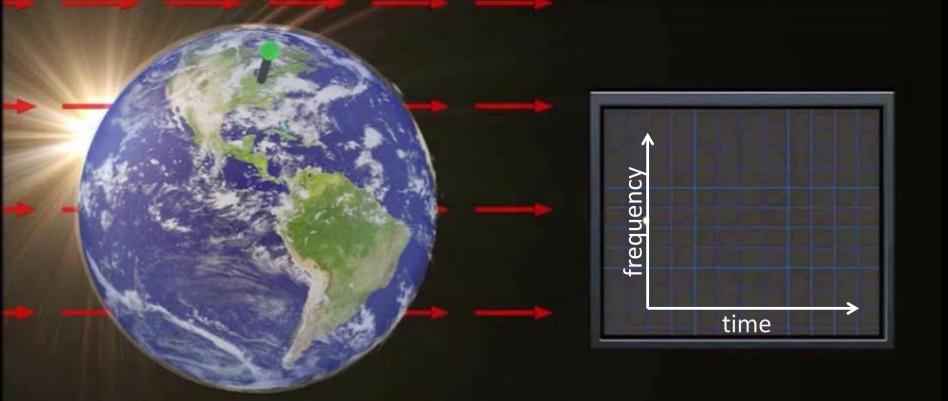
A linear combination of SME coefficients that transform as a spatial vector under rotations of the coordinate system











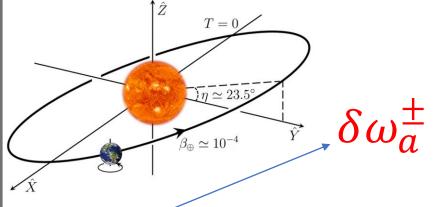
Sidereal variation of the anomalous frequency

Lorentz-violating frequency shift

$$\delta \omega_a = 2 \vec{b}^{\pm} \cdot \hat{B}$$
Lorentz-violating field

B field

Minimal Lorentz-violating terms contain field operator of mass dimensions 3 or 4



Component of the magnetic field in SCF

$$\delta \omega_a^{\pm} \simeq 2 \widecheck{b}^{\pm} \cdot \widehat{B} + \frac{B_I}{B} (\widecheck{T}^{\pm})^{IJ} (\beta_{LF})_J$$

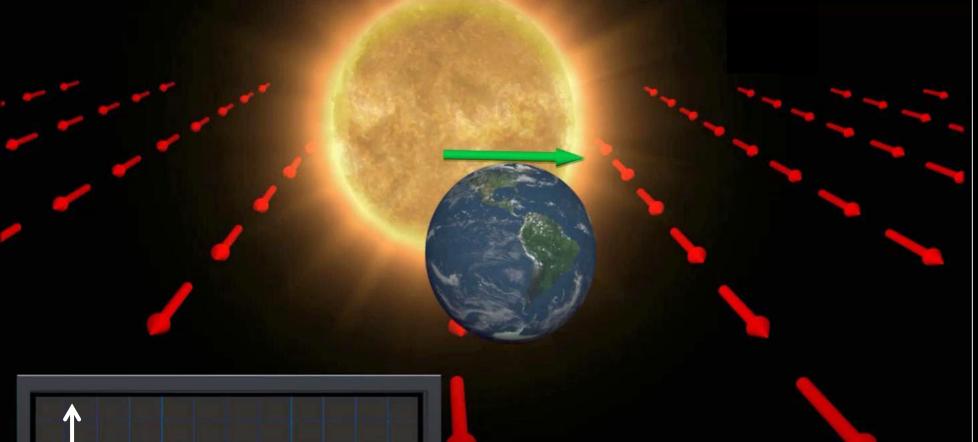
Laboratory's velocity in SCF

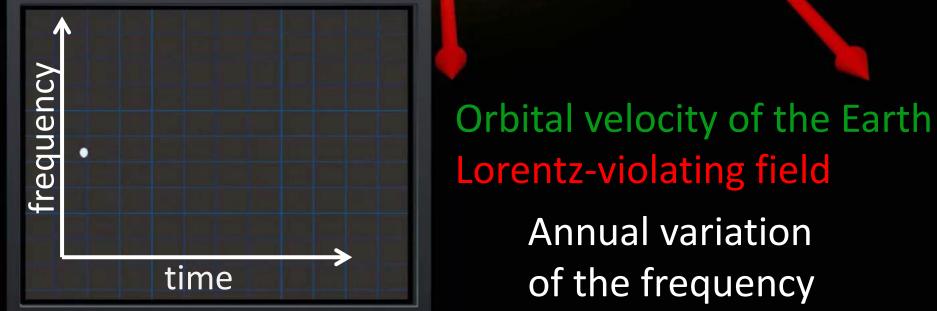
Lorentz-violating anomalous precession frequency shift

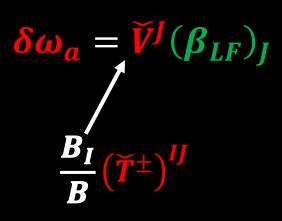
- + for positive muon
- for negative muon

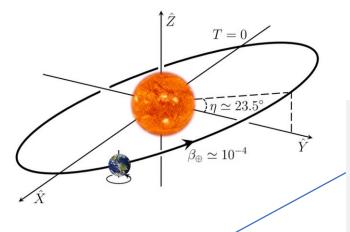
Direction of the magnetic field in SCF

A linear combination of SME coefficients that transform as a spatial vector under rotations of the coordinate system









$$\delta\omega_a^{\pm} \simeq 2\widecheck{b}^{\pm} \cdot \widehat{B} + \frac{B_I}{B} (\widecheck{T}^{\pm})^{IJ} (\beta_{LF})_J$$

Linear combination of SME coefficients

- + for positive muon
- for negative muon

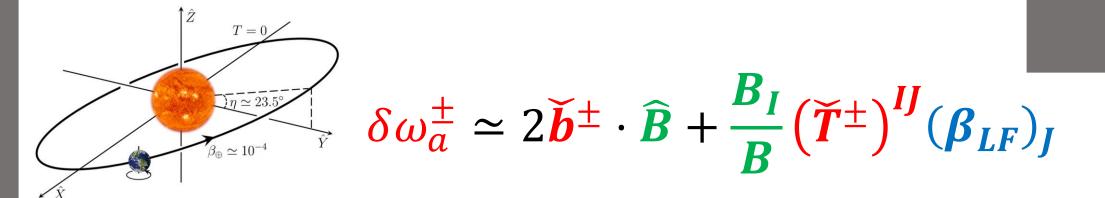
Muon mass

$$\check{b}_{J}^{\pm} = \pm \frac{1}{\gamma} \left(b_{J} - m_{\mu} g_{J}^{(A)} \right) + \epsilon_{JKL} H_{KL} + m_{\mu} d_{JT} \pm \frac{1}{\gamma} \left(1 + \frac{3}{2} \beta^{2} \gamma^{2} \right) m_{\mu} \epsilon_{JKL} g_{KLT}^{(M)}$$

SME coefficients that appear on the Lagrangian

 β velocity of muon relative to the laboratory frame

$$\gamma = (1 - \beta^2)^{-1/2}$$



$$\check{\boldsymbol{b}}_{J}^{\pm} = \pm \frac{1}{\gamma} \left(\boldsymbol{b}_{J} - m_{\mu} \boldsymbol{g}_{J}^{(A)} \right) + \epsilon_{JKL} \boldsymbol{H}_{KL} + m_{\mu} \boldsymbol{d}_{JT} \pm \frac{1}{\gamma} \left(1 + \frac{3}{2} \beta^{2} \gamma^{2} \right) m_{\mu} \epsilon_{JKL} \boldsymbol{g}_{KLT}^{(M)}$$

g-2 experiments with negative muons are sensitive to a different linear combination of SME coefficients than g-2 experiments with positive muons

$$\delta\omega_a^{\pm} \simeq 2\check{b}^{\pm} \cdot \hat{B} + \frac{B_I}{B} (\check{T}^{\pm})^{IJ} (\beta_{LF})_J$$

$$\check{b}_{J}^{\pm} = \pm \frac{1}{\gamma} \left(b_{J} - m_{\mu} g_{J}^{(A)} \right) + \epsilon_{JKL} H_{KL} + m_{\mu} d_{JT} \pm \frac{1}{\gamma} \left(1 + \frac{3}{2} \beta^{2} \gamma^{2} \right) m_{\mu} \epsilon_{JKL} g_{KLT}^{(M)}$$

J-PARC's experiment and Fermilab's experiment are sensitive to slightly different combinations of SME coefficients

J-PARC

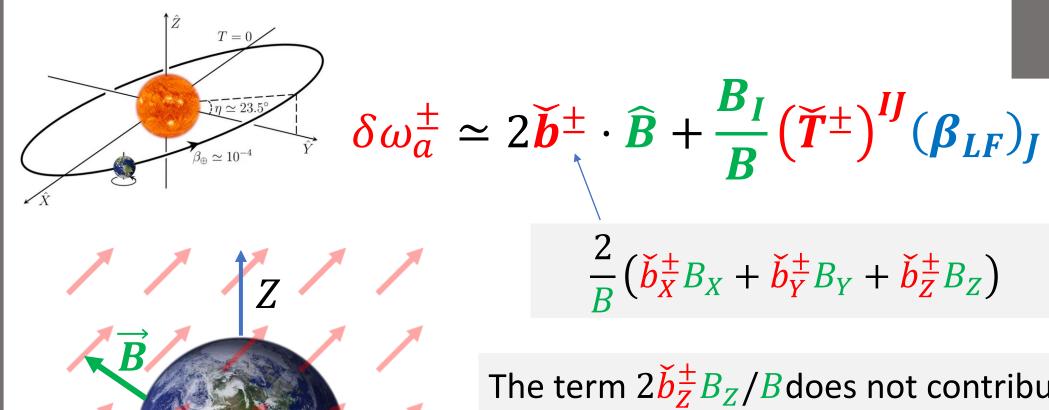
$$\beta \simeq 0.94$$

 $\gamma \simeq 3$

Fermilab and BNL

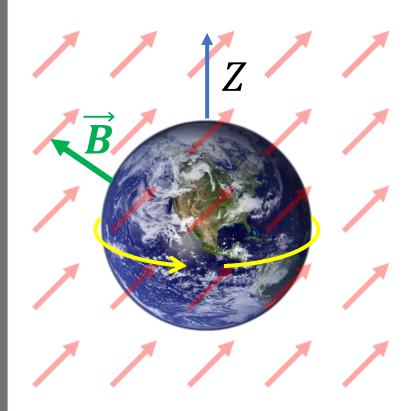
$$\beta \simeq 1$$

$$\gamma \simeq 30$$



The term $2\check{b}_Z^{\pm}B_Z/B$ does not contribute to the sidereal or annual variation of ω_a

b_Z^{\pm} can be studied by comparing the anomalous frequencies for the negative and positive muon

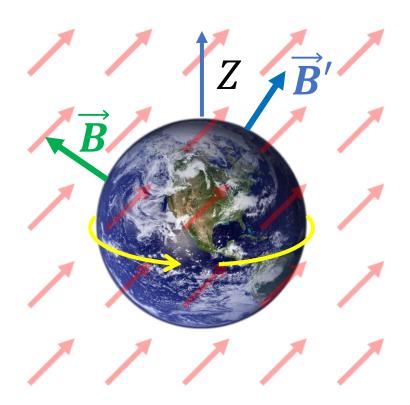


In the same magnetic field

$$\frac{\omega_a^+}{\omega_p} - \frac{\omega_a^-}{\omega_p'} = \dots + 2 \frac{B_z}{B} \left(\frac{\check{b}_z^+}{\omega_p} - \frac{\check{b}_z^-}{\omega_p'} \right) + \dots$$

Proton precession frequency

 \check{b}_Z^{\pm} can be studied by comparing the anomalous frequencies for two orientations of the magnetic field



$$\frac{\omega_a^{\pm}(\hat{B})}{\omega_p} - \frac{\omega_a^{\pm}(\hat{B}')}{\omega_p'}$$
 Proton precession frequency
$$= \cdots + 2\left(\frac{B_Z}{B\omega_p} - \frac{B_Z'}{B'\omega_p'}\right)\check{b}_Z^{\pm} + \cdots$$

Minimal cases

Nonminimal cases

$$2\check{b}_{Z}^{\pm}\hat{B}_{Z}$$

$$2\sum_{dnj} E_{0}^{d-4} G_{j0}(\chi) \left(\check{H}_{nj0}^{(d)Sun} \pm \check{g}_{nj0}^{(d)Sun} \right)$$

Combination of nonminimal SME coefficients

Energy of the muon in the laboratory frame

$$\frac{E_{\rm Fermilab}}{E_{J\text{-}PARC}} \simeq 10$$

Fermilab's experiment is more sensitive to nonminimal SME coefficients than J-PARC by a factor of $(10)^{d-4}$ where d is the mass-dimension of the coefficient

Comparing the anomalous frequency for the negative muon with the positive muon in the Fermilab experiment will result on the best bounds on the effective coefficients $\mathbf{\breve{g}}_{n\,i0}^{(d)\mathrm{Sun}}$ with $d\geq 5$

More information

Laboratory tests of Lorentz and CPT symmetry with muons André H. Gomes, V. Alan Kostelecký, and AJV Phys. Rev. D **90**, 076009 (2014)

