**Solutions for Lecture 1: Introduction to Neutrino Interactions**
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These are the solutions to the worksheet accompanying the Introductions to Neutrino Interactions lecture at the DUNE Neutrino Interactions Summer School.

**Exercise 1** (*lecture slide 3*)

**This is a warm-up exercise, to remind yourself about some of the particles we'll be talking about in the presentation.** *Expected time: 10 minutes.*

| Particle | | Charge | Spin | Is it a…? | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Boson | Fermion | Lepton | Hadron | Meson | Baryon | Constituents |
| Electron | $e^-$ | -1 | ½ | | ✓ | ✓ | | | | - |
| Muon | $\mu^-$ | -1 | ½ | | ✓ | ✓ | | | | - |
| Electron-neutrino | $\nu_e$ | 0 | ½ | | ✓ | ✓ | | | | - |
| Muon-neutrino | $\nu_\mu$ | 0 | ½ | | ✓ | ✓ | | | | - |
| Positron | $e^+$ | +1 | ½ | | ✓ | ✓ | | | | - |
| Proton | $p$ | +1 | ½ | | ✓ | | ✓ | | ✓ | uud |
| Neutron | $n$ | 0 | ½ | | ✓ | | ✓ | | ✓ | udd |
| Photon | $\gamma$ | 0 | 1 | ✓ | | | | | | |
| Pion (3 of them) | $\pi$ | -1, 0, +1 | 0 | ✓ | | | ✓ | ✓ | | $u\bar{d}$ ($\pi^+$) $u\bar{u}/d\bar{u}$ ($\pi^0$) $d\bar{d}$($\pi^-$) |
| W (weak force carrier: 2 of them) | $W$ | +1, -1 | 1 | ✓ | | | | | | - |
| Z (weak force carrier) | | 0 | 1 | ✓ | | | | | | - |
| Delta-1232 resonances (4 of them) | $\Delta$ | -1, 0, +1, +2 | $^3/_2$ | | ✓ | | ✓ | | ✓ | ddd, udd, uud, uuu |

**Exercise 2** (*lecture slide 8*)

**I claim that all DUNE needs to do to search for CP-violation and other neutrino oscillation properties is to:**

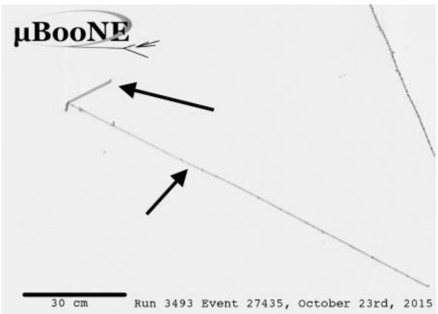
1. **Plot the energies of all the muon-neutrinos at Fermilab**
2. **Plot the energies of all the electron-neutrinos at SURF**
3. **Take the ratio of those two plots**

**Unfortunately, there are several things that make this rather a challenge! Can you think of any reasons why this approach is too simplistic?** *(Expected time: 5-10 minutes)*

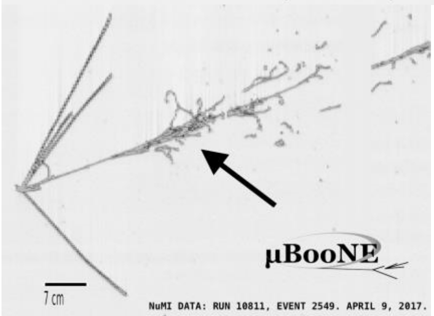
Here are some I thought of:

1. Because neutrinos are uncharged, they can't be directly detected. To determine their flavor, we need them to interact and produce their charged-lepton partners.
2. Interaction cross sections – the probability that neutrinos will interact – are very small. We can never measure "all" the neutrinos at Fermilab. For example, at Fermilab, the "low-energy" NuMI beam provided around $10^{17}$ neutrinos to the MINERvA experiment. Around 200,000 of these were detected – around 2 neutrinos in a trillion. The chance of detecting an individual neutrino is about the same as the chance of catching an individual fish from the entire ocean.
3. Neutrino beams spread as they travel – and the spread is energy-dependent. The number and spectrum of neutrinos arriving at the Fermilab detector will be different to the spectrum at the SURF detector.
4. Not only do we need to know the probability that a neutrino will interact, we need to know the probability that we will detect it. That means understanding our detector efficiency. This is especially important if the near and far detectors have different designs, as they usually do.
5. As we can't directly detect neutrinos, we can't directly measure their energies. Furthermore, it's extremely hard to make a mono-energetic neutrino beam (the beams at Fermilab all have broad energy spectra). We need to reconstruct the energy from the interaction products – and that means we need to understand the interaction.

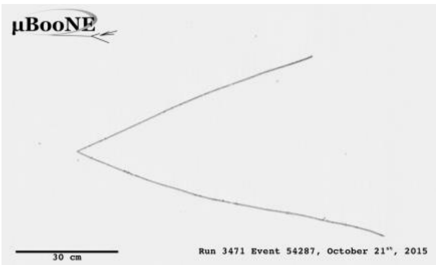
**Exercise 3 (lecture slide 13)**
**What particles are in these event displays?** *Expected time: 5 minutes.*

a) 


The long track is a **muon**. As minimum-ionizing particles, muons make long, straight tracks (or curve in a magnetic field), with a constant rate of energy deposition dE/dx.
The short track is a **proton**. In liquid argon detectors, proton tracks are typically only a few centimeters long.

b) 


This is an **electron.** Electrons experience bremsstrahlung (braking radiation) as they travel, emitting photons (uncharged, so invisible). The photons quickly decay to $e^+$-$e^-$ pairs, which emit more photons, leading to shower-like tracks. A **photon** looks similar, but with a gap between the showering track and the interaction vertex (the photon shower is started by an invisible particle). A $\boldsymbol{\pi^0}$, which decays to two photons, makes two showers like this, though they could overlap.
**Charged pions** make long, straight tracks in liquid argon, like those made by muons. Separating them can be a challenge, and is typically done by different detector components like calorimeters.

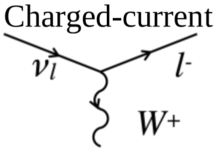
c) 

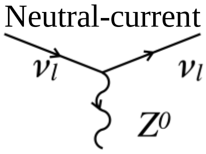

In each of these diagrams, there's also a neutrino. In each diagram, what flavor is the neutrino? Where do you think it is coming from and where does it interact?

a) This is a muon neutrino $\nu_\mu$ (probably not an antineutrino, as it has also produced a proton – to balance the charge, it's probable the muon is a $\mu^-$. More on this later.)
b) This is an electron neutrino (or antineutrino).
c) This a muon neutrino (or antineutrino).

The neutrino beam comes from the left. The neutrino interacts at the "vertex" where the charged-particle tracks converge.

**Quick Question 1:** *(lecture slide 14)*
**Draw a charged- and neutral- current vertex for neutrino scattering. Which helps you tell the neutrino flavor?**

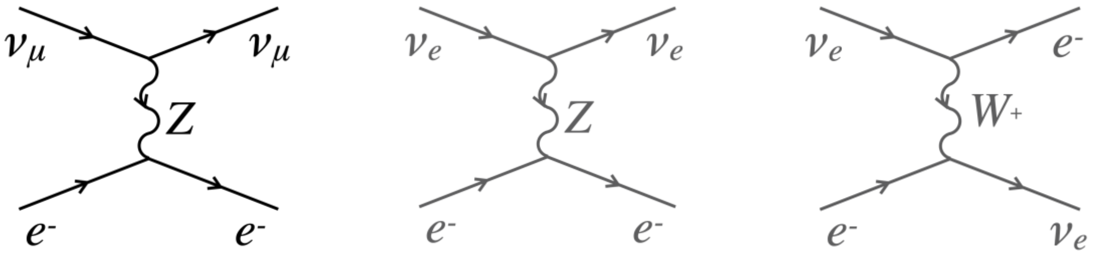
Charged-current



Neutral-current



The charged-current interaction produces a charged lepton that identifies the neutrino flavor.

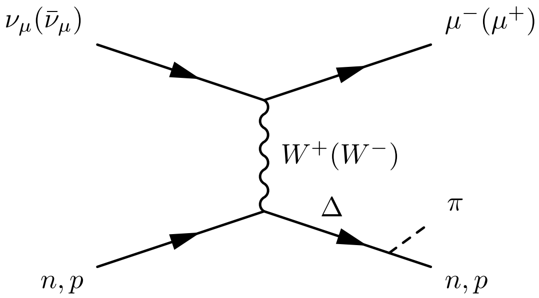
**Exercise 4** *(lecture slide 18)*

**Here's a diagram for neutral-current $\nu_\mu$ scattering from an electron. i) Can you draw the $\nu_e$-$e^-$ equivalent? ii) Can you find another $\nu_e$-$e^-$ diagram that gives an identical final state to the one you drew in i)?**




**Exercise 5** *(lecture slide 31)*

**This is the Feynman diagram for pion production through a Δ resonance:**




**Both neutrinos and antineutrinos can scatter from both neutrons and protons. There are four possible delta resonances. Can you write down the equations for all the allowed neutrino and antineutrino resonant scatters? (Hint, you should find three for each.)**
*(Expected time: 5 minutes)*

$\nu_\mu + n \to \mu^- + \Delta^+ \to \mu^- + n + \pi^+$

$\nu_\mu + n \to \mu^- + \Delta^+ \to \mu^- + p + \pi^0$

$\nu_\mu + p \to \mu^- + \Delta^{++} \to \mu^- + p + \pi^+$

$\bar{\nu}_\mu + p \to \mu^+ + \Delta^0 \to \mu^+ + p + \pi^-$

$\bar{\nu}_\mu + p \to \mu^+ + \Delta^0 \to \mu^+ + n + \pi^0$

$\bar{\nu}_\mu + n \to \mu^+ + \Delta^- \to \mu^+ + n + \pi^-$

**Quick Question 2:** *(lecture slide 35)*

**In deep inelastic neutrinos scatter from an individual quark. In a charged-current interaction, which quarks will neutrinos and antineutrinos interact with?**

Valence quarks are u or d. Sea quarks can be *u*, *d*, *s*, or *c*; or their antiquarks ū, d̄, s̄, c̄.
In neutrino scattering, the exchanged boson is a $W^+$
$(d, s) + W^+ \to (u, c)$
$(\bar{u}, \bar{c}) + W^+ \to (\bar{d}, \bar{s})$
In antineutrino scattering, the exchanged boson is a $W^-$

$(u, c) + W^- \rightarrow (d, s)$
$(\bar{d}, \bar{s}) + W^- \rightarrow (\bar{u}, \bar{c})$

**Quick Question 3:** *(lecture slide 41)*

**The equations for CCQE neutrino and antineutrino scattering from a single nucleon are:**

$$\nu_\mu + n = \mu^- + p \qquad \bar{\nu}_\mu + p = \mu^+ + n$$

**What final states would you expect to see for 2p2h CCQE scattering of $\nu_\mu$ and $\bar{\nu}_\mu$?**
Neutrinos: The initial-state neutron is most likely paired with a proton: $\mu^- + p + p$
It could also be paired with a neutron: $\mu^- + p + n$
Antineutrinos: The initial-state proton is most likely paired with a neutron: $\mu^+ + n + n$
It could also be paired with a proton: $\mu^+ + p + n$

**Exercise 6** *(lecture slide 48) (Estimated time: 5 mins)*

**Match the interaction to the final state:**

| Interaction | Final state |
|---|---|
| $\nu_\mu$ quasi-elastic scattering | $\mu^-$ and 1 proton |
| $\bar{\nu}_\mu$ quasi-elastic scattering | $\mu^+$ and 1 neutron |
| $\nu_\mu$ 2p2h from an n-p pair | $\mu^-$ and 2 protons |
| $\nu_\mu$ 2p2h from an n-n pair | $\mu^-$, 1 neutron, 1 proton |
| $\bar{\nu}_\mu$ 2p2h from a p-p pair | $\mu^+$, 1 neutron, 1 proton |
| $\nu_\mu$ resonant scattering | $\mu^-$, 1 proton, 1 $\pi^0$; $\mu^-$, 1 neutron, 1 $\pi^+$ |
| $\bar{\nu}_\mu$ resonant scattering | $\mu^+$, 1 proton, 1 $\pi^-$; $\mu^+$, 1 neutron, 1 $\pi^0$ (not listed) |
| $\nu_\mu$ DIS | $\mu^-$ and hadron shower |

**Exercise 7** *(lecture slide 51)*

**Match interaction + FSI options that produce each final state.** *(Estimated time 10 mins)*

| Interaction | FSI | Final state |
|---|---|---|
| $\nu_\mu$ quasi-elastic scattering | No FSI | $\mu^-$ + 1 proton |
| $\nu_\mu$ resonant scattering | Pion absorption | |
| $\nu_\mu$ quasi-elastic scattering | No FSI | Muon (charge unknown) + 1 proton |
| $\nu_\mu$ resonant scattering | Pion absorption | |
| $\bar{\nu}_\mu$ resonant scattering | Pion absorption | |
| $\bar{\nu}_\mu$ quasi-elastic scattering | Charge exchange | |
| $\nu_\mu$ MEC | No FSI | $\mu^-$ + 2 protons |
| $\nu_\mu$ quasi-elastic scattering | Elastic scattering | |
| $\nu_\mu$ resonant scattering | Pion absorption | |
| $\nu_\mu$ resonant scattering | No FSI | $\mu^-$ + neutron + 1 $\pi^+$ |
| $\nu_\mu$ quasi-elastic scattering | Pion production & Charge exchange | |
| $\nu_\mu$ resonant scattering | Pion absorption & Nuclear de-excitation | $\mu^-$ + 1 proton + photons |
| $\nu_\mu$ quasi-elastic scattering | Elastic scattering & Nuclear de-excitation | |

| | | |
|---|---|---|
| $\nu_\mu$ resonant scattering | No FSI ($\pi$0 decay to $\gamma\gamma$) | |

**Quick Question 4:** *(lecture slide 59)*

**What do neutrino and electron scattering have in common? What are the differences?**
Neutrino and electron scattering probe the same nuclear physics – initial nuclear state, and final-state interactions. They share the vector part of the interaction, but neutrinos have an additional axial component. However, while neutrinos have a very small cross section, and neutrino beams are typically broad-spectrum, with large uncertainties; electrons have a higher scattering cross section, and mono-energetic electron beams can be produced. While electron-scattering can't fully predict neutrino scattering, we can at least test the vector part of nuclear models with electron beams. If the vector part is wrong, we know that the complete vector + axial model will be wrong.

**Quick Question 5:** *(lecture slide 59)*

**Why are neutrino interactions important?**
Neutrinos can only be detected when they interact. Understanding the cross section is key to turning the spectrum of detected neutrinos into an estimate of the spectrum of neutrinos present. By understanding the models, we can improve our energy reconstruction and to understand when one final state "fakes" another, giving misleading measurements.

**What do we know about interactions?**
Neutrino-electron scattering is well understood. While we have a good understanding of scattering from free nucleons, models for things like the form factors and structure functions are still being improved, and the region of non-resonant scattering between the resonant and DIS regions is just starting to be studied. While we have basic models of the nucleus, these are not yet able to reproduce data.

**What do we still need to understand better?**
Nuclei are extremely complicated. Multinucleon correlations can dramatically affect neutrino scattering. There are several different methods being used to understand these better, from diagrammatic processes like the MEC models of the Valencia and Lyon groups, superscaling and ab initio methods.

**How are we trying to understand neutrino interactions?**
We can test models against data from neutrino- and electron-scattering experiments. For DUNE, we will use simulation of the models and our detectors to determine how sensitive we are to the differences between models, and to investigate the best variables for studying these model differences.