Special thanks to J. Mott, D. Pocanic, M. Passera for discussion and T.Q. Hu and Y.K. Hu for the preparation of this talk.

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Radiative Muon Decay

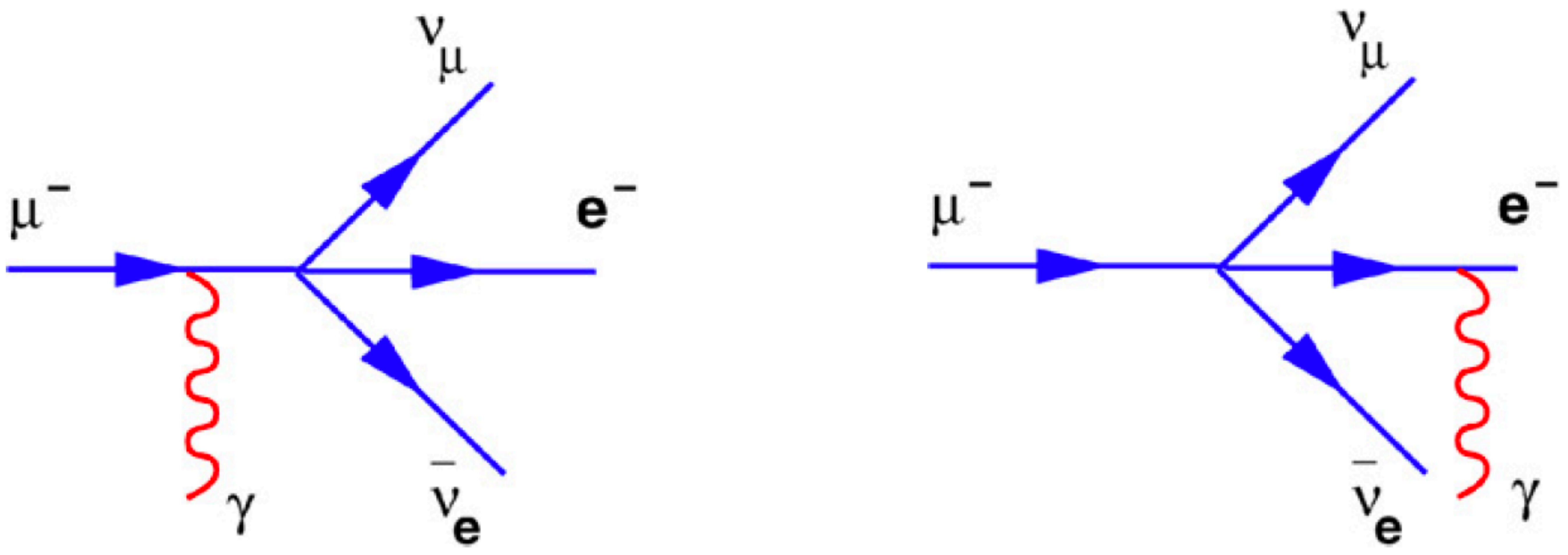
Kim Siang Khaw

Virtual Workshop on Potential Fermilab Muon Campus & Storage Ring Experiments

2021-05-25

Content

- Introduction to Radiative Muon Decay (RMD)
- Current Status: SM prediction and measurement
- Muon g-2 experiment and RMD
- Geant4 study of RMD
- Future Prospects and summary



Muon Decay

- Muon Decay provides a precision test of the SM
  - search for a departure from the weak V-A structure through the famous Michel parameters and decay branching ratios
- Current muon decay branching ratios (from PDG)
  - $\mathcal{B}(\mu^+ \to e^+\nu\bar{\nu})$ (1.0)
  - $\mathcal{B}(\mu^+ \to e^+\nu\bar{\nu}\gamma)$ (~ 0.014)
  - $\mathcal{B}(\mu^+ \to e^+\nu\bar{\nu}e^+e^-)$ (~ 3 x 10^{-5})
  - Any discrepancy in the SM-experiment hints at NP
- Access to $\bar{\eta}$ is possible only via Radiative Muon Decay
  - $\bar{\eta} = 0$ in the SM prediction (any deviation = NP)

$$\frac{d^2\Gamma}{dx\, d(\cos\theta)} = \frac{m_\mu}{4\pi^3} W_{e\mu}^4 G_F^2 \sqrt{x^2 - x_0^2} \times \times \left[\mathbf{F_{IS}}(x) + P_{\mu^+}\cos\theta\, \mathbf{F_{AS}}(x)\right]\left[1 + \vec{P}_{e^+}(x,\theta)\cdot\hat{\zeta}\right]$$

Isotropic part:

$$\mathbf{F_{IS}}(x) = x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta\, x_0(1-x)$$

Anisotropic part:

$$\mathbf{F_{AS}}(x) = \frac{1}{3}\xi\sqrt{x^2 - x_0^2}\left(1 - x + \frac{2}{3}\delta\left[4x - 3 + \left(\sqrt{1 - x_0^2} - 1\right)\right]\right)$$

where $x = \frac{E_e}{E_{max}}$, $x_0 = \frac{m_e}{E_{max}}$, and $E_{max} \simeq \frac{m_\mu}{2}$, .

$$\frac{d^3B(x,y,\theta)}{dx\, dy\, 2\pi\, d(\cos\theta)} = f_1(x,y,\theta) + \bar{\eta} f_2(x,y,\theta) + (1 - \frac{4}{3}\rho) f_3(x,y,\theta)$$

$$\rho = \frac{3}{4} - \frac{3}{4}\Big[|g_{LR}^V|^2 + |g_{RL}^V|^2 + 2|g_{LR}^T|^2 + 2|g_{RL}^T|^2 + \Re(g_{RL}^S g_{RL}^{T*} + g_{LR}^S g_{LR}^{T*})\Big] \overset{\text{SM}}{\equiv} \frac{3}{4},$$

$$\bar{\eta} = \left(|g_{RL}^V|^2 + |g_{LR}^V|^2\right) + \frac{1}{8}\left(|g_{LR}^S + 2g_{LR}^T|^2 + |g_{RL}^S + 2g_{RL}^T|^2\right) + 2\left(|g_{LR}^T|^2 + |g_{RL}^T|^2\right) \overset{\text{SM}}{\equiv} \mathbf{0}.$$

where $x = \frac{E_e}{E_{max}}$ and $y = \frac{E_\gamma}{E_{max}}$.

Courtesy D. Pocanic

Historical remark on RMD branching ratio

- The first analytical expression was derived by A. Lenard in 1953
- The first calculation were performed by Kinoshita(!) and Sirlin in 1959
  - $\mathscr{B}(\mu^+ \to e^+\nu\bar{\nu}\gamma) \sim 4.9\,\%$, for $\mathrm{E}_\gamma > 2\mathrm{m_e}$
  - $\mathscr{B}(\mu^+ \to e^+\nu\bar{\nu}\gamma) \sim 1.2\,\%$, for $\mathrm{E}_\gamma > 20\mathrm{m_e}$

RMD branching ratio is infrared divergent, a well defined measurement requires a region of the phase space which includes a lower limit on the photon energy!

- The first measurement then followed in 1961 by Crittenden *et al.* using a 5-inch freon bubble chamber.
- *Four decay modes were searched for:*
  - $\mu^+ \to e^+\gamma$
  - $\mu^+ \to e^+e^-e^+$
  - $\mu^+ \to e^+\nu\bar{\nu}\gamma$ *(RMD)*
  - $\mu^+ \to e^+\nu\bar{\nu}e^+e^-$ *(Internal Conversion)*

$$R_{\mathrm{IB}} = (\mu^+ \to e^+ + \nu^0 + \bar{\nu}^0 + \gamma)/(\mu^+ \to e^+ + \nu^0 + \bar{\nu}^0),$$
$$R_{\mathrm{IB}} = (1.4 \pm 0.4) \times 10^{-2}, \quad E_{\gamma 0} = 10\ \mathrm{Mev},$$
$$R_{\mathrm{IB}} = (3.3 \pm 1.3) \times 10^{-3}, \quad E_{\gamma 0} = 20\ \mathrm{Mev}.$$

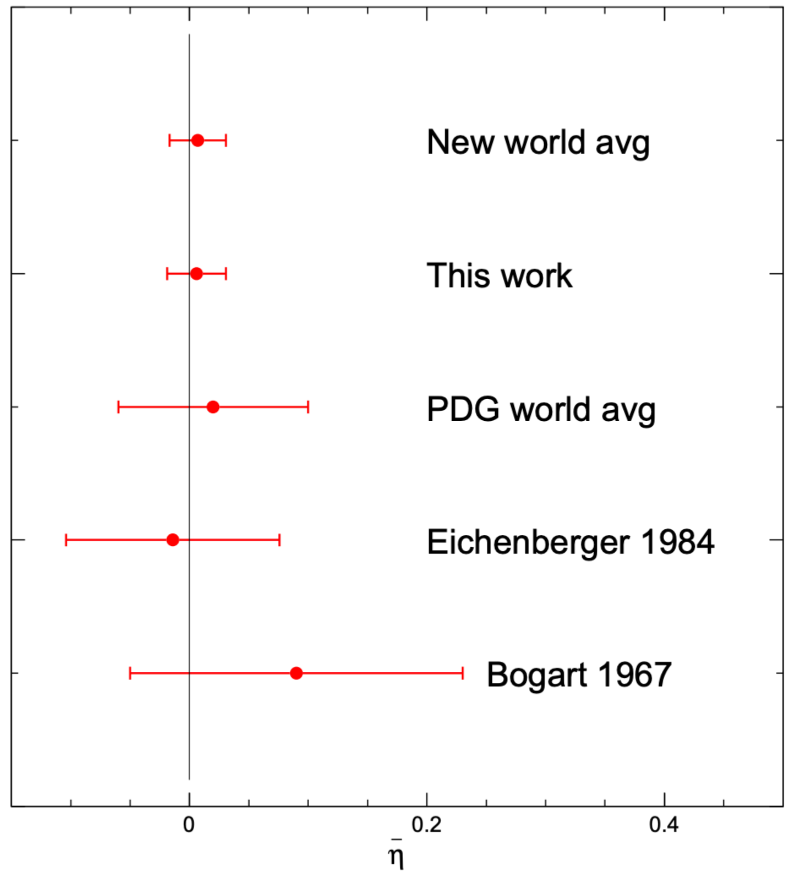
RMD also called internal bremsstrahlung (IB)

Current status of RMD: PIBETA

$$\frac{\mathrm{d}^3 B(x,y,\theta)}{\mathrm{d}x\mathrm{d}y 2\pi \mathrm{d}(\cos\theta)} = f_1(x,y,\theta) + \bar{\eta} f_2(x,y,\theta) + (1 - \frac{4}{3}\rho) f_3(x,y,\theta),$$

$$x = 2E_{e^+}/m_\mu, \quad y = 2E_\gamma/m_\mu, \quad \cos\theta = \hat{p}_{e^+} \cdot \hat{p}_\gamma$$

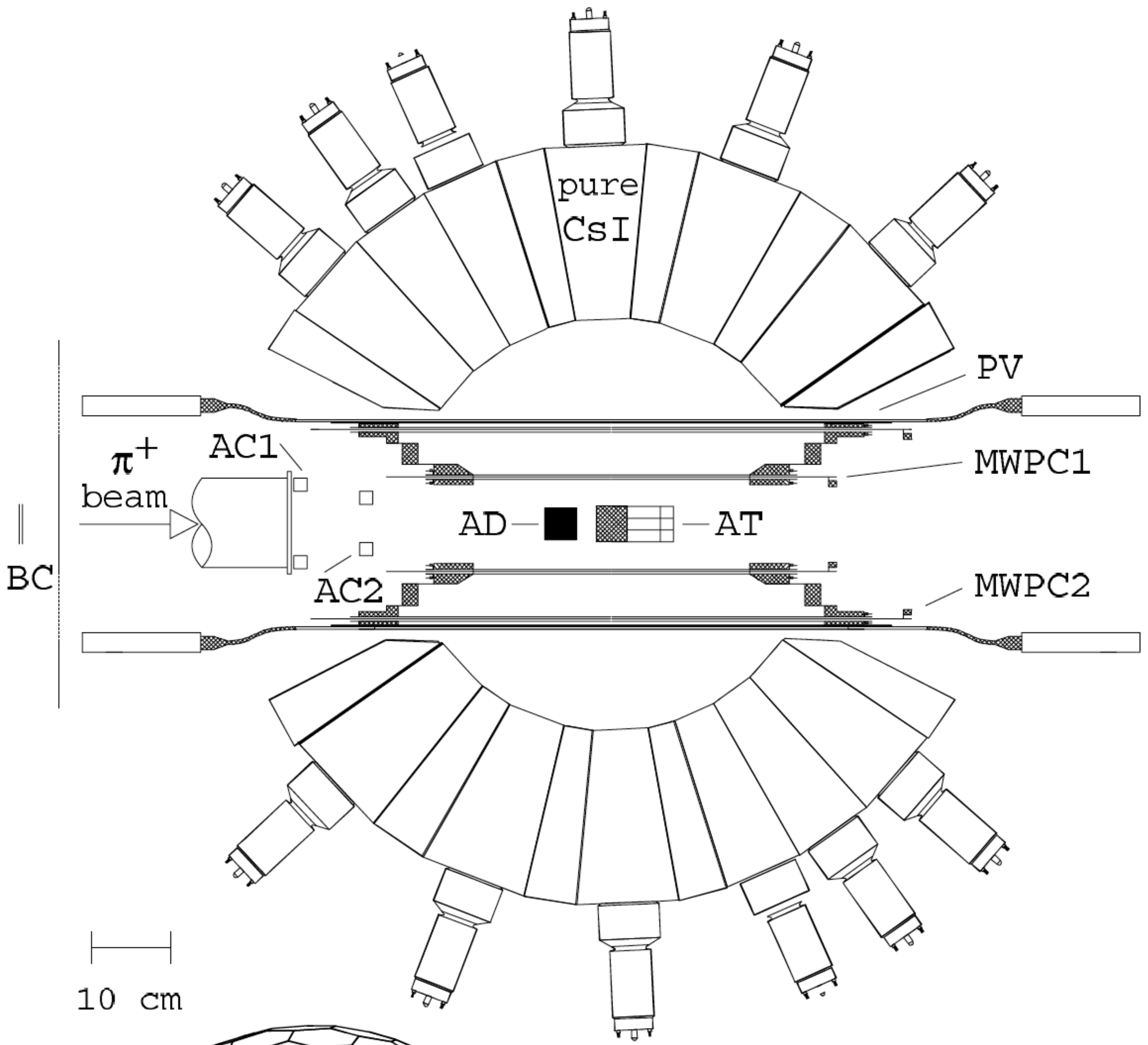
- In 2014, PIBETA collaboration provided a preliminary BR measurement
  - 0.5M events analyzed
  - $\mathscr{B}(\mu^+ \to e^+ \nu\bar{\nu}\gamma) = 4.365(9)_{\mathrm{stat}}(42)_{\mathrm{syst}} \times 10^{-3}$, for $\mathrm{E}_\gamma > 10$ MeV, $\theta_{e\gamma} > 30°$
  - SM (Pruna17): $\mathscr{B}(\mu^+ \to e^+ \nu\bar{\nu}\gamma) = 4.228(1) \times 10^{-3}$
  - There seems to be a tension!
- The value of $\bar{\eta}$ was also measured: $\bar{\eta} = 0.006 \pm 0.017\,(\mathrm{stat.}) \pm 0.018\,(\mathrm{syst.})$ consistent with 0 (SM)

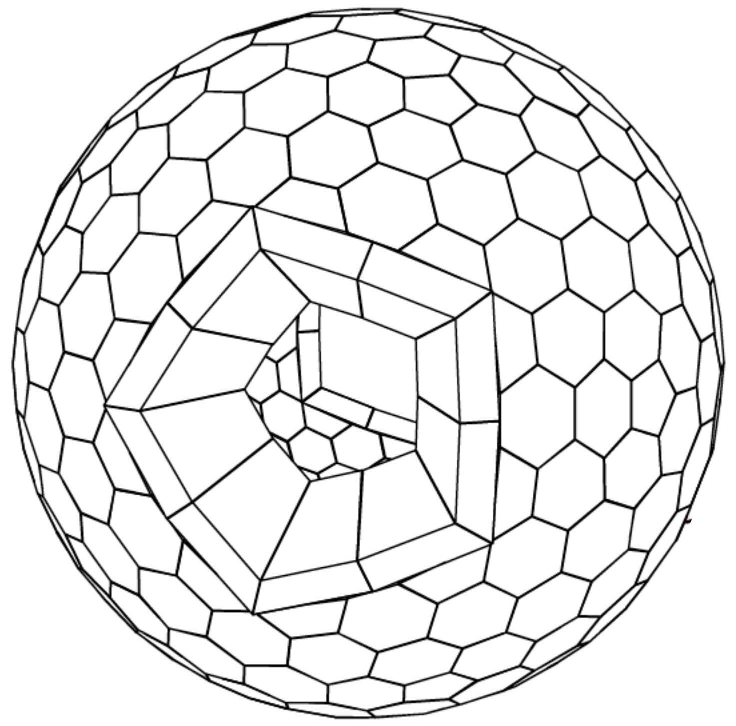


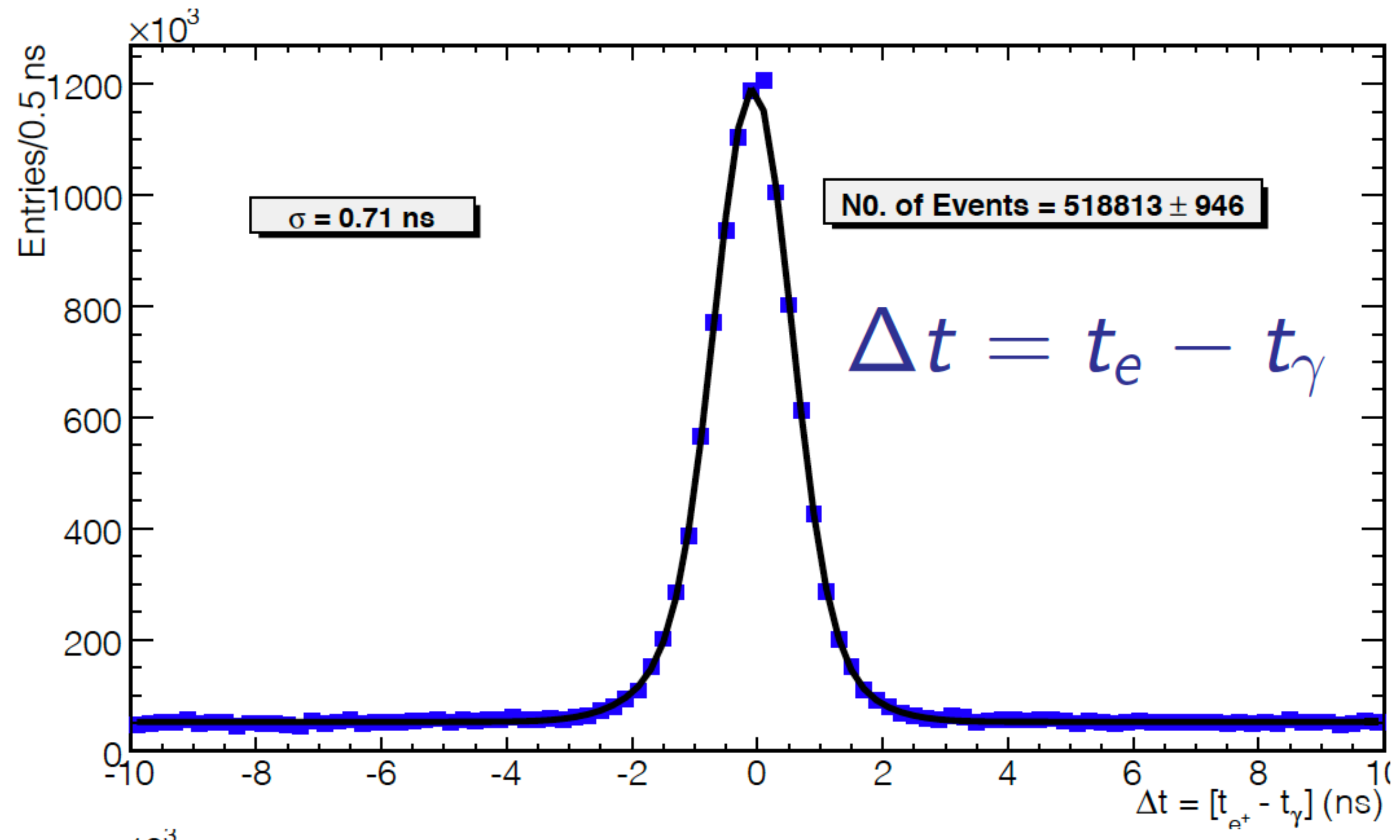
D. Pocanic et al., Int. J. Mod. Phys. Conf. Ser. 35 (2014) 1460437
Pruna et al., Phys. Lett. B 772 (2017) 452-458
Fael et al., JHEP 07 (2015) 153

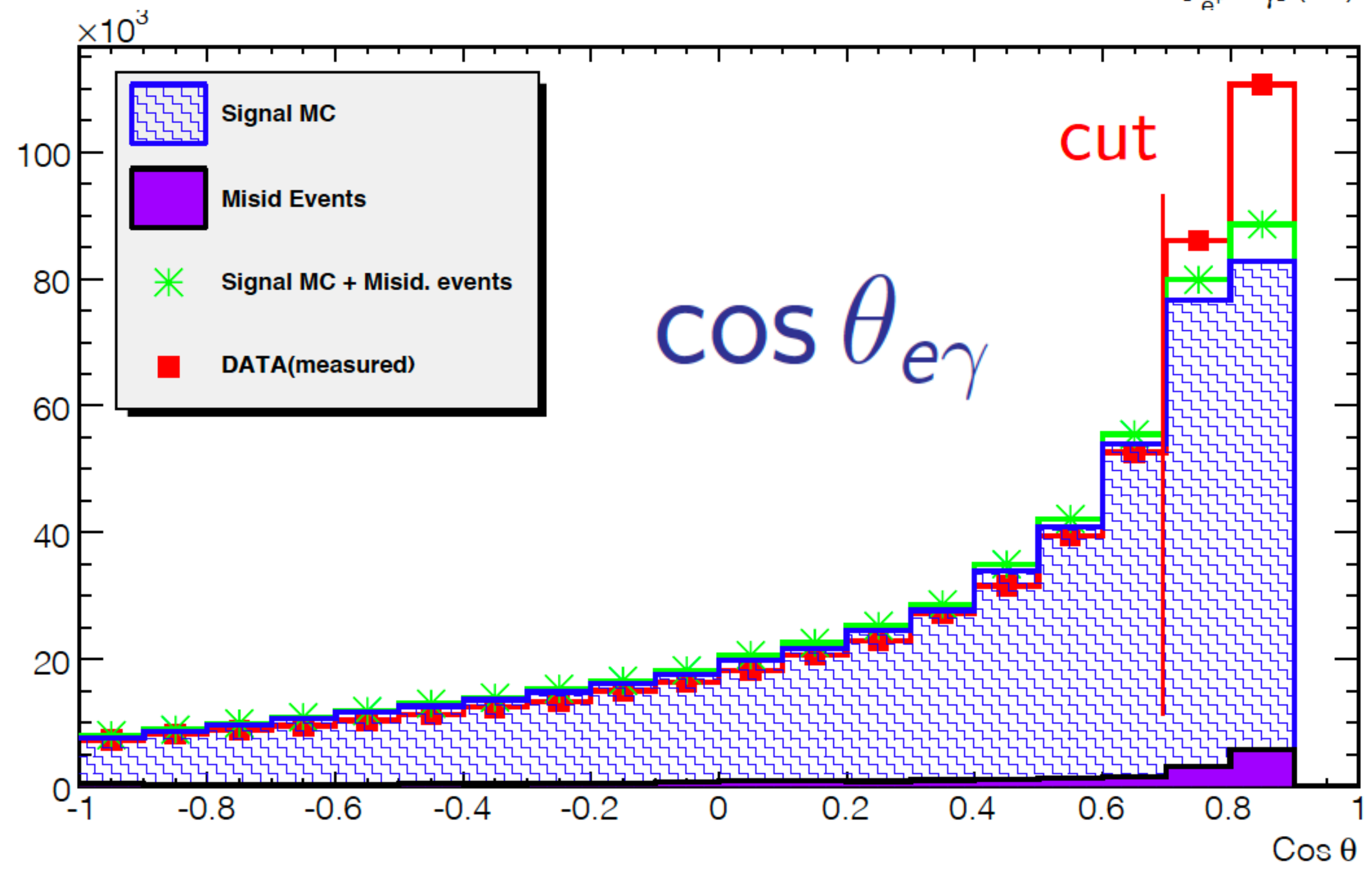
PIBETA measurement

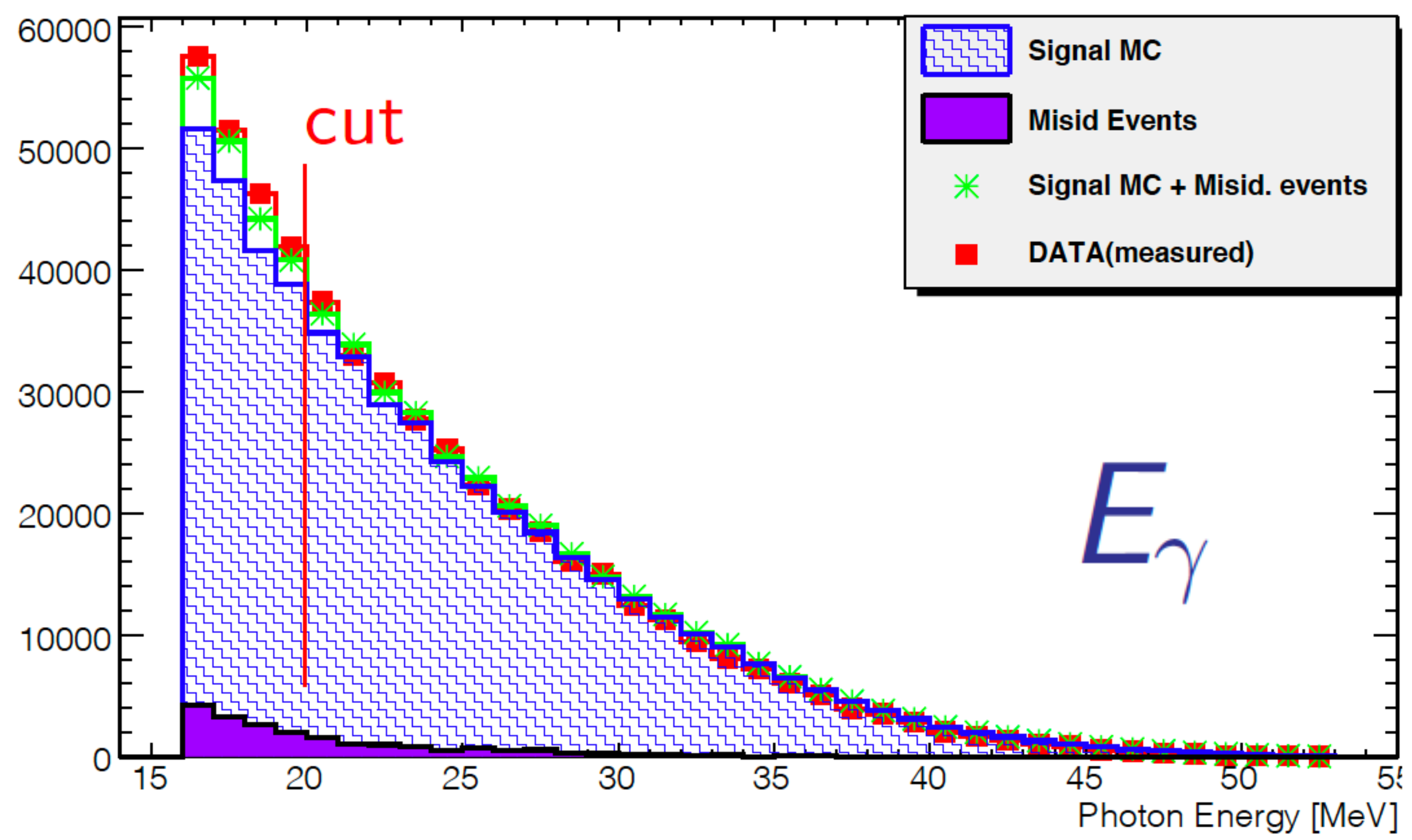
Together with the upgraded version of the experiment, PEN, a final result will be published in the future.

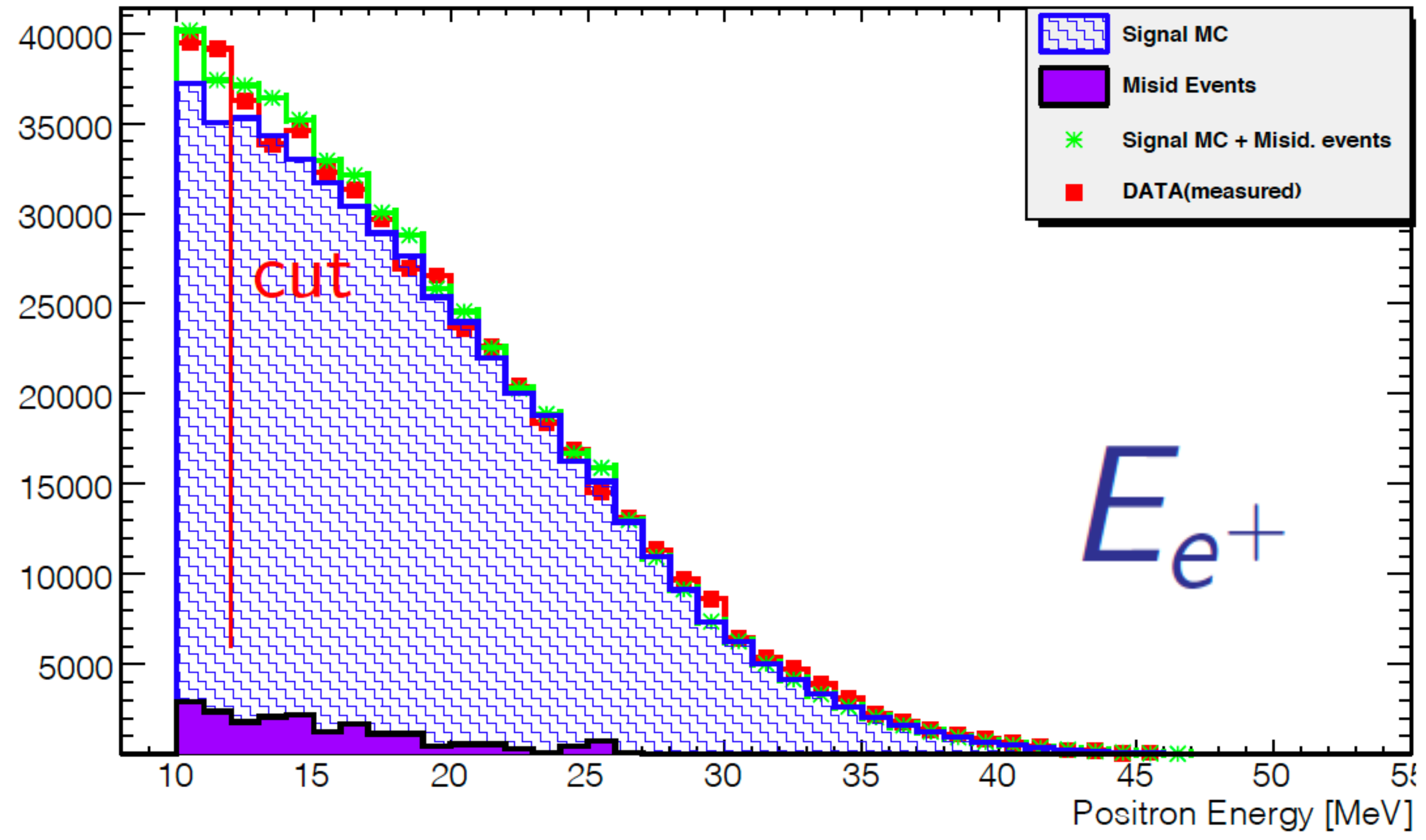












$$\mathscr{B}(\mu^+ \to e^+\nu\bar{\nu}\gamma) = 4.365(9)_{\mathrm{stat}}(42)_{\mathrm{syst}} \times 10^{-3}, \text{ for } E_\gamma > 10 \text{ MeV}, \ \theta_{e\gamma} > 30^\circ$$

D. Pocanic et al., Int. J. Mod. Phys. Conf. Ser. 35 (2014) 1460437

Current status of RMD: MEG

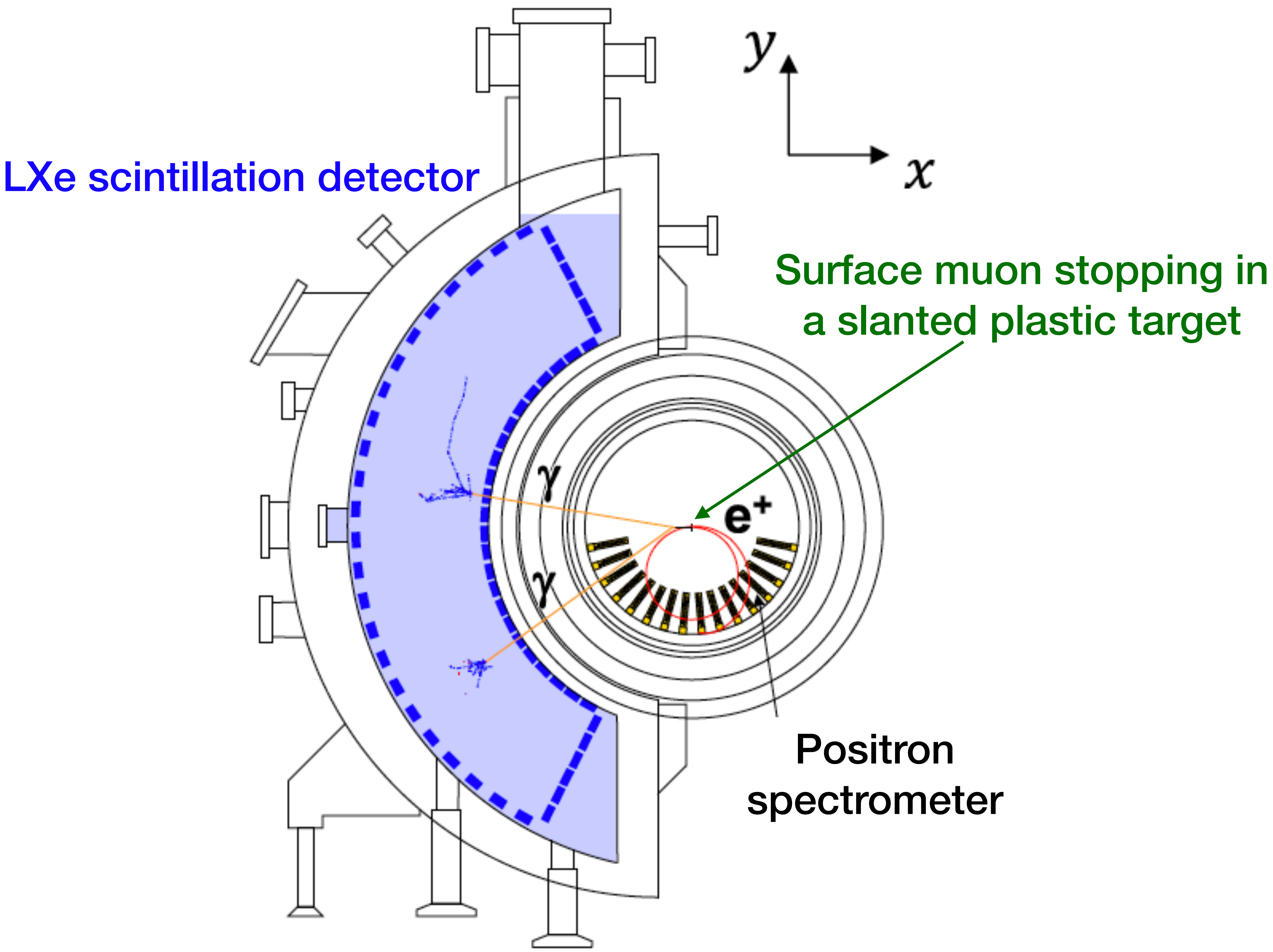
$$\boxed{\frac{\mathrm{d}^3 B(x,y,\theta)}{\mathrm{d}x\mathrm{d}y 2\pi \mathrm{d}(\cos\theta)} = f_1(x,y,\theta) + \bar{\eta} f_2(x,y,\theta) + (1-\frac{4}{3}\rho) f_3(x,y,\theta),}$$

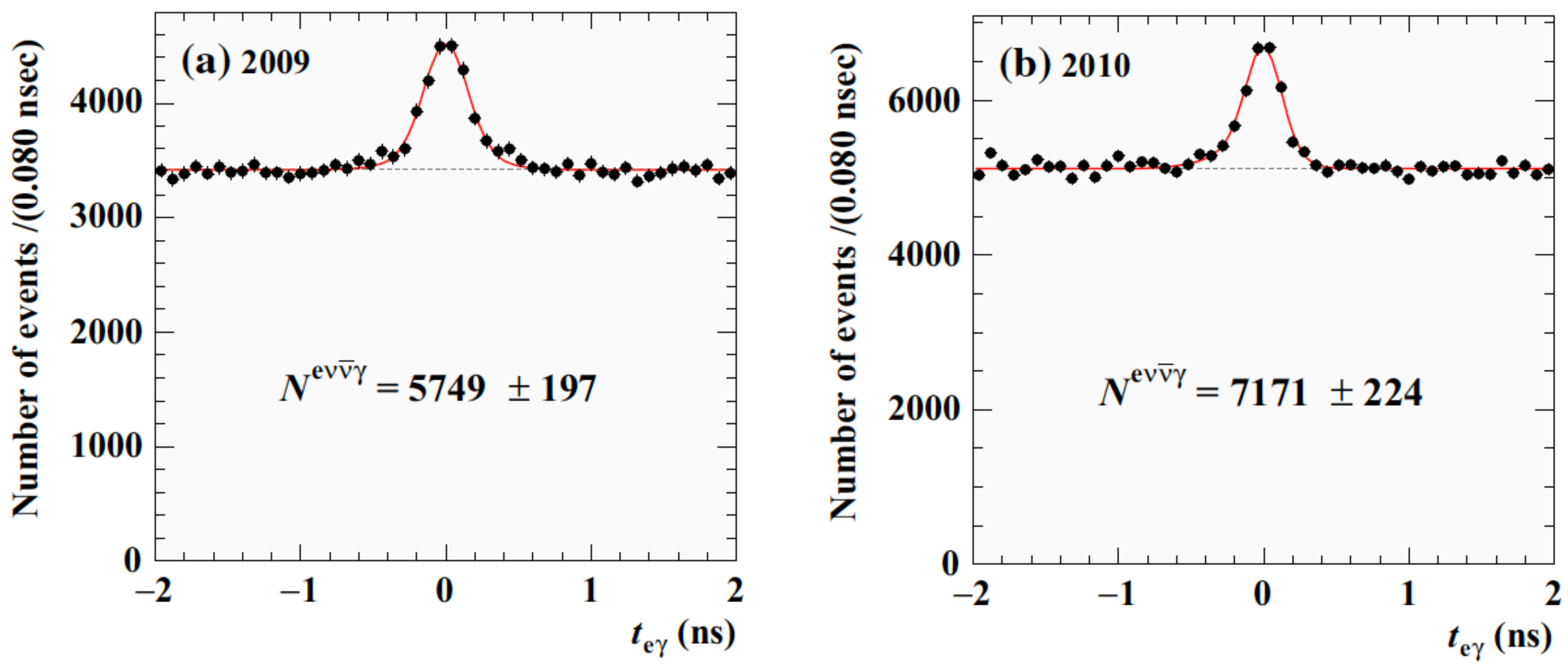
$$x = 2E_{e^+}/m_\mu, \quad y = 2E_\gamma/m_\mu, \quad \cos\theta = \hat{p}_{e^+}\cdot\hat{p}_\gamma$$

- In 2016, MEG collaboration provided a BR in a different kinematic region
  - 13,000 events analyzed (from 1.8 x 10^{14} decays)
  - $\mathscr{B}(\mu^+ \to e^+\nu\bar{\nu}\gamma) = 6.03(14)_{\text{stat}}(53)_{\text{syst}} \times 10^{-8}$, for $E_e > 45$ MeV, $E_\gamma > 40$ MeV
  - SM (Pruna17): $\mathscr{B}(\mu^+ \to e^+\nu\bar{\nu}\gamma) = 5.850(1) \times 10^{-8}$

A.M. Baldini et al., Eur. Phys. J. C (2016) 76:108
Fael et al., JHEP 07 (2015) 153
Pruna et al., *Phys. Lett. B* **772** (2017) 452-458

MEG measurement





$$\mathcal{B}(\mu \to e\nu\bar{\nu}\gamma) = \frac{N^{e\nu\bar{\nu}\gamma}}{N_\mu \times \langle \epsilon^{e\nu\bar{\nu}\gamma} \rangle}$$

$$= N^{e\nu\bar{\nu}\gamma} \times \left(\frac{f^{e\nu\bar{\nu}}_{E_e}}{N^{e\nu\bar{\nu}}} \times \frac{\epsilon^{e\nu\bar{\nu}}_{\mathrm{trg}}}{p^{e\nu\bar{\nu}}} \right) \times \frac{\langle \epsilon^{e\nu\bar{\nu}}_{e} \rangle}{\langle \epsilon^{e\nu\bar{\nu}\gamma}_{e} \rangle}$$

$$\times \frac{1}{\langle \epsilon^{e\nu\bar{\nu}\gamma}_{\gamma} \rangle} \times \frac{1}{\langle \epsilon^{e\nu\bar{\nu}\gamma}_{\mathrm{trg}} \rangle},$$

$\mathscr{B}(\mu^+ \to e^+\nu\bar{\nu}\gamma) = 6.03(14)_{\mathrm{stat}}(53)_{\mathrm{syst}} \times 10^{-8}$, for $E_e > 45$ MeV, $E_\gamma > 40$ MeV

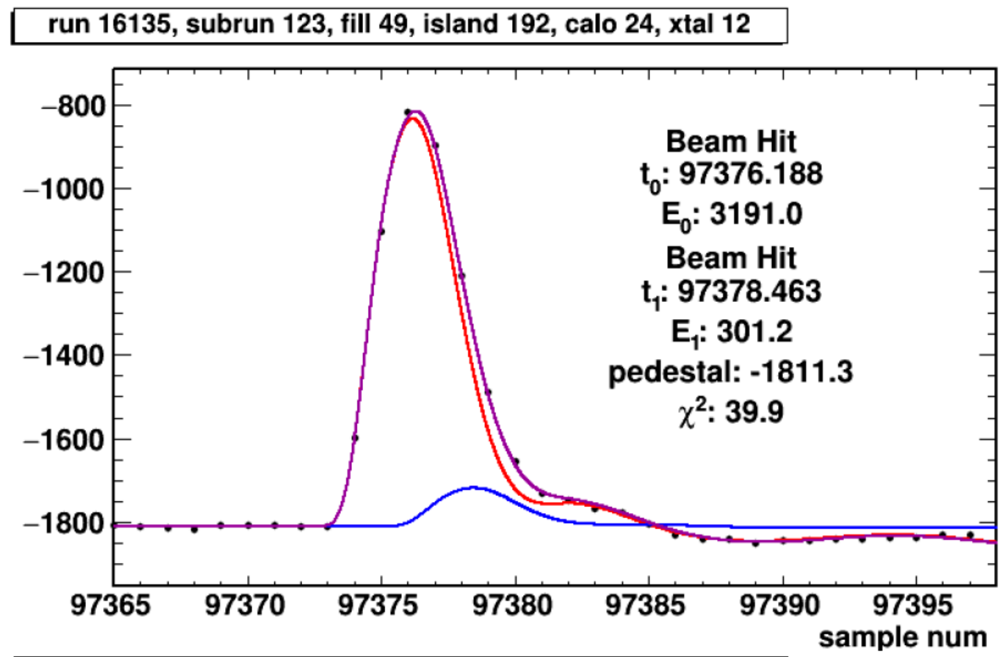
A.M. Baldini et al., Eur. Phys. J. C (2016) 76:108

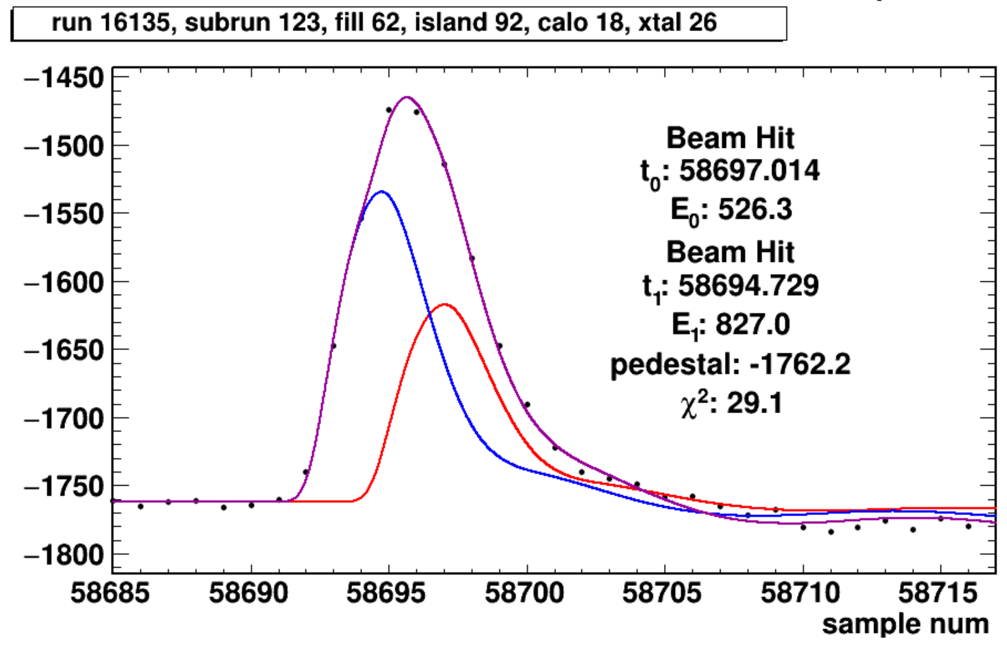
Features of modern RMD experiments

- Large area detectors (large acceptance), e.g. 3π sr (PIBETA/PEN)
- Probed only a tiny fraction of the kinematic phase space of RMD
    - Remember ~5% from Kinoshita prediction
    - Energy and angle cuts from MEG and PIBETA experiments $(10^{-8} - 10^{-3})$
    - Challenging to detect soft electrons or photons
- Is there a way to probe a wider kinematic phase space of RMD?
    - Yes! By using relativistic muons (from Muon g-2 experiment)
    - Both positrons and photons are boosted (gamma 10 MeV —> 600 MeV!)
    - However, acceptance and normalization might be issues for this method
    - In any case, let’s look at the event topology for RMD in the g-2 experiment

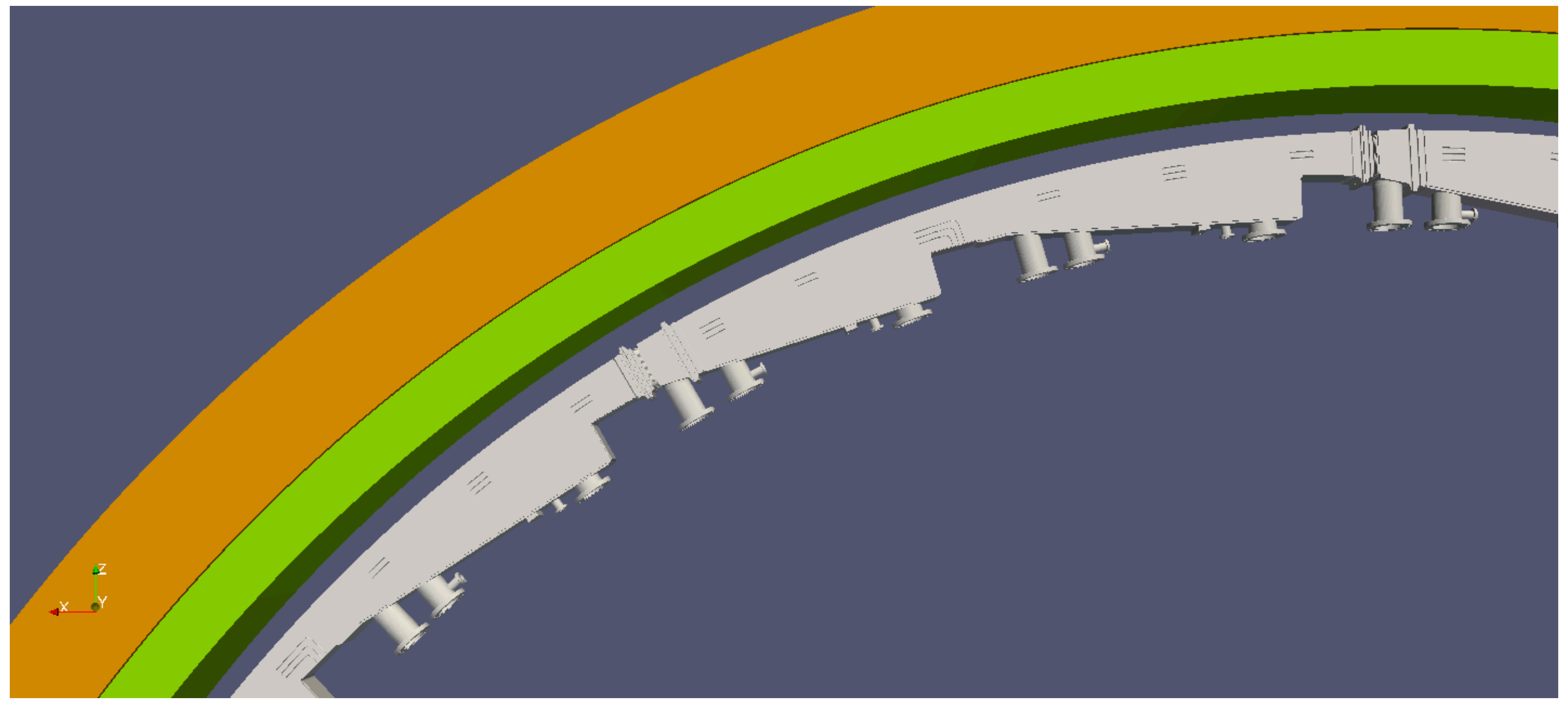
How did RMD came into play?

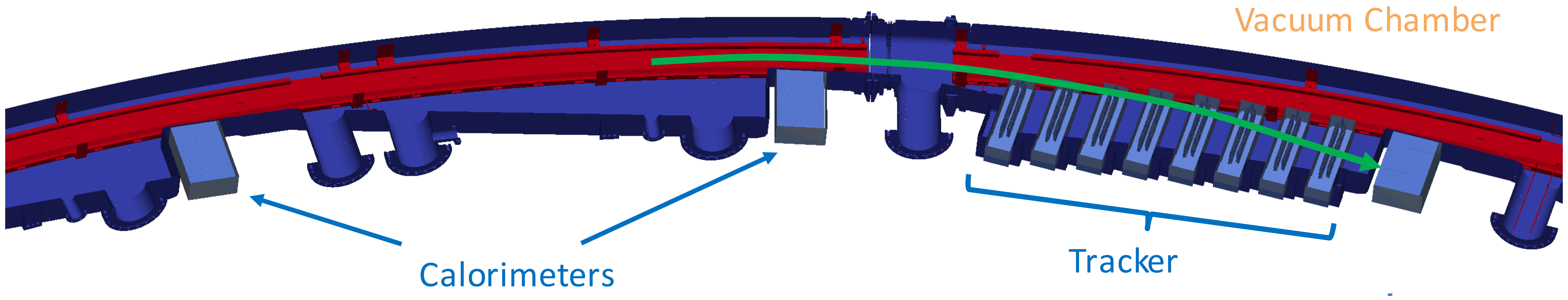
- It all started when I was investigating a data-driven method to extract detector gain vs time in a short time scale (ns) - STDP effect in g-2
    - The standard procedure is to inject two laser pulses close in time to study the effect of the first laser on the second, as a function of time separation
- Found that some “double-pulse” positron events have a lifetime of ~ 64 us
    - Real positron pileup events ~ 32 us
    - These must be originated from the same muon decay
    - Most of the events concentrated at the outer radius of calo
    - Material effect considered
        - Positrons hitting the side aluminum plate before crystals
        - Pre-shower (brems due to positron hitting vacuum chamber, kicker or quad plates, etc)
    - Non-standard decay mode - Radiative Muon Decay

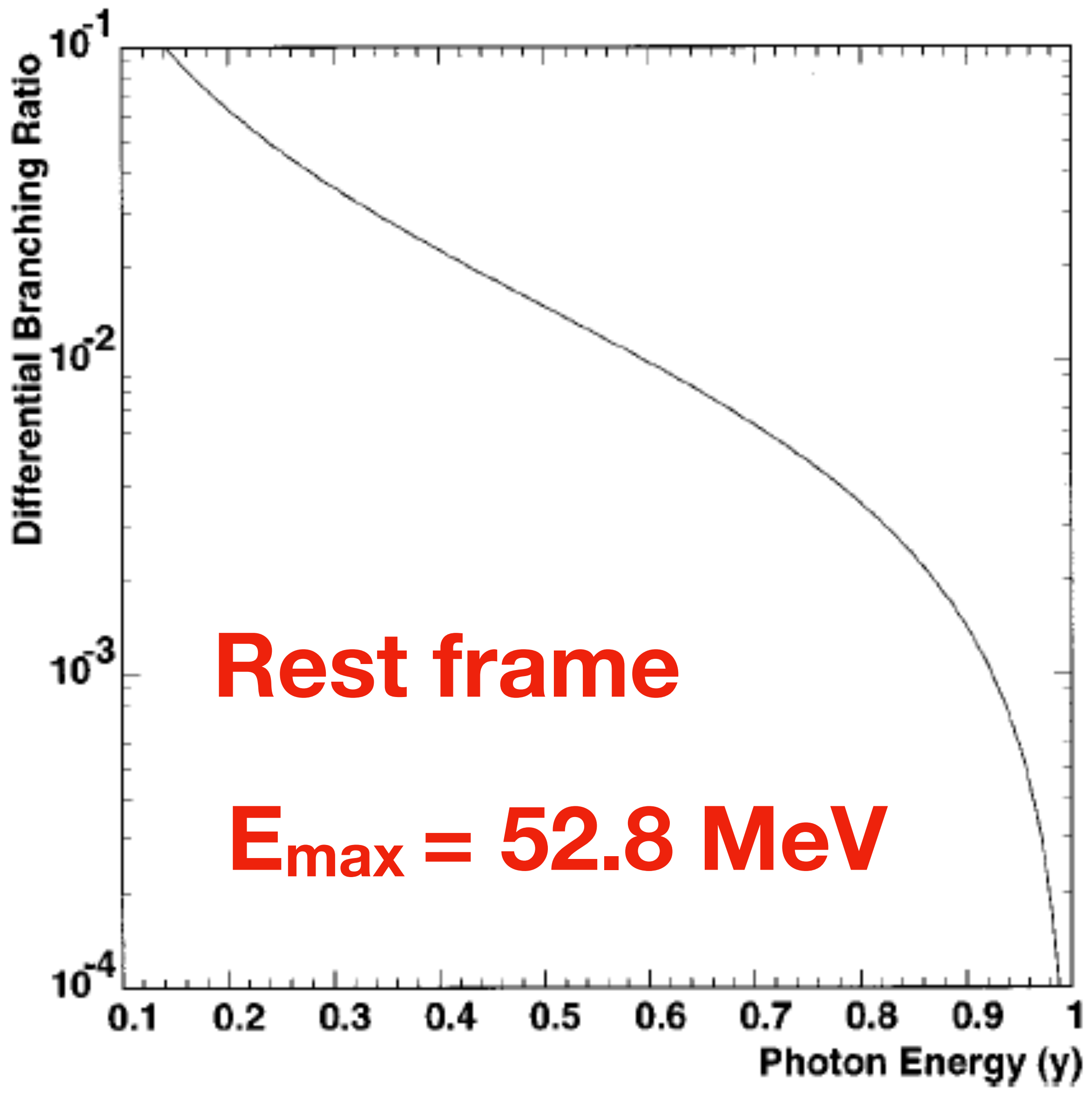


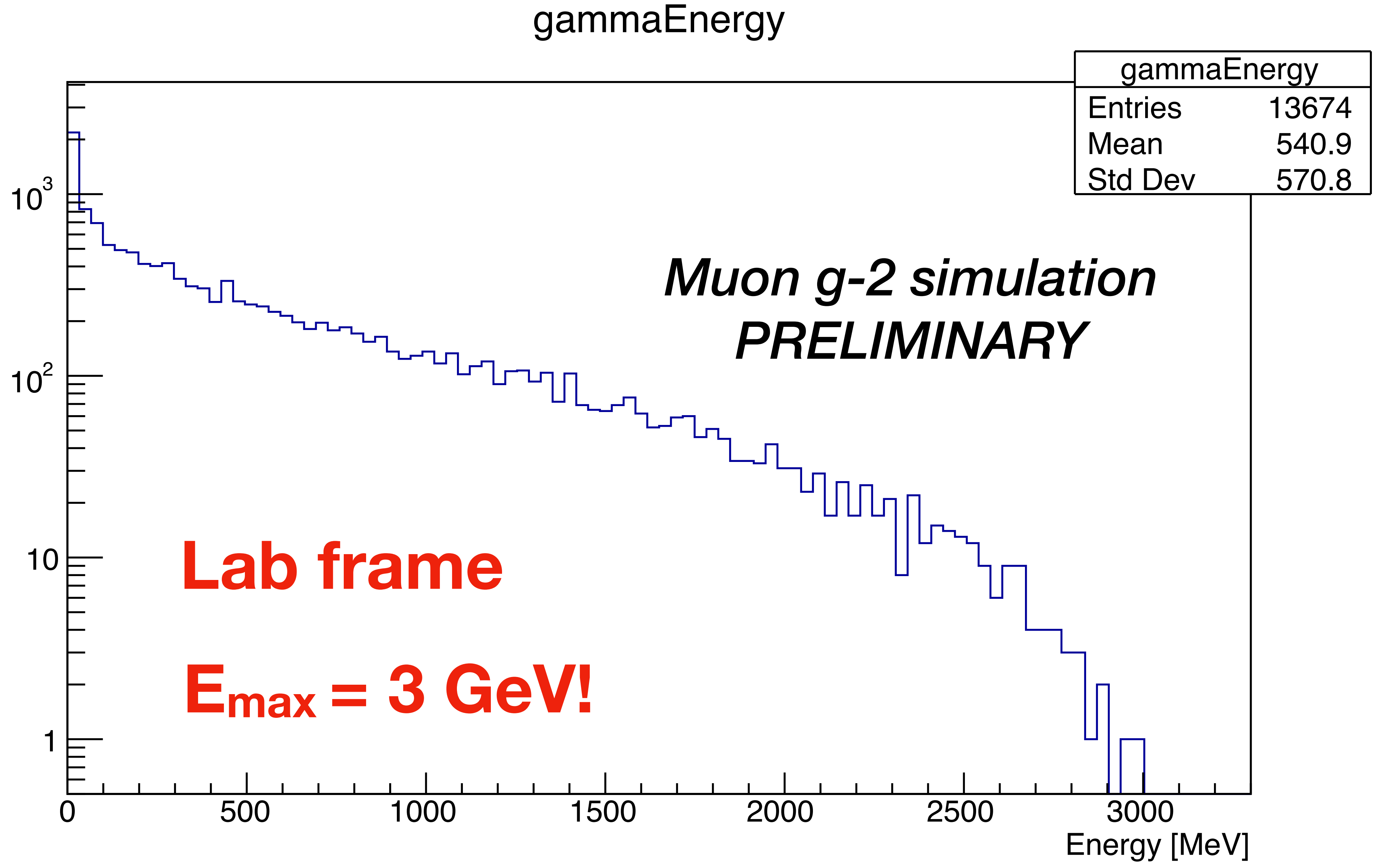


Geant4 simulation for g-2: gm2ringsim

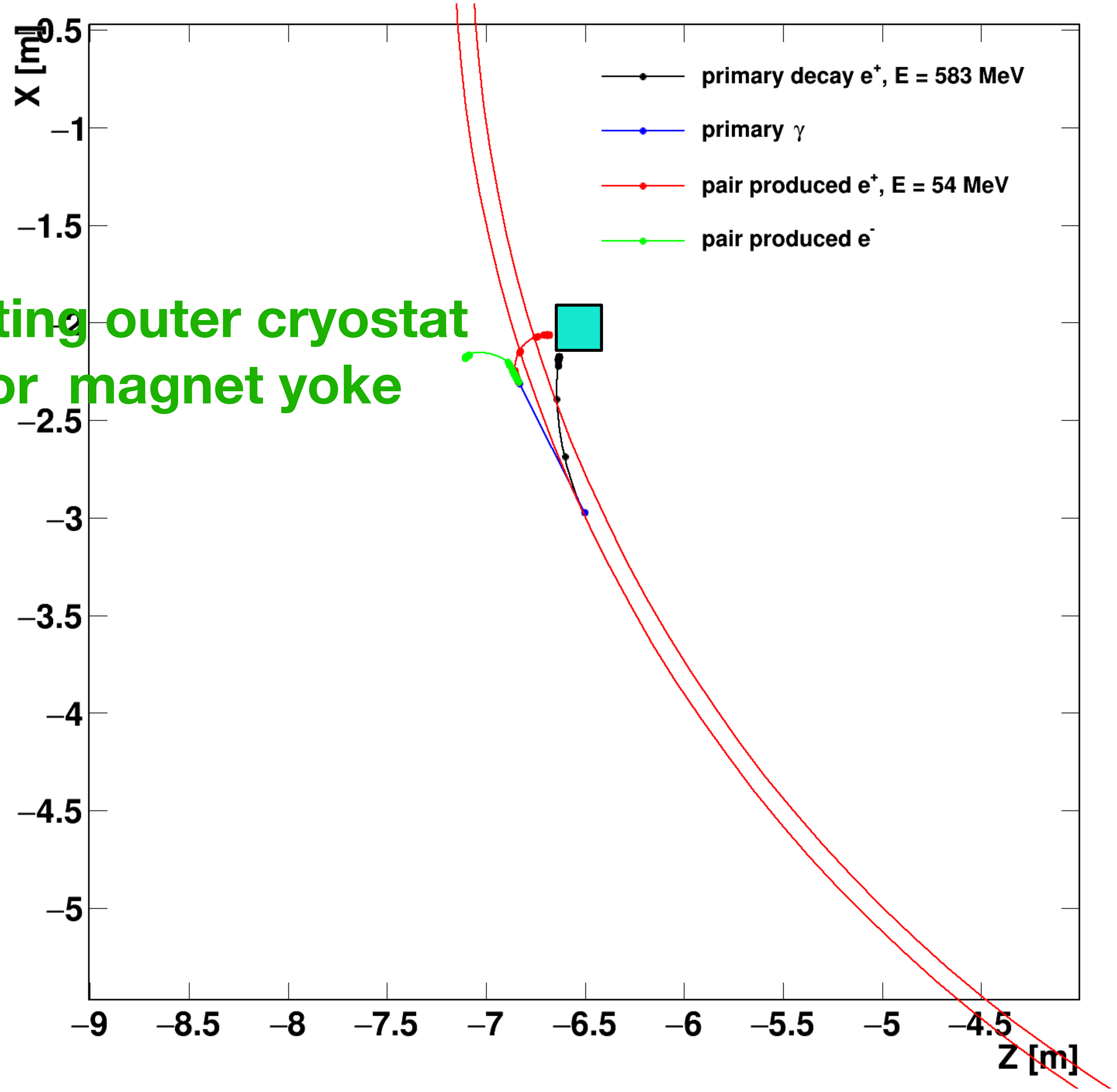




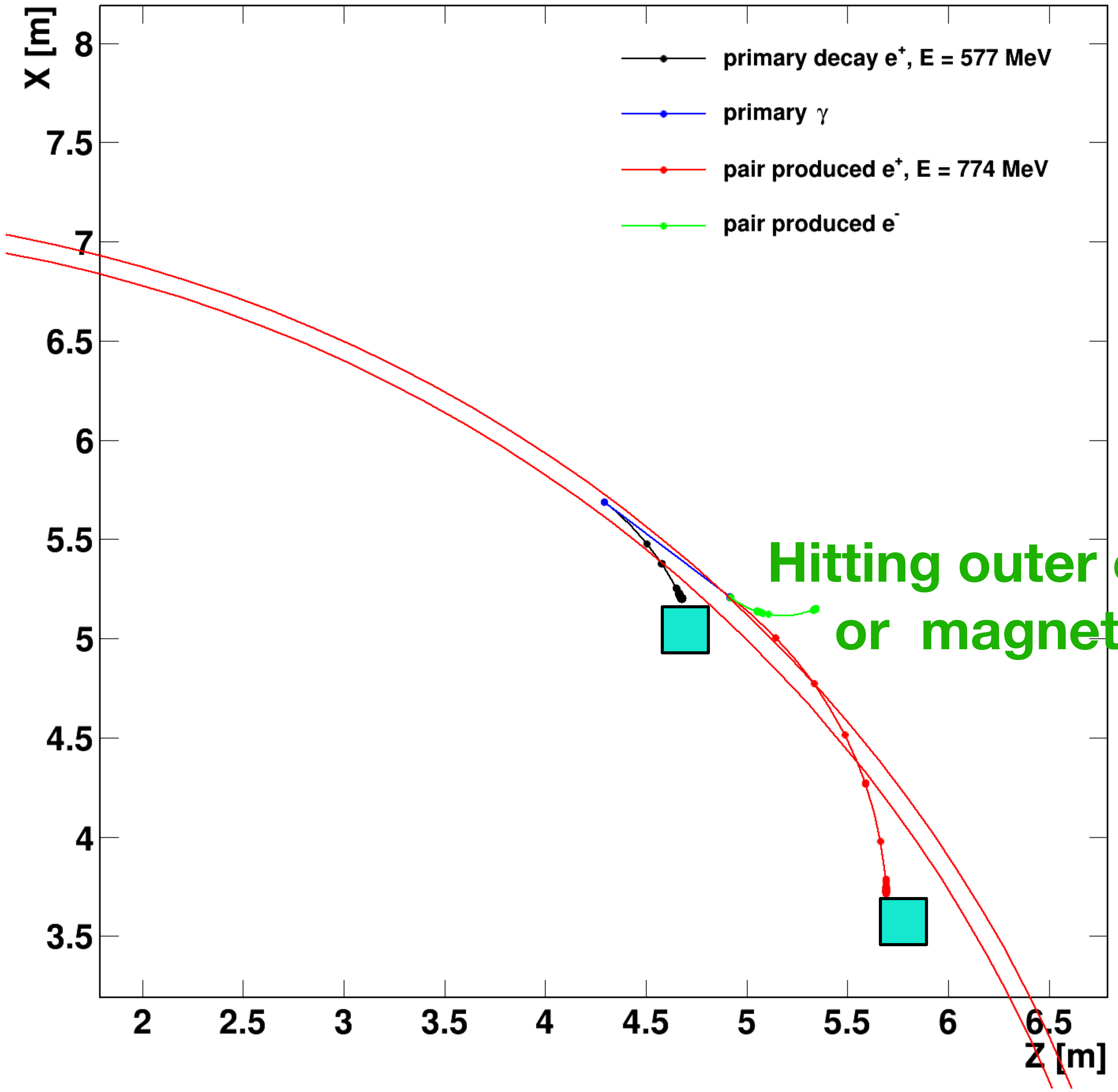




Event topology of RMD



Hitting outer cryostat
or magnet yoke

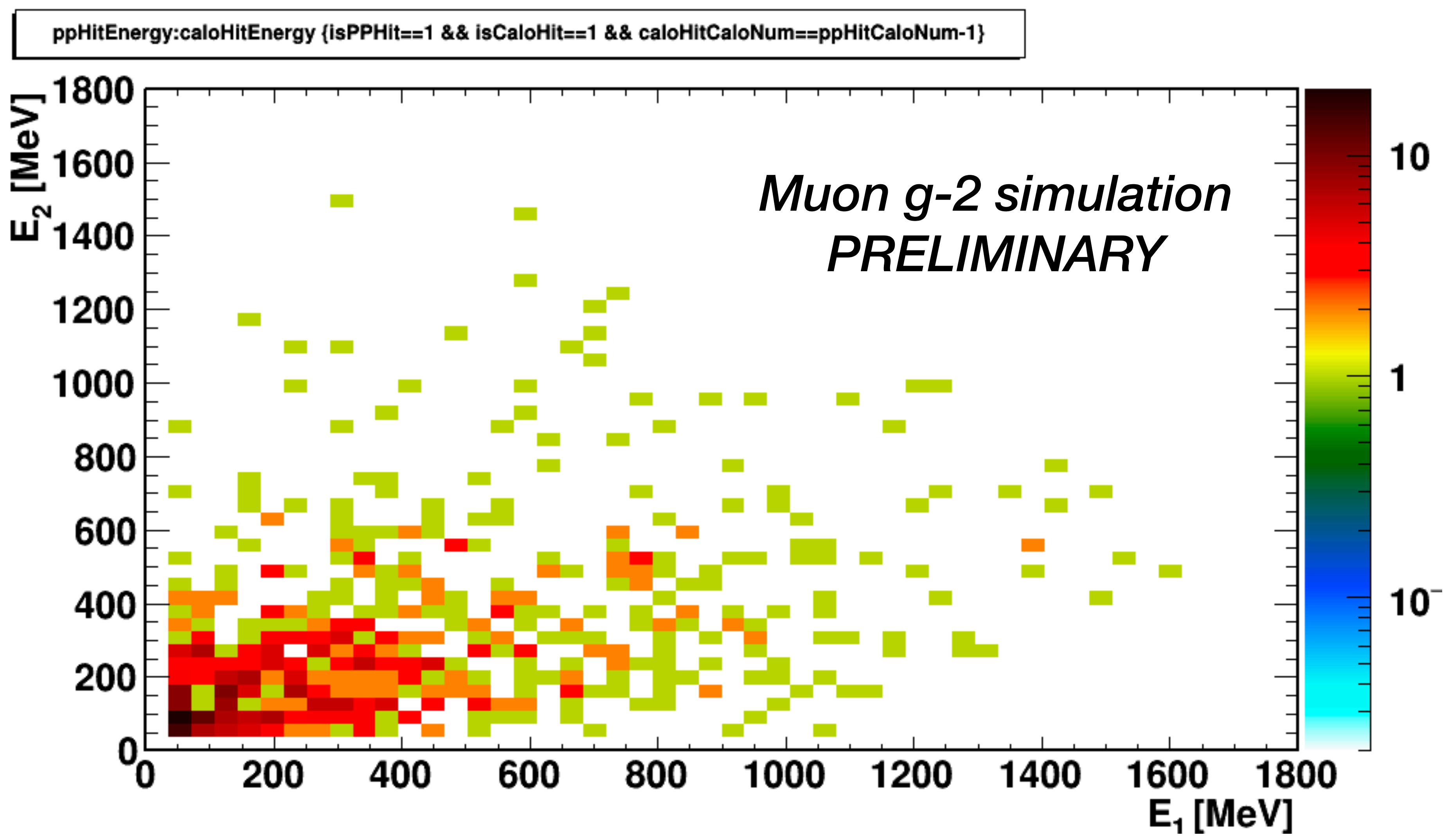


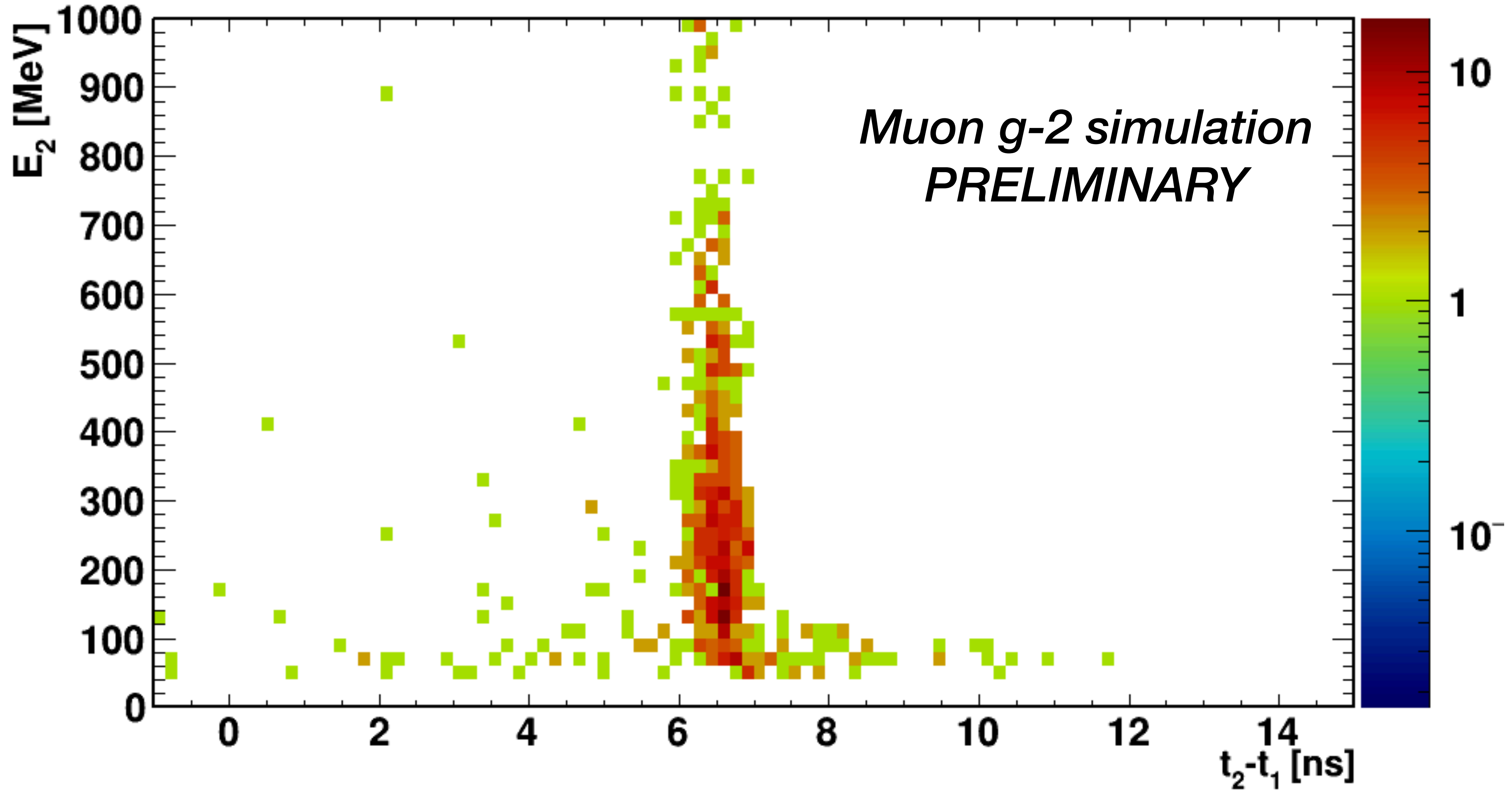
Hitting outer cryostat
or magnet yoke

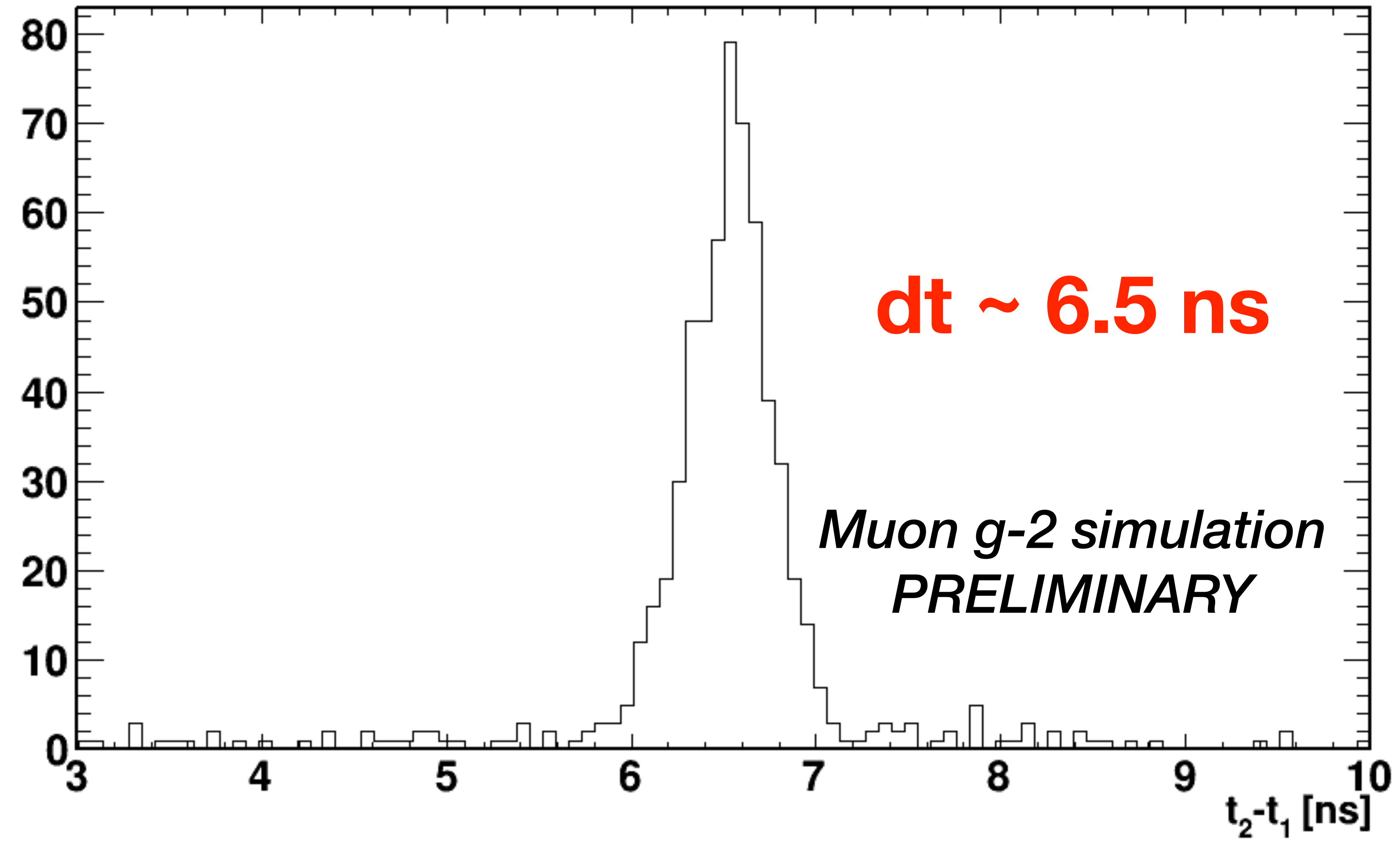
Both positrons hitting the same calo
(~ 50%)

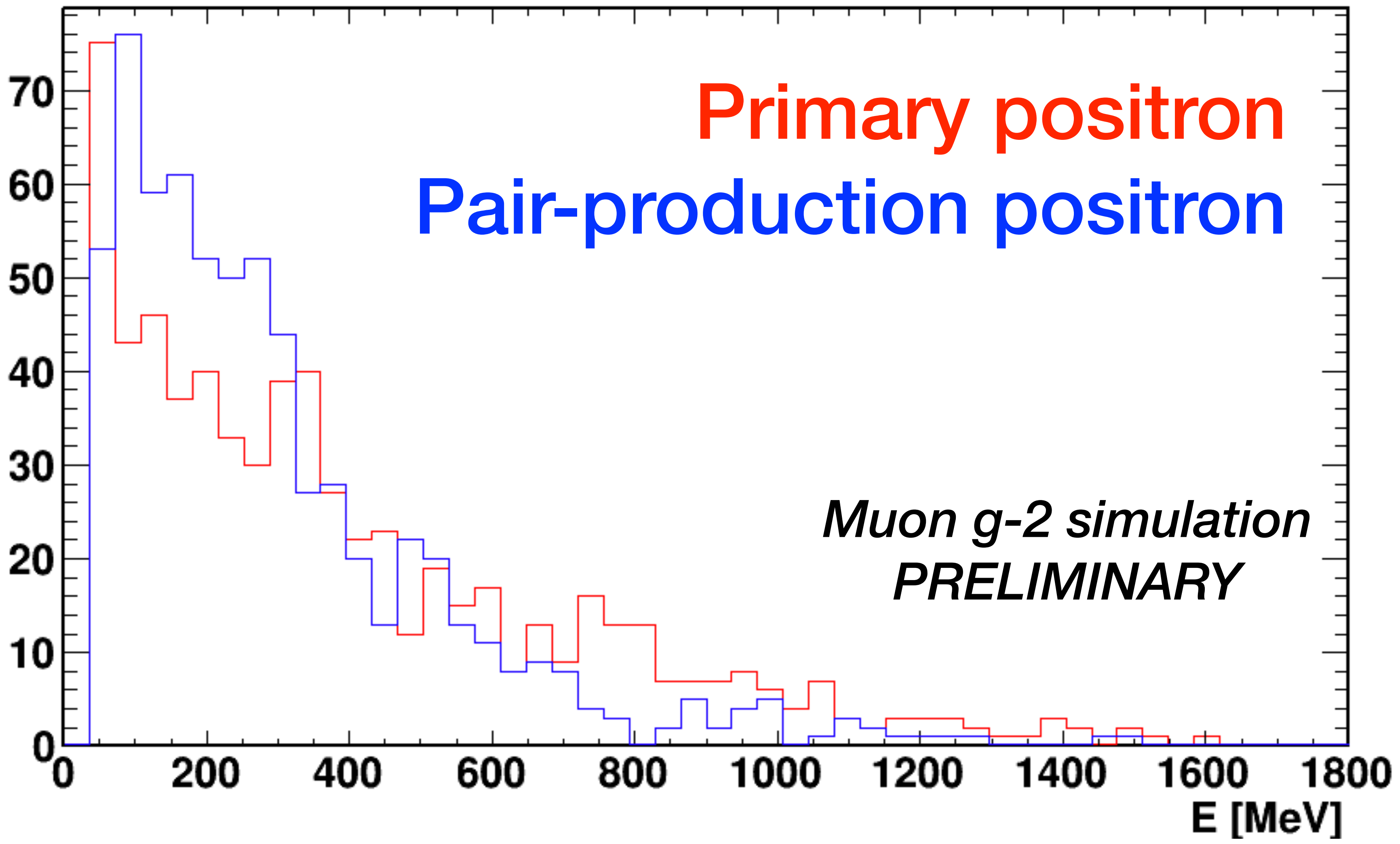
Both positrons hitting different calos
(~ 50%, faking lost muons)

Signature of RMD (E_1, E_2, dt)



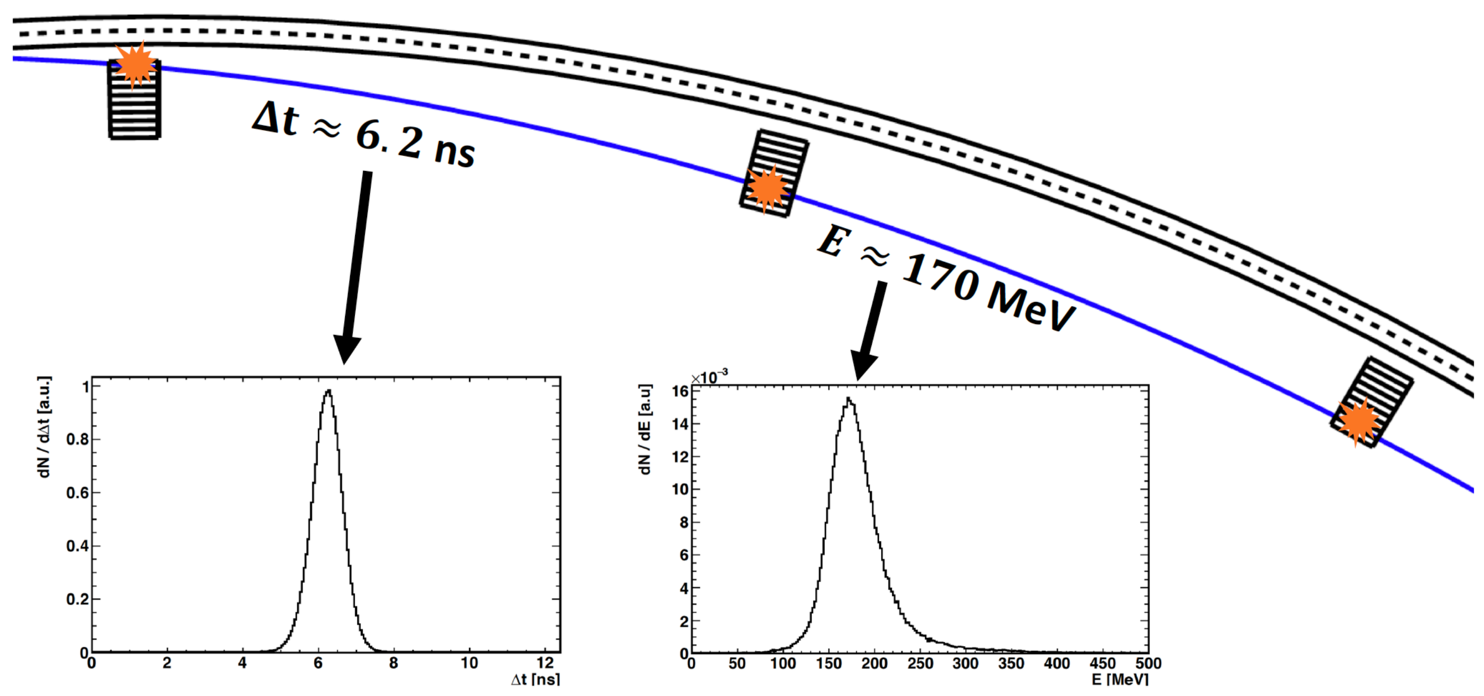




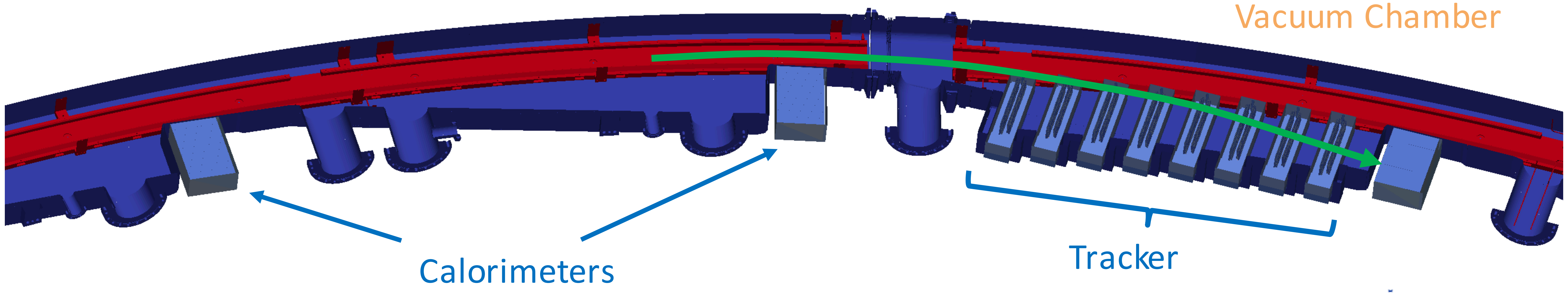


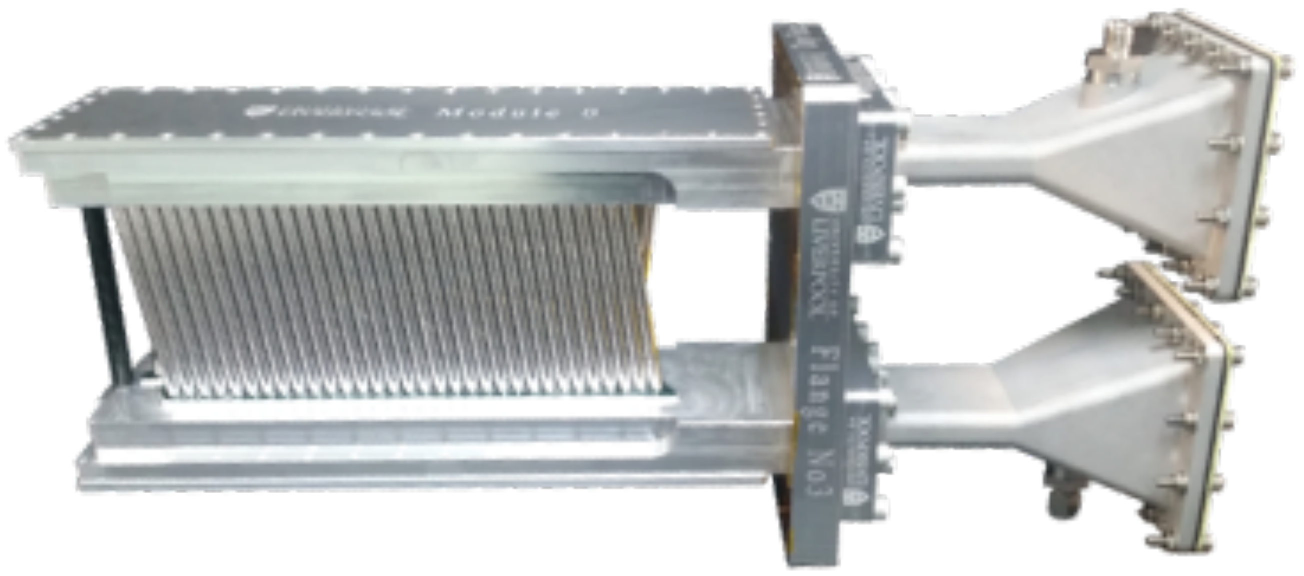
Proposed strategy to search for RMD

c.f. lost muon is 6.2 ns



- Calorimeter data
  - Double coincidence of calorimeter hits
  - 6.5 ns time difference between two calorimeters
  - Exclude 170 MeV energy window (MIP signal of lost muons)
- Tracker data
  - Searching for low momentum tracks “coming out” from the wall
  - Optimize tracking algorithm for lower number of straw hits





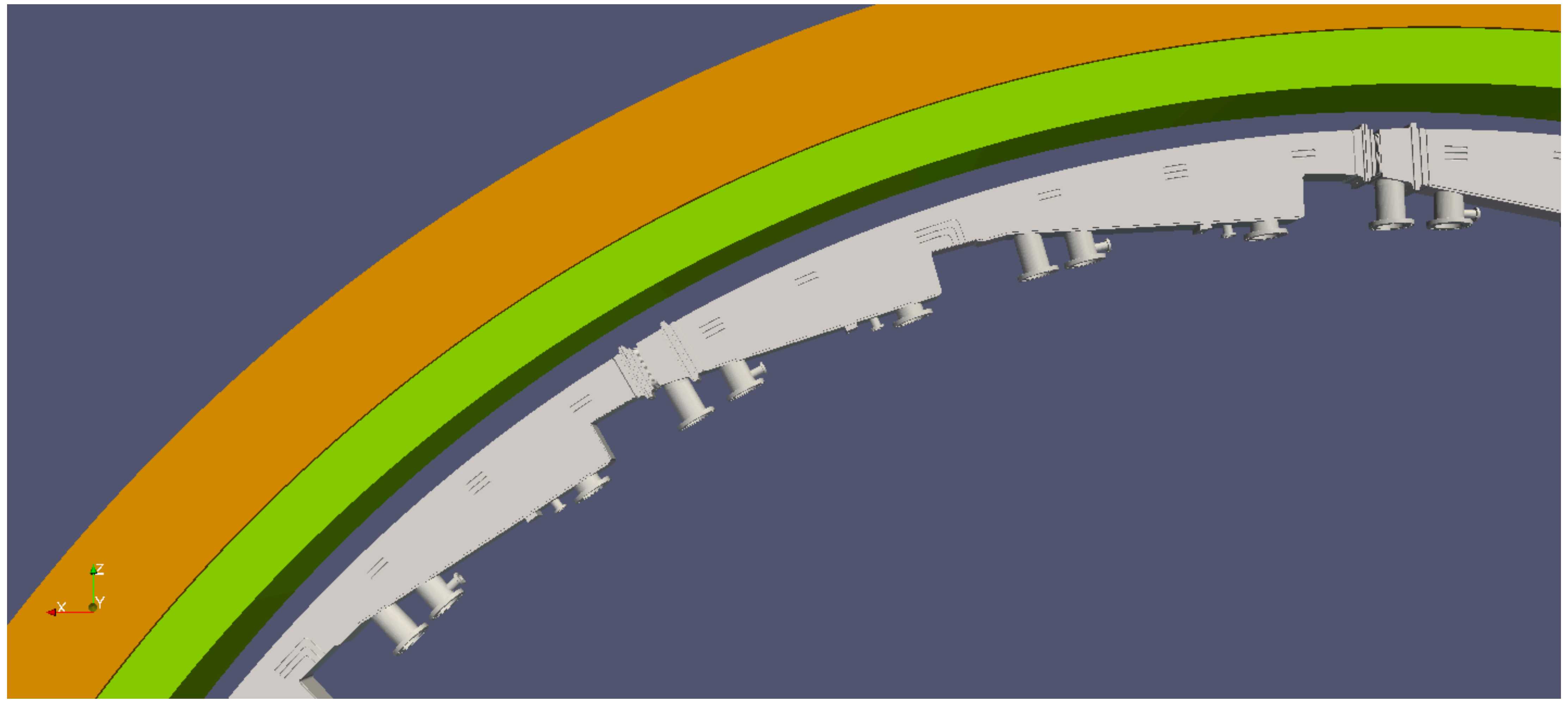
RMD events in Run1-4 of Muon g-2

- Run 1-4 ~ O(10^{11}) detected decays with high energy positrons
- Ordinary muon decays in the storage ring ~ O(10^{12})
- Assuming a 5% branching ratio —> O(10^{10}) decays
  - Estimation of the acceptance after various kinematic cuts is needed
  - # of RMD events potentially larger than 0.5M from PIBETA or 13k from MEG
- Inviting young and smart students or postdocs to work on this measurement
  - Simulation tools available (gm2ringsim with tunable RMD fraction)
  - Calorimeter data available (background to lost muon study)
  - Tracking data available (tracks coming from the wall)
  - SM predictions or analytical formulas available

Fael et al., JHEP 07 (2015) 153
Pruna et al., *Phys. Lett. B* **772** (2017) 452-458

Prospects

- Obviously additional detectors are needed for a dedicated experiment
- The radiated gamma could hit anywhere azimuthally in the storage ring
- Need to place detectors strategically to capture the gamma/e^+e^- pair from RMD events
- Ideally tracking detectors to provide direction and/or charge information



Summary

- Radiative Muon Decay provides a precision test of the SM especially on the V-A structure of weak interaction
- There are only 3 measurements so far and all are in good agreement with the SM except the preliminary PIBETA result (PIBETA/PEN result coming soon)
- The Muon g-2 experiment has many RMD events in the storage ring and they could be searched for using the calorimeter and tracking data
- A dedicated RMD measurement in the future will require a strategic placement of detectors (ideally trackers) to capture the gamma/e^+e^- pair
- Open for collaboration using current data or future dedicated measurement