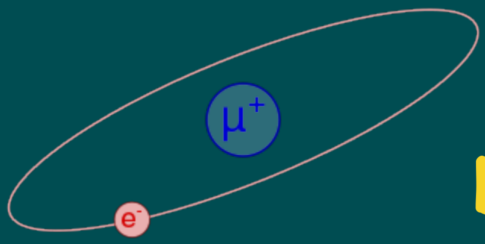


Decays and Oscillations of Muonium

Alexey A. Petrov
Wayne State University

Table of Contents:

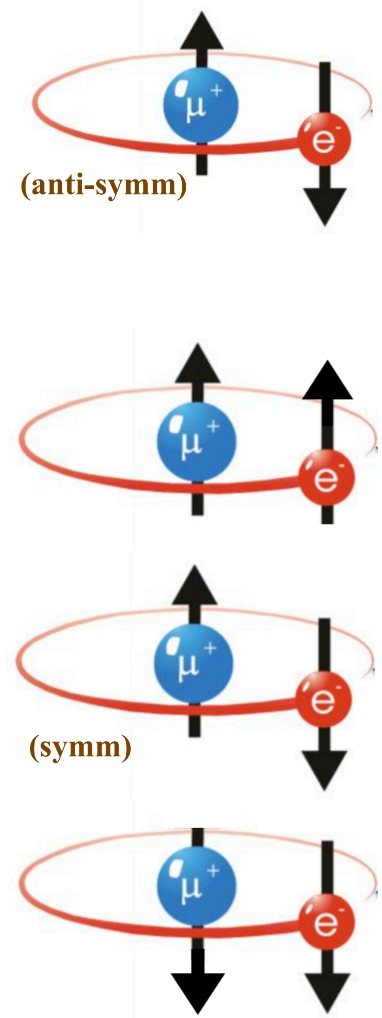
Mainly based on R. Conlin and AAP, Phys.Rev.D 102 (2020) 9, 095001
e-Print: 2005.10276 [hep-ph]

Muonium: just like hydrogen, but simpler!

- Muonium: a bound state of μ^+ and e^-
  - $(\mu^+\mu^-)$ bound state is *true muonium*

- Muonium lifetime $\tau_{M_\mu} = 2.2\ \mu s$
  - main decay mode: $M_\mu \to e^+e^-\bar{\nu}_\mu\nu_e$
  - annihilation: $M_\mu \to \bar{\nu}_\mu\nu_e$

- Muonium's been around since 1960's
  - used in chemistry
  - QED bound state physics, etc.
  - New Physics searches (oscillations)



Spin-0 (singlet) paramuonium

Spin-1 (triplet) orthomuonium

Hughes (1960)

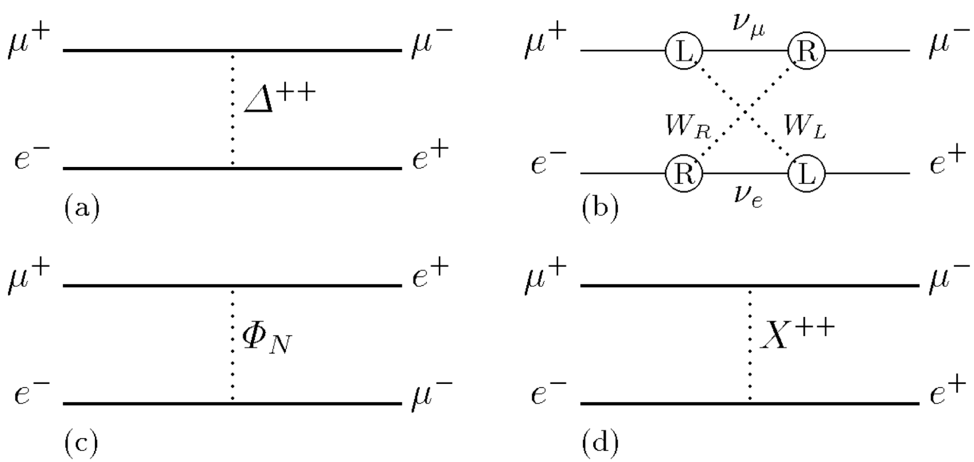
The masses of singlet and triplet are almost the same!

Muonium oscillations: just like $B^0\bar{B}^0$ mixing, but simpler!

★ Lepton-flavor violating interactions can change $M_\mu \to \overline{M}_\mu$

Pontecorvo (1957)
Feinberg, Weinberg (1961)

- Such transition amplitudes are tiny in the Standard Model
  - … but there are plenty of New Physics models where it can happen



$$\sim (\bar{\mu}\Gamma e)(\bar{\mu}\Gamma e)$$

effective operator

  - theory: compute transition amplitudes for ALL New Physics models!
  - experiment: produce M_μ but see for decay products of $\overline{M}_\mu$

Combined evolution = flavor oscillations

- If there is an interaction that couples M_μ and $\overline{M}_\mu$ (both SM or NP)
  - combined time evolution: non-diagonal Hamiltonian!

$$i\frac{d}{dt}\begin{pmatrix} |M(t)\rangle \\ |\overline{M}(t)\rangle \end{pmatrix} = \left(m - i\frac{\Gamma}{2}\right)\begin{pmatrix} |M(t)\rangle \\ |\overline{M}(t)\rangle \end{pmatrix}$$

  - diagonalization: new mass eigenstates:

$$|M_{\mu_{1,2}}\rangle = \frac{1}{\sqrt{2}}\left[|M_\mu\rangle \mp |\overline{M}_\mu\rangle\right]$$

  - new mass eigenstates: mass and lifetime differences

$$\left.\begin{aligned} \Delta m &\equiv M_1 - M_2, \\ \Delta\Gamma &\equiv \Gamma_2 - \Gamma_1. \end{aligned}\right\} \quad x = \frac{\Delta m}{\Gamma}, \qquad y = \frac{\Delta\Gamma}{2\Gamma} \quad \text{(small)}$$

These mass and width difference are observable quantities

Combined evolution = flavor oscillations

- Study oscillations via decays: amplitudes for $M_\mu \to f$ and $\overline{M}_\mu \to \bar{f}$
  - possibility of flavor oscillations ($M_\mu \to \overline{M}_\mu \to \bar{f}$)

$$\begin{aligned} |M(t)\rangle &= g_+(t)\,|M_\mu\rangle + g_-(t)\,|\overline{M}_\mu\rangle\,, \\ |\overline{M}(t)\rangle &= g_-(t)\,|M_\mu\rangle + g_+(t)\,|\overline{M}_\mu\rangle\,, \end{aligned}$$

with

$$\begin{aligned} g_+(t) &= e^{-\Gamma_1 t/2} e^{-i m_1 t}\left[1 + \frac{1}{8}(y - ix)^2 (\Gamma t)^2\right], \\ g_-(t) &= \frac{1}{2} e^{-\Gamma_1 t/2} e^{-i m_1 t}(y - ix)(\Gamma t)\,. \end{aligned}$$

  - time-dependent width: $\Gamma(M_\mu \to \overline{f})(t) = \dfrac{1}{2} N_f \, |A_f|^2 \, e^{-\Gamma t} (\Gamma t)^2 \, R_M(x,y)$
  - oscillation probability: $\boxed{P(M_\mu \to \overline{M}_\mu) = \dfrac{\Gamma(M_\mu \to \overline{f})}{\Gamma(M_\mu \to f)} = R_M(x,y) = \dfrac{1}{2}\left(x^2 + y^2\right)}$

Oscillation parameters: introduction

- Mixing parameters are related to off-diagonal matrix elements
  - heavy and light intermediate degrees of freedom

$$\left(m-\frac{i}{2}\Gamma\right)_{12}=\frac{1}{2M_M}\left\langle\overline{M}_\mu\left|\mathcal{H}_{\text{eff}}\right|M_\mu\right\rangle+\frac{1}{2M_M}\sum_n\frac{\left\langle\overline{M}_\mu\left|\mathcal{H}_{\text{eff}}\right|n\right\rangle\left\langle n\left|\mathcal{H}_{\text{eff}}\right|M_\mu\right\rangle}{M_M-E_n+i\epsilon}$$

Local at scale $\mu = M_\mu$: only Δm
lepton number change $\Delta L_\mu = 2$

Bi-local at scale $\mu = M_\mu$: both Δm and $\Delta\Gamma$
lepton number changes: $(\Delta L_\mu = 1)^2$
or $(\Delta L_\mu = 0)(\Delta L_\mu = 2)$

  - each term has contributions from different effective Lagrangians

$$\mathcal{L}_{\text{eff}}=\mathcal{L}_{\text{eff}}^{\Delta L_\mu=0}+\mathcal{L}_{\text{eff}}^{\Delta L_\mu=1}+\mathcal{L}_{\text{eff}}^{\Delta L_\mu=2}$$

  - … all of which have a form $\mathcal{L}_{\text{eff}}=-\frac{1}{\Lambda^2}\sum_i c_i(\mu)Q_i,$ with $\Lambda\sim\mathcal{O}(TeV)$

Mass difference = real (dispersive) part; width difference: imaginary (absorptive) part

Mass difference

- Mass difference comes from the dispersive part

$$x = \frac{1}{2M_M\Gamma}\mathrm{Re}\left[2\langle\overline{M}_\mu\left|\mathcal{H}_{\rm eff}\right|M_\mu\rangle + \cancel{\langle\overline{M}_\mu\left|i\int d^4x\ \mathrm{T}\left[\mathcal{H}_{\rm eff}(x)\mathcal{H}_{\rm eff}(0)\right]\right|M_\mu\rangle}\right]$$

  – consider only $\Delta L_\mu = 2$ Lagrangian contributions (largest?)

$$\mathcal{L}_{\rm eff}^{\Delta L_\mu=2} = -\frac{1}{\Lambda^2}\sum_i C_i^{\Delta L=2}(\mu)Q_i(\mu)$$

  – leading order: all heavy New Physics models are encoded in (the Wilson coefficients of) the five dimension-6 operators

$$\begin{aligned}
Q_1 &= (\overline{\mu}_L\gamma_\alpha e_L)\,(\overline{\mu}_L\gamma^\alpha e_L)\,, \quad Q_2 = (\overline{\mu}_R\gamma_\alpha e_R)\,(\overline{\mu}_R\gamma^\alpha e_R)\,,\\
Q_3 &= (\overline{\mu}_L\gamma_\alpha e_L)\,(\overline{\mu}_R\gamma^\alpha e_R)\,, \quad Q_4 = (\overline{\mu}_L e_R)\,(\overline{\mu}_L e_R)\,,\\
Q_5 &= (\overline{\mu}_R e_L)\,(\overline{\mu}_R e_L)\,.
\end{aligned}$$

  – need to compute matrix elements for both singlet and triplet states

Effective Lagrangians and particular models

- Effective Lagrangian approach encompasses all models
  - lets look at an example of a model with a doubly charged Higgs Δ^{--}
  - this is common for the left-right models, etc.

$$\mathcal{L}_R = g_{\ell\ell} \overline{\ell}_R \ell^c \Delta + H.c.,$$

  - integrate out Δ^{--} to get

$$\mathcal{H}_\Delta = \frac{g_{ee} g_{\mu\mu}}{2M_\Delta^2} \left(\overline{\mu}_R \gamma_\alpha e_R\right) \left(\overline{\mu}_R \gamma^\alpha e_R\right) + H.c.,$$

  - match to $\mathscr{L}_{\rm eff}^{\Delta L=2}$ to see that $M_\Delta = \Lambda$ and

$$C_2^{\Delta L=2} = g_{ee} g_{\mu\mu}/2.$$

Mass difference: matrix elements

- QED bound state: know leading order wave function!
  - spacial part is the same as in Hydrogen atom

$$\varphi(r) = \frac{1}{\sqrt{\pi a_{M_\mu}^3}} e^{-\frac{r}{a_{M_\mu}}}$$

  - can unambiguously compute decay constants and mixing MEs (QED)

$$\langle 0 | \overline{\mu}\gamma^\alpha\gamma^5 e \left| M_\mu^P \right\rangle \; = \; i f_P p^\alpha, \quad \langle 0 | \overline{\mu}\gamma^\alpha e \left| M_\mu^V \right\rangle = f_V M_M \epsilon^\alpha(p),$$
$$\langle 0 | \overline{\mu}\sigma^{\alpha\beta} e \left| M_\mu^V \right\rangle \; = \; i f_T \left(\epsilon^\alpha p^\beta - \epsilon^\beta p^\alpha\right),$$

  - in the non-relativistic limit all decay constants $f_P = f_V = f_T = f_M$

$$f_M^2 = 4\frac{|\varphi(0)|^2}{M_M}$$ (QED version of Van Royen-Weisskopf)

  - NR matrix elements: “vacuum insertion” = direct computation

Mass difference: results

- Spin-singlet muonium state:
  - matrix elements:

$$\langle \bar{M}_\mu^P | Q_1 | M_\mu^P \rangle = f_M^2 M_M^2, \quad \langle \bar{M}_\mu^P | Q_2 | M_\mu^P \rangle = f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^P | Q_3 | M_\mu^P \rangle = -\frac{3}{2} f_M^2 M_M^2, \quad \langle \bar{M}_\mu^P | Q_4 | M_\mu^P \rangle = -\frac{1}{4} f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^P | Q_5 | M_\mu^P \rangle = -\frac{1}{4} f_M^2 M_M^2.$$

$$\boxed{x_P = \frac{4(m_{red}\alpha)^3}{\pi\Lambda^2\Gamma}\left[C_1^{\Delta L=2} + C_2^{\Delta L=2} - \frac{3}{2}C_3^{\Delta L=2} - \frac{1}{4}\left(C_4^{\Delta L=2} + C_5^{\Delta L=2}\right)\right]}$$

- Spin-triplet muonium state:
  - matrix elements

$$\langle \bar{M}_\mu^V | Q_1 | M_\mu^V \rangle = -3 f_M^2 M_M^2, \quad \langle \bar{M}_\mu^V | Q_2 | M_\mu^V \rangle = -3 f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^V | Q_3 | M_\mu^V \rangle = -\frac{3}{2} f_M^2 M_M^2, \quad \langle \bar{M}_\mu^V | Q_4 | M_\mu^V \rangle = -\frac{3}{4} f_M^2 M_M^2,$$
$$\langle \bar{M}_\mu^V | Q_5 | M_\mu^V \rangle = -\frac{3}{4} f_M^2 M_M^2.$$

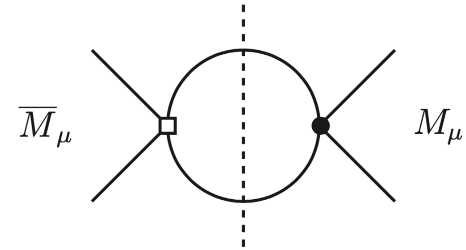
$$\boxed{x_V = -\frac{12(m_{red}\alpha)^3}{\pi\Lambda^2\Gamma}\left[C_1^{\Delta L=2} + C_2^{\Delta L=2} + \frac{1}{2}C_3^{\Delta L=2} + \frac{1}{4}\left(C_4^{\Delta L=2} + C_5^{\Delta L=2}\right)\right]}$$

Experimental constraints on x result on experimental constraints on Wilson coefficients $C_k^{\Delta L=2}$ that encode all information about possible New Physics contributions

R. Conlin and AAP, arXiv: 2005.10276

Width difference and muonium decays

- Width difference comes from the absorptive part
  - light SM intermediate states ($e^+e^-, \gamma\gamma, \bar{\nu}\nu, etc.$)
  - $\bar{\nu}\nu$ state gives parametrically largest contribution



- Muonium two- and there-body decays
  - two-body decays ($M_\mu^{V,P} \to e^+e^-, \gamma\gamma, etc$) are dominated by New Physics
  - probe different combinations of SM EFT Wilson coefficients
    - e.g. $\mu \to 3e$ vs. $M_\mu \to e^+e^-$ (also phase space enhancement)

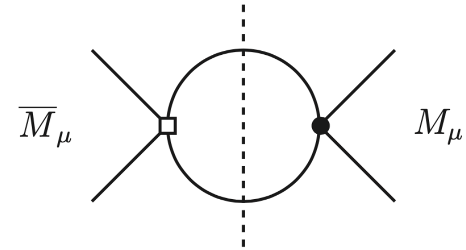
R. Conlin and AAP

  - can $M_\mu \to invisible$ (SM: $M_\mu \to \nu_e \bar{\nu}_\mu$) be measured?

Gninenko, Krasnikov, Matveev.
Phys.Rev. D87 (2013) 015016

Width difference

- Width difference comes from the absorptive part
  - light SM intermediate states ($e^+e^-, \gamma\gamma, \bar{\nu}\nu, etc.$)
  - $\bar{\nu}\nu$ state gives parametrically largest contribution



$$y = \frac{1}{2M_M\Gamma}\mathrm{Im}\left[\langle\overline{M}_\mu\left| i\int d^4x\ \mathrm{T}\left[\mathcal{H}_{\text{eff}}(x)\mathcal{H}_{\text{eff}}(0)\right]\right| M_\mu\rangle\right]$$

$$= \frac{1}{M_M\Gamma}\mathrm{Im}\left[\langle\overline{M}_\mu\left| i\int d^4x\ \mathrm{T}\left[\mathcal{H}_{\text{eff}}^{\Delta L_\mu=2}(x)\mathcal{H}_{\text{eff}}^{\Delta L_\mu=0}(0)\right]\right| M_\mu\rangle\right]$$

New Physics $\Delta L_\mu = 2$ contribution

$$\mathcal{L}_{\text{eff}}^{\Delta L_\mu=2} = -\frac{1}{\Lambda^2}\sum_i C_i^{\Delta L=2}(\mu)Q_i(\mu)$$

$$Q_6 = \left(\overline{\mu}_L\gamma_\alpha e_L\right)\left(\overline{\nu_\mu}_L\gamma^\alpha\nu_{eL}\right),$$

$$Q_7 = \left(\overline{\mu}_R\gamma_\alpha e_R\right)\left(\overline{\nu_\mu}_L\gamma^\alpha\nu_{eL}\right)$$

Standard Model $\Delta L_\mu = 0$ contribution

$$\mathcal{L}_{\text{eff}}^{\Delta L_\mu=0} = -\frac{4G_F}{\sqrt{2}}\left(\overline{\mu}_L\gamma_\alpha e_L\right)\left(\overline{\nu_e}_L\gamma^\alpha\nu_{\mu L}\right)$$

Width difference: results

- Spin-singlet muonium state:

$$y_P = \frac{G_F}{\sqrt{2}\Lambda^2} \frac{M_M^2}{\pi^2 \Gamma} (m_{red}\alpha)^3 \left(C_6^{\Delta L=2} - C_7^{\Delta L=2}\right)$$

- Spin-triplet muonium state:

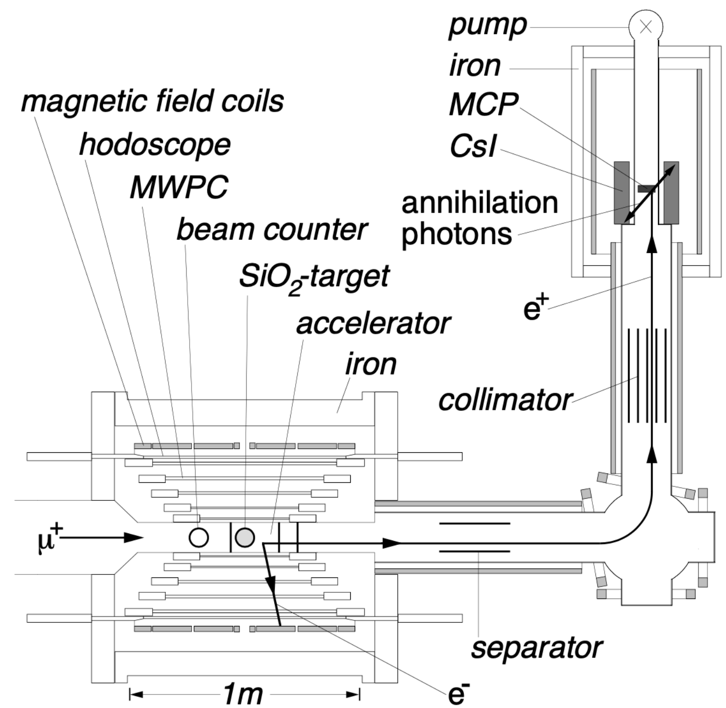
$$y_V = -\frac{G_F}{\sqrt{2}\Lambda^2} \frac{M_M^2}{\pi^2 \Gamma} (m_{red}\alpha)^3 \left(5C_6^{\Delta L=2} + C_7^{\Delta L=2}\right)$$

- Note: y has the same $1/\Lambda^2$ suppression as the mass difference!

R. Conlin and AAP, arXiv: 2005.10276

Experimental setup and constraints

- Similar experimental set ups for different experiments
  - example: MACS at PSI
  - idea: form M_μ by scattering muon (μ^+) beam on SiO_2 target
- A couple of "little inconveniences":
  - ➡ how to tell f apart from $\bar{f}$?
    - $M_\mu \to f$ decay: $M_\mu \to e^+ e^- \bar{\nu}_\mu \nu_e$
    - $\overline{M}_\mu \to \bar{f}$ decay: $\overline{M}_\mu \to e^+ e^- \bar{\nu}_e \nu_\mu$
    - $\bar{f}$: fast e^- (~53 MeV), slow e^+ (13.5 eV)
  - ➡ oscillations happen in magnetic field
    - … which selects M_μ vs. $\overline{M}_\mu$



Muonium-Antimuonium Conversion Spectrometer (MACS)

L. Willmann, et al. PRL 82 (1999) 49

The most recent experimental data comes from 1999! Time is ripe for an update!

Experimental results

- MACS: observed 5.7×10^{10} muonium atoms after 4 months of running
  - magnetic field is taken into account (suppression factor)

Interaction type	2.8 μT	0.1 T	100 T
SS	0.75	0.50	0.50
PP	1.0	0.9	0.50
$(V \pm A) \times (V \pm A)$ or $(S \pm P) \times (S \pm P)$	0.75	0.35	0.0
$(V \pm A) \times (V \mp A)$ or $(S \pm P) \times (S \mp P)$	0.95	0.78	0.67

L. Willmann, et al. PRL 82 (1999) 49

  - no oscillations have been observed

Experimental constraints

- We can now put constraints on the Wilson coefficients of effective operators from experimental data (assume single operator dominance)
  - presence of the magnetic field

$$P(M_\mu \to \overline{M}_\mu) \leq 8.3 \times 10^{-11}/S_B(B_0)$$

  - no separation of spin states: average

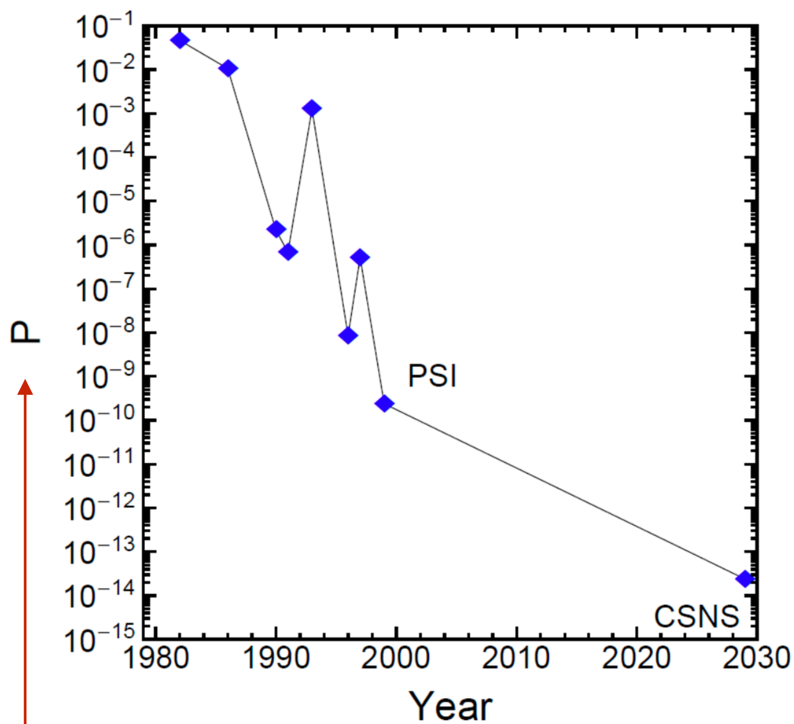
$$P(M_\mu \to \overline{M}_\mu)_{\exp} = \sum_{i=P,V} \frac{1}{2S_i+1} P(M_\mu{}^i \to \overline{M}_\mu{}^i)$$

  - set Wilson coefficients to one, set constraints on the scale probed

Operator	Interaction type	$S_B(B_0)$ (from [9])	Constraints on the scale Λ, TeV
Q_1	$(V-A)\times(V-A)$	0.75	5.4
Q_2	$(V+A)\times(V+A)$	0.75	5.4
Q_3	$(V-A)\times(V+A)$	0.95	5.4
Q_4	$(S+P)\times(S+P)$	0.75	2.7
Q_5	$(S-P)\times(S-P)$	0.75	2.7
Q_6	$(V-A)\times(V-A)$	0.75	0.58×10^{-3}
Q_7	$(V+A)\times(V-A)$	0.95	0.38×10^{-3}

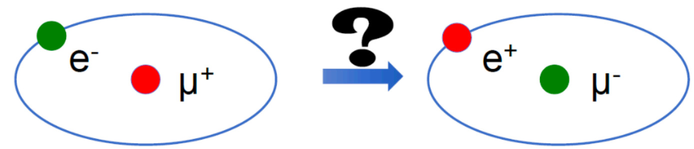
R. Conlin and AAP, arXiv: 2005.10276

Fundamental science with EMuS (China)



$$P(M_\mu \to \overline{M}_\mu) = \frac{\Gamma(M_\mu \to \overline{f})}{\Gamma(M_\mu \to f)} = R_M(x,y)$$

$$R_M(x,y) = \frac{1}{2}\left(x^2 + y^2\right)$$



- The latest bound was done at PSI more than 20 years ago with a muon intensity $8 \times 10^6 \mu^+/s$ and high-precision magnetic spectrometer.
- Timing resolution in detector: ~ ns
- Position resolution in detector: ~ mm
- EMuS plan to offer $10^9 \mu^+/s$
- Current timing resolution in detector: ~ ps
- Current position resolution in detector:~μs
- Expect to be improved by > O(10^2)?

MACE experiment at EMuS (Chinese SNS)
Jian Tang, talk at RPPM meeting (Snowmass 2021)

Conclusions and things to take home

- Muonium oscillations
  - a heavy-light state that can exhibit flavor oscillations (like K, B, and D mesons)
  - oscillations probe New Physics without complications of QCD
  - results can be matched to particular models of New Physics
  - found that both Δm and $\Delta\Gamma$ parametrically scale as $\mathcal{O}(\Lambda^{-2})$
- Muonium decays
  - the only SM-dominated annihilation mode is $\nu\bar{\nu}$ (not observed)
  - lifetime differences $\Delta\Gamma$ (SM intermediate state, NP in ΔL_μ operators)
  - two-body decays: different constraints on SM EFT Wilson coefficients
- Muonium is the simplest atom: atomic physics
  - level splitting (Lamb shift): probe NP w/out QCD complications

MuSEUM experiment (J-PARC)

- Last experimental data is from 1999! Need new data!
  - we already probe the LHC energy domain!


EXTRA!

- A possibility of using muon beams at CMP facilities

Jian Tang, talk at RPPM meeting (Snowmass 2021)

	Proton driver [MW]	Surface muons			Decay muons		
		Intensity [1E6/s]	Polarization [%]	Spread [%]	energy [MeV/c]	Intensity [1E6/s]	Spread [%]
PSI	1.3	420	90	10	85-125	240	3
ISIS	0.16	1.5	95	<15	20-120	0.4	10
RIKEN/RAL	0.16	0.8	95	<15	65-120	1	10
JPARC	1	100	95	15	33-250	10	15
TRIUMF	0.075	1.4	90	7	20-100	0.0014	10
EMuS	0.005	83	50	10	50-450	16	10
Baby EMuS	0.005	1.2	95	10			

$\times$5 CSNS-II upgrade

Facility	Source Type	Intensity (μ+/sec)*
ISIS	pulsed	1.5×10^6
J-PARC	continuous	1.8×10^6
PSI	continuous	7.0×10^4
TRIUMF	pulsed	5.0×10^6
SEEMS	pulsed	1.9×10^8

- Muonium Antimuonium Conversion Experiment (MACE) EMuS at CSNS

Effective Lagrangians and lifetime difference

- Effective Lagrangians for $\Delta L_\mu = 0$, $\Delta L_\mu = 1$, and $\Delta L_\mu = 2$

$$\mathcal{L}_{\rm eff}^{\Delta L_\mu=0} = -\frac{4G_F}{\sqrt{2}} \left(\overline{\mu}_L \gamma_\alpha e_L\right) \left(\overline{\nu_e}_L \gamma^\alpha \nu_{\mu L}\right)$$

$$\begin{aligned}
\mathcal{L}_{\rm eff}^{\Delta L_\mu=1} = & -\frac{1}{\Lambda^2} \sum_f \Big[\left(C_{VR}^f \, \overline{\mu}_R \gamma^\alpha e_R + C_{VL}^f \, \overline{\mu}_L \gamma^\alpha e_L \right) \, \overline{f} \gamma_\alpha f \\
& + \left(C_{AR}^f \, \overline{\mu}_R \gamma^\alpha e_R + C_{AL}^q \, \overline{\mu}_L \gamma^\alpha e_L \right) \, \overline{f} \gamma_\alpha \gamma_5 f \\
& + m_e m_f G_F \left(C_{SR}^f \, \overline{\mu}_R e_L + C_{SL}^f \, \overline{\mu}_L e_R \right) \, \overline{f} f \\
& + m_e m_f G_F \left(C_{PR}^f \, \overline{\mu}_R e_L + C_{PL}^f \, \overline{\mu}_L e_R \right) \, \overline{f} \gamma_5 f \\
& + m_e m_f G_F \left(C_{TR}^f \, \overline{\mu}_R \sigma^{\alpha\beta} e_L + C_{TL}^f \, \overline{\mu}_L \sigma^{\alpha\beta} e_R \right) \, \overline{f} \sigma_{\alpha\beta} f + h.c. \Big],
\end{aligned}$$

$$\mathcal{L}_{\rm eff}^{\Delta L_\mu=2} = -\frac{1}{\Lambda^2} \sum_i C_i^{\Delta L=2}(\mu) Q_i(\mu)$$

$$Q_6 = \left(\overline{\mu}_L \gamma_\alpha e_L\right) \left(\overline{\nu_\mu}_L \gamma^\alpha \nu_{eL}\right), \quad Q_7 = \left(\overline{\mu}_R \gamma_\alpha e_R\right) \left(\overline{\nu_\mu}_L \gamma^\alpha \nu_{eL}\right)$$

- $\Delta\Gamma$: naively $\mathcal{O}(\Lambda^{-4})$ from double $\Delta L_\mu = 1$ insertion! But not always…