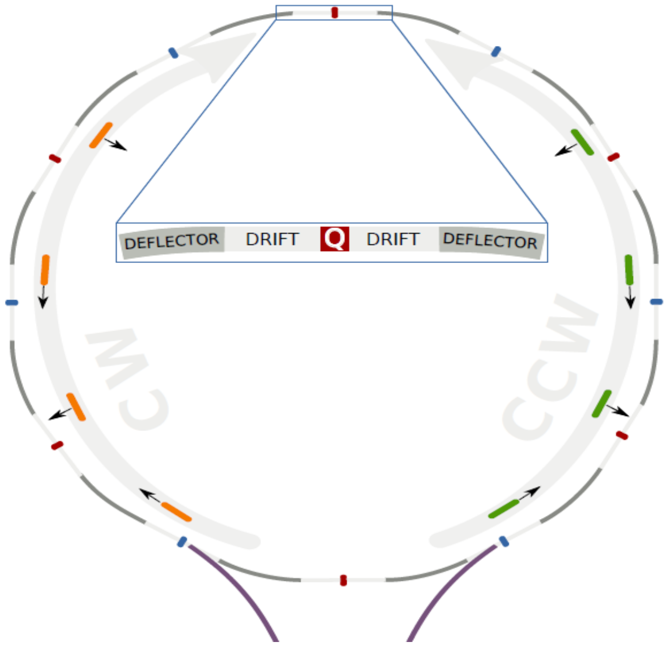
Storage Ring Proton EDM Experiment

William Morse – BNL

May 25, 2021

- Proton edm sensitivity $10^{-29}\mathrm{e}\cdot\mathrm{cm}$.
- Improves the sensitivity to QCD CP violation (θ_{QCD}) by three orders of magnitude.
- New Physics reach at 10^3 TeV mass scale.
- Highly symmetric, magic momentum storage ring lattice in order to control systematics.
  - Proton magic momentum $= 0.7\,\mathrm{GeV/c}$.
  - Proton polarimetry peak sensitivity at the magic momentum.
  - Electric bending, magnetic focusing is optimal.
  - 3×10^{10} polarized protons per fill.
  - Stores simultaneously CW and CCW bunches.
  - Stores simultaneously positive and negative helicity bunches.
  - 24-fold symmetric storage ring.
  - Change sign of the focusing/defocusing quadrupoles.
  - Circumference = 800m with $E = 4.4\,\mathrm{MV/m}$.
  - Conservative electric field strength.
- After proton edm, add magnetic bending for Deuteron/He3 edm measurements.
  - Deuteron and He3 edm measurements complementary physics to proton edm.



Storage Ring Proton EDM

- History of Storage Ring EDM
- Studies with Polarized Beams
- PEDM Ring Design
- Systematics
- Dark Matter/Dark Energy

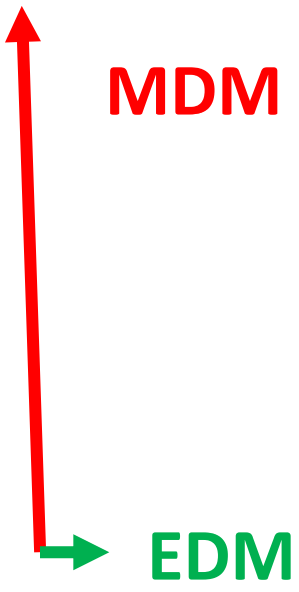
Idea came from BNL E821 muon $g-2$ experiment

- MDM Magic β $\qquad a = G = (g-2)/2$

$$\frac{d\left(\hat{\beta}\bullet\vec{S}\right)}{dt} = \frac{e}{mc}\vec{S}_T \bullet \left[a\hat{\beta}\times\vec{B} + \left(\frac{g\beta}{2} - \frac{1}{\beta}\right)\vec{E}\right]$$

- EDM

$$\frac{d\vec{S}}{dt} = d_\mu\, \hat{S} \times \left(\vec{E} + \vec{v} \times \vec{B}\right)$$



- Parasitic measurement BNL E821: $d_\mu < 1.9 \times 10^{-19} ecm$
- FNAL E989 should do up to $100\times$ better.

Yannis Semertzidis Idea 24 years ago

- Dedicated edm experiment: turn off the MDM spin precession.
- MDM

$$\frac{d\hat{\beta}\cdot\vec{s}}{dt} = -\frac{e}{m}\vec{s}_P \cdot \left[G\hat{\beta}\times\vec{B} + \left(\frac{g\beta}{2} - \frac{1}{\beta}\right)\frac{\vec{E}}{c}\right] = 0$$

- E821 was our "day job". Thinking about sredm "for the fun of it".

Storage Ring **EDM**
Technical Review – 12/7/2009

Polarimeter Development
Requirements

BNL Director's Review 2009

1. Measure a change in the vertical polarization with a sensitivity of 10^{-6}.
   Provide a continuous record with time.
   Reduce systematic errors to below the sensitivity limit.
2. Track the magnitude of the polarization with time.
3. Provide transverse (X) asymmetry data continuously for control.
   Operate at high efficiency.

Development proposal made to COSY-Jülich in 2007.
Ring design was for 1 GeV/c deuteron beam (250 MeV)

Best scheme requires deuteron scattering from carbon.

Conduct study using as much existing equipment as possible.

(Begin studies of production/preservation of horizontal polarization.)

Deuteron and proton polarimeters are similar.

Edward J. Stephenson, IUCF

Experiments with Polarized Beams

- We have done polarized beam experiments at:
- KVI, Groningen, Netherlands,
- COSY, Julich, Germany,
- AGS, BNL, USA.

Polarized Beam Experiments Publications

G. Guidoboni, E. Stephenson, A. Wronska *et al.*,	Connection between zero chromaticity and long in-plane polarization lifetime in a magnetic storage ring	Phys. Rev. ST Accel Beams 22, 024201 (2018)
A. Saleev, N. Nikolaev, F. Rathmann *et al.*,	Spin tune mapping as a novel tool to probe the spin dynamics in storage rings	Phys. Rev. ST Accel. Beams 20, 072801 (2017)
Nils Hempelmann	Phase Locking the Spin Precession in a Storage Ring	Phys. Rev. Lett, 119, 014801 (2017)
G. Guidoboni, E.J. Stephenson, *et al.*	How to reach a thousand-second in-plane polarization lifetime with 0.97-GeV/c deuterons in a storage ring	Phys. Rev. Lett. 117, 054801 (2016)
D. Eversmann *et al.*	New method for a continuous determination of the spin tune in storage rings and implications for precision experiments	Phys. Rev. Lett. 115, 094801 (2015)
Z. Bagdasarian *et al.*	Measuring the polarization of a rapidly precessing deuteron beam	Phys. Rev. ST Accel. Beams 17, 052803 (2014)
N. Brantjes *et al.*	Correcting Systematic Errors in High Sensitivity Deuteron Polariztion Measurements	Nucl. Inst. Meth. A664, 49 (2012)
P. Benati *et al.*	Synchrotron oscillation effects on an rf-solenoid spin resonance	Phys. Rev. ST Accel. Beams 15, 124202 (2012)

1 - 3

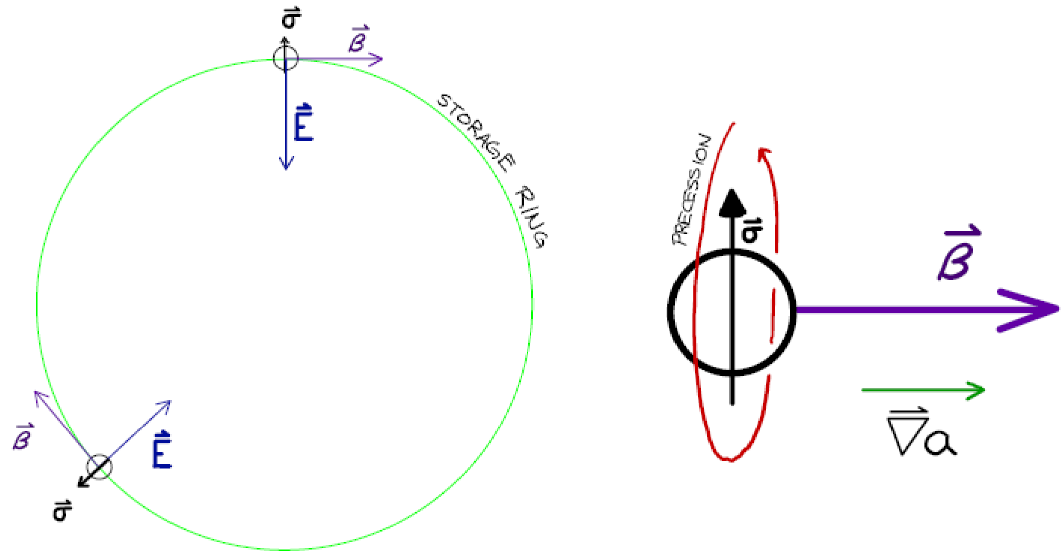
FNAL E989

Y. Orlov Edm Note

Symmetries

- $\frac{d\vec{S}_y}{dt} = d_p\, \vec{S}_z \times \left(\vec{E}_x + \vec{v} \times \vec{B}\right)$
- Radial $= x$, vertical $= y$, longitudinal $= z$.
- Electric bending, with strong alternating gradient magnetic focusing.
- Simultaneous CW/CCW.
- Symmetric lattice.
- Longitudinal (sensitive to edm) and radial (not sensitive to edm) polarized bunches.

P. Graham et al. Dark matter, dark energy



$\boldsymbol{\beta = 0.6}$

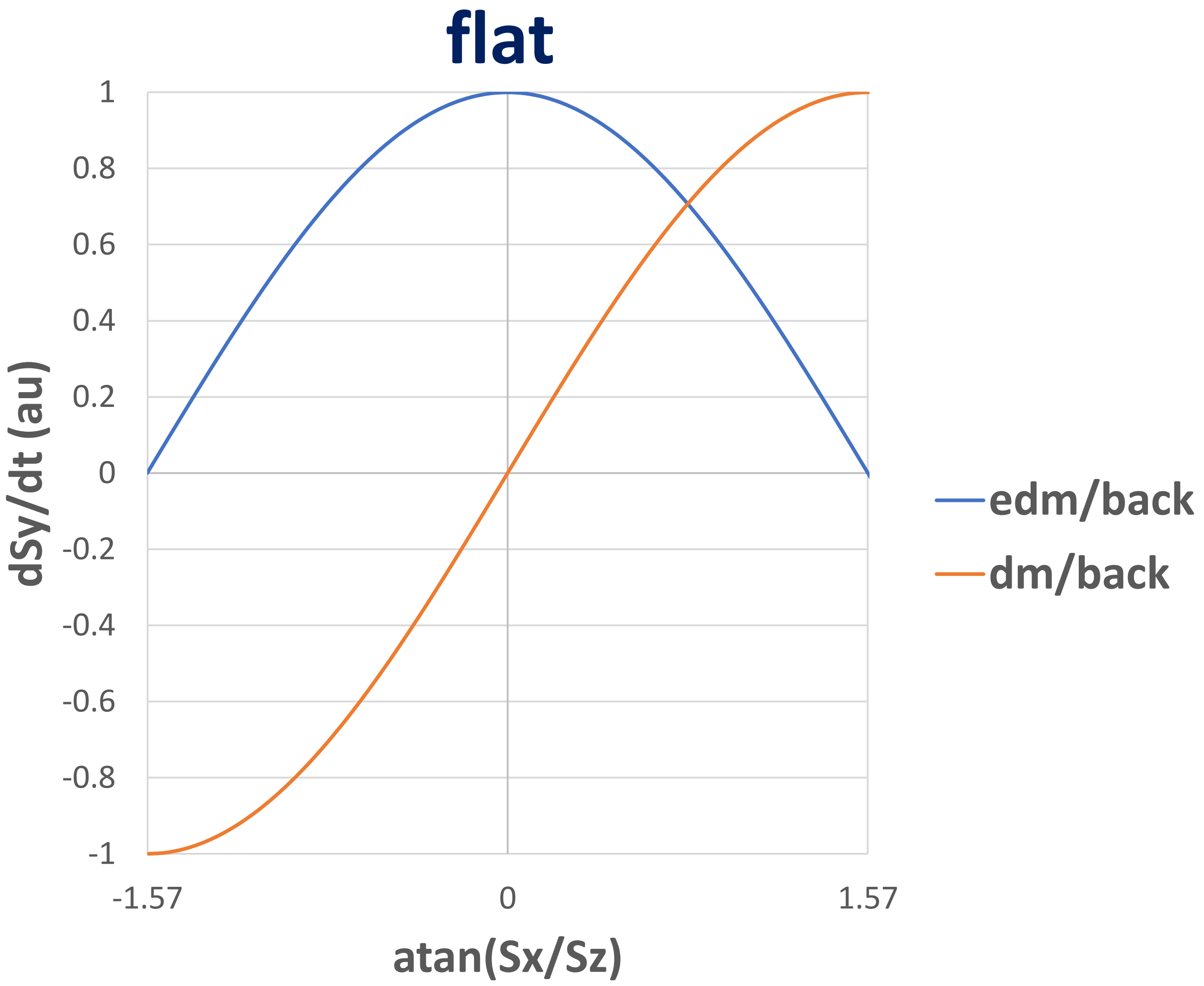
Figure 1: A sketch of the geometry for this storage ring proposal (left figure) and the directions of the proton's spin $\vec{\sigma}$, velocity $\vec{\beta}$ and precession, as well as the axion field gradient seen by the proton (right figure). The proton's spin must be oriented radially and will then precess around its velocity (out of the plane of the ring).

P. Graham et al. Dark matter, dark energy

Inspired by these designs, we evaluate the feasibility of storage rings as a way to search for ultra-light dark matter and dark energy. We will show that storage rings can have sensitivities comparable to atomic co-magnetometer techniques for pseudo-scalar interactions. For vector backgrounds, due to the relativistic nature of the beam, these rings have enhanced sensitivity to magnetic dipole interactions and can thus distinguish between electric and magnetic dipole interactions. Storage ring techniques are thus complementary to atomic co-magnetometer searches — the combination of both techniques can be used to extract the underlying nature of any new physics discovered in such experiments. The rest of this paper is organized as follows:

Longitudinal (edm) and radial (DM/DE) polarized bunches.

- Longitudinal spin: edm and edm-like background is max.
- Radial spin: DM and DM-like background is max.
- DM-like background can not imitate edm – wrong symmetry.

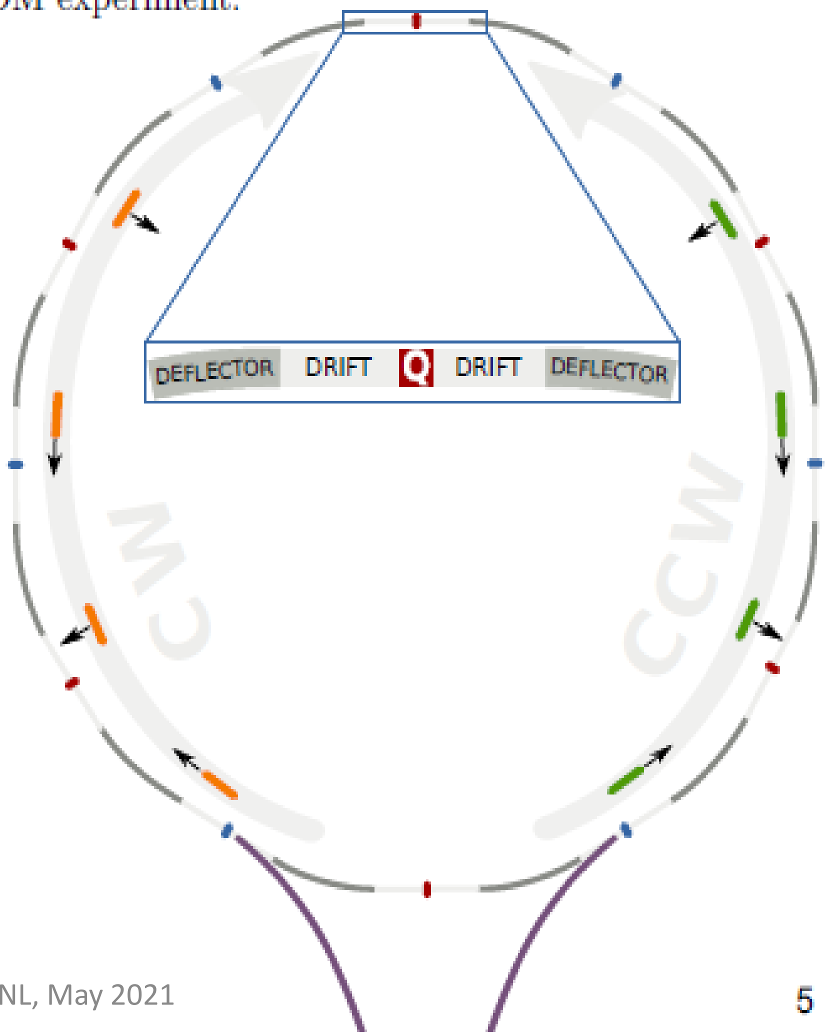


EDM, Dark Matter/Dark Energy, with the hybrid, symmetric ring lattice

arXiv:2005.11867v1 [hep-ph] 25 May 2020

Table 1: The lattice parameters for the storage-ring proton EDM experiment.

Parameter	Magnitude	Description
p_0	0.71 GeV/c	Magic momentum
β	0.59	$= v/c$, the particle speed
R_0	95.5 m	Deflector radius
C	800 m	Ring circumference
f_c	0.22 MHz	Cyclotron frequency
f_x	0.51 MHz	Horizontal betatron frequency
Q_x	2.3	Horizontal betatron tune
f_y	0.49 MHz	Vertical betatron frequency
Q_y	2.2	Vertical betatron tune
E_0	4.4 MV/m	Deflector electric field
k	0.2 T/m	Quadrupole strength
L_{quad}	40 cm	Quadrupole length
L_{str}	4.6 m	Straight section length (incl. quad.)
N	48	Number of cells



CW,CCW

Stray Magnetic Field < 0.1mG

Flux gates and trim coils

Discuss the two most difficult systematic effects:

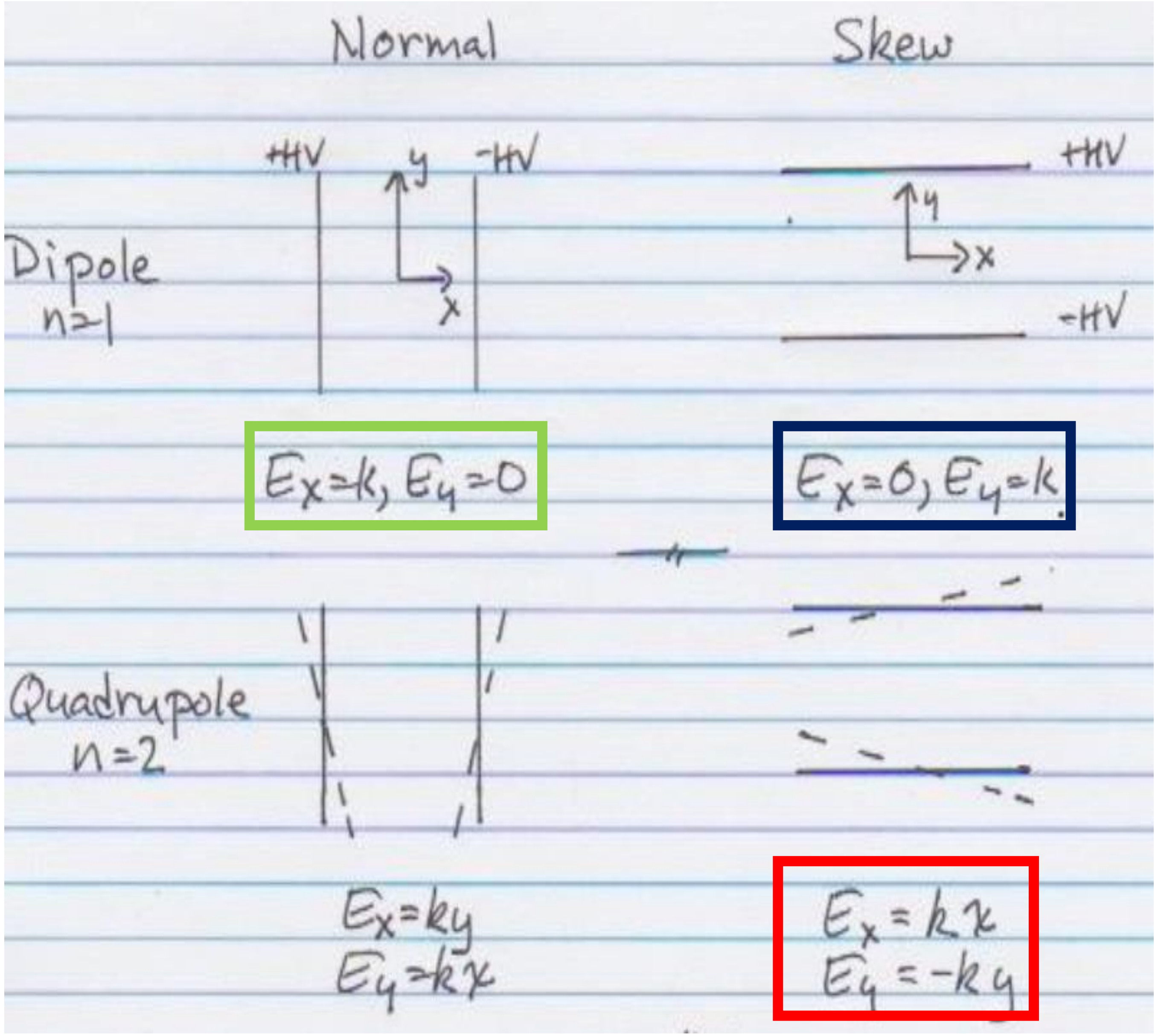
- EDM-like systematic effect.
- Stray radial magnetic field plus unwanted electric quadrupole multipole.
- DM-like effect.
- Vertical velocity effect.

EDM like systematic

- For a storage ring with dipole electric bending field, magnetic quadrupole focusing, and a stray radial magnetic field:
- Stray radial magnetic field gives a distortion of the ideal orbit, in such a way that the total radial magnetic field (stray plus focusing) = 0 on the closed orbit.
- Self fixing!
- Problem comes if we have an unwanted electric quadrupole multipole:
- $E_y = -ky$, then the radial magnetic field is not zero on the closed orbit.

$$E_y = -ky$$

- Normal dipole.
- What we want for bending.
- Skew dipole.
- Plates are rotated by ϑ.
- Spin effect canceled by CW/CCW symmetry.
- Skew quadrupole.
- Skew dipole plus misaligned by angle α. $\propto \vartheta\alpha$.
- Ronald Reagan: Trust but verify.



Closed Orbit due to Field Error

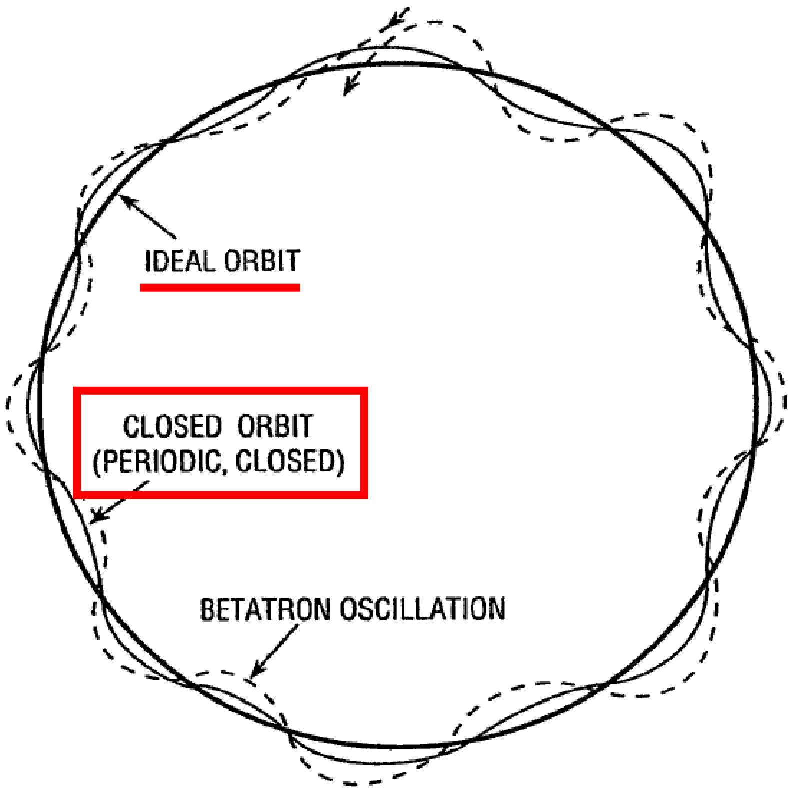


Fig. 2.1 The ideal orbit and closed orbit close after one turn. The betatron oscillation is quasiperiodic and always open.

3.2 Orbit Distortion due to Field Error

In the presence of a field error ΔB in the dipoles, the actual closed orbit deviates from the ideal orbit,

$$\frac{d^2x}{ds^2} + k(s)x = -\frac{\Delta B}{B\rho} \equiv F(s)\,. \qquad (3.2)$$

Since the field error is in the dipoles, ΔB depends on s, *but not on* x. The Courant-Snyder transformation yields

$$\frac{d^2\eta}{d\phi^2} + \nu^2\eta = \nu^2\beta^{3/2}F(\phi)\,. \qquad (3.3)$$

The Green function method gives the periodic solution

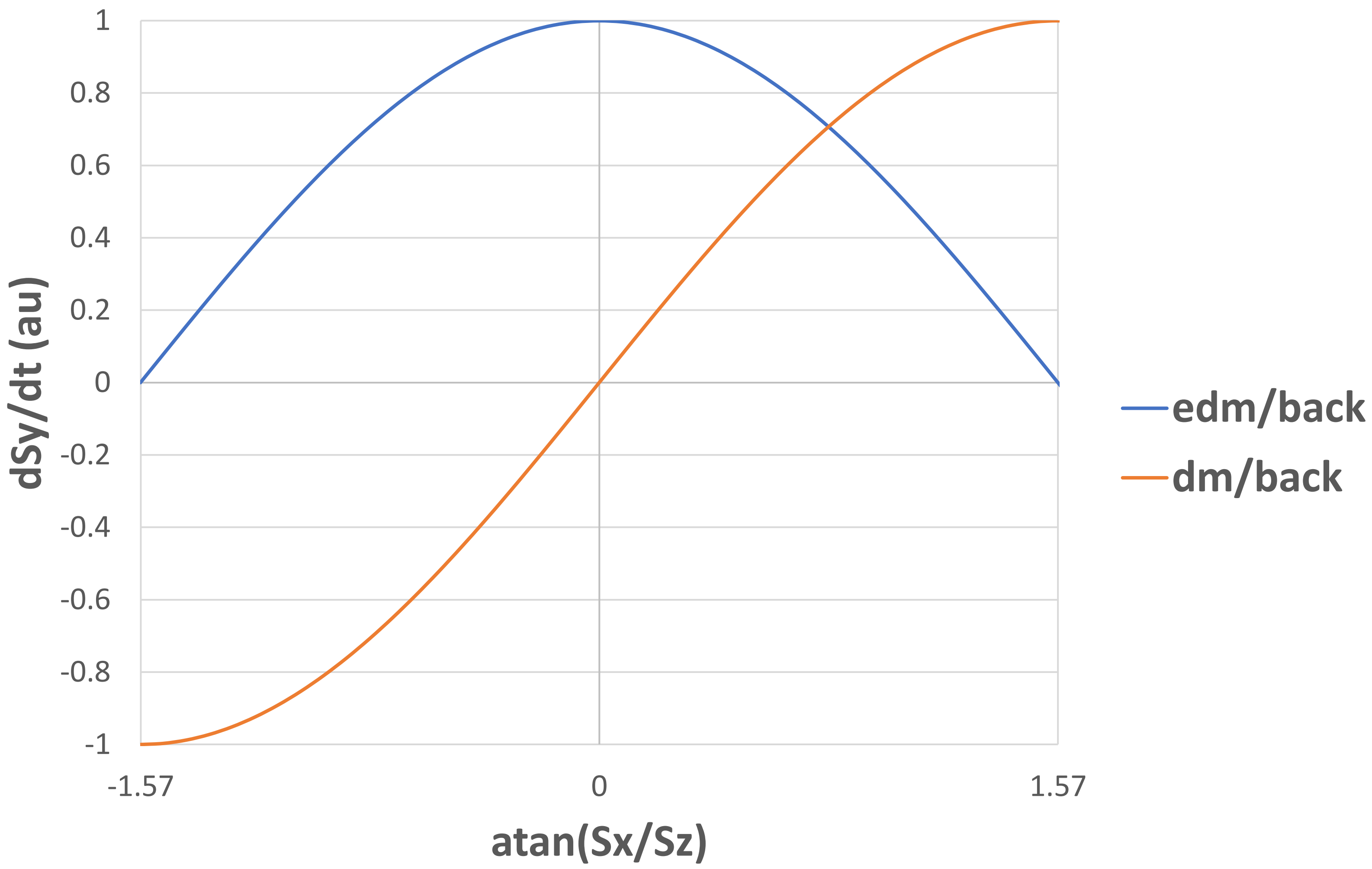
$$\eta(\phi) = \frac{\nu}{2\sin(\pi\nu)}\int_{\phi}^{\phi+2\pi} \beta^{3/2}F(\phi')\cos[\nu(\pi+\phi-\phi')]\,d\phi' \qquad (3.4)$$

as long as $\nu \neq$ integer. We see that if $\nu =$ integer, then the closed orbit $\rightarrow \infty$, i.e. even a small field error ΔB will cause a large closed orbit distortion. Since small errors ΔB are always present, this makes it undesirable to choose a tune close to an integer.

Electric quadrupole multipole

- Create a large radial magnetic field with trim coils.
- $\approx 10^2 \times$ larger than stray radial magnetic fields.
- See how much dS_y/dt changes.
- Adjust trim electric quads around the ring to get acceptable dS_y/dt .

DM-like background



Four Straight Sections – Minimum Number

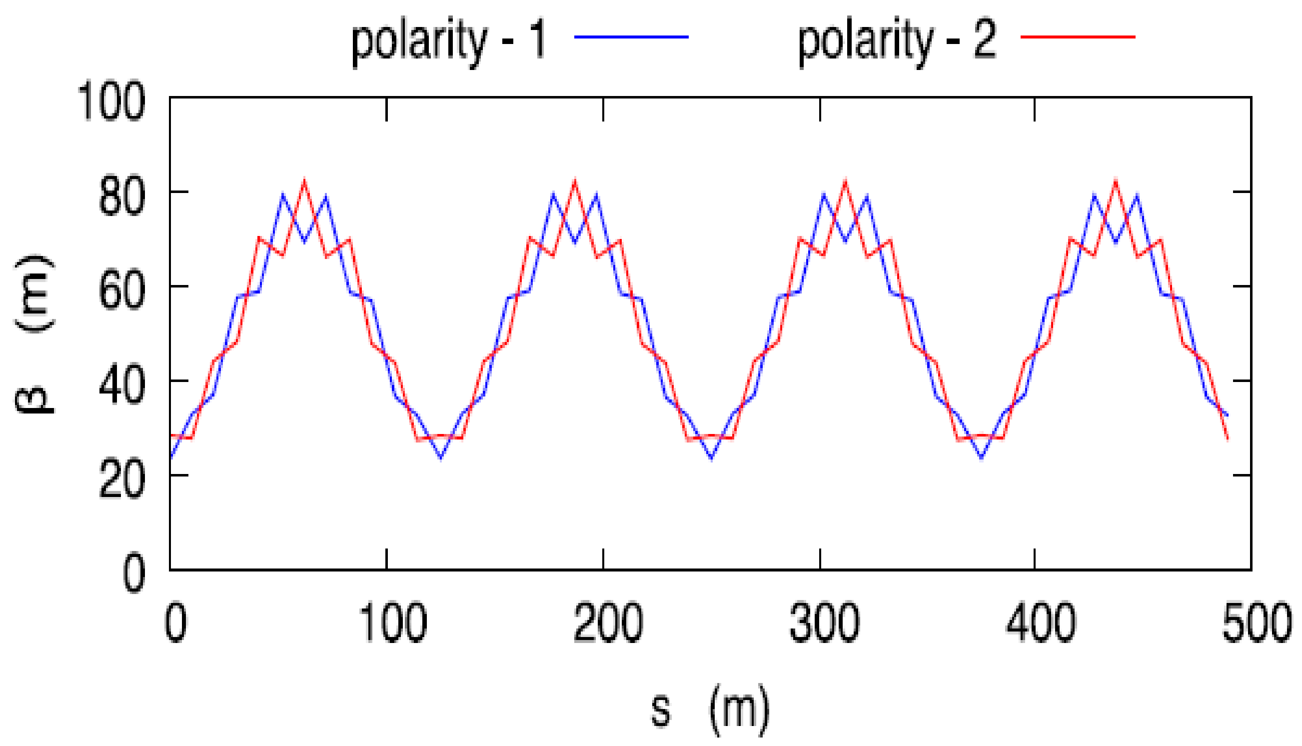
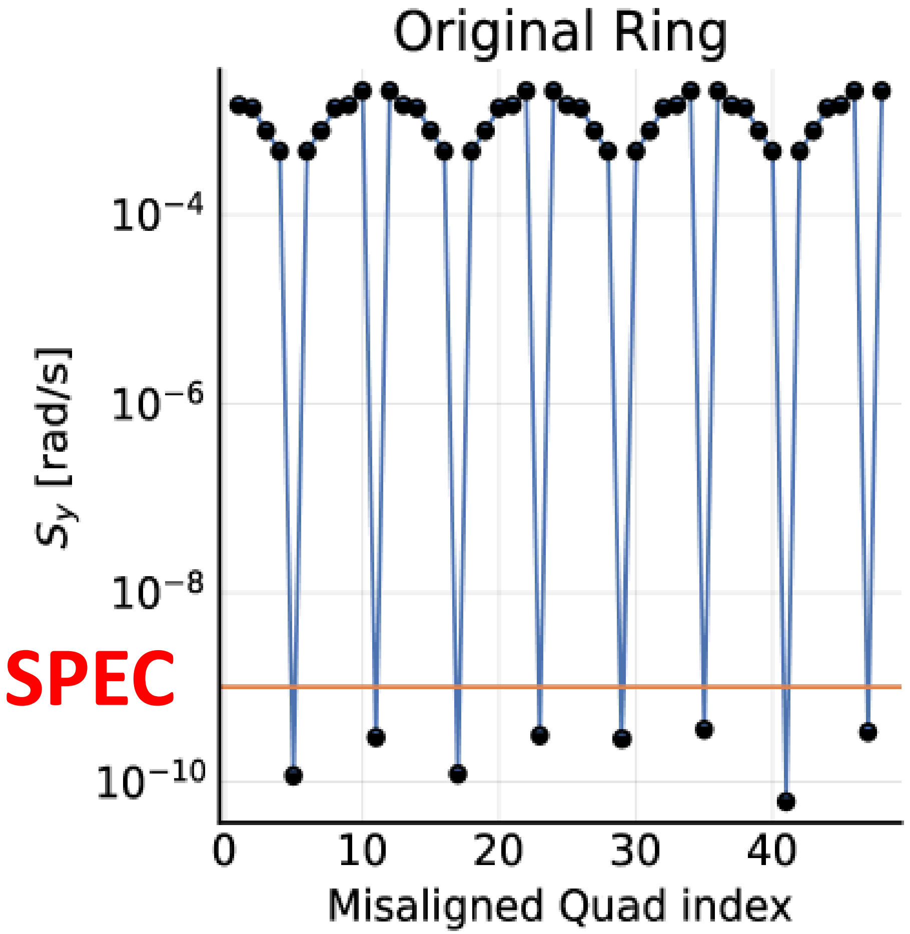


FIG. 3. The horizontal beta-function values around the ring for CW and CCW operations.

Scale of the problem in the 4-fold ring

- Radially polarized beam
- Misalign one quadrupole at a time by $1\,\mu m$
- 8 islands of high symmetry



48 cells – more symmetry

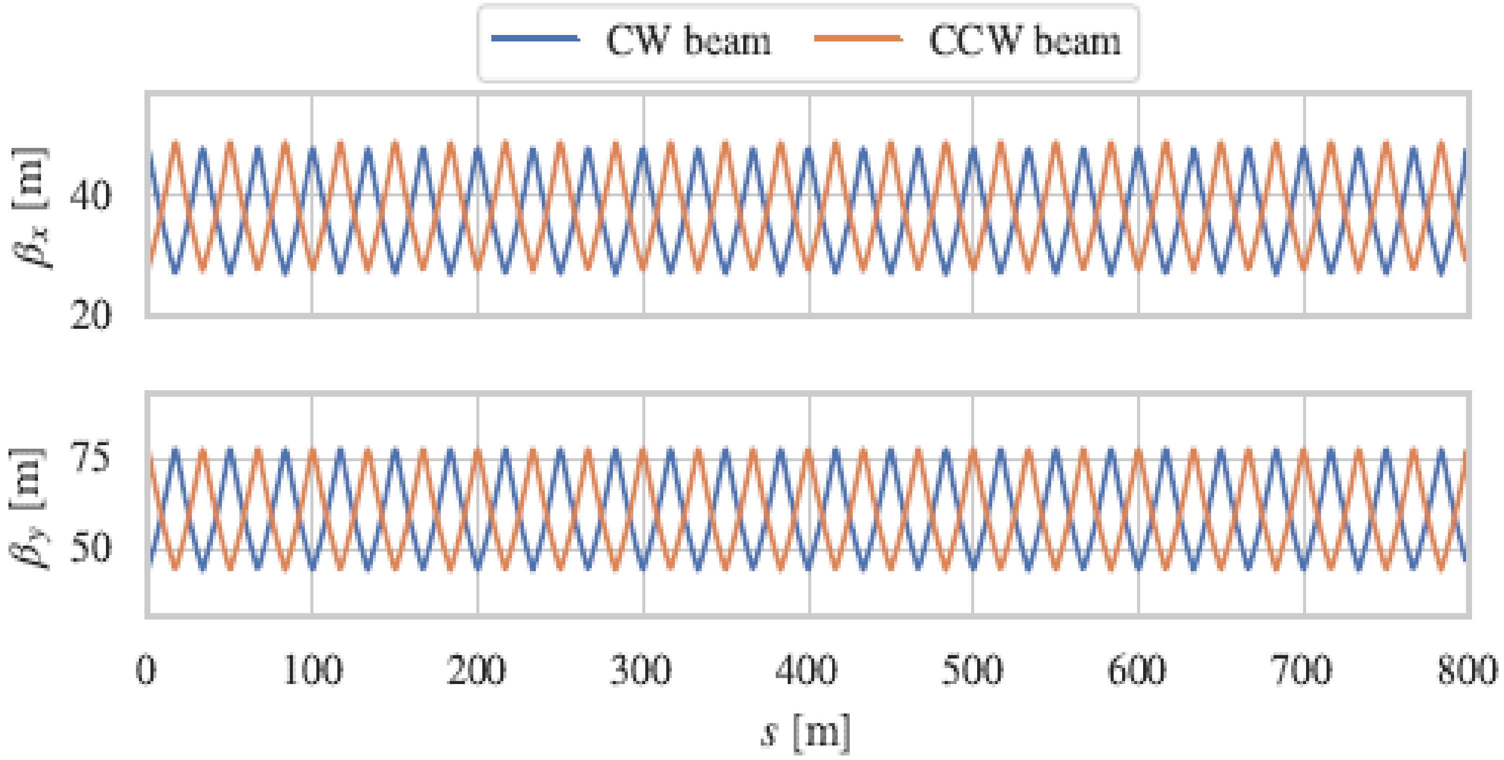


Figure 4: Beta functions along the ring. The results were obtained via numerical tracking.

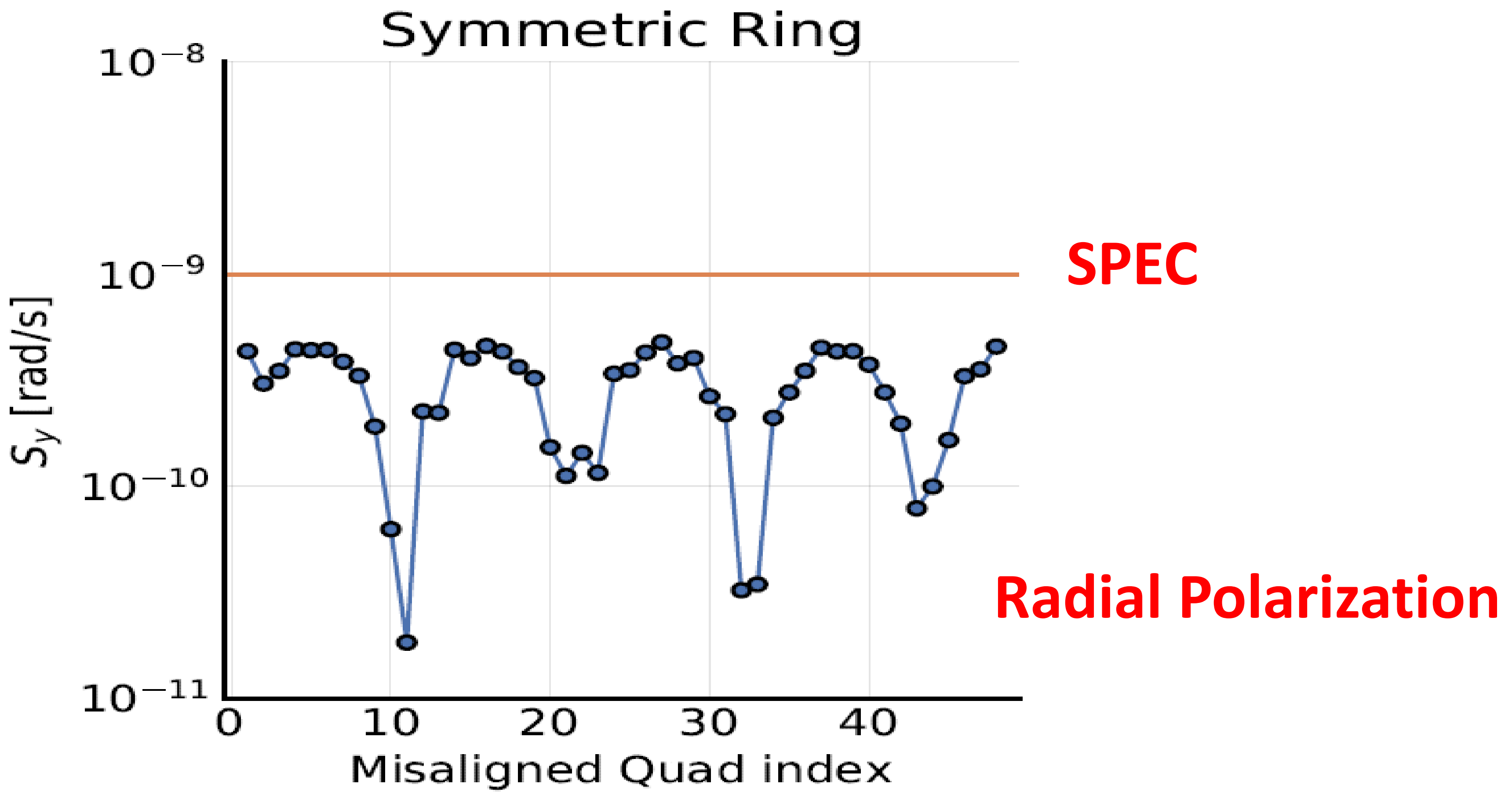


Figure 8: Vertical spin precession rate vs. index of the 1 µm misaligned quad along the azimuth. Irregularities of the points are due to the inability to determine the exact precession rate from the simulation results. Hence, the points only show upper limit of the possible vertical precession rate, actual rates could be lower. The orange line corresponds to the target EDM sensitivity

Vertical Velocity

Zanibek Omarov Symmetry

- Vertical velocity β_y combined with E_x:

$$\left(\alpha \vec{S} \times (\vec{\beta} \times \vec{E})_s\right)_y = \alpha S_x \beta_y E_x$$

- Intuitively $\langle \beta_y \rangle \equiv 0$ by definition
- However,

$$\langle \beta_y \rangle_{\text{straight}} + \langle \beta_y \rangle_{\text{bending}} \equiv 0$$

- Hence,

$$\langle \beta_y \times E_x \rangle \neq 0$$

Beam Based Alignment/Spin Based Alignment

- BBA developed in the 1990s for ILC, NLC, …
- They needed the beam to go through the center of hundreds of quads in LINAC to $1\mu m$ in order to get the correct final focus.
- In order to avoid using spin based alignment, need $0.01mm$ BBA.
- Can't use spin based alignment for DM/DE.
- RHIC achieved $0.1\ mm$ BBA [1].
- This was plenty good enough for RHIC.

1. BNL CA Dept. Note 83395.

Beam Based Alignment/Spin Based Alignment

- We will have:
- SQUID BPM resolution: $0.01\mu m$ [1].
- BMP at every quad location.
- AC modulation of closed orbit, instead of DC.
- Correct higher order multipoles in quads with correction windings.
- After BBA, use transverse polarization, and adjust with spin based alignment.

1. S. Haciomeroglu, ICHEP 2018, Proc. Sci. (2018).

Statistics

P Magic Energy: 232 MeV

- 3×10^{10}polarized protons per fill.
- Excellent polarimeter analyzing power.
- $10^{-29} ecm/6 \times 10^{7} s$

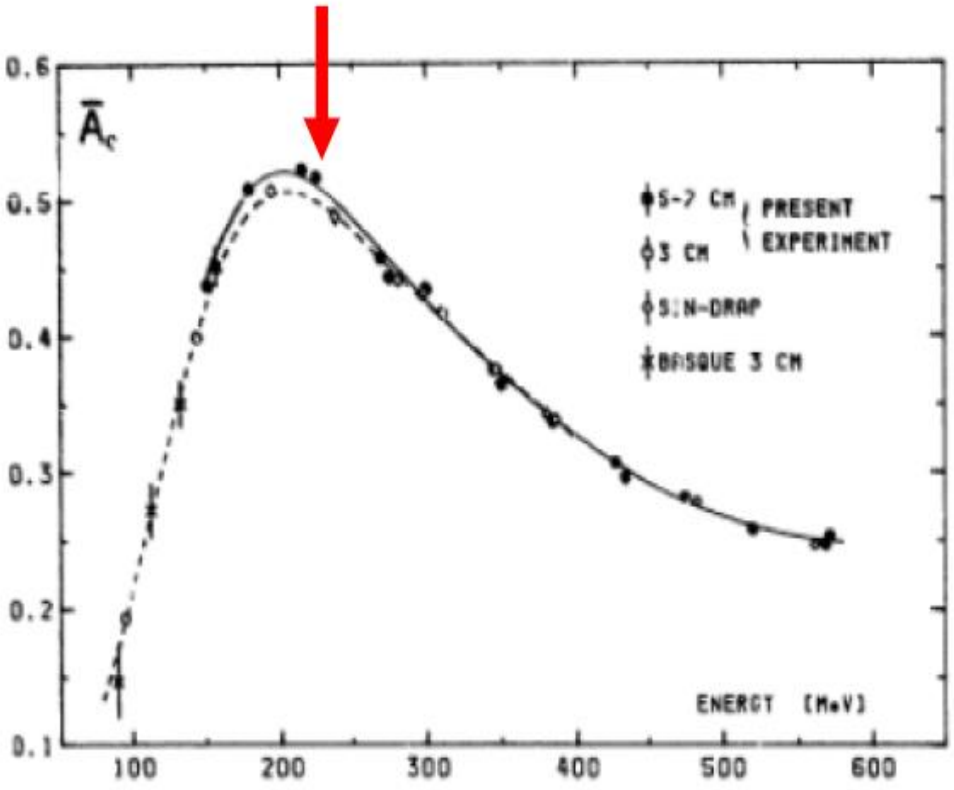
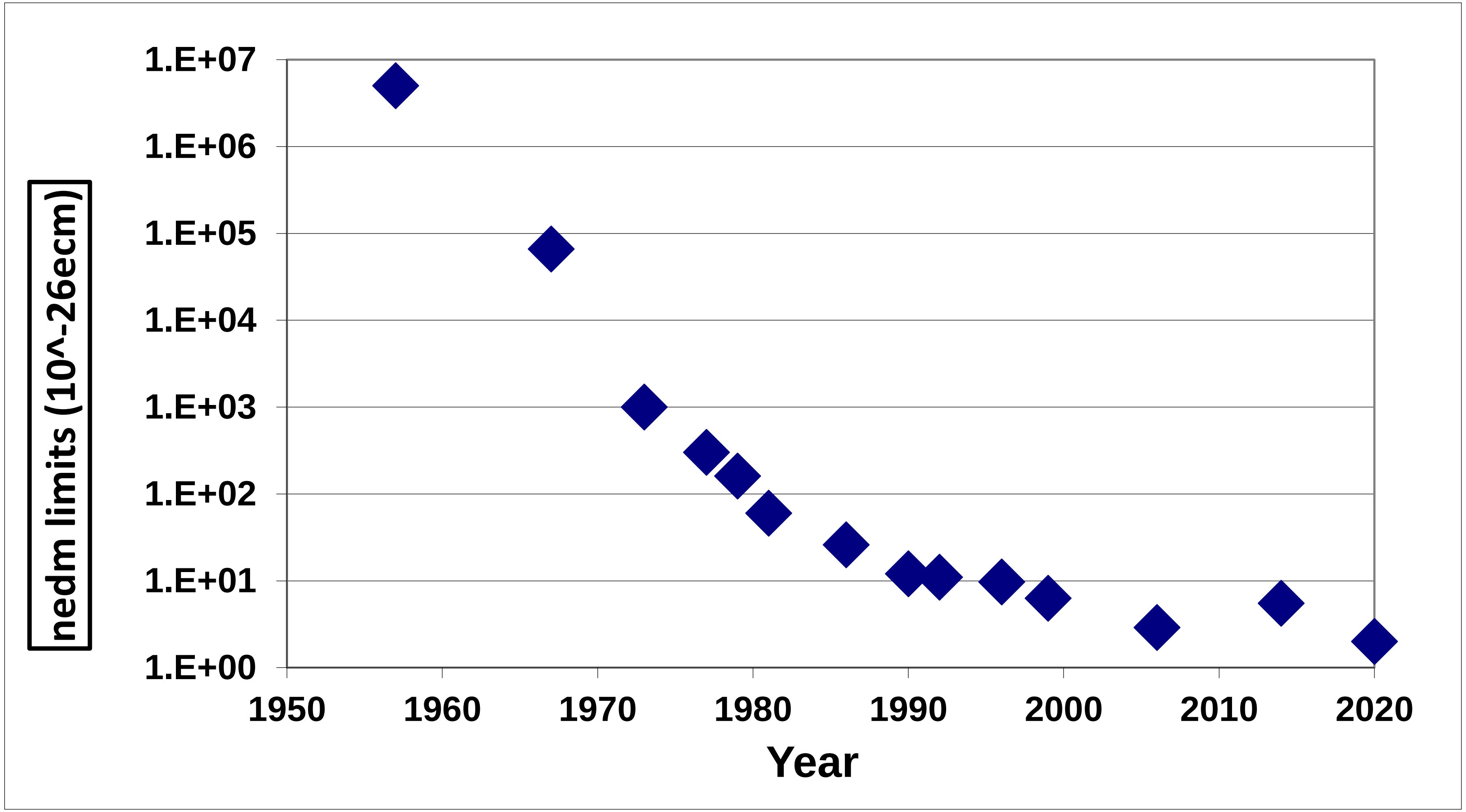


Fig. 4. Angle-averaged effective analyzing power. Curves show our fits. Points are the data included in the fits. Errors are statistical only

Fig.4. The angle averaged effective analyzing power as a function of the proton kinetic energy. The magic momentum of 0.7GeV/c corresponds to 232MeV.

PEDM limited by statistics at $10^{-29} e \cdot cm$.



nedm limited by statistics at $10^{-26} e \cdot cm$

LOI

Jim Alexander,[7] Vassilis Anastassopoulos,[34*] Rick Baartman,[26*] Stefan Baessler,[37*] Franco Bedeschi,[19] Martin Berz,[17*] Michael Blaskiewicz,[4*] Themis Bowcock,[31*] Kevin Brown,[4*] Dmitry Budker,[9,29*] Sergey Burdin,[31] Gianluigi Casse,[31*] Giovanni Cantatore,[36*] Timothy Chupp,[32*] Hooman Davoudiasl,[4*] Milind V. Diwan,[4*] George Fanourakis,[20*] Antonios Gardikiotis,[28,34*] Claudio Gatti,[18*] James Gooding,[31*] Renee Fatemi,[30] Wolfram Fischer,[4*] Peter Graham,[25*] Frederick Gray,[22*] Selcuk Haciomeroglu,[6*] Georg H. Hoffstaetter,[7*] Haixin Huang,[4*] Marco Incagli,[19*] Hoyong Jeong,[16*] David Kaplan,[13*] On Kim,[6,15*] Ivan Koop,[5*] Marin Karuza,[35*] David Kawall,[27*] Valeri Lebedev,[8*] MyeongJae Lee,[6*] Soohyung Lee,[6*] Alberto Lusiani,[24,19*] William J. Marciano,[4*] Marios Maroudas,[34*] Andrei Matlashov,[6*] Francois Meot,[4*] James P. Miller,[3*] William M. Morse,[4*] James Mott,[3,8] Zhanibek Omarov,[6,15*] Yuri F. Orlov,[7*] Cenap Ozben,[11*] SeongTae Park,[6*] Giovanni Maria Piacentino,[33*] Boris Podobedov,[4*] Matthew Poelker,[12] Dinko Pocanic,[37*] Joe Price,[31*] Deepak Raparia,[4*] Surjeet Rajendran,[13*] Sergio Rescia,[4*] B. Lee Roberts,[3*] Yannis K. Semertzidis,[6,15*] Alexander Silenko,[14*] Edward Stephenson,[10*] Riad Suleiman,[12*] Michael Syphers,[21*] Pia Thoerngren,[23*] Volodya Tishchenko,[4*] Nikolaos Tsoupas,[4*] Spyros Tzamarias,[1*] Alessandro Variola,[18*] Graziano Venanzoni,[19*] Eva Vilella,[31*] Joost Vossebeld,[31*] Peter Winter,[2] Eunil Won,[16*] Konstantin Zioutas.[34*]

Summary

- Exciting sredm physics.
- CP violation beyond the SM and/or QCD CP violation, i.e., Peccei-Quinn symmetry axion physics.
- EDM has a spin flip, so proportion to mass.
- Physics is complementary to electron, neutron, Hg, etc., searches.
- PEDM systematics are OK. E821 and E989 found new beam/spin dynamics systematics at the level of sensitivity. Need statistics.

Extra

Quantity	Value
Bending Radius R_0	95.49 m
Number of periods	24
Electrode spacing	4 cm
Electrode height	20 cm
Deflector shape	cylindrical
Radial bending E-field	4.4 MV/m
Straight section length	4.16 m
Quadrupole length	0.4 m
Quadrupole strength	±0.21 T/m
Bending section length	12.5 m
Bending section circumference	600 m
Total circumference	800 m
Cyclotron frequency	224 kHz
Revolution time	4.46 µs
$\beta_x^{\max}$, $\beta_y^{\max}$	64.54 m, 77.39 m
Dispersion, $D_x^{\max}$	33.81 m
Tunes, Q_x, Q_y	2.699, 2.245
Slip factor, $\frac{dt}{t}/\frac{dp}{p}$	-0.253
Transition energy, $\gamma_{tr} = 1/\sqrt{\alpha}$	566 MeV
Momentum acceptance, (dp/p)	5.2×10^{-4}
Horizontal acceptance [mm mrad]	4.8
RMS emittance [mm mrad], ϵ_x, ϵ_y	0.214, 0.250
RMS momentum spread	1.177×10^{-4}
Particles per bunch	1.17×10^{8}
RF voltage	1.89 kV
Harmonic number, h	80
Synchrotron tune, Q_s	3.81×10^{-3}
Bucket height, $\Delta p/p_{\text{bucket}}$	3.77×10^{-4}
Bucket length	10 m
RMS bunch length, σ_s	0.994 m

Storage Ring Probes of Dark Matter and Dark Energy

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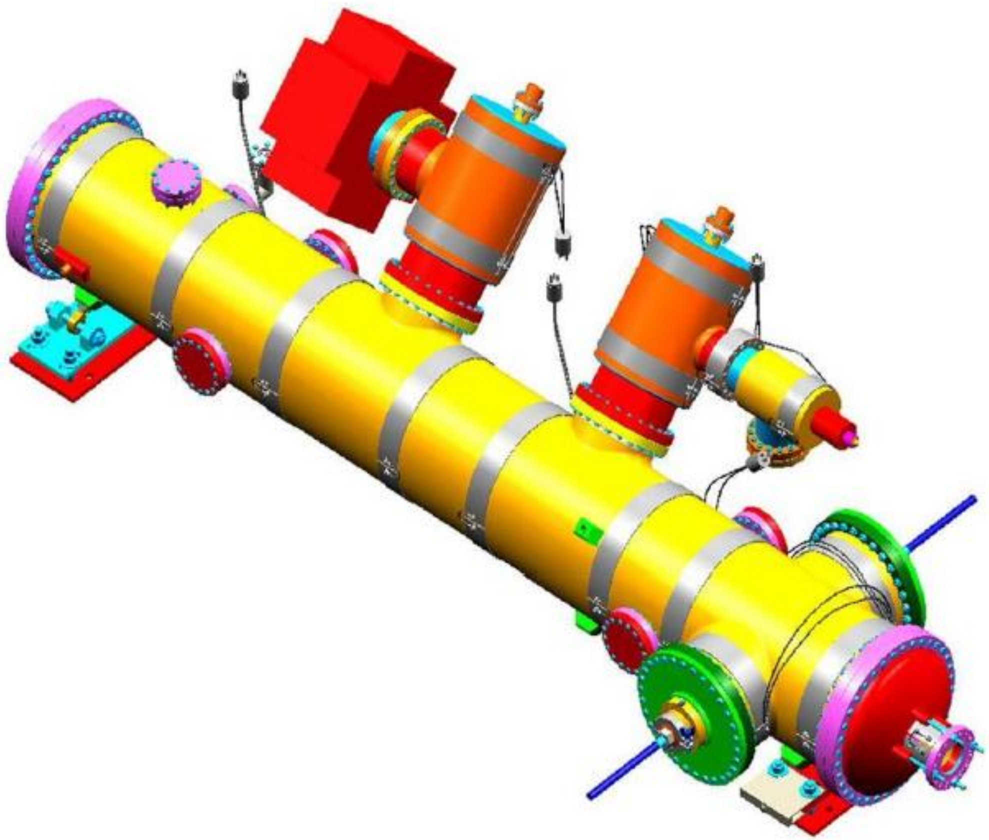
May 26, 2020

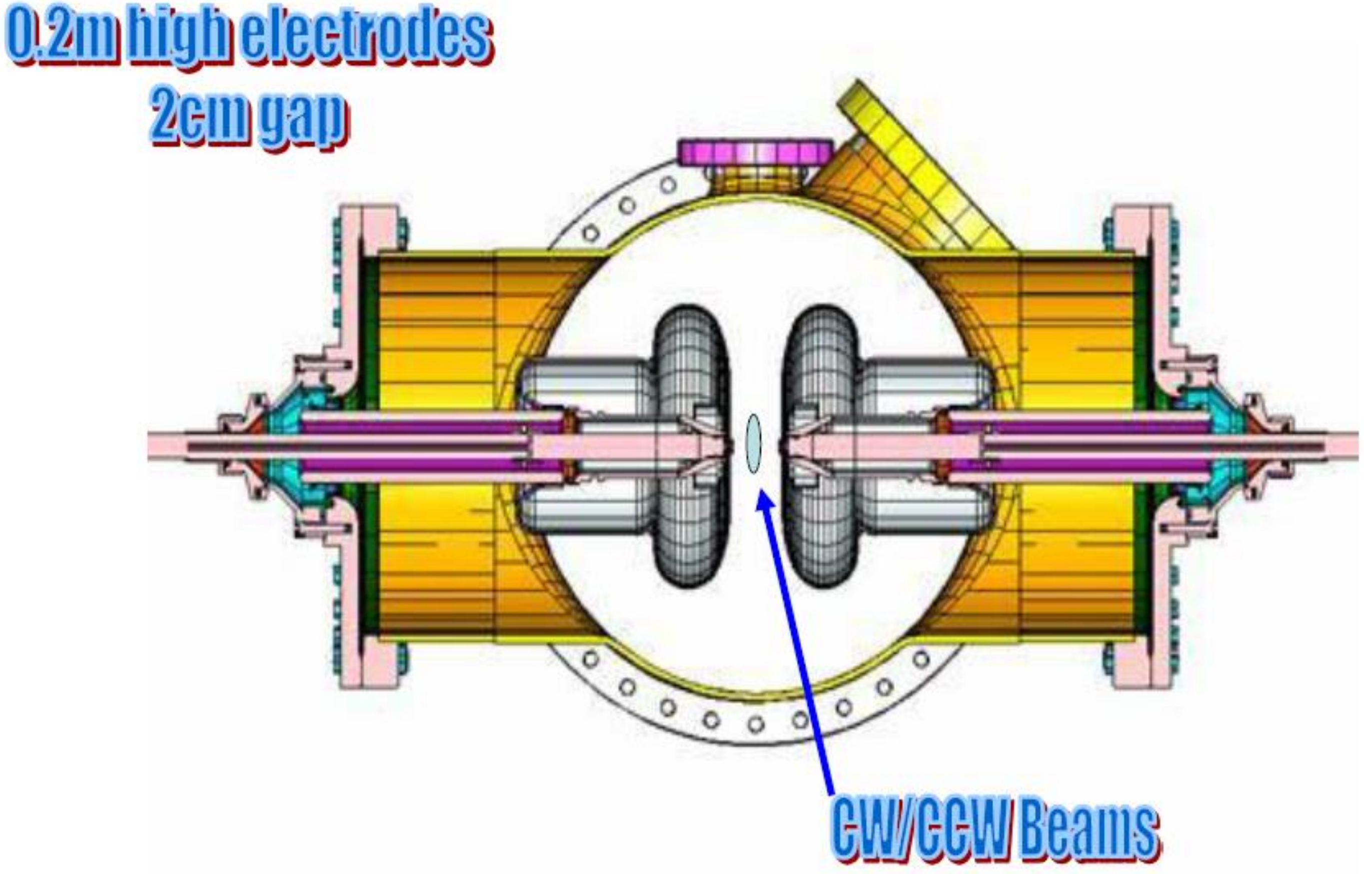
Abstract

We show that proton storage ring experiments designed to search for proton electric dipole moments can also be used to look for the nearly dc spin precession induced by dark energy and ultra-light dark matter. These experiments are sensitive to both axion-like and vector fields. Current technology permits probes of these phenomena up to three orders of magnitude beyond astrophysical limits. The relativistic boost of the protons in these rings allows this scheme to have sensitivities comparable to atomic co-magnetometer experiments that can also probe similar phenomena. These complementary approaches can be used to extract the micro-physics of a signal, allowing us to distinguish between pseudo-scalar, magnetic and electric dipole moment interactions.

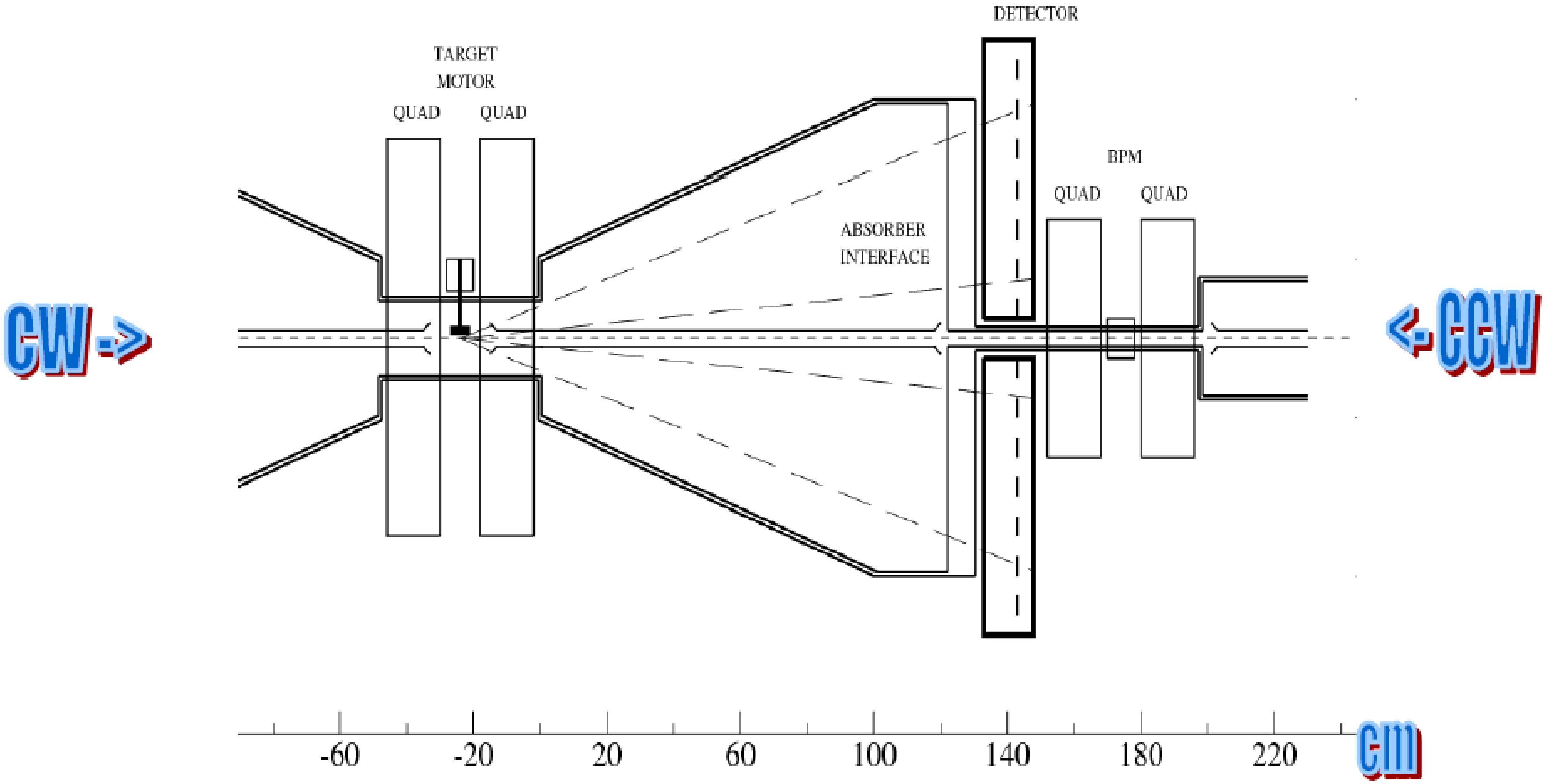
Extras

3m long Tevatron pbar-p Separator




0.2m high electrodes
2cm gap
CW/CCW Beams

½ of PEDM Polarimeter



It turns out that the vertical electric field dipole multipole gets canceled CW/CCW [1]. Briefly, the radial focusing magnetic field flips sign CW/CCW:

$$< F_y > = e < E_y + vB_x >= 0 \quad (2)$$

$$< B_x >= -< \frac{E_y}{v} > =< k_{2B} y > \quad (3)$$

In equ. 3, k_{2B} gives the strength of the magnetic focusing quadrupole = 0.2T/m.

$$y_{co} = -\frac{< E_y >}{k_{2B}\, v} \quad (4)$$

The MDM spin precession flips sign CW/CCW:

$$\frac{d\hat{s}}{dt} = \frac{e}{mc}\hat{s}\times\left\{\left(\frac{g}{2}-1+\frac{1}{\gamma}\right)\vec{B} - \left(\frac{g}{2}-1\right)\frac{\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left(\frac{g}{2}-\frac{\gamma}{\gamma+1}\right)\vec{\beta}\times\vec{E}\right\} \quad (5)$$

The EDM spin precession from the bending electric field doesn't flip sign CW/CCW:

$$\frac{d\vec{s}}{dt} = d_p\, \vec{s} \times \left(\vec{E} + \vec{v} \times \vec{B}\right) \quad (6)$$

The accuracy limitation of BBA on RHIC is about 0.1 mm because of the following factors:

1. BPM accuracy is temperature dependent. Each Δ10°F temperature change results in ~Δ1ns change in trigger time which adds an uncertainty of ~ +/-0.1mm to ~1mm in BPM reading. BPM resolution is ~0.01mm at a constant temperature;
2. The physical misalignment of quadrupoles relative to each other is ~ +/-0.06mm;
3. BPM position with respect to the outside fiducials is ~ +/-0.13mm;
4. Unknown beam positions and angles at the first corrector (beam angle at the triplet). It was estimated to have an effect ~ +/-0.1mm;
5. Repeatability under the same machine settings is found by experiment to be ~ +/-0.1mm.

CW/CCW

- The major conceptual and technological strengths of the srEDM method render it ready for technical evaluation. Its critical conceptual strength is the realization that a ring with purely electric bending sections and alternate magnetic focusing (a hybrid-ring lattice) permits simultaneous clock-wise (CW) and counter-clock-wise (CCW) storage, thus *eliminating firstorder systematic error sources*, i.e., out-of-plane electric fields, *as well as the need to significantly shield the ring from external magnetic fields*.

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Resolution and systematic limitations in beam-based alignment

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Beam-based alignment of quadrupoles by variation of quadrupole strength is a widely used technique in accelerators today. We describe the dominant systematic limitation of such algorithms, which arises from the change in the center position of the quadrupole as the strength is varied, and derive expressions for the resulting error. In addition, we derive an expression for the statistical resolution of such techniques in a periodic transport line, given knowledge of the line's transport matrices, the resolution of the beam position monitor system, and the details of the strength variation procedure. These results are applied to the Next Linear Collider main linear accelerator, an 11 km accelerator containing 750 quadrupoles and 5 000 accelerator structures. We find that, in principle, a statistical resolution of 1 μm is easily achievable, but the systematic error due to variation of the magnetic centers could be several times larger.

Effect	Remediation
Radial B-field.	Magnetic focusing.
Unwanted vertical forces when other than magnetic focusing is present.	Vary the magnetic focusing strength and fit for the DC offset in the vertical precession rate.[14]
Dipole vertical E-fields.	Cancel exactly with CW and CCW beam storage.
Quadrupole E-field in the electric bending sections.	Probe it by locally splitting the counter-rotating beams and cancel it with trim E-fields. Finally, keep the counter-rotating beams at the same position to S-BPM resolution.
Corrugated (non-planar) orbit.	Minimize effect with symmetric lattice design. Finally, keep the CR beams at same position, at the electric field bending sections, using beam-based alignment.
Longitudinal B-field.	Small effect.
Geometrical phase effect due to lattice elements imperfections.	Equivalent to a spin resonance due to lattice elements imperfections. Magnetic quadrupoles: beam-based alignment to 1μm rms. E-field sections: Absolute beam position monitors to $<$0.1mm.
Geometrical phase effect due to external magnetic fields.	Equivalent to a spin resonance due to external magnetic interference coupled with electric field bending section misplacement.[24, 26] When the local spin effects are kept below 1nT B-field equivalent, the effect is negligible even for one directional (CW or CCW only) storage.
RF cavity vertical and horizontal misalignment.	Vary the longitudinal lattice impedance to probe the effect of the cavity's vertical and horizontal angular misalignments. The vertical and horizontal offsets are much smaller effects.

BEAM BASED ALIGNMENT

Derivation of formula for orbit change

$$\Delta x(s) = \left(\frac{\Delta k \cdot x(\bar{s}) l}{B\rho}\right)\left(\frac{1}{1 - k\frac{l\beta(\bar{s})}{2B\rho\tan\pi\nu}}\right)\frac{\sqrt{\beta(s)}\sqrt{\beta(\bar{s})}}{2\sin\pi\nu}\cos(\phi(s) - \phi(\bar{s}) - \pi\nu)$$

- Δx = orbit change
- s = measurement position
- $\bar{s}$ = position of quadrupole
- Δk = change of quadrupole strength
- $x(\bar{s})$ = position of beam inside the quadrupole
- β = beta function
- ν = tune
- ϕ = betatron phase
- k = quadrupole strength
- l = length of quadrupole
- $B\rho$ = magnetic rigidity of the beam

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Resolution and systematic limitations in beam-based alignment

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Beam-based alignment of quadrupoles by variation of quadrupole strength is a widely used technique in accelerators today. We describe the dominant systematic limitation of such algorithms, which arises from the change in the center position of the quadrupole as the strength is varied, and derive expressions for the resulting error. In addition, we derive an expression for the statistical resolution of such techniques in a periodic transport line, given knowledge of the line's transport matrices, the resolution of the beam position monitor system, and the details of the strength variation procedure. These results are applied to the Next Linear Collider main linear accelerator, an 11 km accelerator containing 750 quadrupoles and 5 000 accelerator structures. We find that, in principle, a statistical resolution of 1 μm is easily achievable, but the systematic error due to variation of the magnetic centers could be several times larger.

G. K modulation

Finally, another approach that is worth mentioning is the strength modulation technique. In K modulation, the strength of a quadrupole is varied harmonically at a frequency that is low compared to the betatron and synchrotron frequencies of a storage ring; this generates oscillations in the closed orbit at the same frequency. Because K modulation causes narrow band orbit oscillations, it is possible to use lock-in amplifiers and other techniques to vastly improve the signal-to-noise performance of the measurement [12]. This technique has been used in a number of storage rings where it is not possible to make large changes to the quadrupole strengths because of the

Field Gradients OK for E989
Eric Metodiev

Thomas-BMT equation generalized to electric dipole moments and field gradients

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An expression is presented for the relativistic equations of motion, including field gradients, of a particle and its spin with electric and magnetic dipole moments aligned along the spin axis. An electromagnetic duality transformation is used to generalize a Thomas-BMT equation with gradient terms. Corrections to particle dynamics in storage rings for precision $(g-2)$ and electric dipole moment measurements are calculated, and applications to precision particle tracking programs are considered.

I. INTRODUCTION

A detailed knowledge of the spin dynamics of particles with non-zero electric dipole moments (EDMs) and magnetic dipole moments (MDMs) is necessary for precision EDM and $(g-2)$ measurements using spin precession in storage rings [1–5]. The Thomas-Bargmann-Michel-Telegdi (T-BMT) equation [6, 7] governs the classical spin dynamics of a particle with a non-zero MDM in electric and magnetic fields, neglecting field gradients. Recently, derivations have been presented which generalize the T-BMT equation to include a non-zero particle EDM based on duality transformations [8, 9] and explicit relativistic constructions [10].

The spin equation of motion of a particle with a non-zero MDM including first order field gradients has also been established [11]. By making use of an electromagnetic duality transformation on these equations of motion, a generalization of the T-BMT equation for non-zero particle EDMs and MDMs and first order field gradients is determined. The corrections to the spin and particle equtions of motion are then studied. We find that typical experimental methods in storage ring EDM methods are robust to higher order than previously demonstrated. Higher-order corrections to the dynamical equations used in many precision particle tracking programs [12, 13] are also presented.

II. DYNAMICS WITH A NON-ZERO MAGNETIC DIPOLE MOMENT

R.H. Good [11] determined the classical equations of motion for a particle and its spin with a MDM μ along its spin direction to first order in the field gradients. For a particle of mass m, electric charge e, spin angular momentum $I\hbar$, and velocity $\mathbf{v} = c\beta$, he derived that:

$$mc\frac{d(\gamma\beta)}{dt} = e\mathbf{E} + ec\beta \times \mathbf{B} + \frac{\mu\gamma}{cI\hbar}[\nabla + \beta \times (\beta \times \nabla) + \frac{1}{c}\beta\partial_t]\mathbf{s} \cdot \mathbf{R} \quad (1)$$

and

$$\frac{d\mathbf{s}}{dt} = \frac{\mu}{cI\hbar}\frac{1}{\gamma}\mathbf{s} \times \mathbf{R} - \frac{e}{mc}\mathbf{s} \times \mathbf{N} + \frac{\mu}{I\hbar mc^2}\frac{1}{\gamma+1}\mathbf{s} \times (\beta \times \nabla)[\mathbf{s} \cdot \mathbf{R}], \quad (2)$$

where:

$$\mathbf{M} = c\mathbf{B} - \frac{\gamma}{\gamma+1}\beta \times \mathbf{E}$$
$$\mathbf{N} = \frac{\gamma}{\gamma+1}(\mathbf{E} + c\beta \times \mathbf{B}) \times \beta \quad (3)$$
$$\mathbf{R} = \mathbf{M} + \gamma\mathbf{N}.$$

A non-zero quadrupole moment q is also considered in the solution, but we omit it here for clarity. This method can also be used to extend the T-BMT equation to electric quadrupole moments. We define the EDM $\mathbf{d}$ and MDM $\boldsymbol{\mu}$ in terms of the rest frame spin $\mathbf{s}$ as:

$$\mathbf{d} = \frac{\eta}{2}\frac{e}{mc}\mathbf{s}, \quad \boldsymbol{\mu} = \frac{g}{2}\frac{e}{m}\mathbf{s}, \quad (4)$$

where these relations define g and η.

We can also write the Equation 1 as an equation for β in the form:

$$\gamma mc\frac{d\beta}{dt} = e\mathbf{E} + ec\beta \times \mathbf{B} - e\beta(\beta \cdot \mathbf{E}) + \frac{\mu\gamma}{cI\hbar}[\nabla + \beta \times (\beta \times \nabla) + \frac{1}{c}\beta\partial_t]\mathbf{s} \cdot \mathbf{R}. \quad (5)$$

To write the equations in covariant form, we use the four-velocity u^ν and define the spin 4-pseudovector which takes the form $a^\mu = (0, \mathbf{s})$ in the particle rest frame. Further we use the electromagnetic field strength tensor $F^{\mu\nu}$ and its dual $F^{*\mu\nu}$. In these terms, the equations become:

$$m\frac{du_\nu}{d\tau} = eF^{\nu\rho}u_\rho + \frac{\mu}{I\hbar c}u_\sigma\left(\partial_\nu + \frac{1}{c^2}u_\nu u^\gamma\partial_\gamma\right)F^{*\sigma\rho}a_\rho \quad (6)$$

$$\frac{da_\nu}{d\tau} = \frac{\mu}{I\hbar}F_{\nu\sigma}a^\sigma - \frac{1}{c^2}\left(\frac{\mu}{I\hbar} - \frac{e}{m}\right)u_\nu u_\mu F^{\mu\sigma}a_\sigma + \frac{\mu}{I\hbar mc^2}u_\nu u_\mu a^\gamma\partial_\gamma F^{*\mu\sigma}a_\sigma. \quad (7)$$

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