MUON DECAY IN ORBIT

Robert Szafron

BROOKHAVEN NATIONAL LABORATORY

Mu2e-II Snowmass22 Workshop (viii)
28 April 2021

THEORY WORKING GROUP

LORENZO CALIBBI, JULIAN HEECK, ROBERT SZAFRON, YUICHI UESAKA

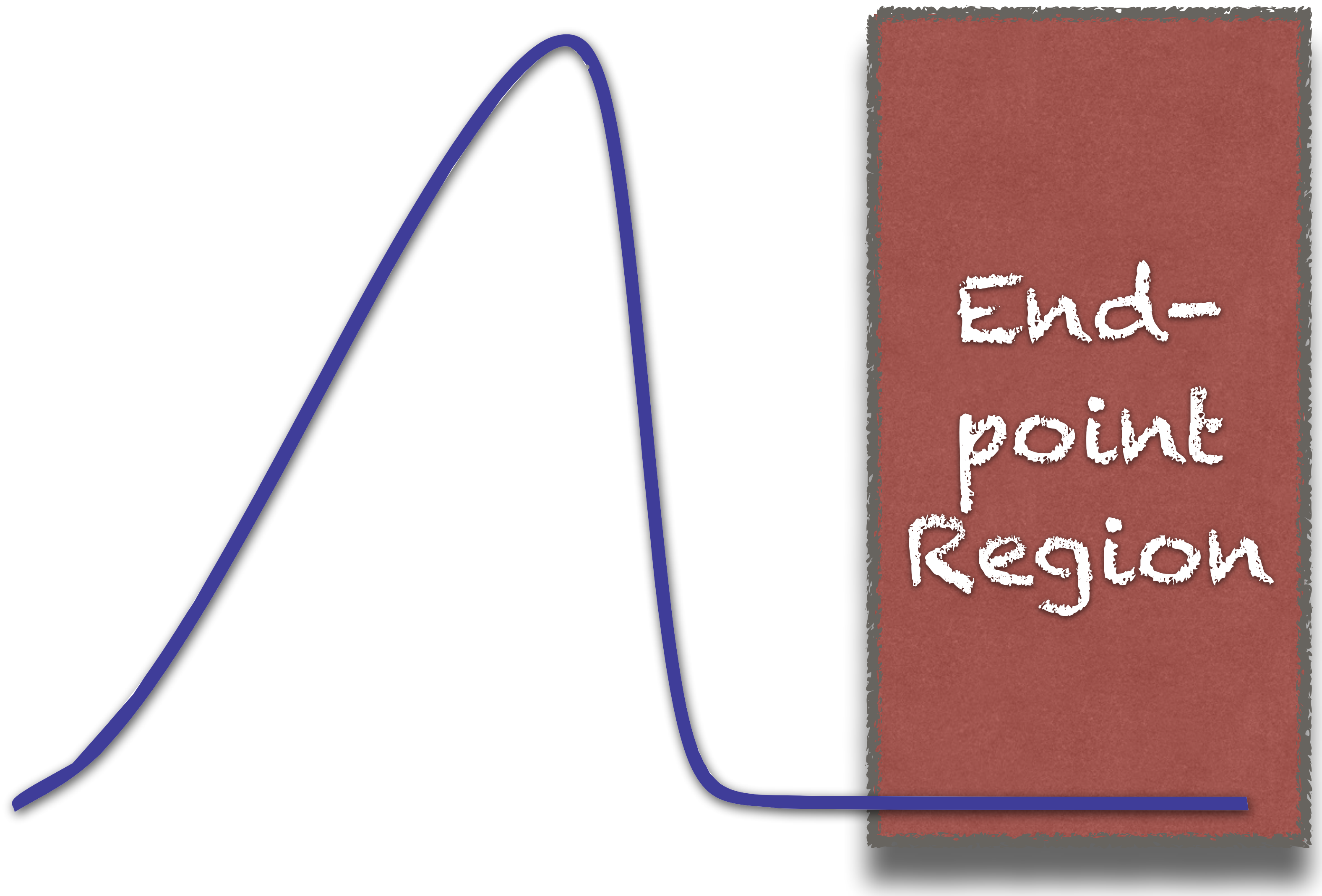
Goal: Provide good and simple approximation to the endpoint DIO spectrum near to endpoint to allow easy studies for different targets

This requires:

- *Good understanding of the spectrum near the endpoint*
- *Control of various effects to achieve the desired precision*
- *Computation of overlap integrals* ⟶ Yuichi

ENDPOINT REGION

$$E_e \sim m_\mu$$



- Free muon spectrum is nonexistent in this region
- Binding effects constitute the LO terms
- Typical momentum transfer between the nucleus and the muon is large $q^2 \sim m_\mu^2$
- Both wave functions and propagators can be expanded in powers of $Z\alpha$ — non-relativistic expansion

ENDPOINT ENERGY

$$E_{\max} = m_\mu + E_b + E_{\text{rec}}$$

$$E_b \approx -m_\mu \frac{(Z\alpha)^2}{2}$$

Binding energy

(+ higher orders)

In practice: computed numerically by solving Dirac equation

$$E_{\text{rec}} \approx -\frac{m_\mu^2}{2m_N}$$

Recoil energy

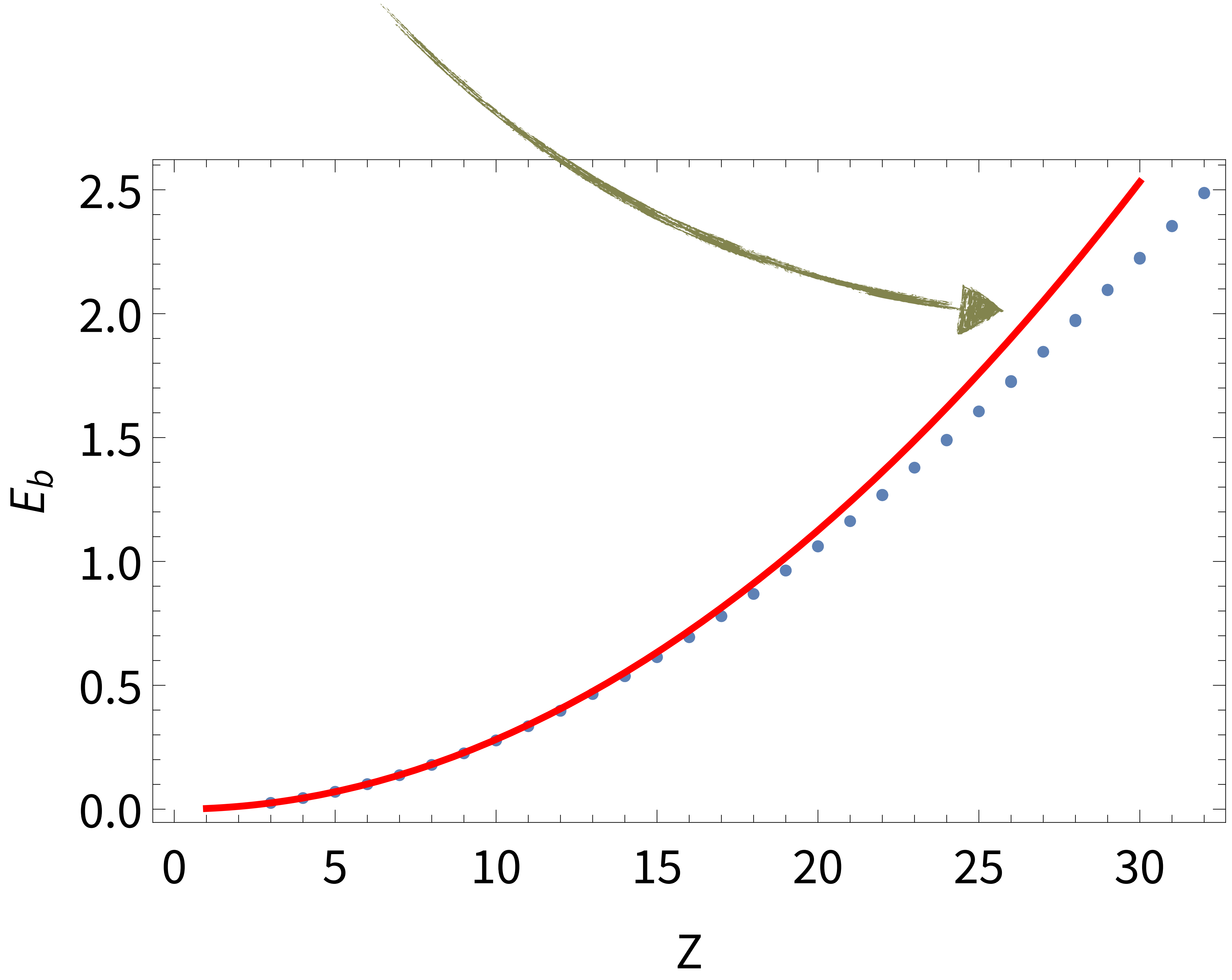
(kinetic energy of the nucleus)

Both corrections decrease the endpoint energy

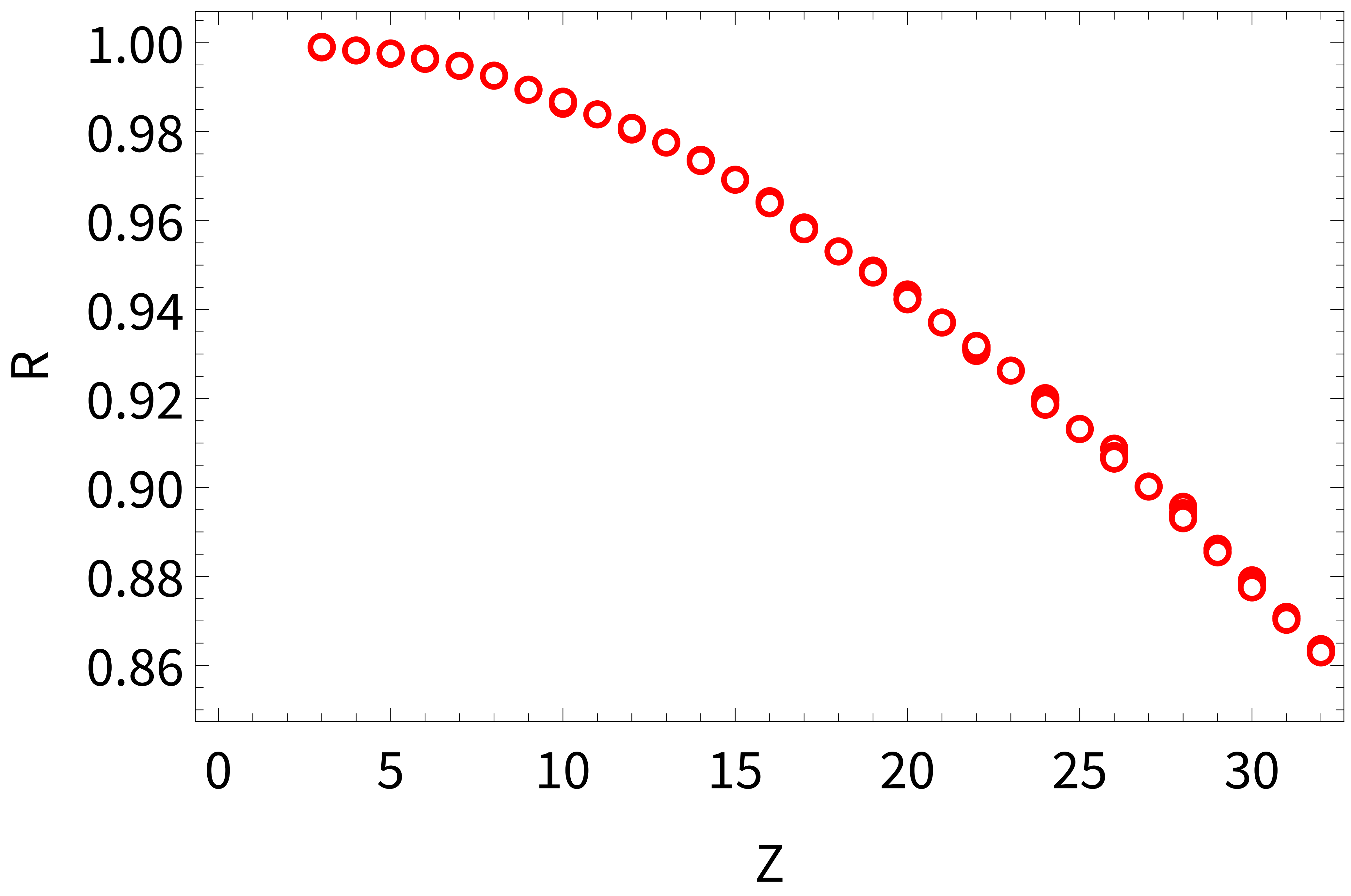
$|E_b|$ grows with Z, $|E_{\text{rec}}|$ decreases with Z

ENDPOINT ENERGY

$$E_b \approx -m_\mu \frac{(Z\alpha)^2}{2}$$



$$R = \frac{E_b^{\text{Dirac}}}{-m_\mu (Z\alpha)^2/2}$$



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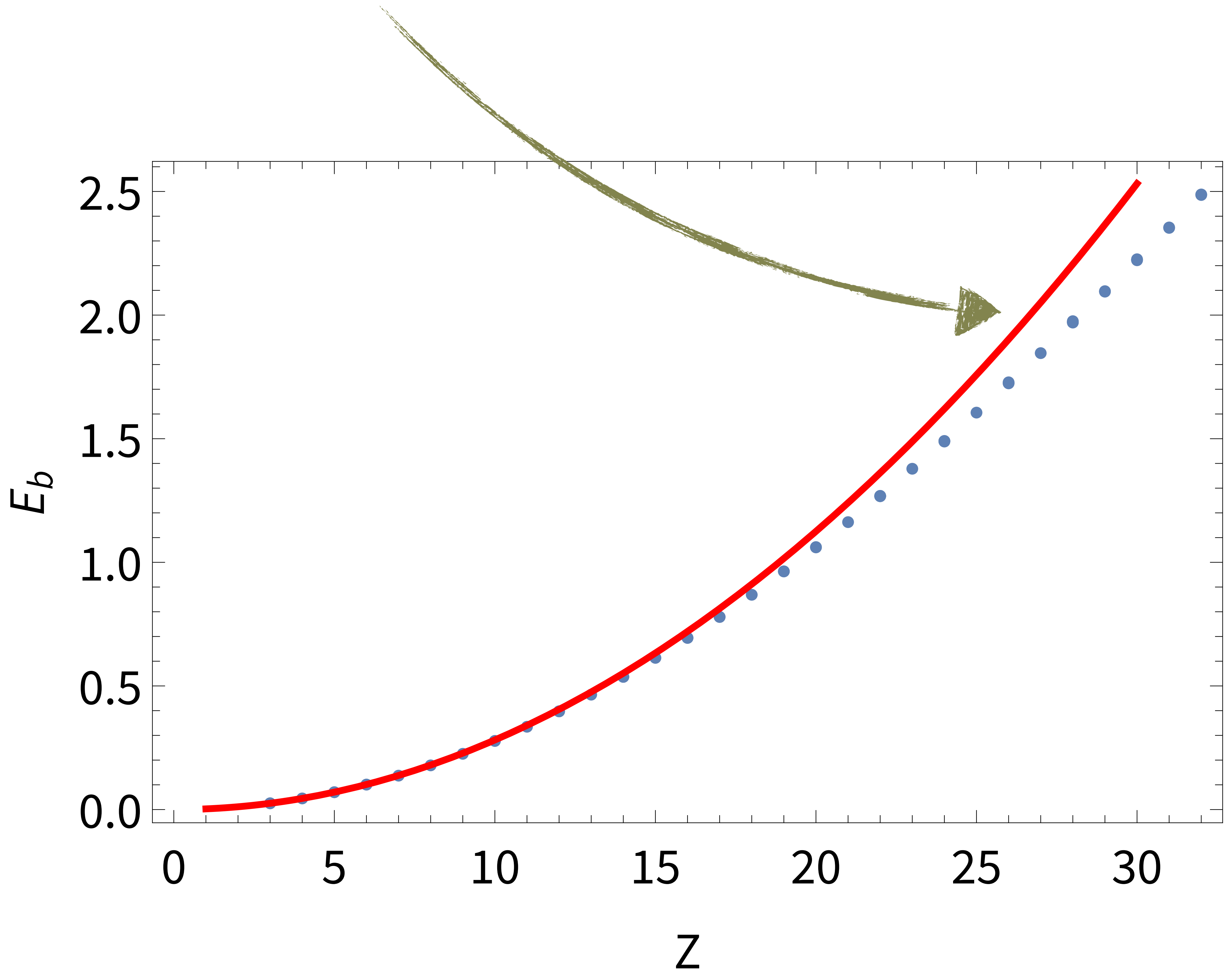
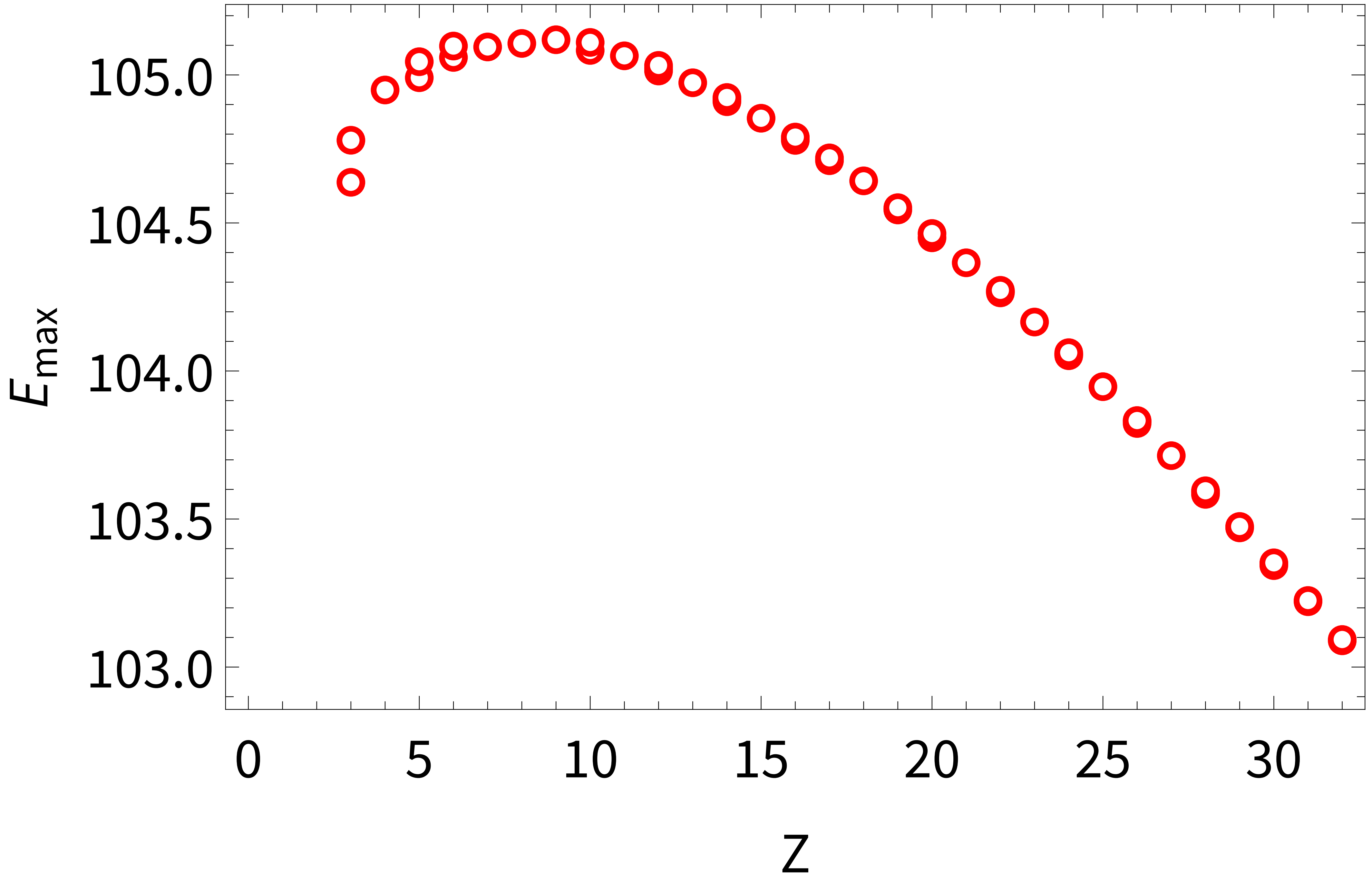


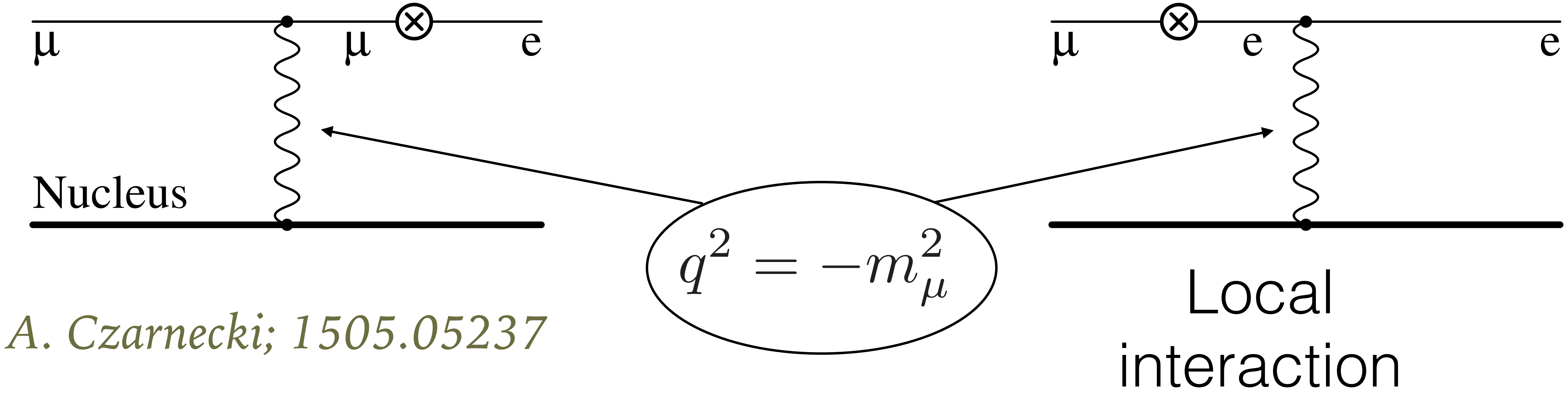


ENDPOINT EXPANSION

Near the endpoint, the dominant contribution comes from the exchange of hard virtual photons

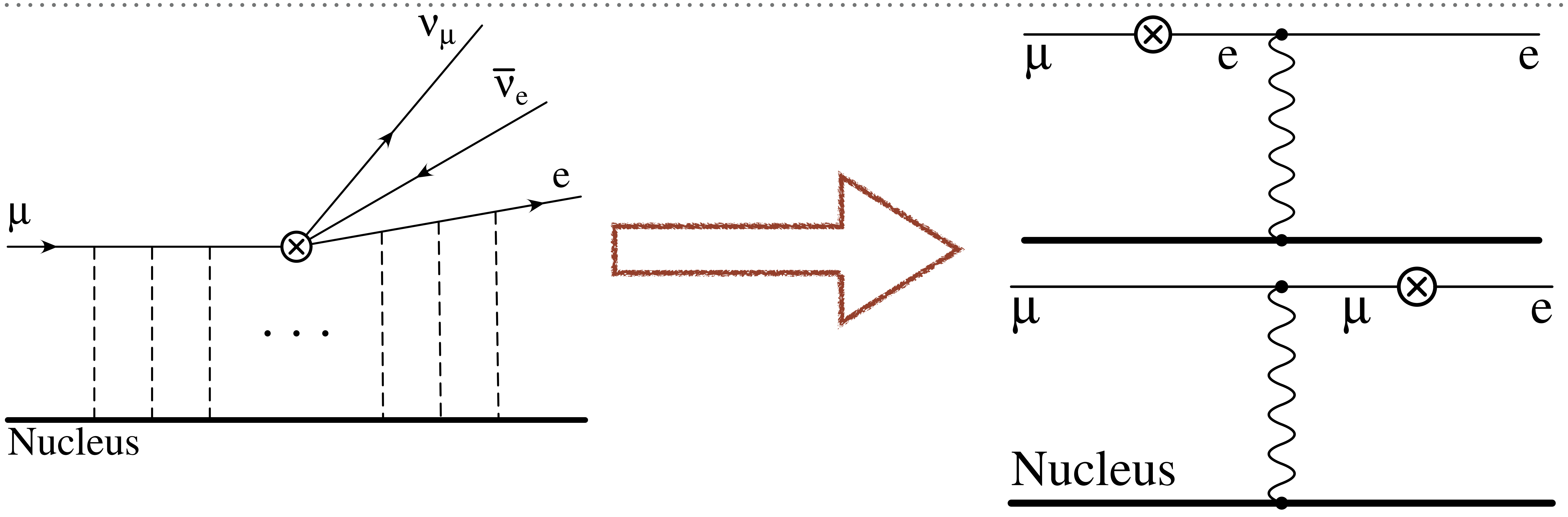
$$\frac{m_\mu}{\Gamma_{Free}} \frac{d\Gamma}{dE_e} \approx \frac{1024}{5\pi} (Z\alpha)^5 \left(\frac{\Delta}{m_\mu} \right)^5$$

$$\Delta = E_{max} - E_e$$



R. Szafron and A. Czarnecki; 1505.05237

PHASE SPACE SUPPRESSION



Each neutrino gives 3 powers of Δ

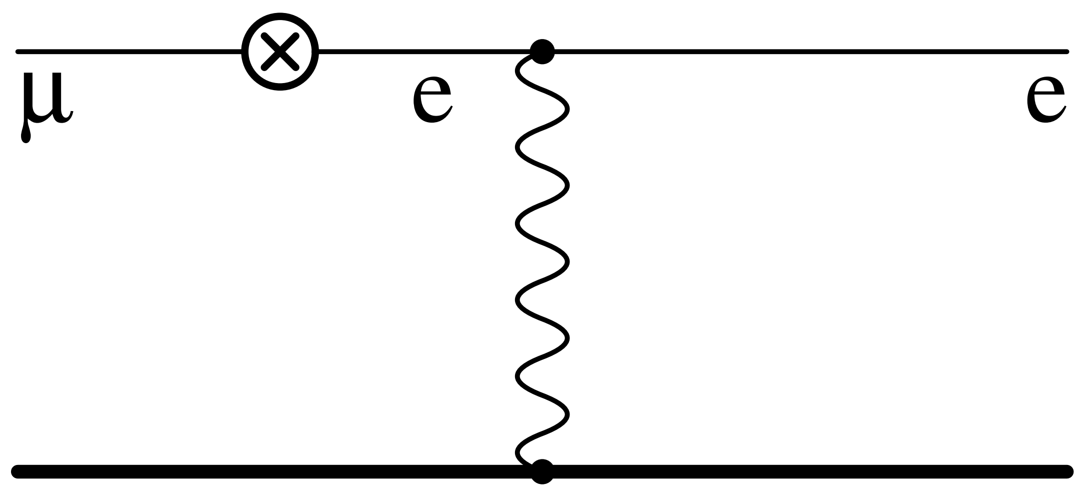
$$\int \frac{\mathrm{d}^3\nu}{\nu_0}\frac{\mathrm{d}^3\overline{\nu}_0}{\overline{\nu}_0}\delta\left(\Delta-\nu_0-\overline{\nu}_0\right)\dots\not{\psi}\dots\overline{\not{\psi}}\sim\Delta^5$$

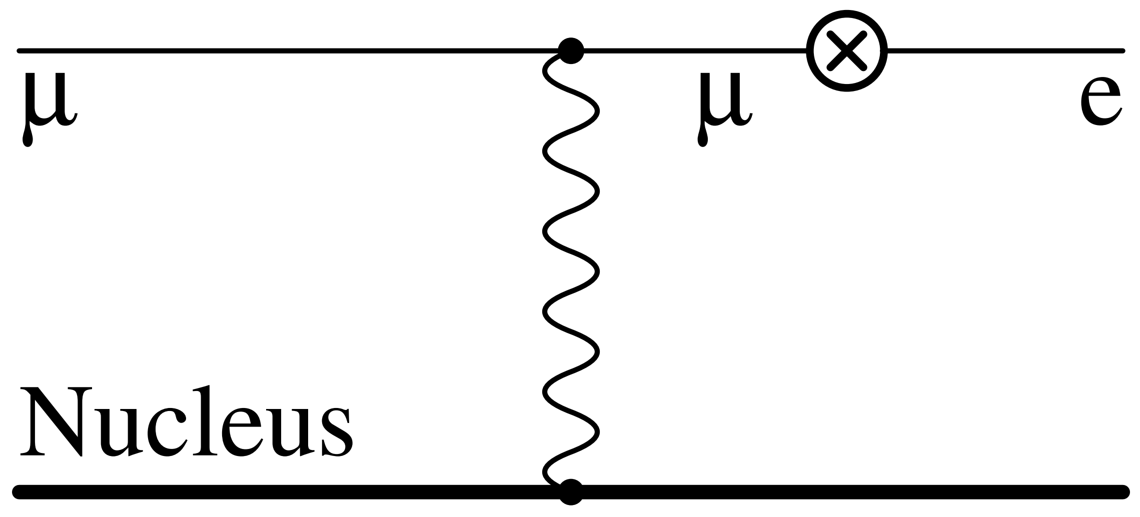
Can be used to constrain effective BSM operators

— e.g. scalar: $\mathscr{L}\sim\partial_\mu s\,\overline{\psi}_i\gamma^\mu\psi_j$

$$\int\frac{d^3s}{s_0}\delta(\Delta-s_0)\dots s_\mu\dots s_\nu\dots\sim\Delta^3$$

BINDING SUPPRESSION

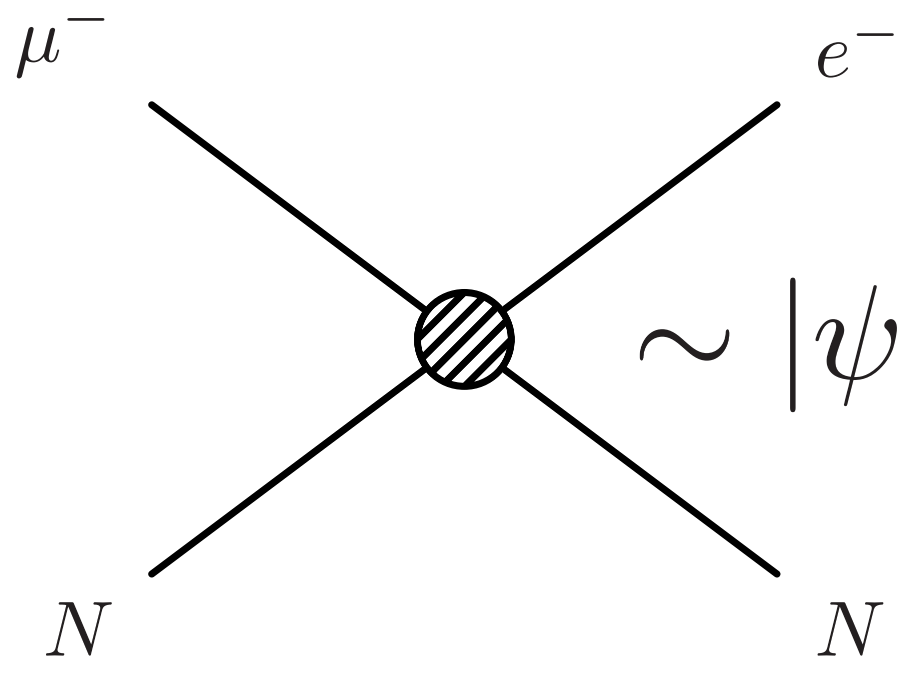




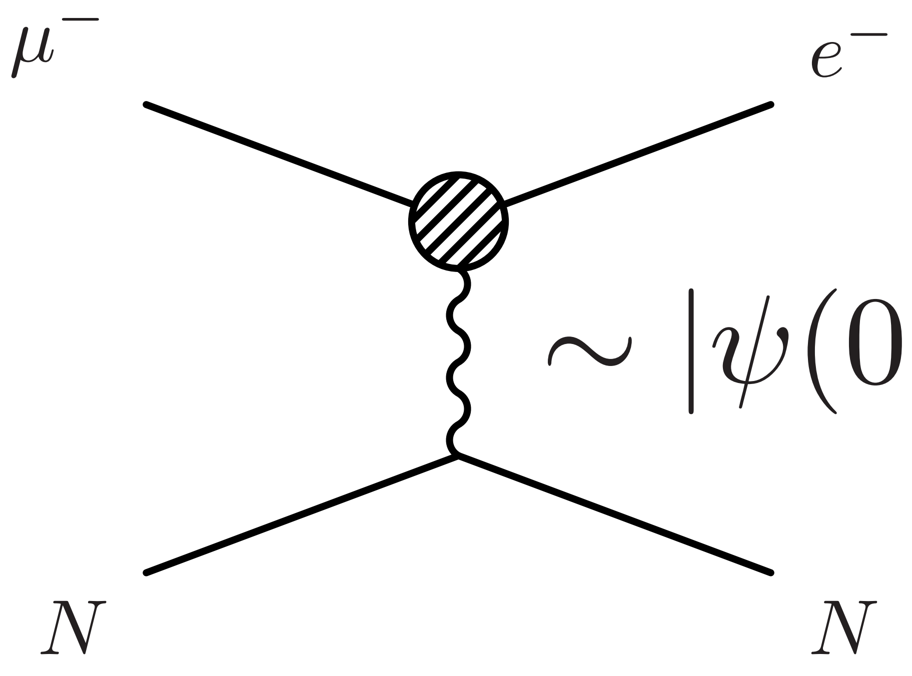
$$|\mathcal{M}|^2 \sim |\psi(0)|^2 \times \left|V(m_\mu^2)\right|^2 \sim (Z\alpha)^3 \times (Z\alpha)^2$$

$$|\psi(0)|^2 \sim (Z\alpha)^3$$

$$V(k^2) \sim -\frac{Z\alpha}{k^2}$$



$$\sim |\psi(0)|^2$$



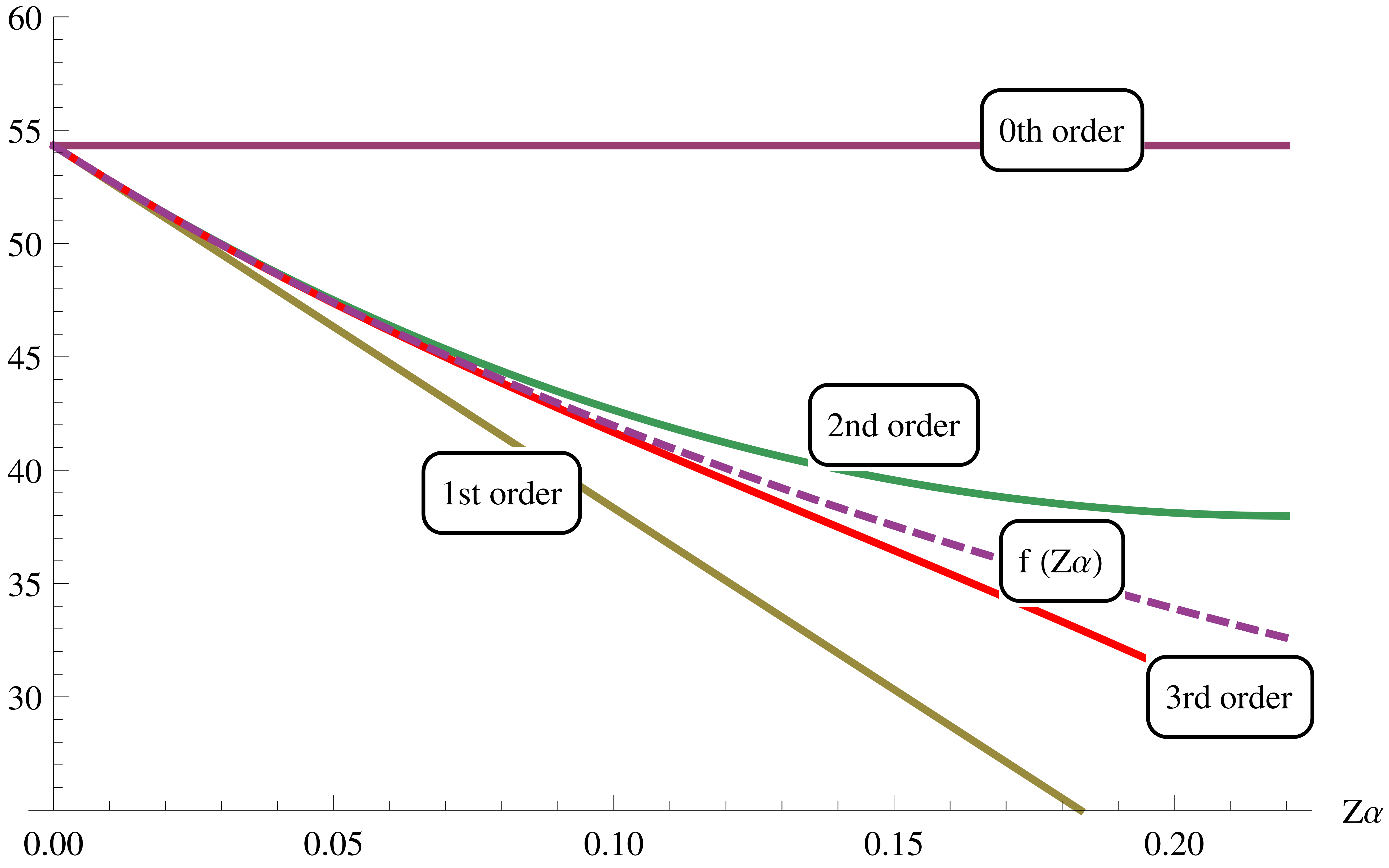
$$\sim |\psi(0)|^2 \times \left|V(m_\mu^2)\right|^2$$

NON-RELATIVISTIC EXPANSION?

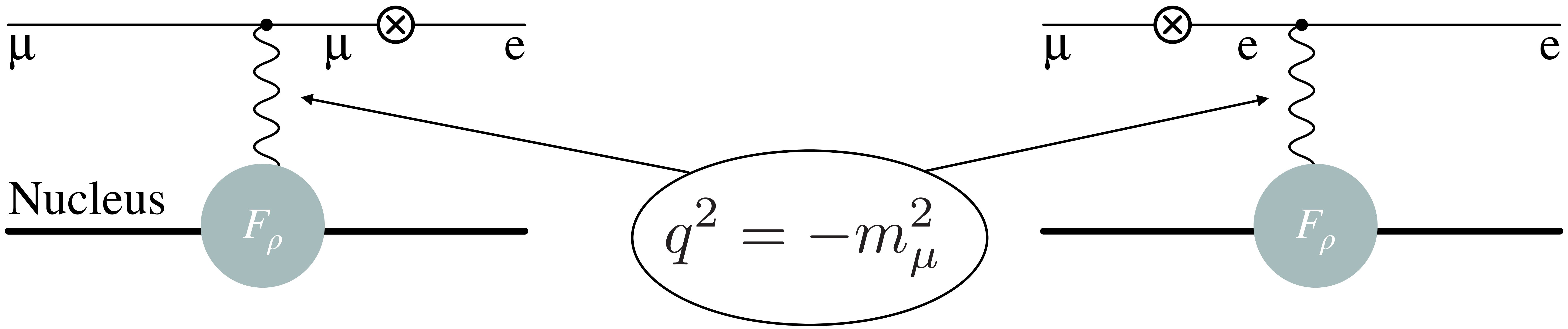
$$f(Z\alpha) \approx \frac{512}{3\pi} - 160\,Z\alpha + \frac{6064 + 473\pi^2 - 2944\log(2) - 1536\log(Z\alpha)}{9\pi}(Z\alpha)^2$$

Toy model: $\mu^- \to s\, e^-$

$$\frac{m_\mu}{\Gamma_0}\frac{d\Gamma}{dE_e} \approx (Z\alpha)^5 (E_e - E_{max})^3 f(Z\alpha)$$



FINITE NUCLEAR SIZE EFFECTS ARE ALSO IMPORTANT



$F_\rho(-q^2)$ Nuclear charge form factor $\quad \dfrac{d\Gamma}{dE_e} \sim F_\rho^2(m_\mu^2)$

For example, for Al — $F_\rho \approx 0.63$; Ti —$F_\rho \approx 0.53$

Similar size effect is related to the $\psi(0)$ factor

Nuclear size effects are $\mathcal{O}(1)$ near the endpoint

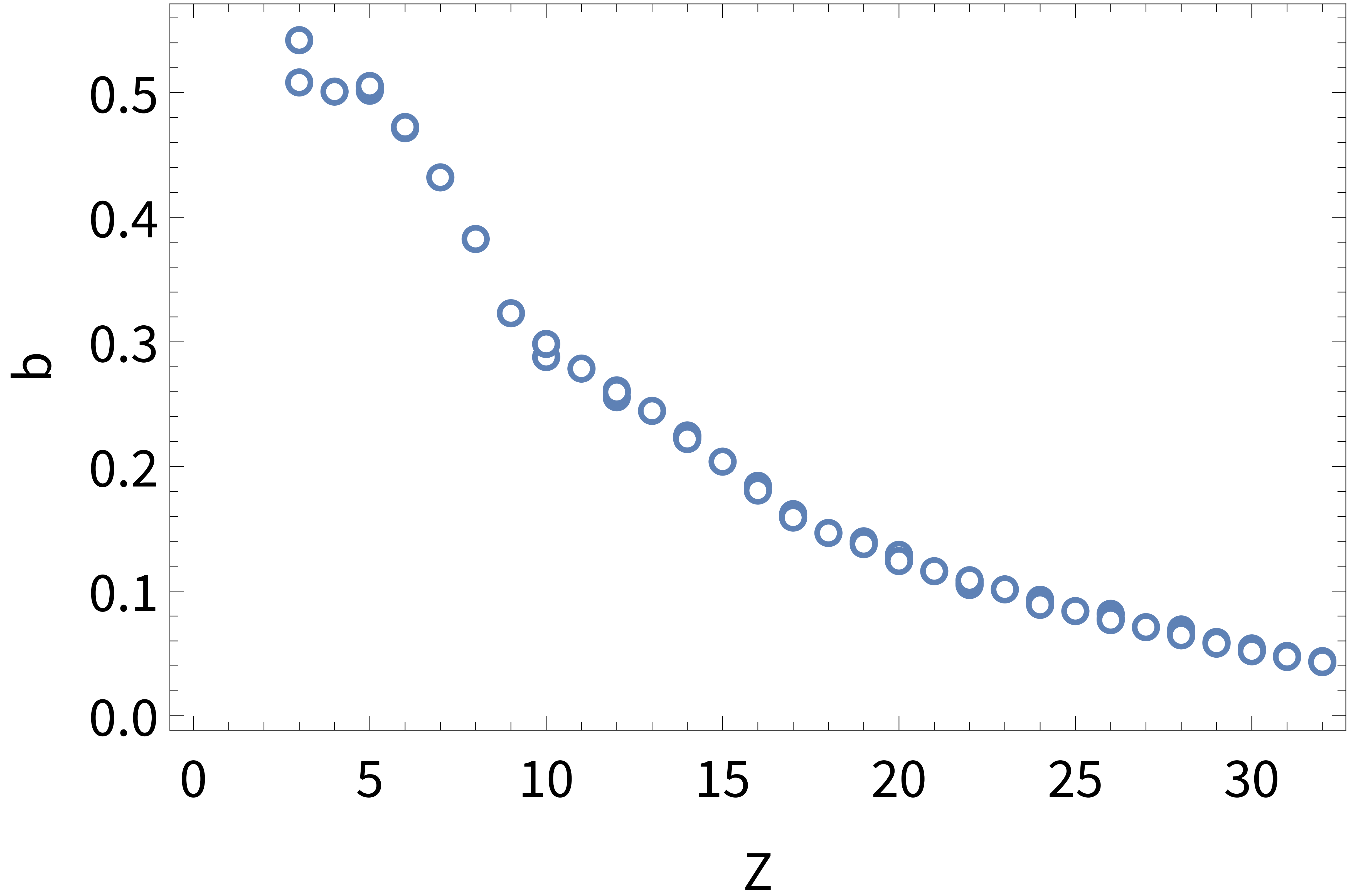
NUMERICAL RESULTS

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_e} = B\left(E_{\max} - E_e\right)^5$$

$$b = B/B_0 \qquad B_0 = \frac{1024}{5\pi}\frac{(Z\alpha)^5}{m_\mu^6}$$

$$B = \frac{64}{5\pi m_\mu^5}\left(p_1^2 + \frac{s_1^2}{3} + \frac{2}{3}r_2^2\right)$$

Overlap integrals of electron and muon wave-functions

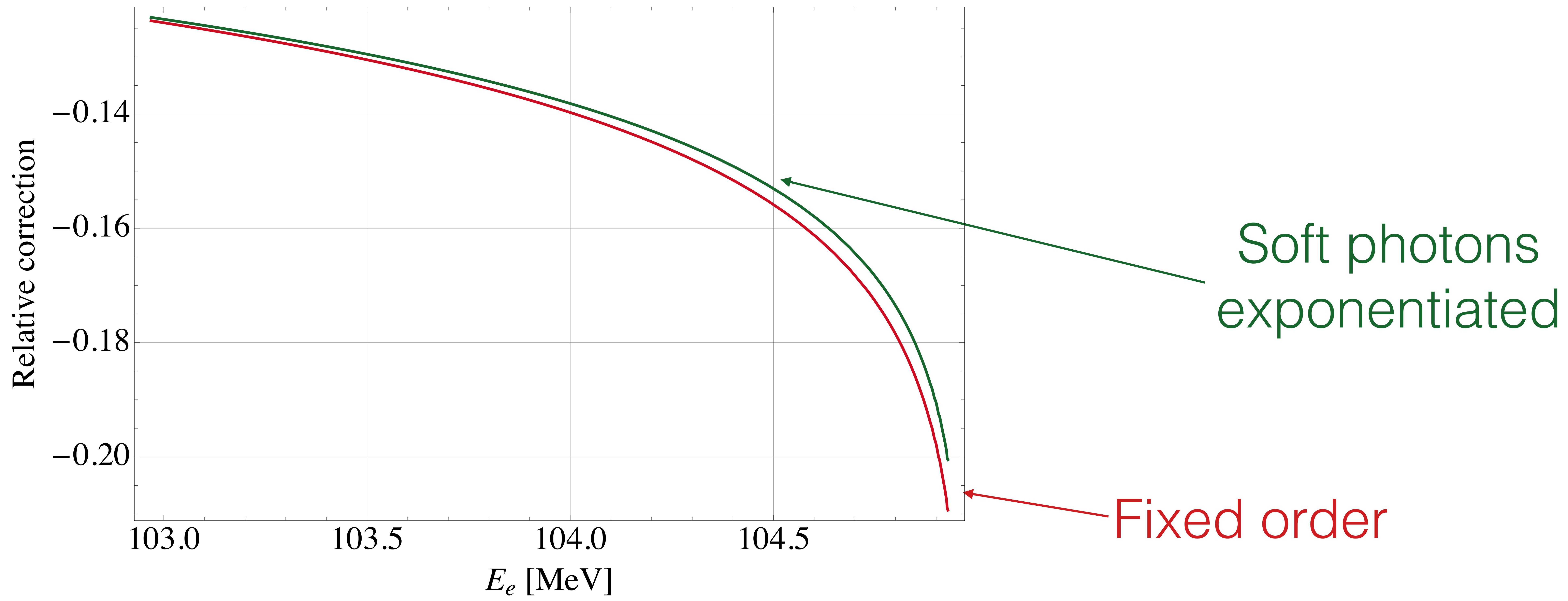


LEADING QED EFFECTS

R. Szafron and A. Czarnecki; 1505.05237

Near the endpoint emission of soft photons is logarithmically enhanced — effects is large $\sim 10\,\%$ but universal and known to exponentiate

$$\frac{\alpha}{\pi}\delta_S \ln\left(\frac{E_{\max}-E_e}{E_{\max}}\right) \to \left(\frac{E_{\max}-E_e}{E_{\max}}\right)^{\frac{\alpha}{\pi}\delta_S}$$



PRESCRIPTION FOR HIGHER ORDER CORRECTIONS

R. Szafron and A. Czarnecki; 1505.05237, 1608.05447

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_e} = B\left(E_{\max} - E_e\right)^5 \to B\left(E_{\max} - E_e\right)^{5+\frac{\alpha}{\pi}\delta_S}$$

$$\delta_S = 2\ln\frac{2m_\mu}{m_e} - 2$$

Good approximation for $E_e \sim E_{\max}$, error $\sim \frac{E_{\max} - E_e}{E_{\max}}$

Theoretical uncertainty:

We included dominant, universal correction $\sim 10\,\%$

We neglected corrections in $B \sim 2\,\%$ (increase with Z)

Remaining not-included corrections $\sim 1\,\%$

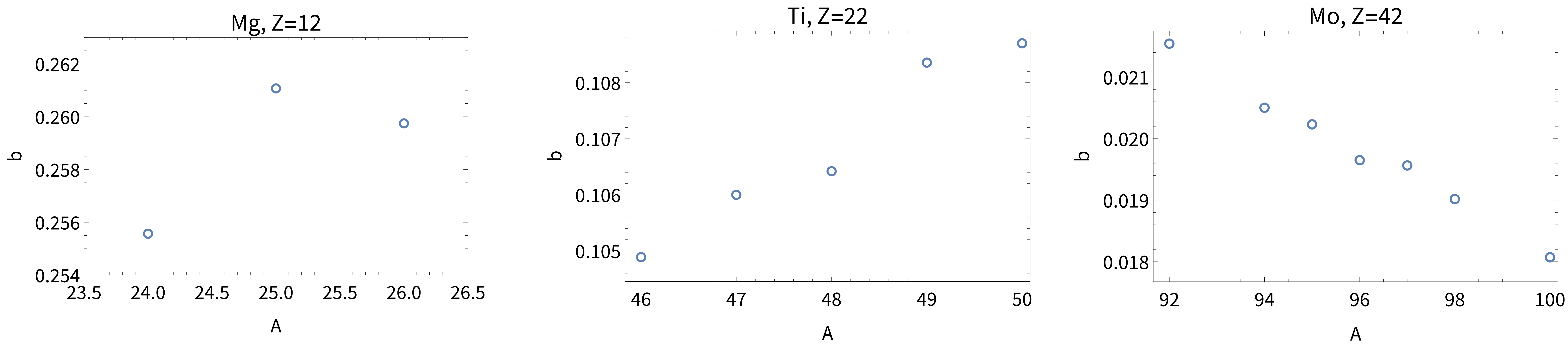
Corrections to $E_b \sim$ several keV

$$\Delta E_b = \frac{\alpha}{\pi}(Z\alpha)^2 m_\mu \left(\frac{11}{9} - \frac{2}{3}\log\left[\frac{2m_\mu Z\alpha}{m_e}\right]\right)$$

Parametric uncertainty:

Uncertainty due to finite size parameters $\sim$ several %

ISOTOPE DEPENDENCE



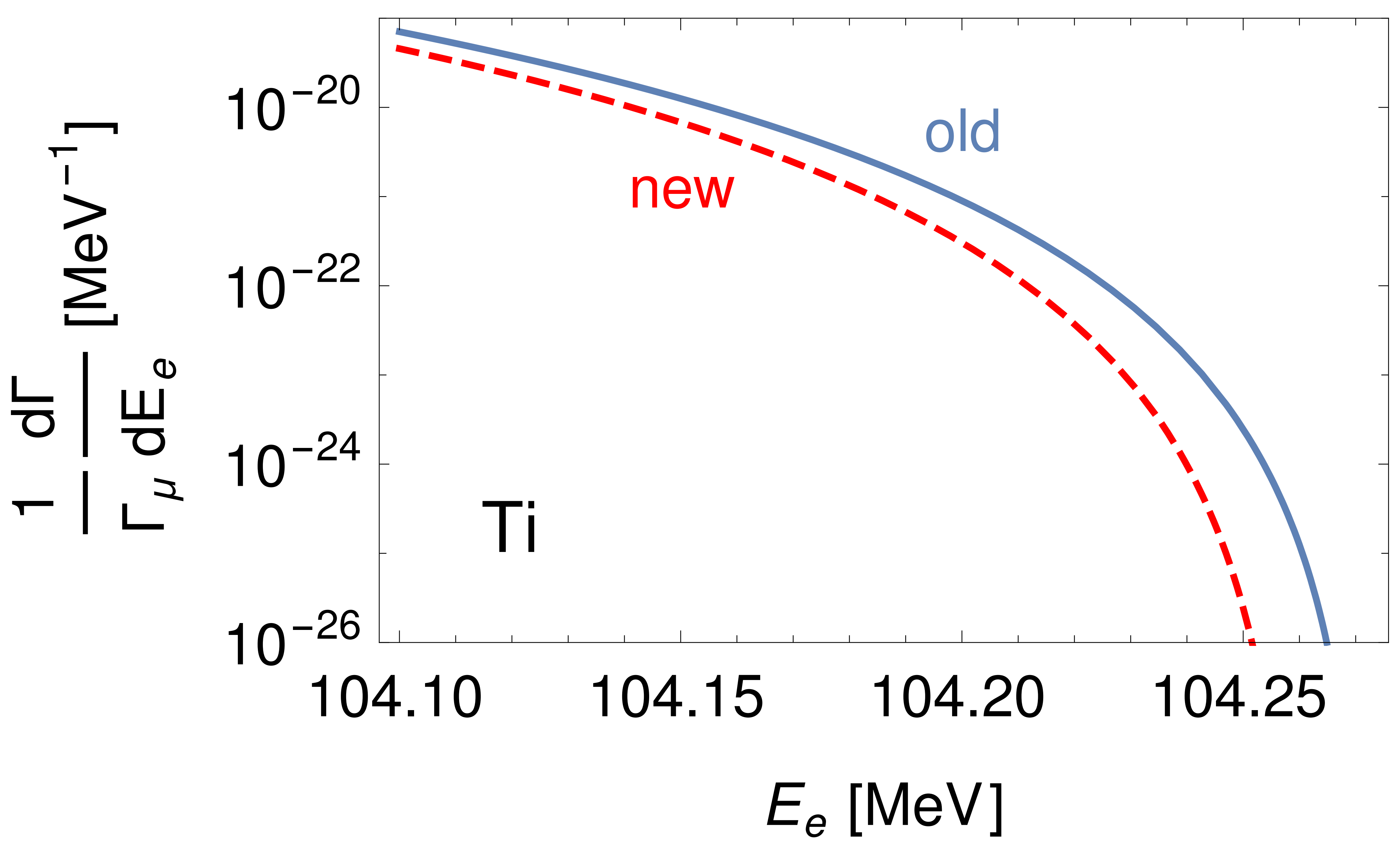
~2%

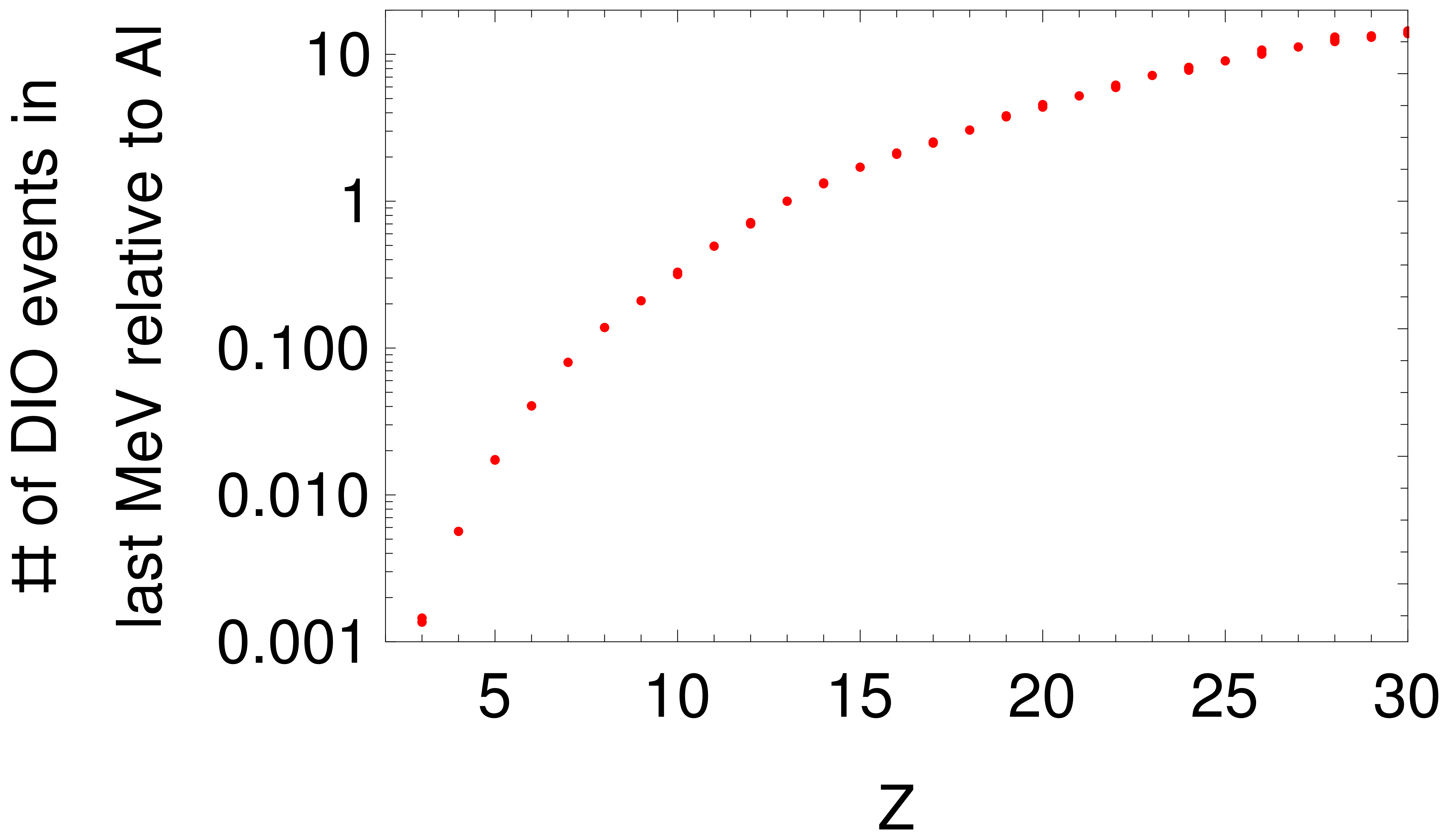
~4%

~16%

For light targets isotope dependence is comparable with the parametric uncertainty due to the finite size effects

ENDPOINT SPECTRUM AND EVENTS





Difference mostly due to the radiative corrections

Old result: A. Czarnecki, X. Garcia i Tormo, and W. J. Marciano; 1106.4756

SUMMARY

- *New data for DIO endpoint — included dominant radiative effects*
- *Nuclear size effects need to be further studied*
- *Isotope dependence under control for light targets*

OUTLOOK

- *For selected targets one should study full DIO spectrum with complete set of corrections*
- *Detailed study of the signal must also be performed using the same nuclear charge distribution data and isotope dependence*

DIO REFERENCES

R. Szafron, A. Czarnecki	Phys.Rev.D 94 (2016) 5, 051301	Most up to date spectrum with radiative corrections and endpoint energy for Al
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A. Czarnecki, M. Dowling, X. Garcia i Tormo, W. Marciano, R, Szafron	Phys.Rev.D 90 (2014) 9, 093002	Central region, radiative corrections
A. Czarnecki, X. Garcia i Tormo, W. Marciano	Phys.Rev.D 84 (2011) 013006	Numerical evaluation of the LO spectrum