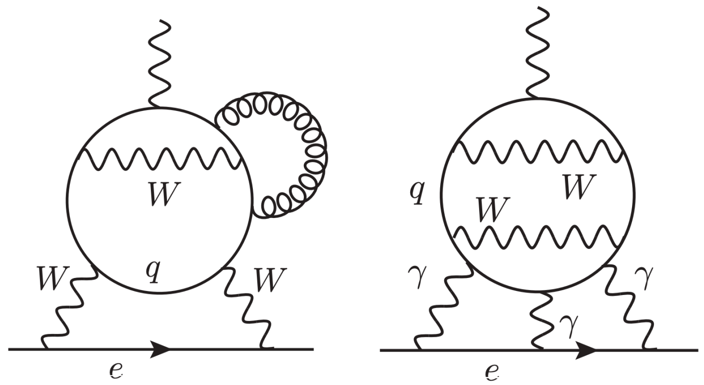
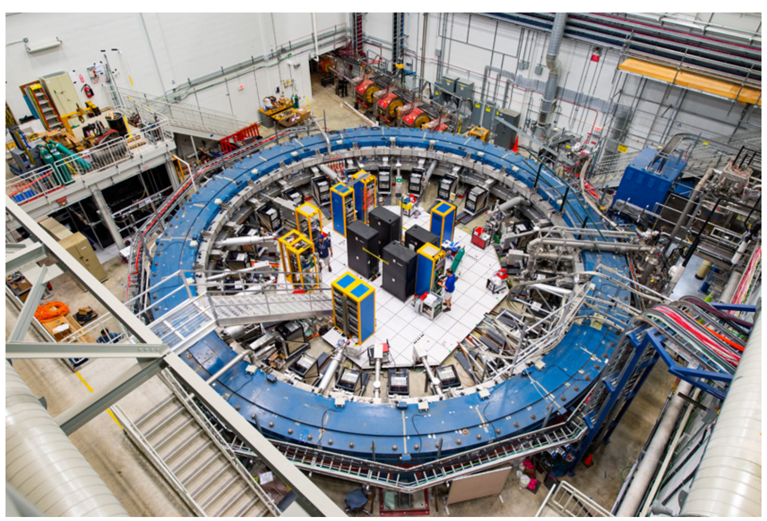
A Dedicated Muon EDM Experiment in the `g-2’ Storage Ring - A Beam Dynamics Perspective

Aakaash Narayanan and Mike Syphers

Potential Fermilab Muon Campus & Storage Ring Experiments Workshop
24 May 2021

Motivation for EDM at Fermilab and in the `g-2' Ring





- A non-zero EDM value is an indication of combined CPV violation.
- EDM of a muon is heavily suppressed in the Standard Model unlike in few other BSM models - an excellent probe for new physics.
- The current muon EDM limit of $d_\mu < 1.8 \times 10^{-19}$ is the the only EDM of fundamental particle probed directly on the bare particle, that too done using the same `g-2' storage ring!
- We have the combined wisdom of operating the `g-2' storage ring for over two decades.

Fermilab

Thomas-BMT Equation

The total precession frequency $\vec{\Omega}_S$ of the spin in the presence of $\vec{B}$ and $\vec{E}$ (both $\perp \vec{p}$) would be the net sum of MDM precession and the EDM precession, given by the Thomas-BMT equation:

$$\vec{\Omega}_S = \underbrace{-\frac{q}{m}\left[G\vec{B} - \left(G - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}\right]}_{MDM} + \underbrace{\frac{\eta q}{2mc}\left[\vec{E} + c\vec{\beta} \times \vec{B}\right]}_{EDM}$$

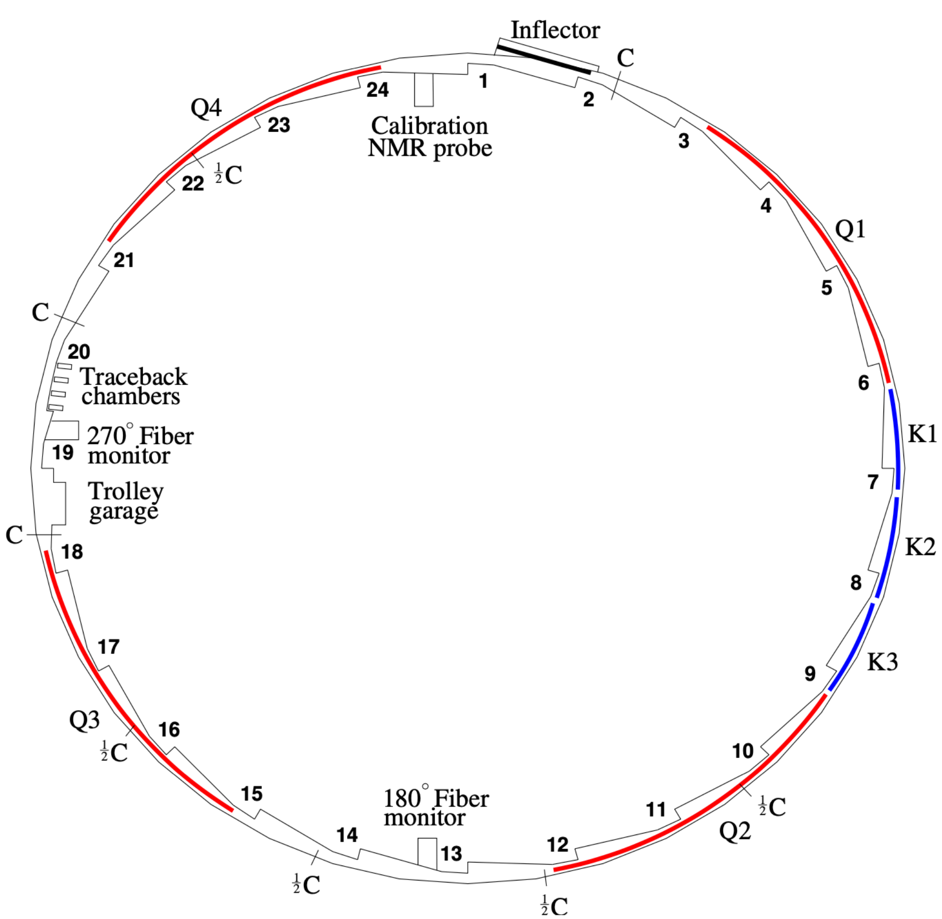
If the magnetic field is purely vertical and the electric field is purely radial,

- MDM spin precession would be about the vertical axis in the plane of the ring
- EDM spin precession would be about the radius tipping vertically out of the plane of the ring

The `g-2' Storage Ring (at present)

- The current `g-2' storage ring is being operated for measuring the magnetic dipole moment (MDM) of the muon primarily caused by the vertical magnetic field $\vec{B}$.

The precession frequency Ω_{MDM} is given by:



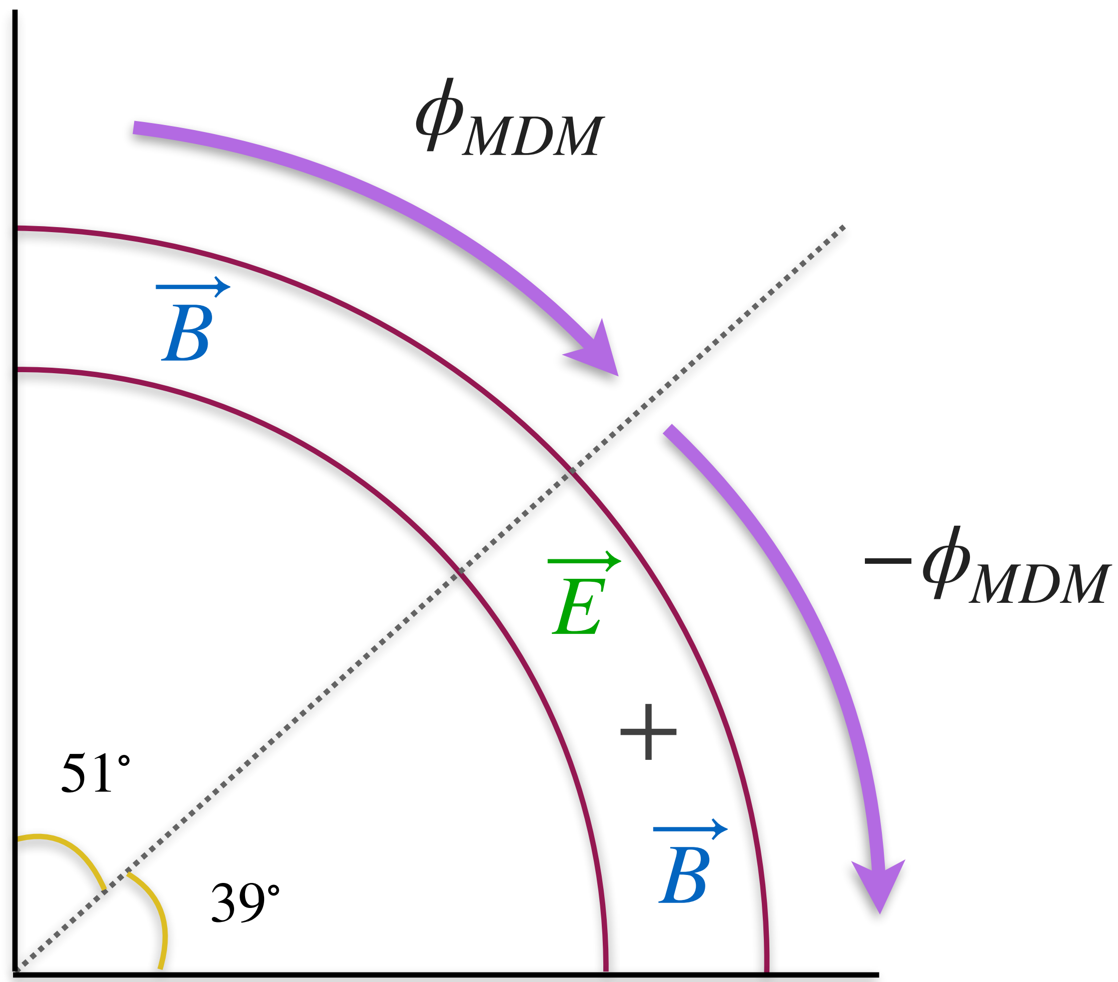
$$\vec{\Omega}_{MDM} = -\frac{q}{m}\left[G\vec{B} - \cancel{\left(G - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}}\right]$$

- The relativistic γ is chosen such that the second term is zero, operating at muon's 'magic momentum' of 3.09 GeV/c.
- The ring has a radius of 7.112 meters and is four-fold symmetric.
- It has a highly purified constant vertical dipole magnetic field throughout and four isometrically placed electrostatic quadrupoles for vertical focussing.
- Each 90 degree section consists of:
  - 51 degrees of dipole $\vec{B}$-field only region
  - 39 degrees of (dipole $\vec{B}$ + quadrupole $\vec{E}$) region
- There are no dipole electric fields in the `g-2' storage ring at present.

Proposed Scheme - A Hybrid Storage Ring

In this study, we investigate the idea of freezing the MDM precession and enhancing the EDM signal by introducing a dipole electric field in the electrostatic quadrupole sections.

The idea:



1. The μ^+ traverses through 51° $\vec{B}$-only section.

2. The MDM component of the spin precession increases by an amount ϕ_{MDM} due to the $\vec{B}$-field.

3. The μ^+ then enters the 39° section $\vec{E} + \vec{B}$ section.

4. The dipole $\vec{E}$ field (along with $\vec{B}$) in the 39° section is chosen such that the MDM precesses the spin in the opposite direction by the same amount $-\phi_{MDM}$

Freezing the MDM Precession

The amount of spin's MDM precession in the 51° of $\vec{B}$-only region is given by:

$$\phi_{MDM,\,B} = \frac{q}{m} GB \,.\, \frac{51^\circ}{90^\circ} \frac{T_{\mathrm{rev}}}{4}$$

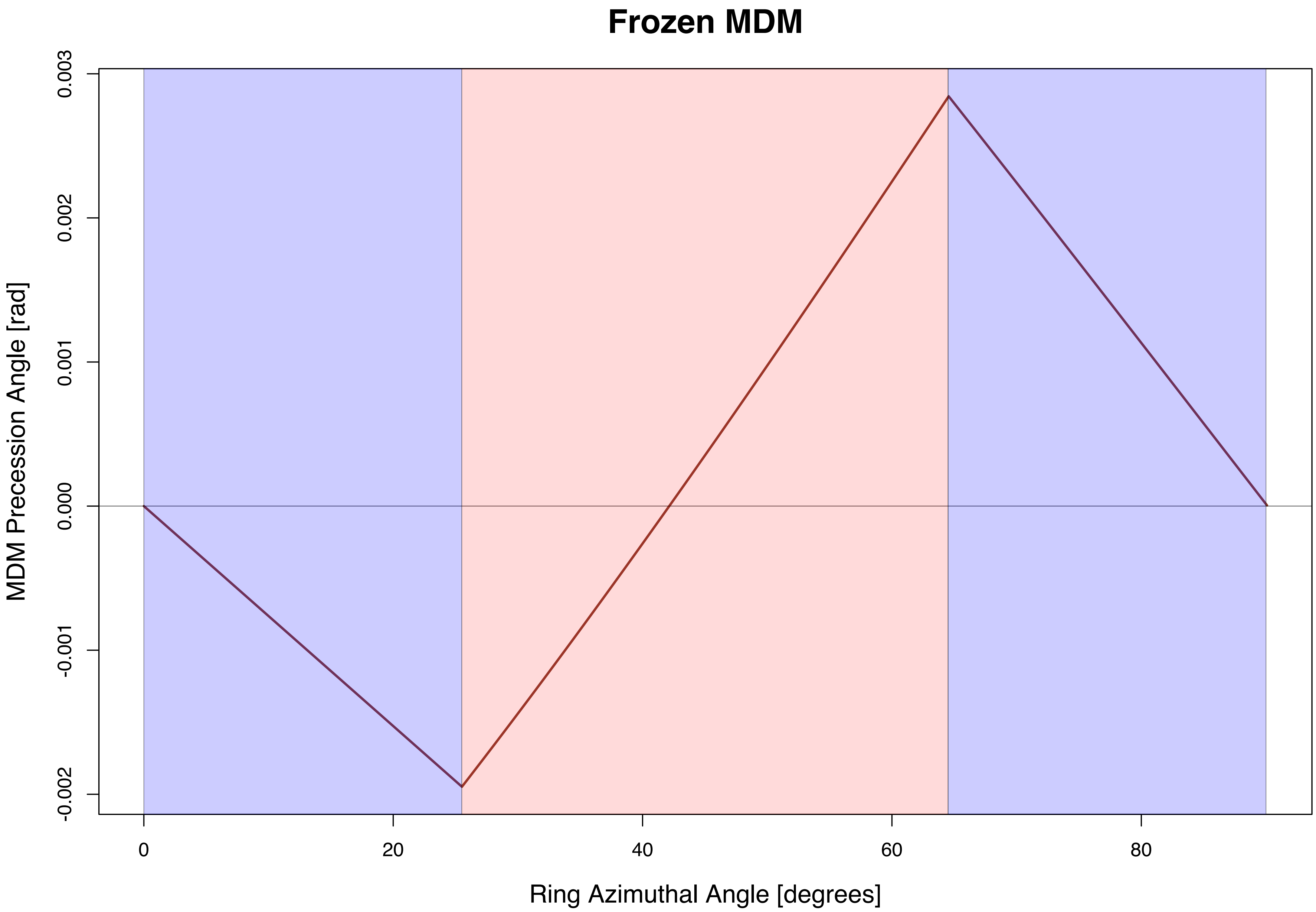
The amount of spin's MDM precession in the $\vec{E} + \vec{B}$-only region is given by:

$$\phi_{MDM,\,E+B} = \frac{q}{m} \left[G\vec{B} - \left(G - \frac{1}{\gamma^2 - 1} \right) \frac{\beta \,.\, E}{c} \right] \,.\, \frac{39^\circ}{90^\circ} \frac{T_{\mathrm{rev}}}{4}$$

Equating them both, we can solve for the electric field value needed to cancel the MDM precession accumulated in the $\vec{B}$-only section.

Fermilab

Freezing the MDM Precession



Finding the $\vec{E}$ and $\vec{B}$ field values

Simplifying the equation for frozen MDM precession, we have a linear equation in E and B:

$$\vec{E} - \vec{B} . \left[\frac{Gc}{|\beta|} \frac{90^\circ}{39^\circ} \left(G - \frac{1}{\gamma^2 - 1} \right)^{-1} \right] = 0$$

CONSTRAINT #1

Since we look to re-use the `g-2' storage ring, the radius of the ring imposes a condition via the centripetal Lorentz force required to keep the muons on the 7.112 meter orbit:

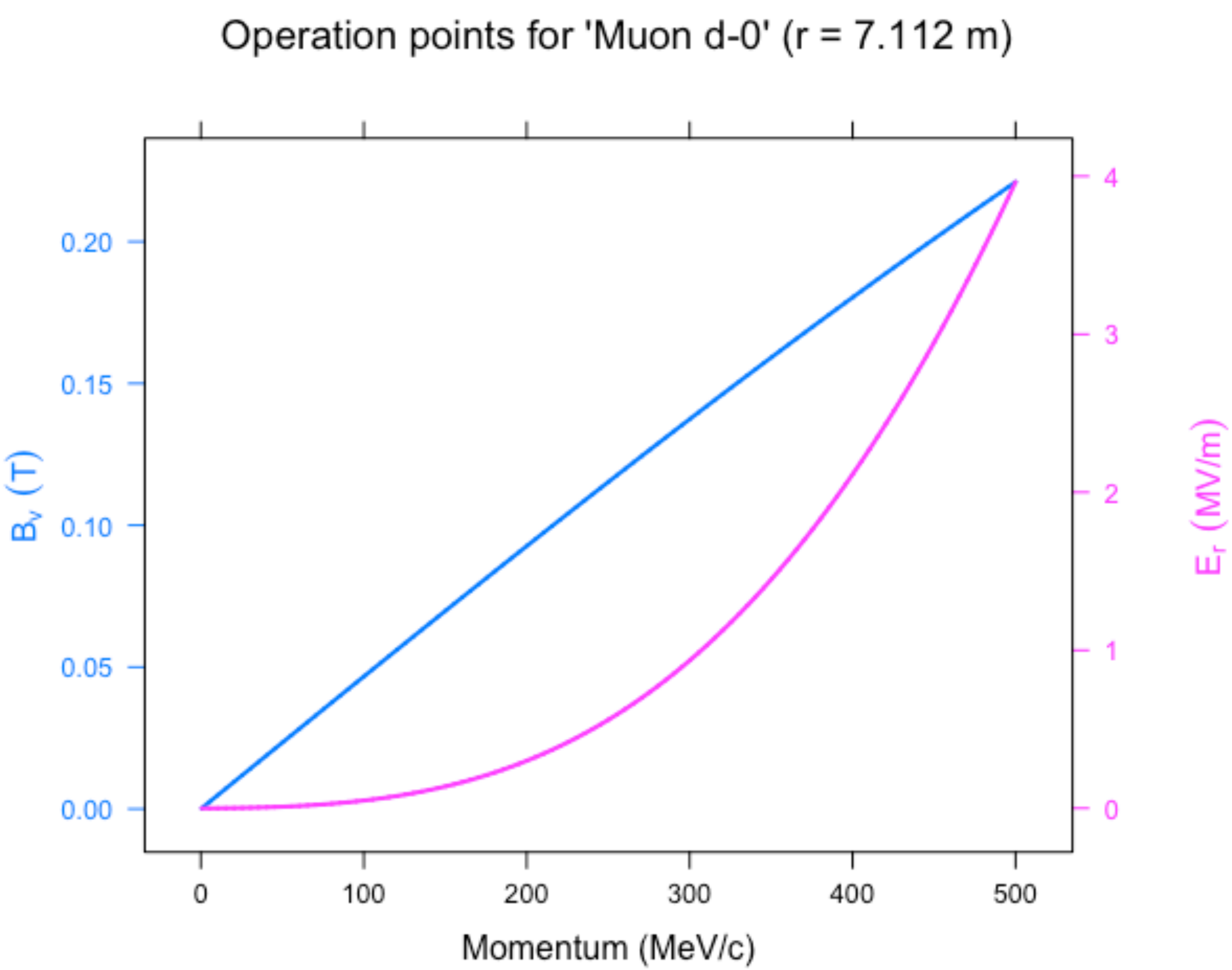
$$\vec{E} + v\vec{B} = \gamma \frac{mv^2}{qr} \hat{r}$$

CONSTRAINT #2

(Since we could use both E and B-fields in the ring, there is no constraint to operate on the magic momentum anymore.)

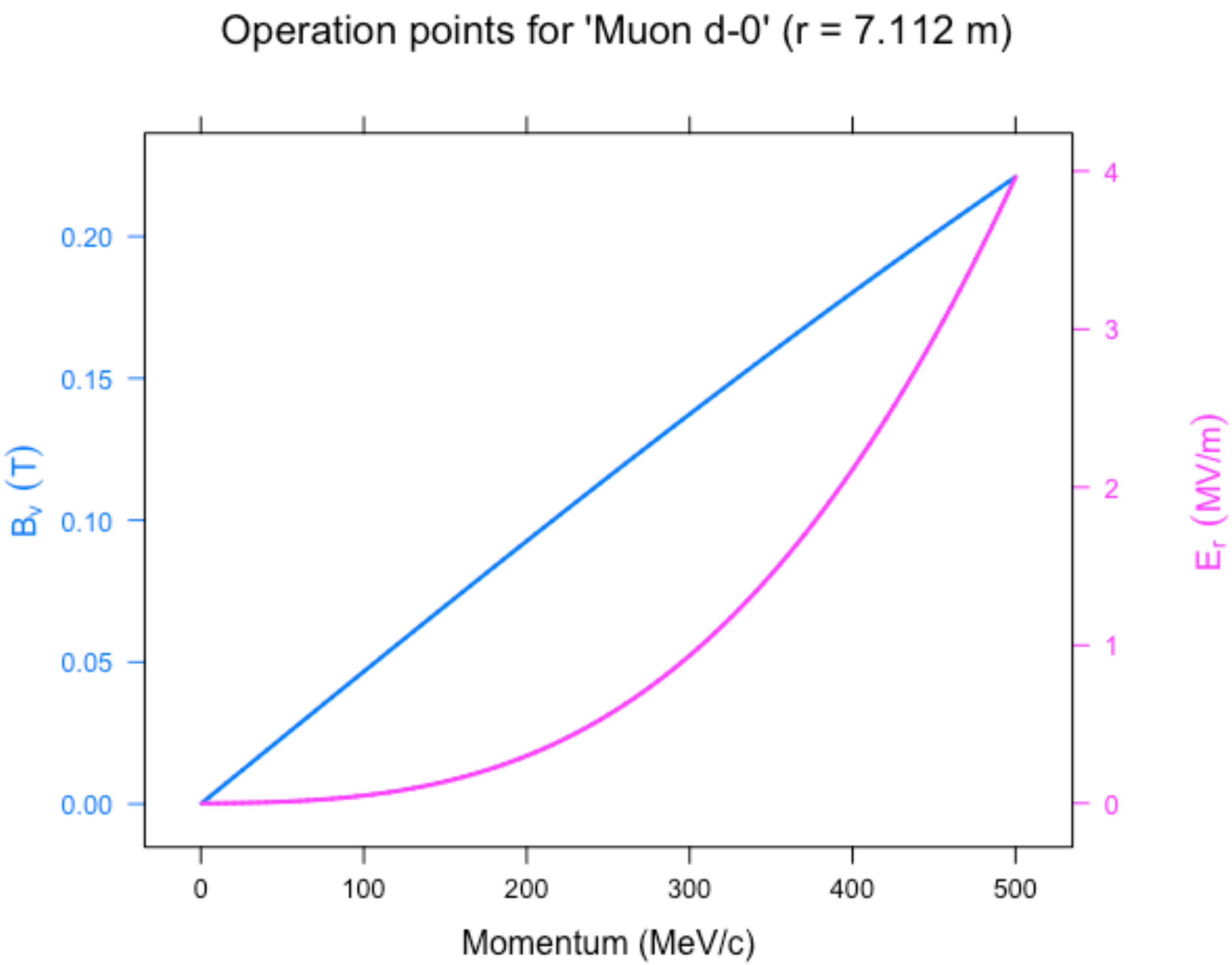
The above two constraints thus give us two linear equations in $\vec{E}$ and $\vec{B}$ which we can solve for various values of momentum (γ) for possible operational value of fields.

Possible field values for frozen MDM condition:



Momentum (MeV/c)	Vertical Magnetic Field (Tesla)	Radial Electric Field (MV/m)
100	0.046	0.048
200	0.092	0.300
300	0.137	0.935
400	0.180	2.113
500	0.220	3.963

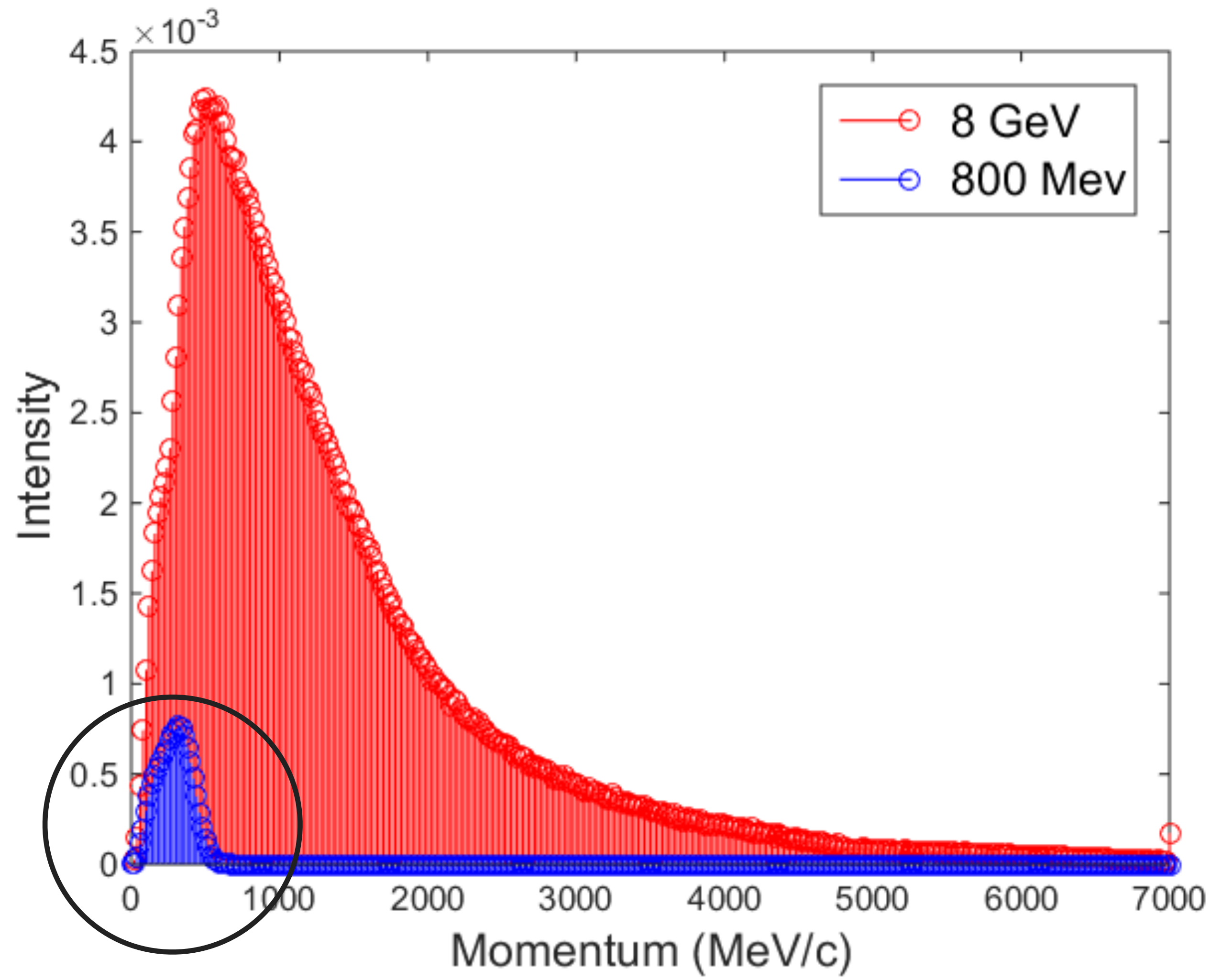
Which momentum to choose?



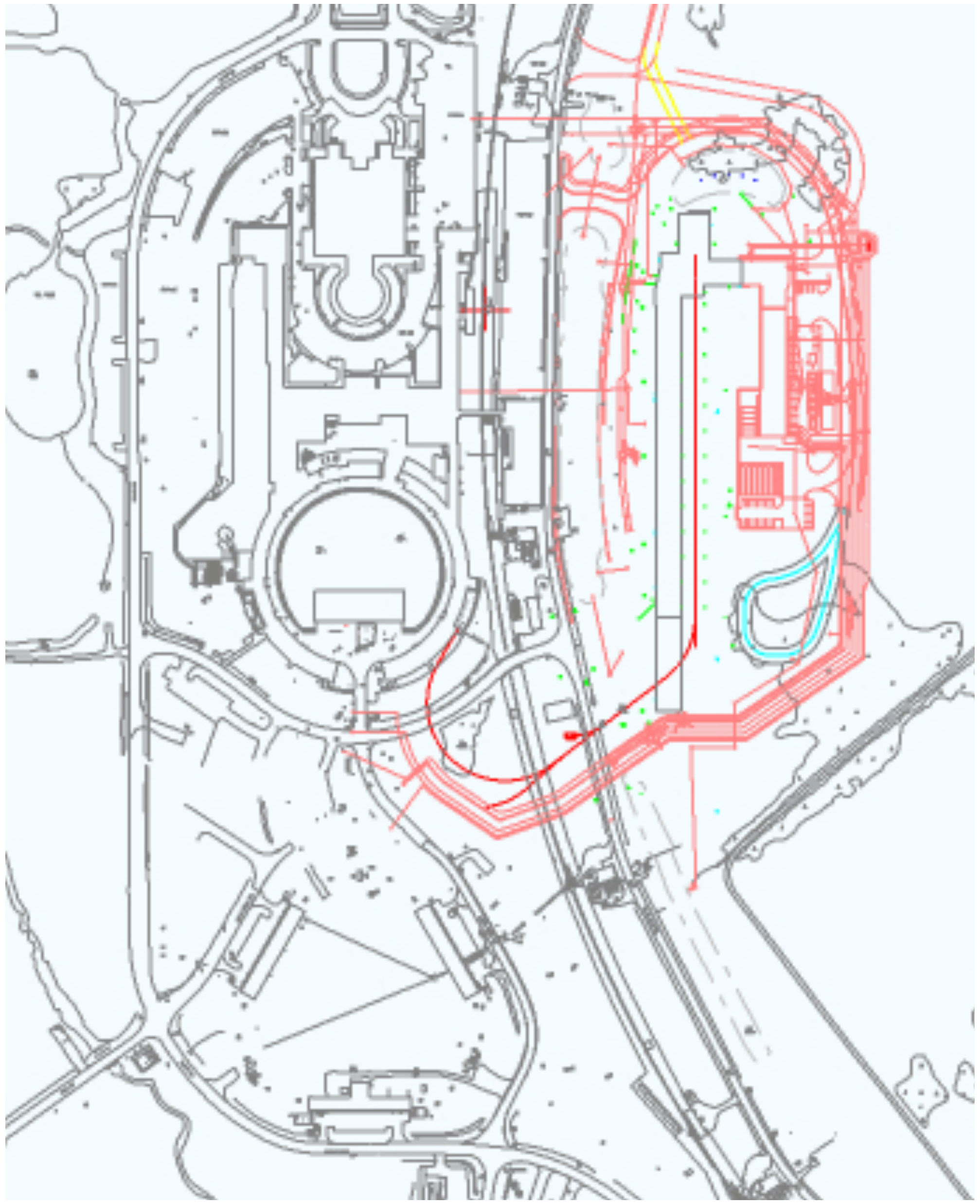
Momentum (MeV/c)	Vertical Magnetic Field (Tesla)	Radial Electric Field (MV/m)
100	0.046	0.048
200	0.092	0.300
300	0.137	0.935
400	0.180	2.113
500	0.220	3.963

Making use of PIP-II Protons

- PIP-II is the proposed Proton Improvement Plan where a new superconducting linac shall be built with the capability to deliver high-power 800 MeV proton beam.
- A preliminary study of muon production by PIP-II protons on target has been performed by Diktys Stratkis 👉🏼
- We see that the peak intensity of muon distribution is exactly around the 300 MeV/c range!

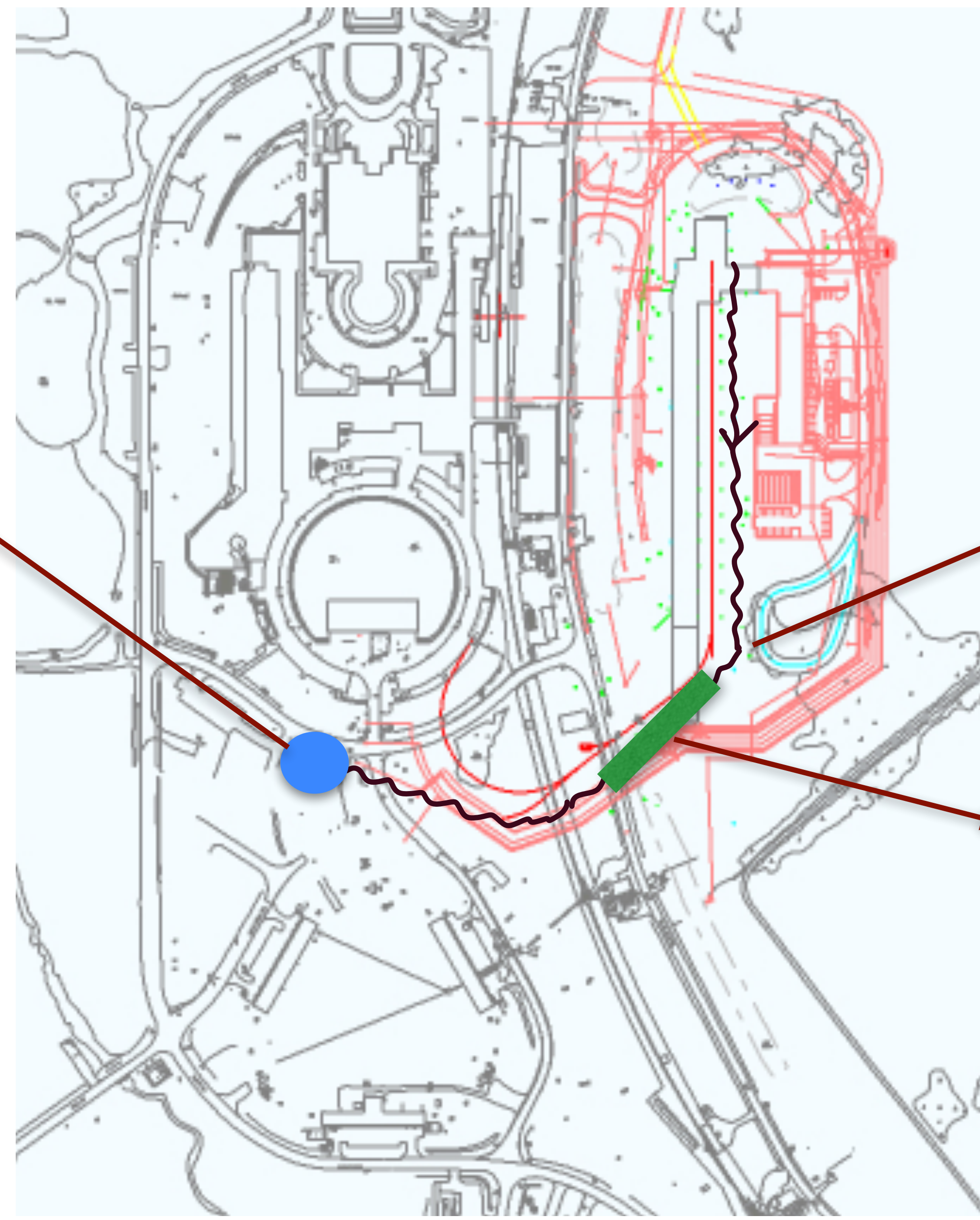


PIP-II Layout



PIP-II Beam Potential for a ‘d-0’ EDM Experiment

300 MeV/c
muons



800 MeV/c
protons

Muon Production
Target

Particle Rates

RMS Current from PIP-II Linac	0.002 A
Pulse length for \`g-2’ storage ring	120 ns
No. of PIP-II bunches per EDM pulse	19.5
Protons per EDM pulse	1.5×10^{9}
Good’ pions/muons off the target	18000
Muon storage duration	10 lifetimes = 83 microseconds
Muons stored per year	9×10^{11}

Fermilab

Beam Dynamics

The next question: with both electric and magnetic dipole fields, can we have a stable closed orbit inside the ring with frozen MDM precession conditions?

The answer is: YES!

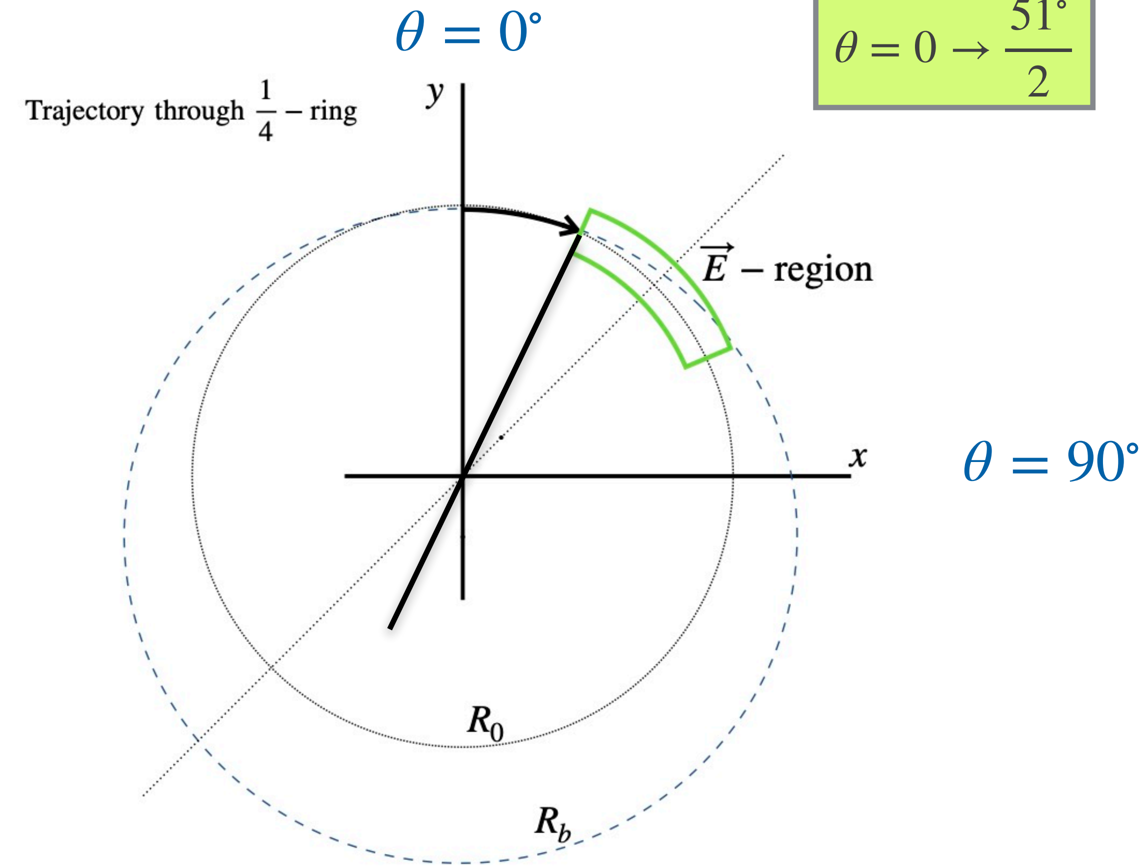
Only that the stable closed orbit will not be a perfect circle anymore.

Let us assume our 'quarter section' starts with the second half of a '$\vec{B}$ -only' section.

Since there is only $\vec{B}$-field, the particle will orbit with a radius R_B , given by

$$R_B = \frac{\gamma m v^2}{q v B}$$

$\theta = 0 \rightarrow \frac{51^\circ}{2}$



Closed Orbit - Geometric Analysis

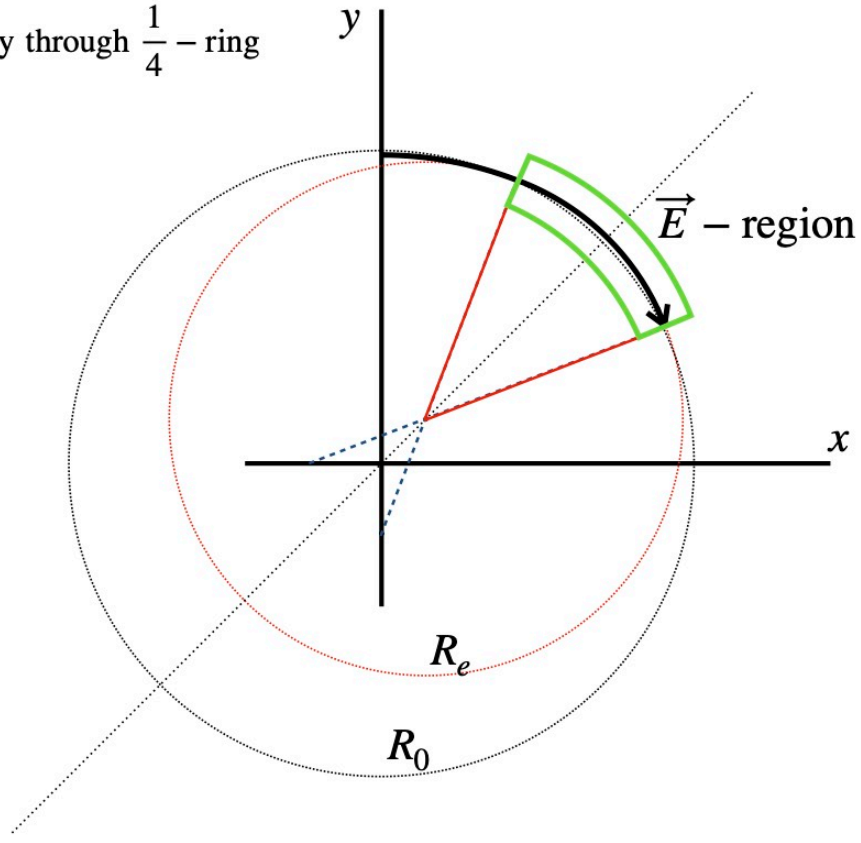
As the particle next enters the $(\vec{E} + \vec{B})$ region, it is going to orbit in a circle again but of a tighter radius R_E , given by,

$$R_e = \frac{\gamma m v^2}{qvB + qE}$$

$\theta = 0^\circ$

$$\boxed{\theta = \frac{51^\circ}{2} \rightarrow \frac{51^\circ}{2} + 39^\circ}$$

Trajectory through $\frac{1}{4}$ – ring



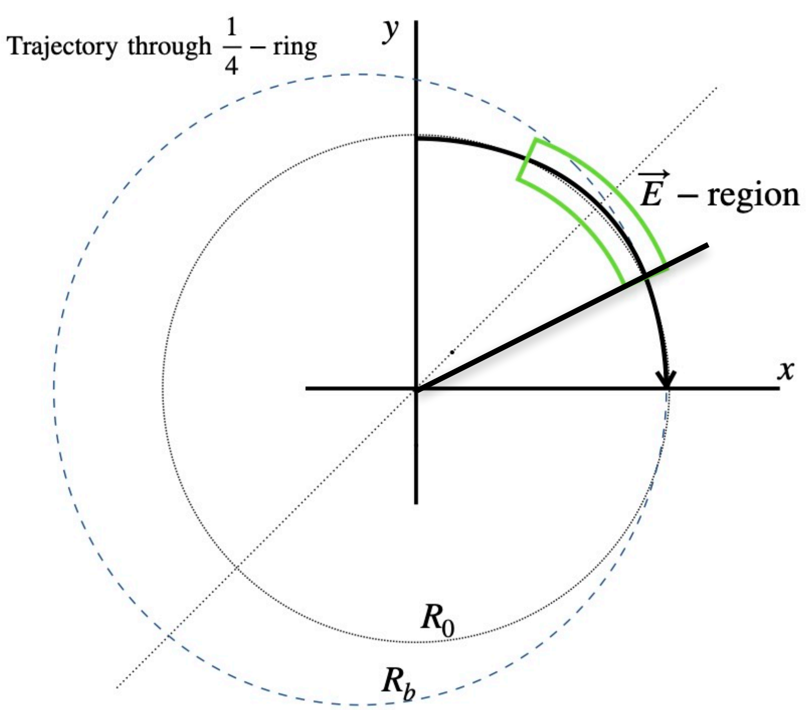
$\theta = 90^\circ$

Because of the tighter radius, the particle will 'curve in' with respect to the 7.112 m orbit.

Closed Orbit - Geometric Analysis

As the particle comes out of the $\vec{E} + \vec{B}$ region and enters the $\vec{B}$ only region, it follows a circle of radius R_b once again.

$\theta = 0^\circ$



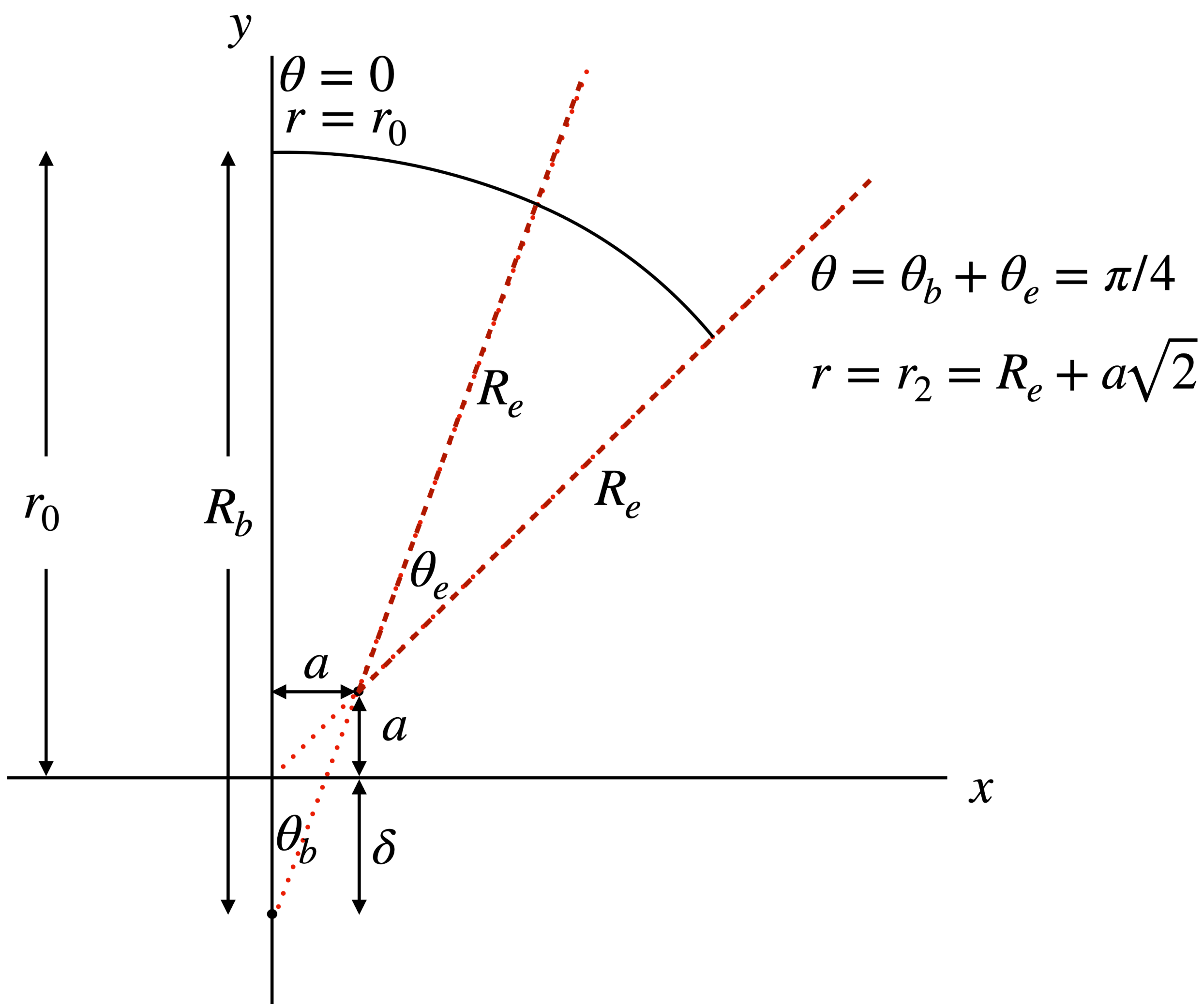
$$\theta = \frac{51^\circ}{2} + 39^\circ \rightarrow 90^\circ$$

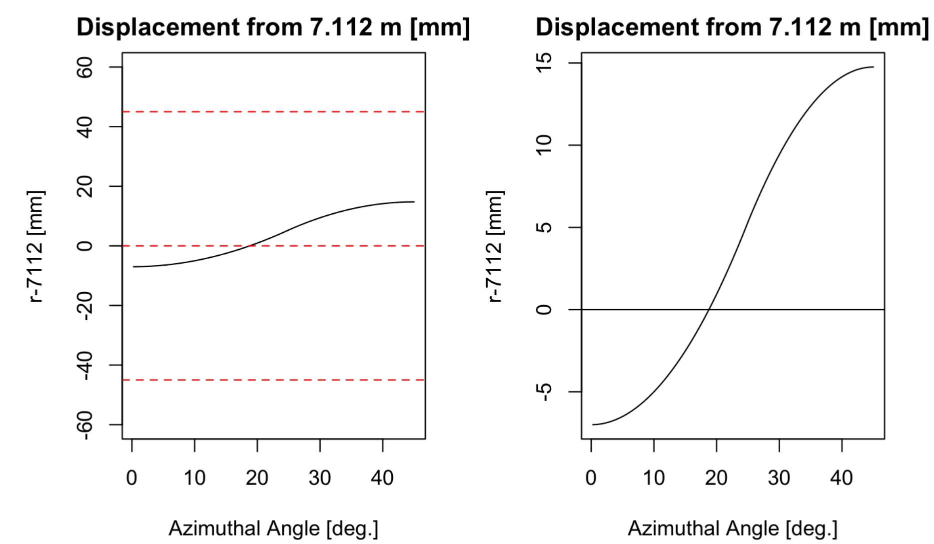
$\theta = 90^\circ$

If we were to plot the closed orbit with respect to the 7.112 m orbit, we would have a 'wiggle'.

But how large are these deviations from the 7.112 meter orbit?

Closed Orbit - Geometric Analysis





With some geometric analysis, we find that,

$$r_0 = R_b[1 - \cos(\theta_b) + \sin(\theta_b)] + R_e[\cos(\theta_b) - \sin(\theta_b)]$$

$$r_2 = \sqrt{2}R_b \sin\theta_b + R_e(1 - \sqrt{2}\sin\theta_b)$$

For example, with the parameters of

$\vec{p} = 387$ MeV/c,
$\vec{E} = 1.98$ MV/m,
$\vec{B} = 0.178$ T,

the maximum radial orbital variations from the 7.112 meters circular orbit are only ± 10.9 mm!

Closed Orbit - 4th Order Runge-Kutta simulation

One could verify the previous geometric analysis with actual particle tracking to see if we indeed can have a stable closed orbit.

A particle tracking simulation was thus done by solving the coupled differential Lorentz equations in the $\overrightarrow{B}$ only region and $(\overrightarrow{E} + \overrightarrow{B})$-region for various momenta values at a time step of 1 nanosecond.

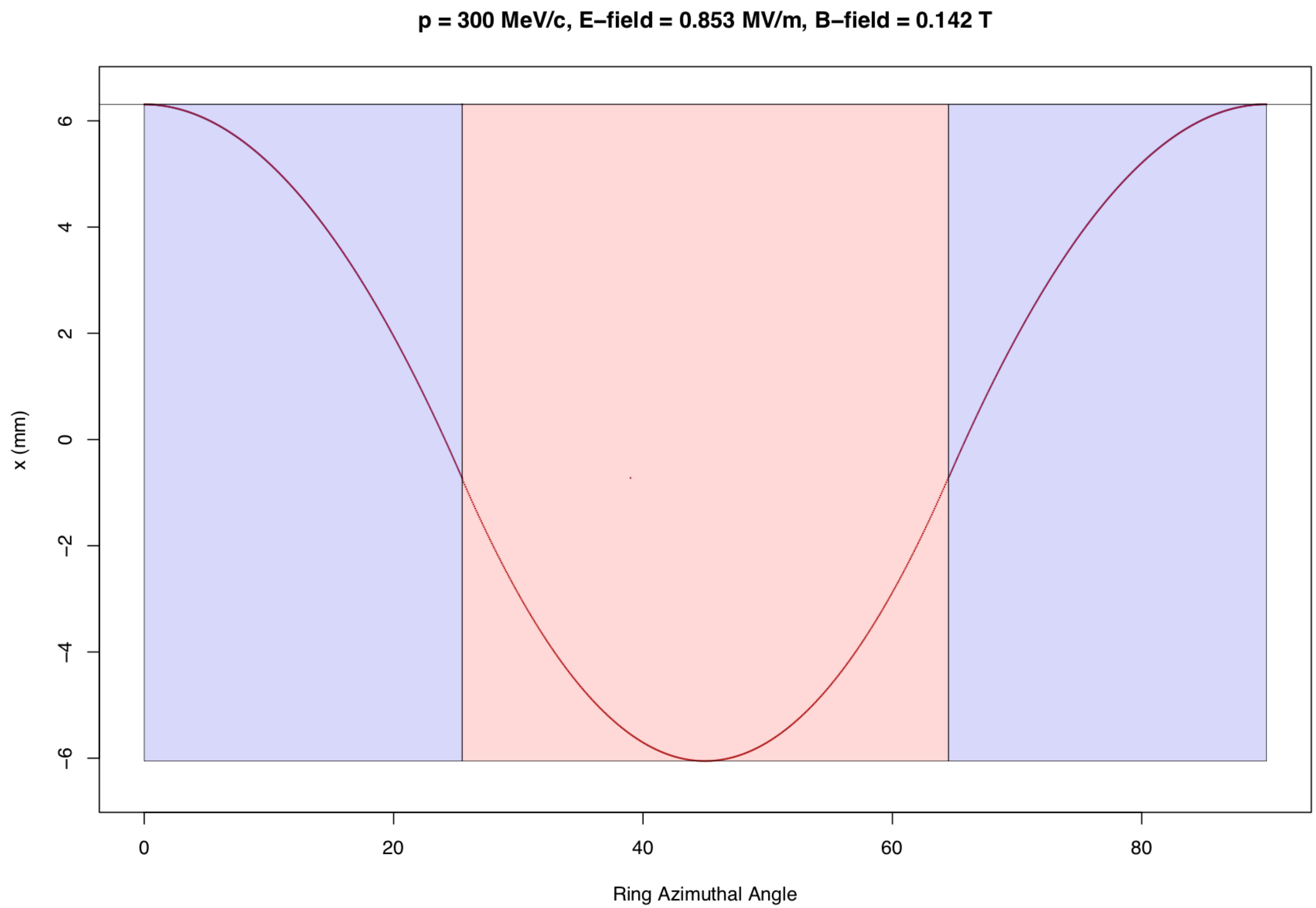
$$\frac{dx_\circ}{dt} = v_{x_\circ} \qquad \frac{dy_\circ}{dt} = v_{y_\circ}$$

$$\frac{dv_{x_\circ}}{dt} = \frac{q}{m}E\cos\theta + \frac{q}{m}v_{y_\circ}B_z \qquad \frac{dv_{y_\circ}}{dt} = \frac{q}{m}E\sin\theta - \frac{q}{m}v_{x_\circ}B_z$$

where $x_\circ$ and $y_\circ$ are the coordinates in the horizontal plane of the ring with (0,0) being the centre of the ring..

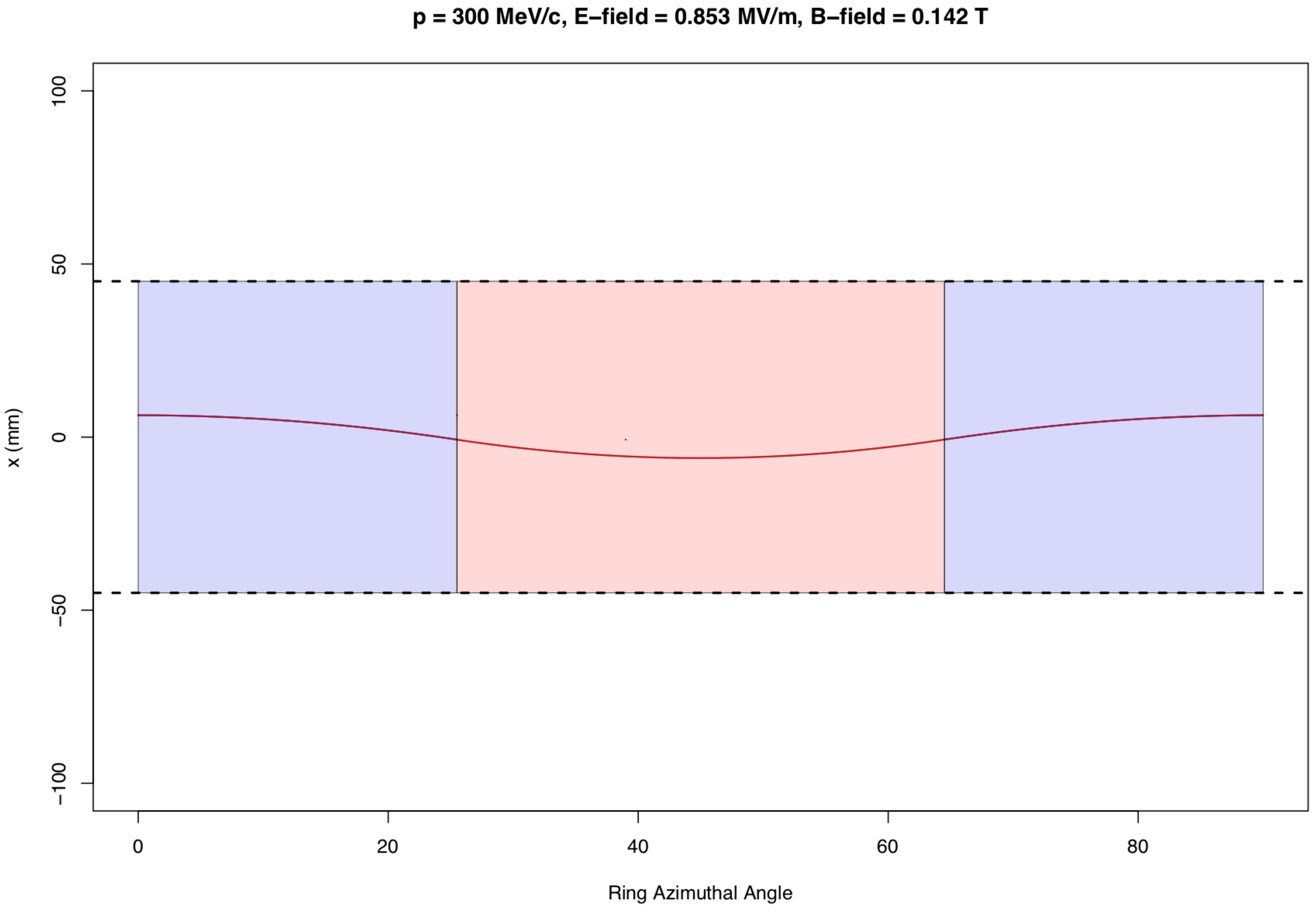
Closed Orbit - 4th Order Runge-Kutta simulation - Results

RK4 simulation validated our geometric analysis!

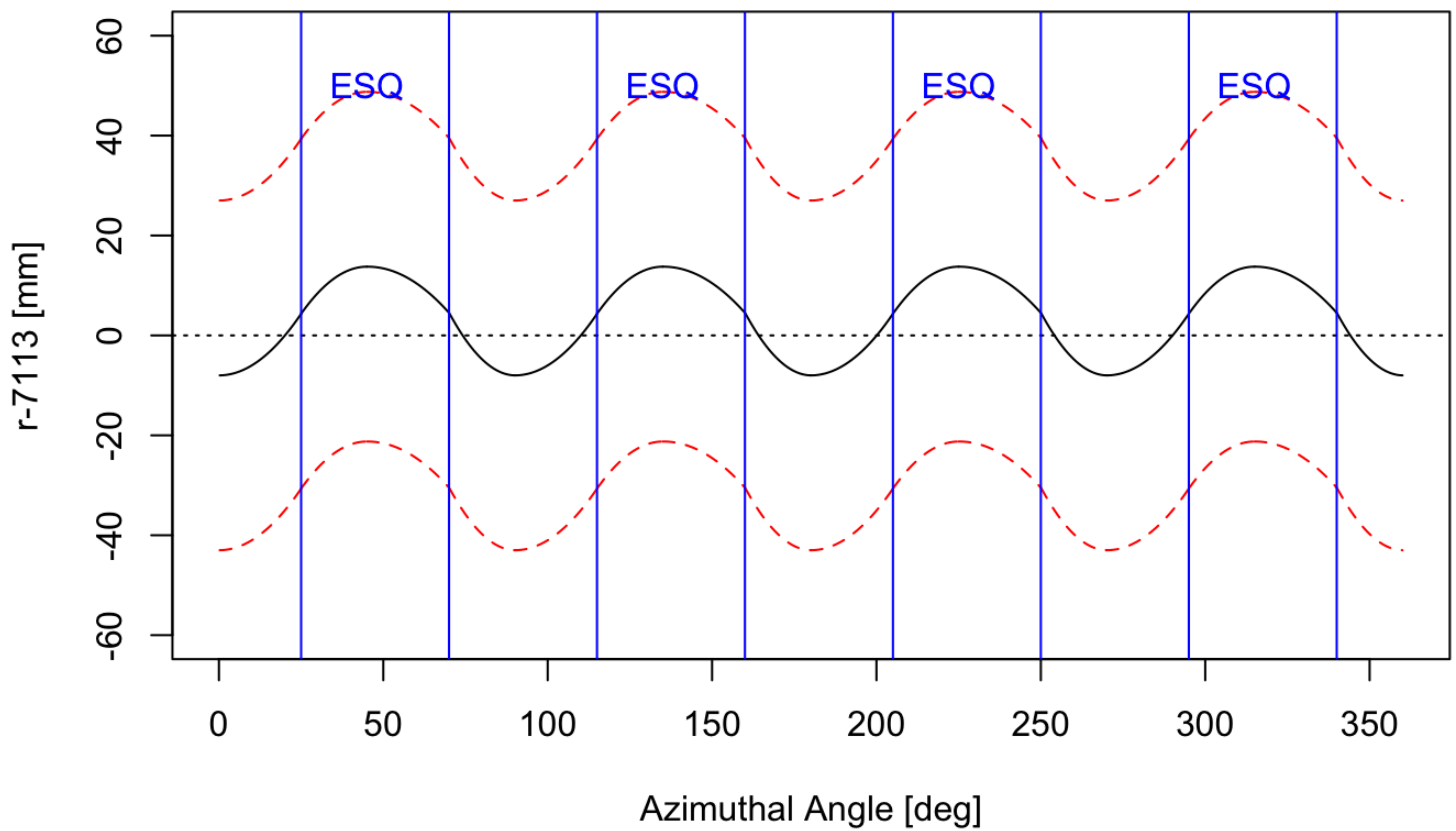


Closed Orbit - 4th Order Runge-Kutta simulation

With a ±50 mm aperture, the scale of a typical closed orbit would look like:



Possible rearrangement of cylindrical vacuum chamber



EDM Precession Estimates

Unlike the MDM, the precession due to EDM at a given point in the ring is going to keep constantly building up until the muon decays.

Since we have two distinct regions within a quarter section, the rate of precession will *slightly* vary within the $\overrightarrow{B}$-only section and the $(\overrightarrow{E} + \overrightarrow{B})$-section, albeit by a small factor.

The total precession through a half-quadrant will be:

$$\Delta\phi_{EDM} = \frac{d}{S} . B_0 \left[\ell_b + \ell_e \left(1 + \frac{E_0}{B_0 \beta c} \right) \right]$$

If we plug in the an appropriate set of field values for a momentum range of 300 MeV/c and the path lengths in the respective sections, we see that the EDM precesses in the order of 10 mrad for 5 muon lifetimes!

Fermilab

Quick Summary

We could envision having an EDM experiment in the `g-2 storage ring by constructing a new quadrupole system to create a radial electric dipole field pointing radially inward.

The new system would not be significantly different from the current set-up, other than

- the radius of curvature for the plates would be different,
- the inner/outer plates would be at higher potentials than the upper/lower plates in order to create the electric dipole field.

Electric field levels of 1 MV/m are not technically difficult with potentials of about ±40 kV for plate separations on the scale of 30 to 50 mm.

PIP-II could provide intense high-power 300 MeV/c range muons, facilitating higher EDM statistics.

Since the muons' central momentum could be around 300 MeV/c, it makes the requirements for the existing magnetic dipole field, inflector system, and kicker system reduce by a factor of ten!

Muons' central orbit would still remain 7112 mm but with deviations of just ±7 mm.

Quick Summary

Parameter	Value	Unit
Muon Momentum	387	MeV/c
Magnetic Field	0.178	T
Radial Electric Field	-1.98	MV/m
Plate Separation	$\pm$ 35	mm
Plate Voltage	$\pm$ 69.283	kV
Quadrupole Gradient	TBD	MV/m/m
Central Orbit Radius	7112	mm
Radial Orbit Deviations	$\pm$ 10.9	mm
Ring Admittance (Horiz., central momentum)	153	π mm-mrad

Next up…

- What is the natural first-order focusing due to set of curved ‘dipole’ plates?
- What quadrupole gradient would we need? What tunes to choose (especially vertical tune)?
- What would be the expansion coefficients of the E-field due to plate distortions and misalignments? What are its effect on EDM measurement?
- How bad can the radial and azimuthal magnetic field be?
- To what accuracy must the E-field be measured in the ring? And how to do it?
- Detector related and other systematics.