



# Theoretical Overview

## Gordan Krnjaic

# Overview

Current Status of  $a_\mu$

Model Survey

Dark Matter Connection

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Model Survey

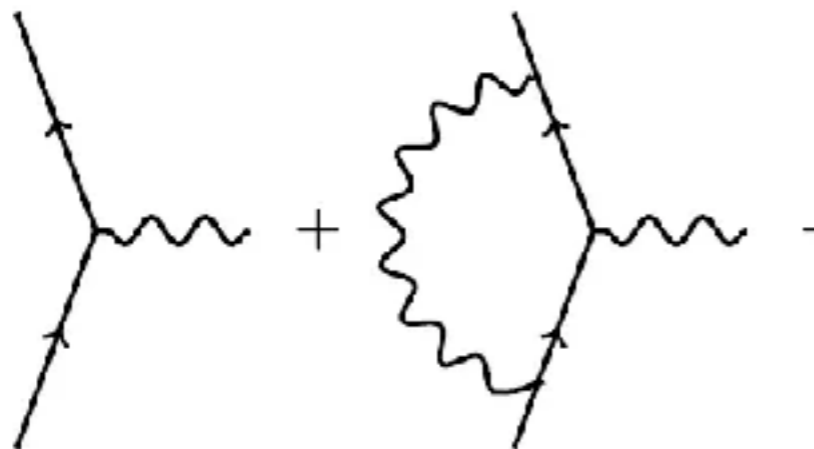
Dark Matter Connection

# Muon Anomalous Magnetic Moment

Lepton dipole moment  $\vec{\mu}_\ell = \pm g_\ell \frac{e}{2m_\ell} \vec{S}$   $a \equiv \frac{g - 2}{2}$

Tree level QED prediction:  $a = 0$

Quantum loop corrections:  $a \neq 0$



The diagram shows two Feynman diagrams for the lepton magnetic moment. The first diagram is a tree-level vertex where a lepton line (solid line with an arrow) meets a photon line (wavy line). The second diagram is a one-loop correction where a lepton line forms a loop with a photon line (wavy line) and a fermion line (solid line with an arrow) inside the loop. The diagrams are separated by a plus sign and followed by an ellipsis. To the right of the diagrams is the equation  $g = 2 + \frac{\alpha}{2\pi} + \dots$ .

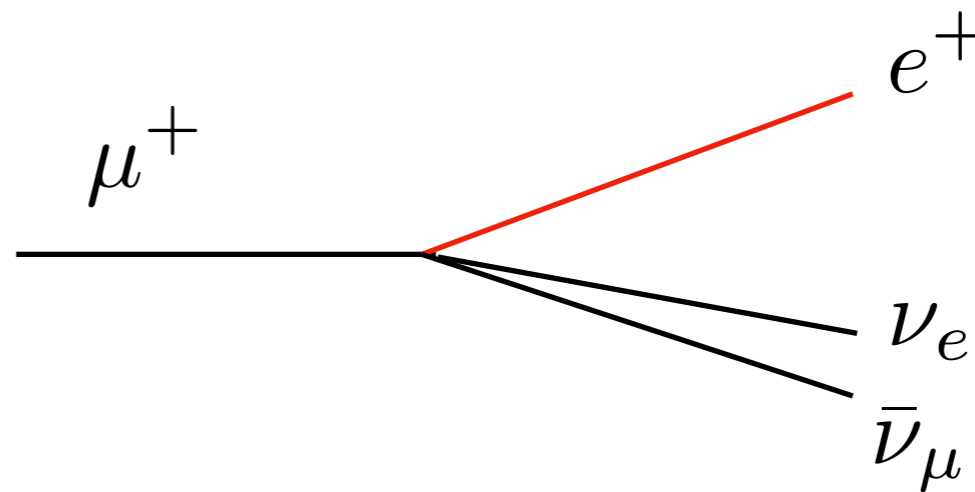
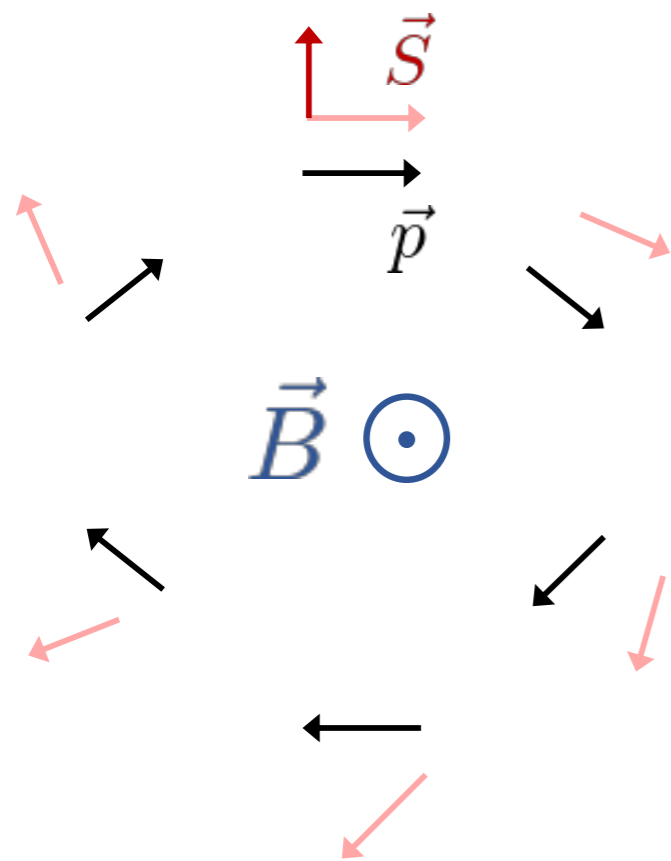
$$\rightarrow g = 2 + \frac{\alpha}{2\pi} + \dots$$

Sensitive to all known *and unknown* particles coupled to leptons  
For electrons agrees SM to  $\sim 12$  decimals, best prediction in history

# Spin precession in a uniform B field

$$\omega_a = a_\mu \frac{eB}{2m_\mu}$$

Lab Frame



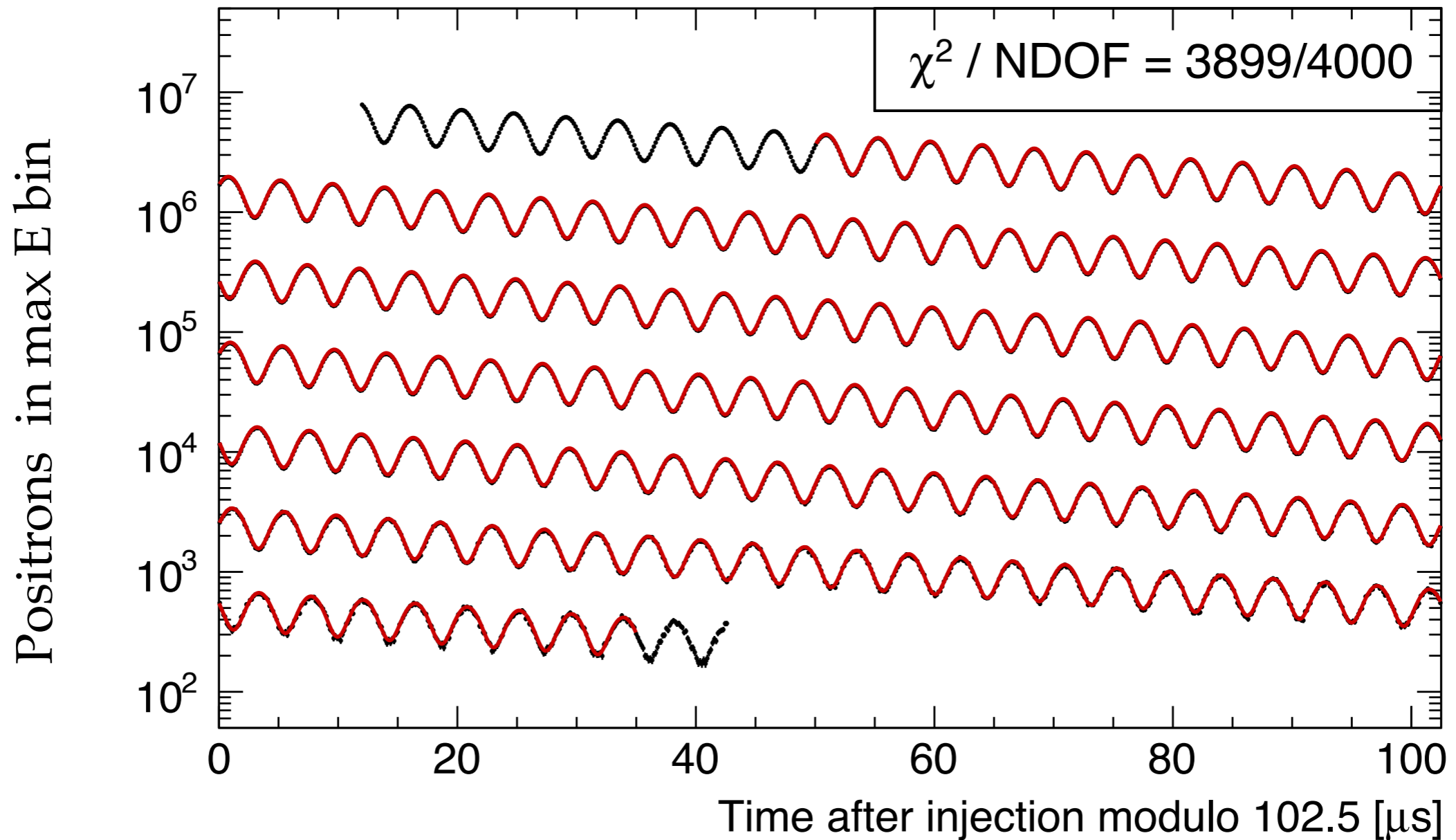
$$\frac{d\Gamma}{dE} = A [1 - B(\vec{s} \cdot \vec{p})]$$

$$\vec{s} \cdot \vec{p} \propto \cos \omega_a t$$

Upon  $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$  decay  $e^+$  emitted preferentially along  $\vec{S}$   
 Asymmetry in  $e^+$  energy distribution measures  $\omega_a$

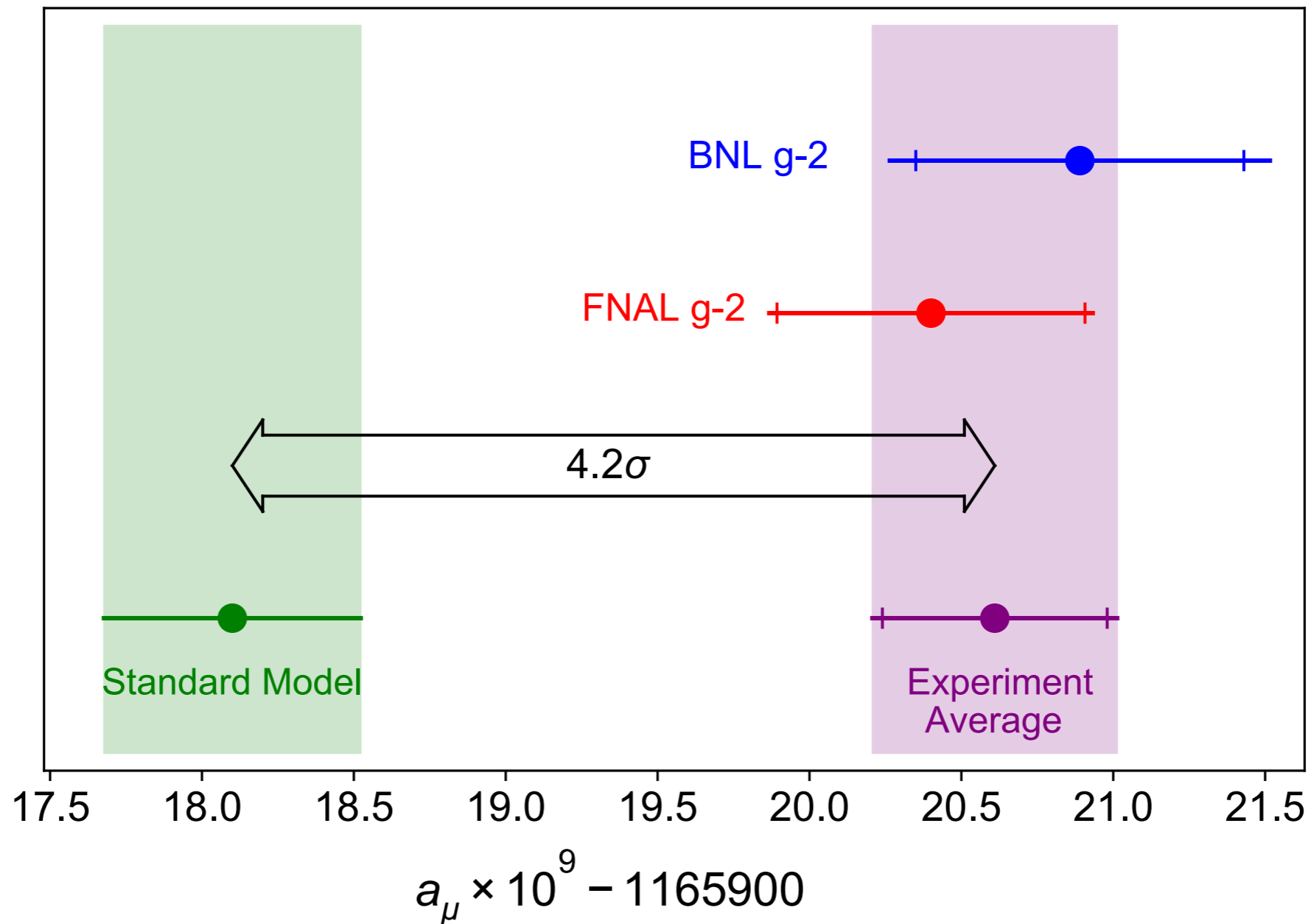
# Spin precession in a uniform B field

$$\omega_a = a_\mu \frac{eB}{2m_\mu}$$

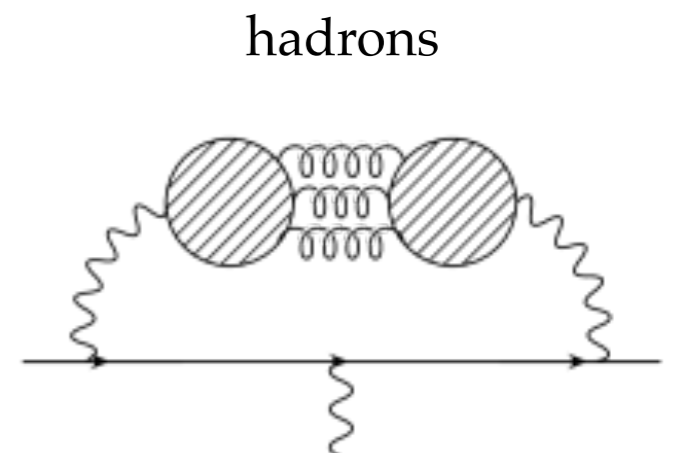


$$N_e(t) \propto e^{-t/\tau_\mu} (1 + B' \cos \omega_a t)$$

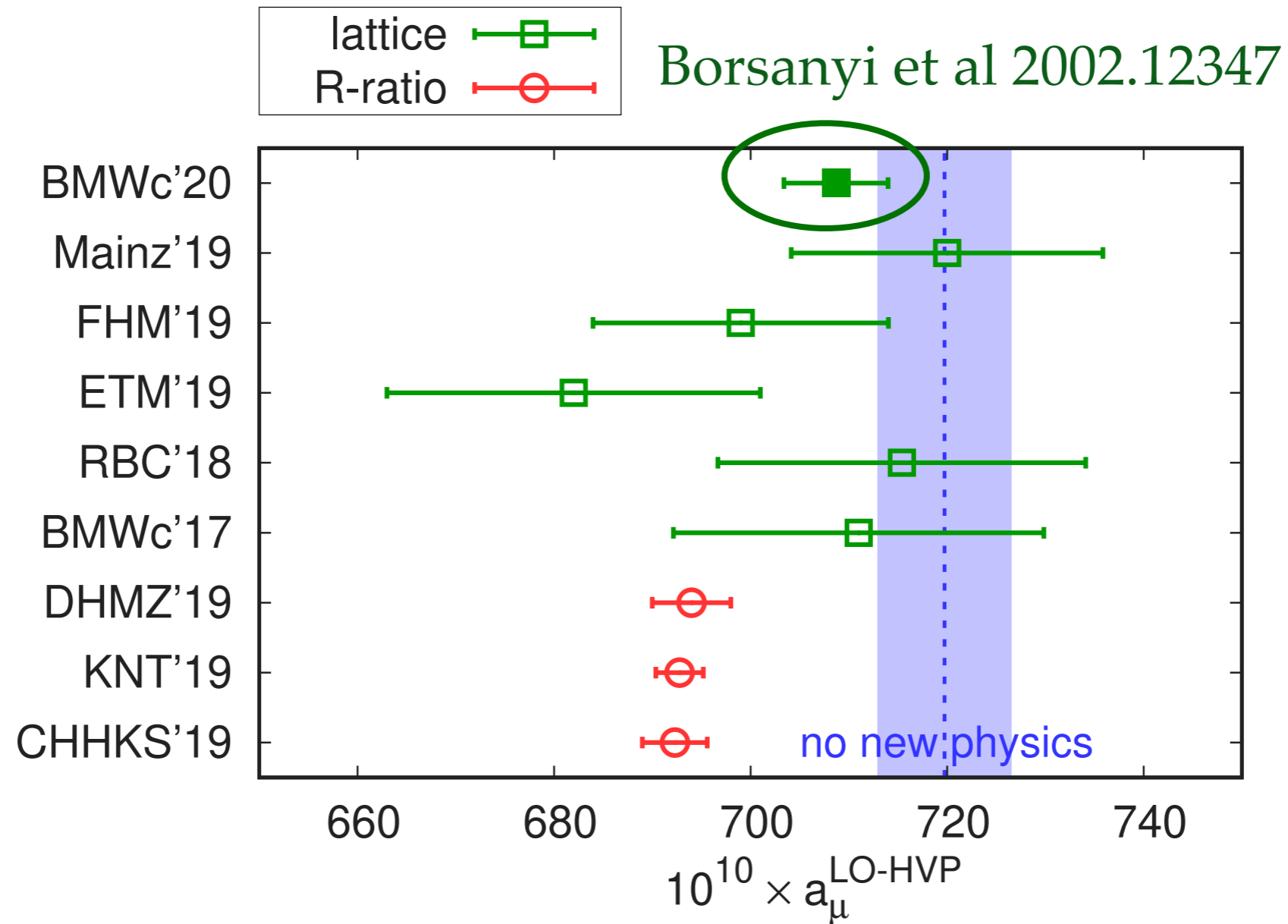
# Theory vs. Experiment



Theory uncertainty driven by  
non-perturbative QCD corrections



# Comparing SM Theory Calculations

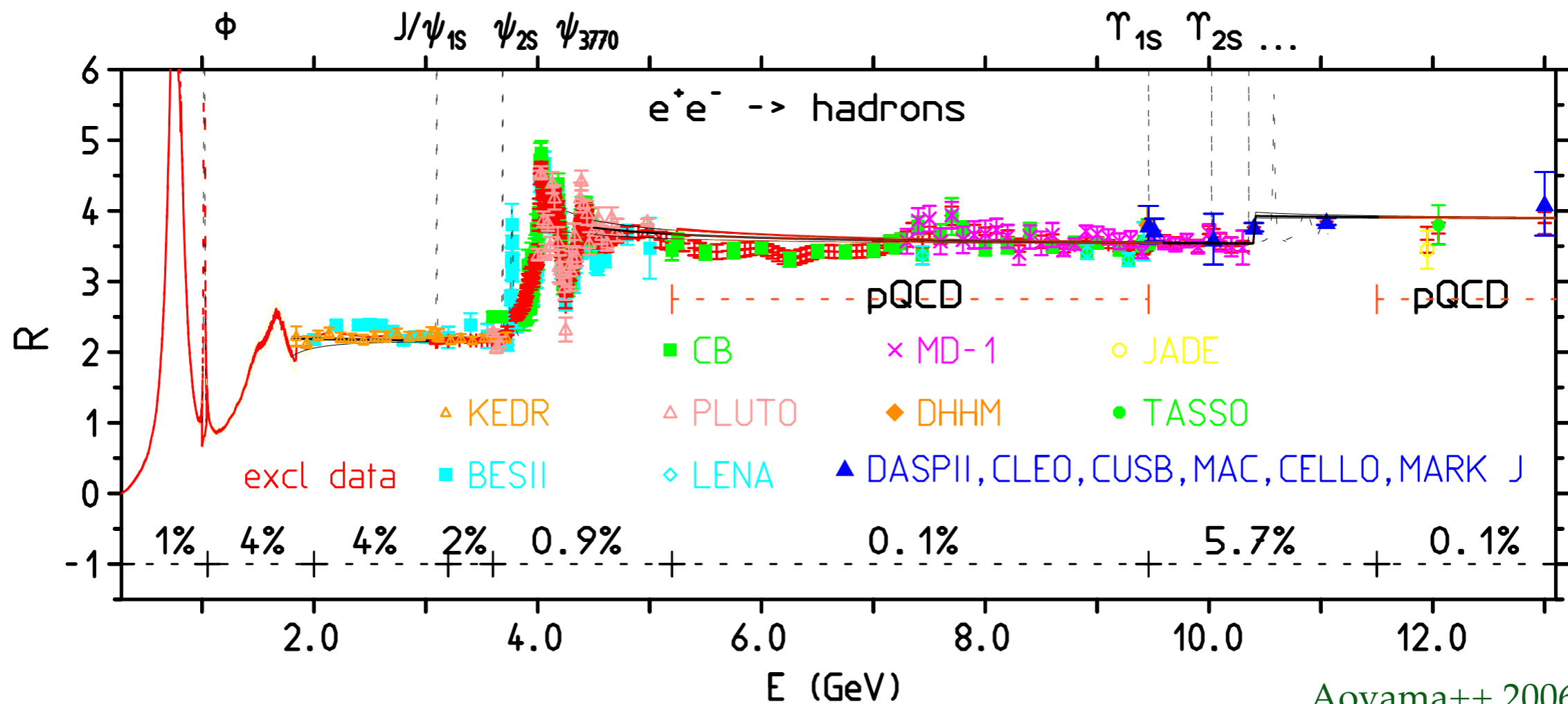


Recent lattice BMWc result in tension with data driven R-ratio method  
... but it's closer to experiment



# R-Ratio Calculations

Hadronic contributions can be extracted from  $e^+e^- \rightarrow \text{hadrons}$  data

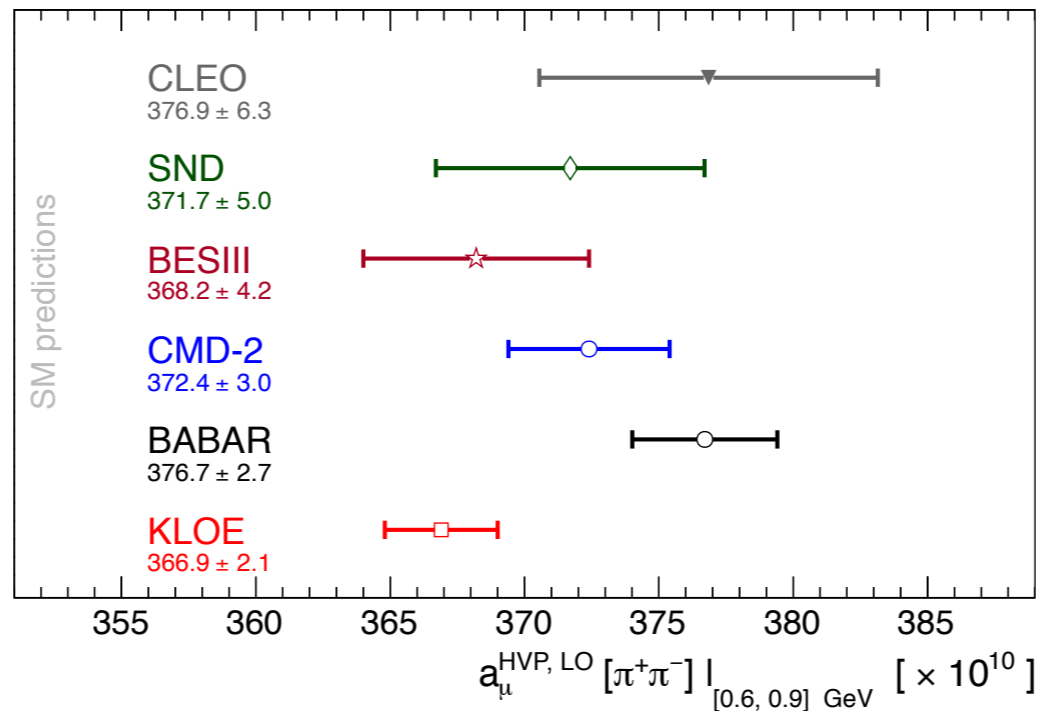


Aoyama++ 2006.04822

$$a_{\mu}^{\text{HVP, LO}} = \frac{\alpha^2}{3\pi^2} \int_{M_{\pi}^2}^{\infty} \frac{K(s)}{s} R(s) ds$$

$$R(s) \propto \sigma(e^+e^- \rightarrow \text{hadrons})$$

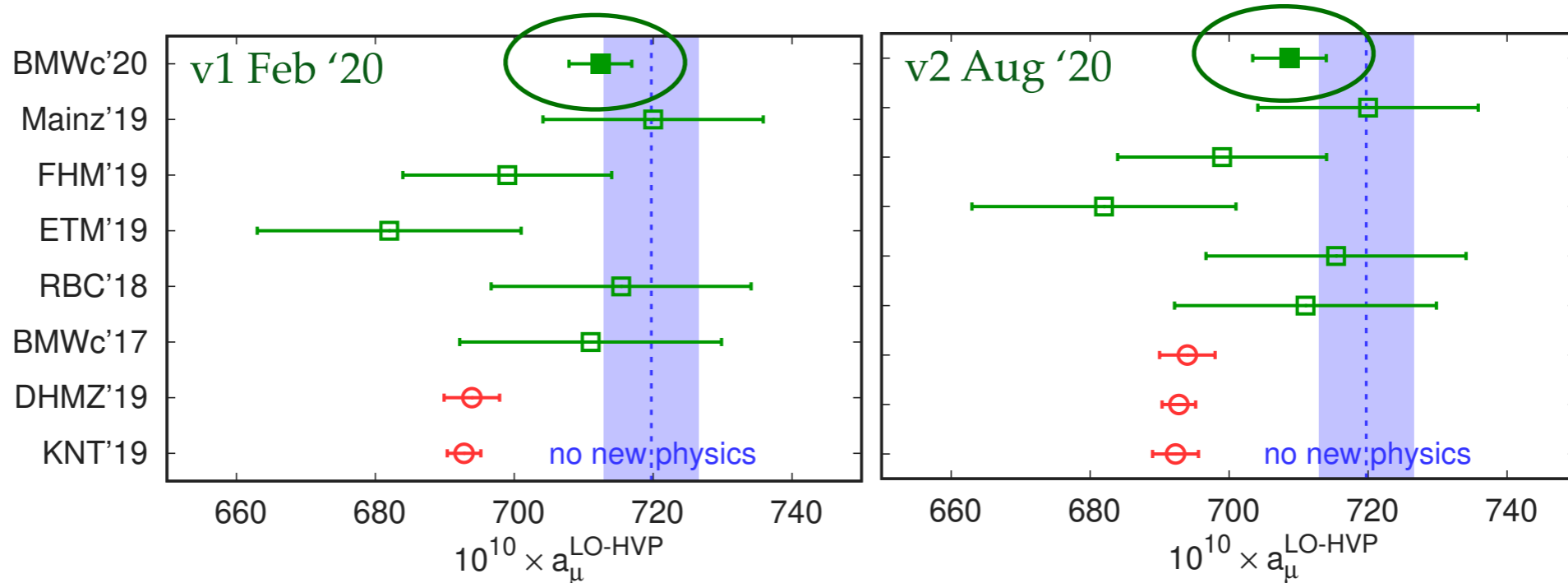
# R-Ratio Possible issue of internal consistency across full data set?



Aoyama++ 2006.04822

Borsanyi++ 2002.12347

# Lattice



Possible issue of extrapolating to continuum limit?

BMW also makes electroweak fit worse and in tension with  $e^+e^- \rightarrow \pi\pi$

# What should we believe?

## 1) Issue with with R-ratio calculations?

Possible, but nothing obvious (maybe tension in data?)

## 2) Issue with lattice calculations?

Also possible, need confirmation from other groups

## 3) R-ratio correct, but unknown experimental systematic?

After new data, this is extremely unlikely

This is the main new thing we have learned

## 4) New BSM particles contributing to loops?

# Overview

Current Status of  $a_\mu$

**Model Survey**

**Dark Matter Connection**

## What's the highest BSM scale?

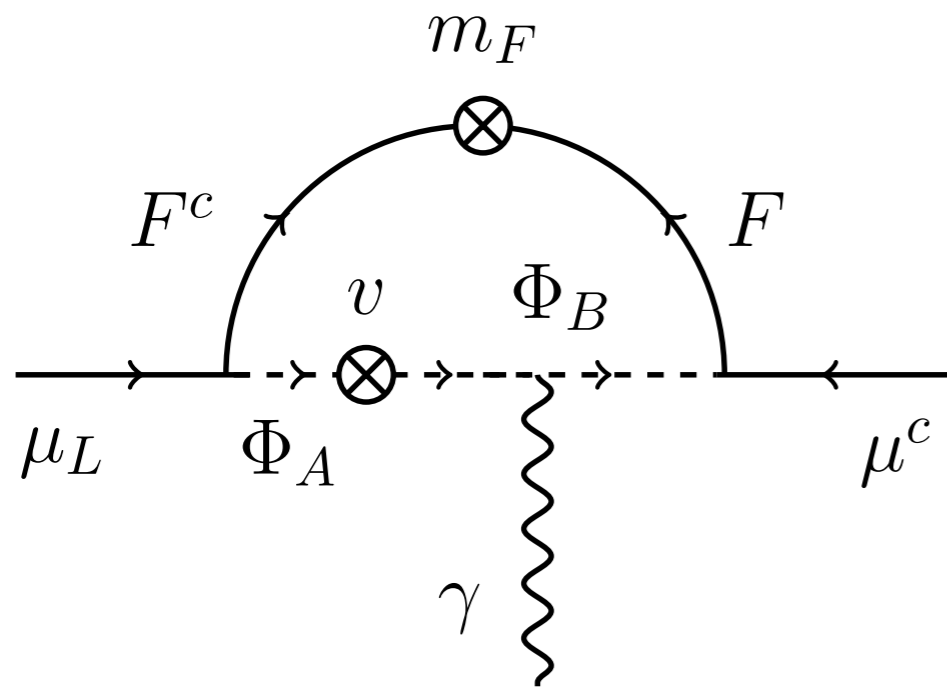
$$\mathcal{L}_{\text{eff}} = C_{\text{eff}} \frac{v}{M^2} (\mu_L \sigma^{\nu\rho} \mu^c) F_{\nu\rho} + \text{h.c.}$$

Contributing to  $g-2$  requires EWSB insertion and chiral flip

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$$\mathcal{L}_{\text{eff}} = C_{\text{eff}} \frac{v}{M^2} (\mu_L \sigma^{\nu\rho} \mu^c) F_{\nu\rho} + \text{h.c.}$$

Contributing to  $g-2$  requires EWSB insertion and chiral flip  
 If both arise from BSM vertices, highest scale  $\sim$  few 10s TeV



**Nightmare Heavy BSM**  
 Muon-philic  $\sim 50$  TeV  
 with  $O(1)$  couplings

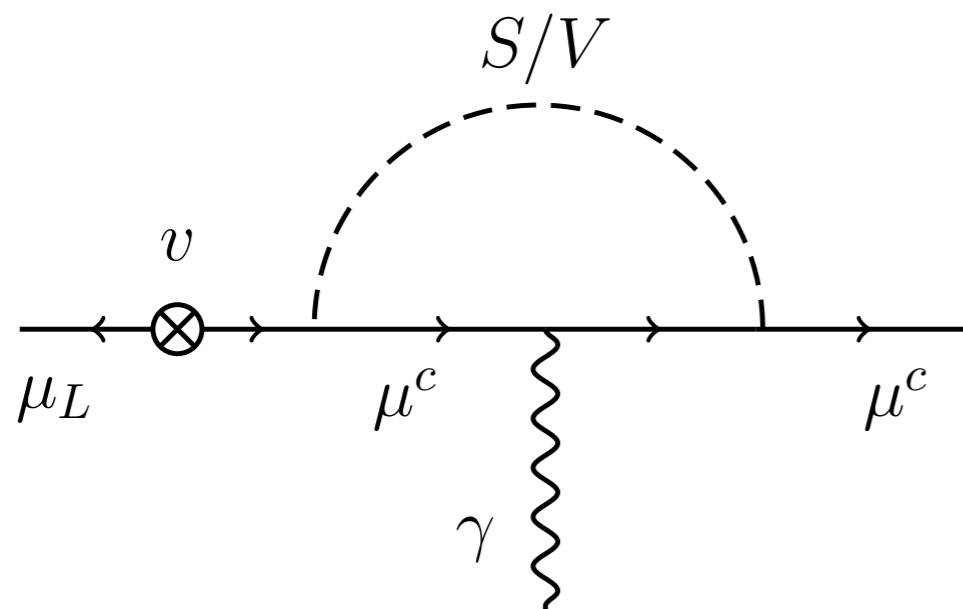
Requires future *muon* collider to test “nightmare” scenario

Capdevilla, Curtin, Kahn, GK 2006.16277

## What about light (< GeV) new physics?

$$\mathcal{L}_{\text{eff}} = C_{\text{eff}} \frac{v}{M^2} (\mu_L \sigma^{\nu\rho} \mu^c) F_{\nu\rho} + \text{h.c.}$$

Chiral flip and EWSB on muon line



### Simpler BSM Landscape

Must be scalar (S) or vector (V)

Must be SM gauge singlet

Must be  $> \text{MeV}$  (cosmology)

$$\Delta a_{\mu}^V = \frac{g_V^2}{4\pi^2} \int_0^1 dz \frac{m_{\mu}^2 z(1-z)^2}{m_{\mu}^2(1-z)^2 + m_V^2 z} \simeq 1.3 \times 10^{-10} \left( \frac{g_V}{10^{-4}} \right)^2 \quad (m_V \ll m_{\mu})$$

# Options For Light BSM?

- 1) Mix  $S/V$  with existing SM particles
- 2) Couple  $S/V$  to heavy states that mix with the muon
- 3)  $V$  is the gauge boson of a new  $U(1)$  SM extension

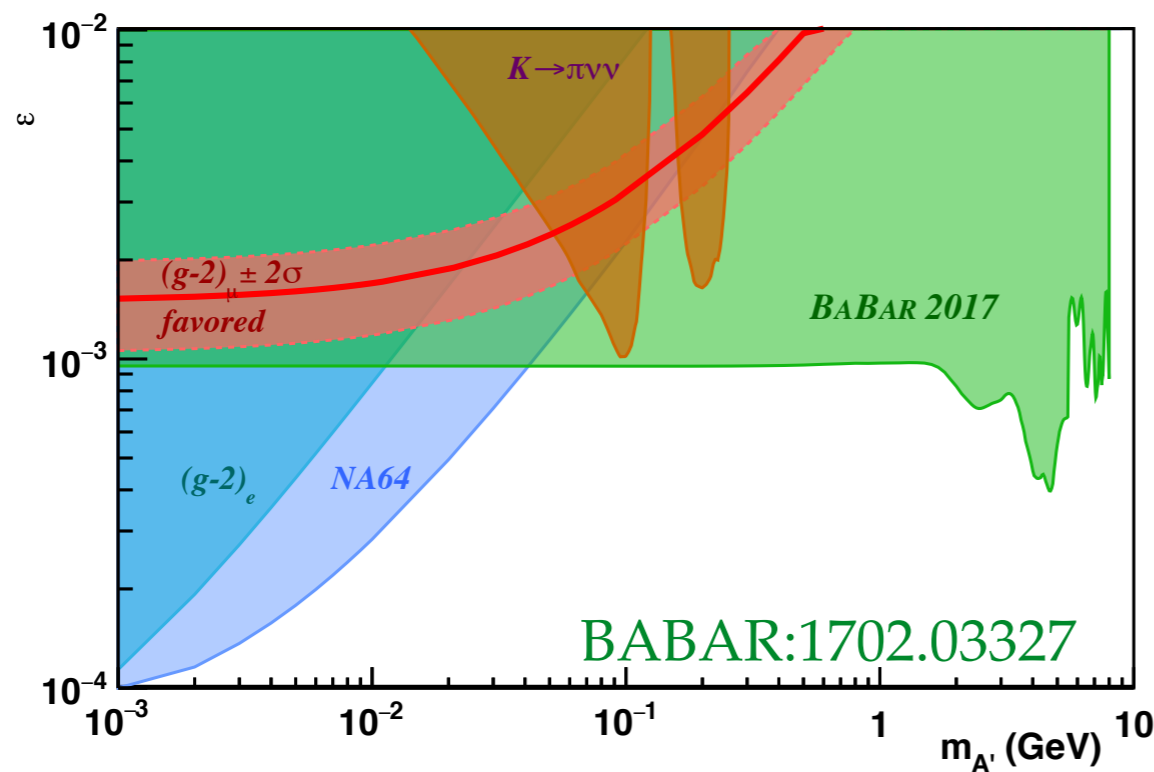


# Options For Light BSM?

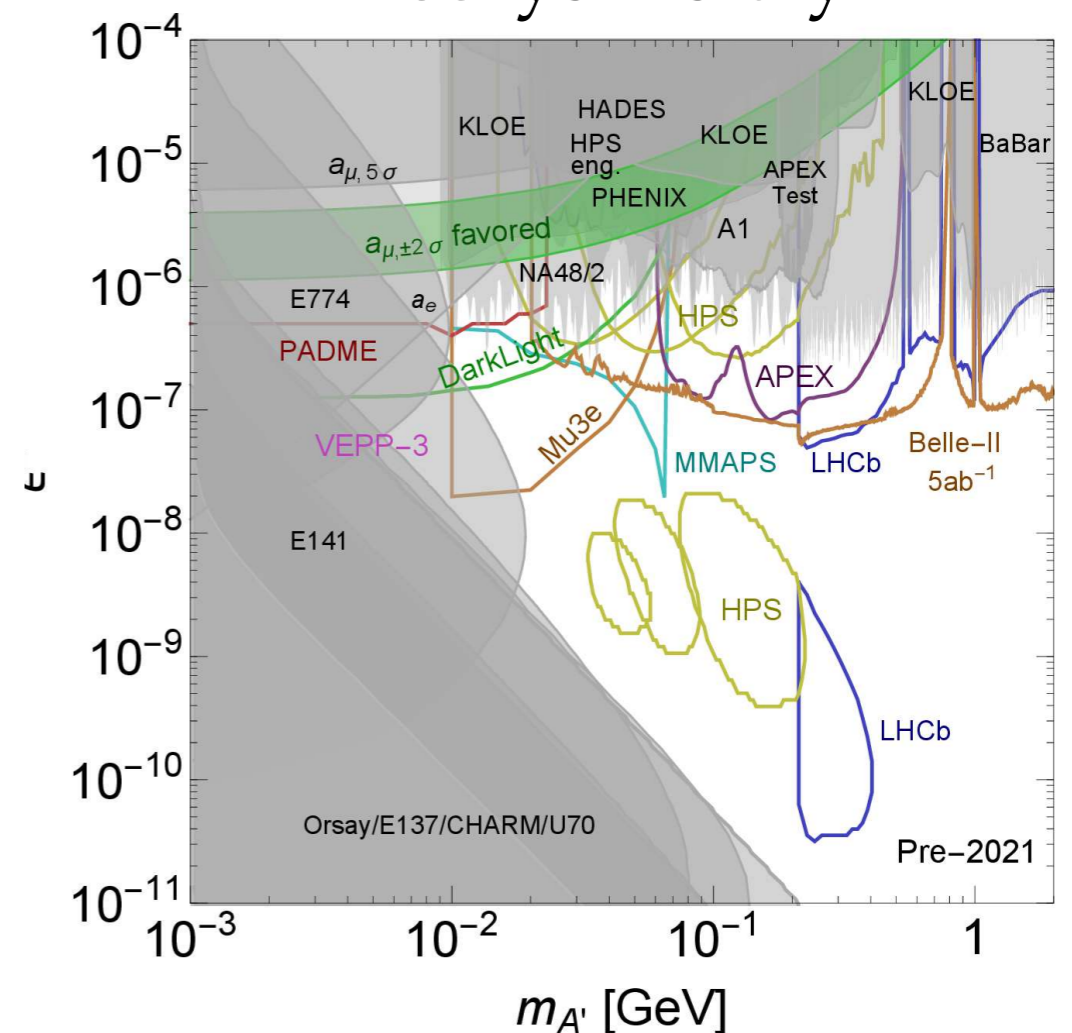
1) Mix S/V with existing SM particles

Kinetically mixed dark photon  $A'$  ruled out  $\mathcal{L}_{\text{int}} = \epsilon e A'_\mu J_{\text{EM}}^\mu$

Decays invisibly



Decays visibly



See Mohlabeng for semi-visible decays 1809.07768

1707.04591

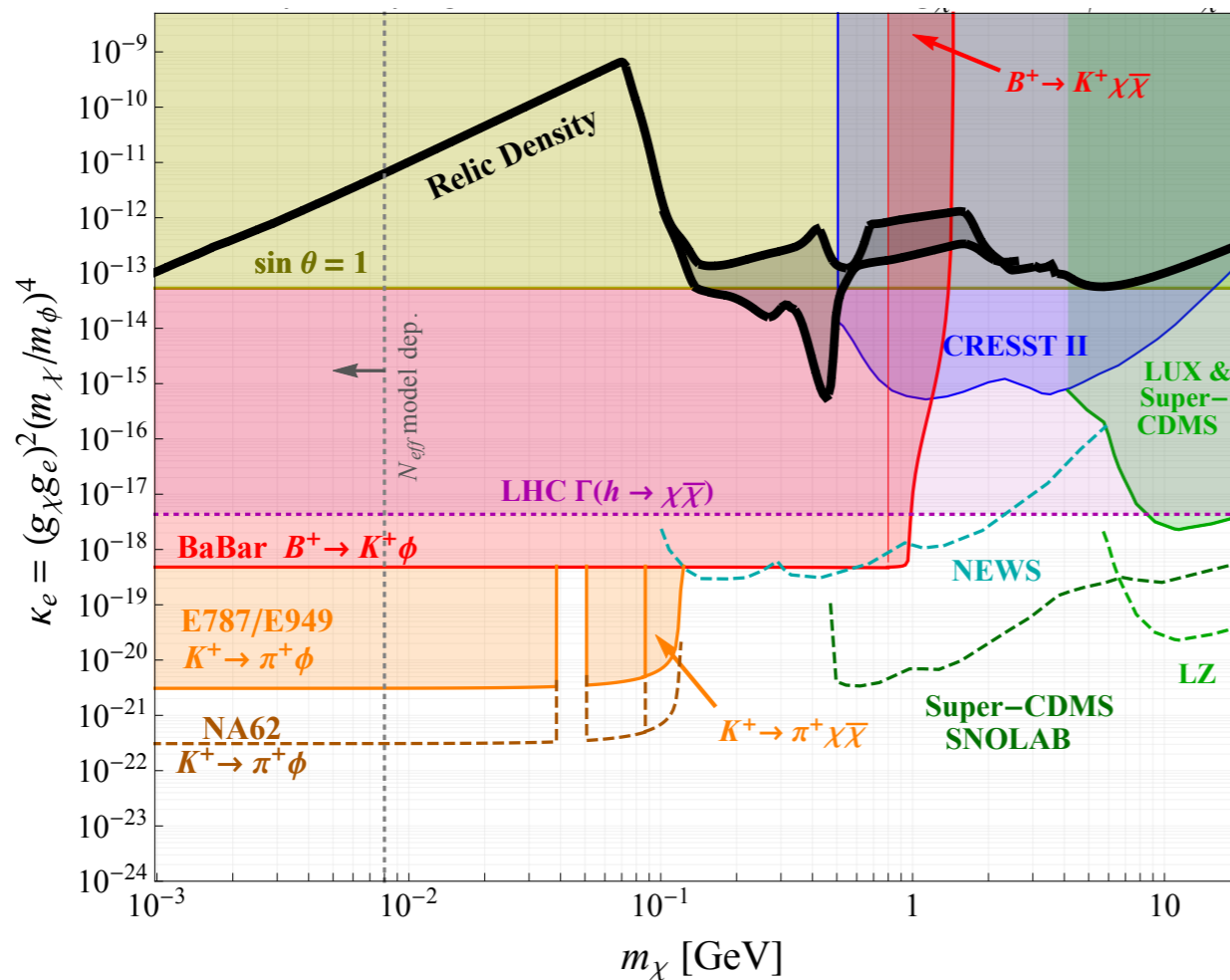
# Options For Light BSM?

1) Mix S/V with existing SM particles

Higgs mixed scalar  $\phi$  ruled out

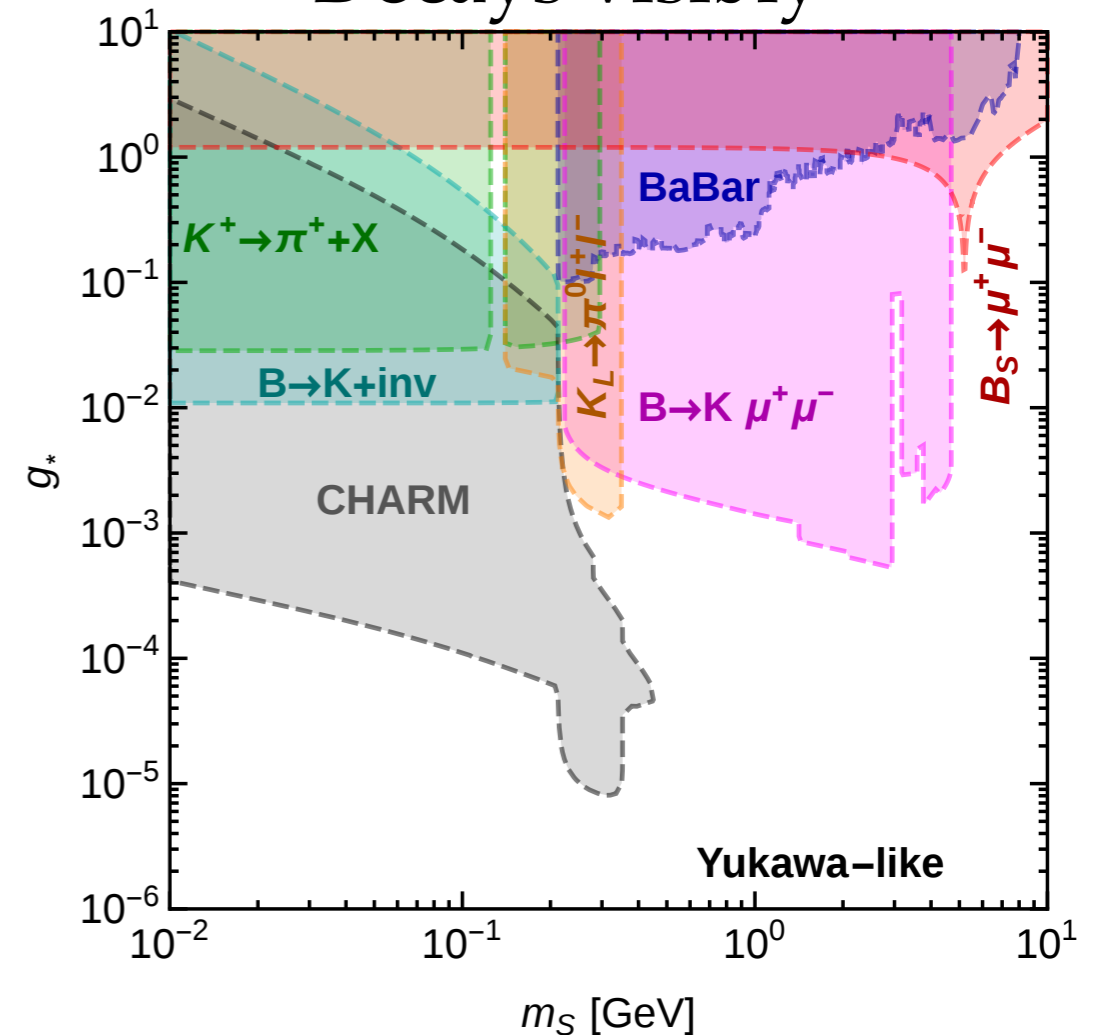
$$\mathcal{L}_{\text{int}} = \sin \theta \phi \frac{m_f}{v} \bar{f} f$$

Decays invisibly



GK 1512.04119

Decays visibly



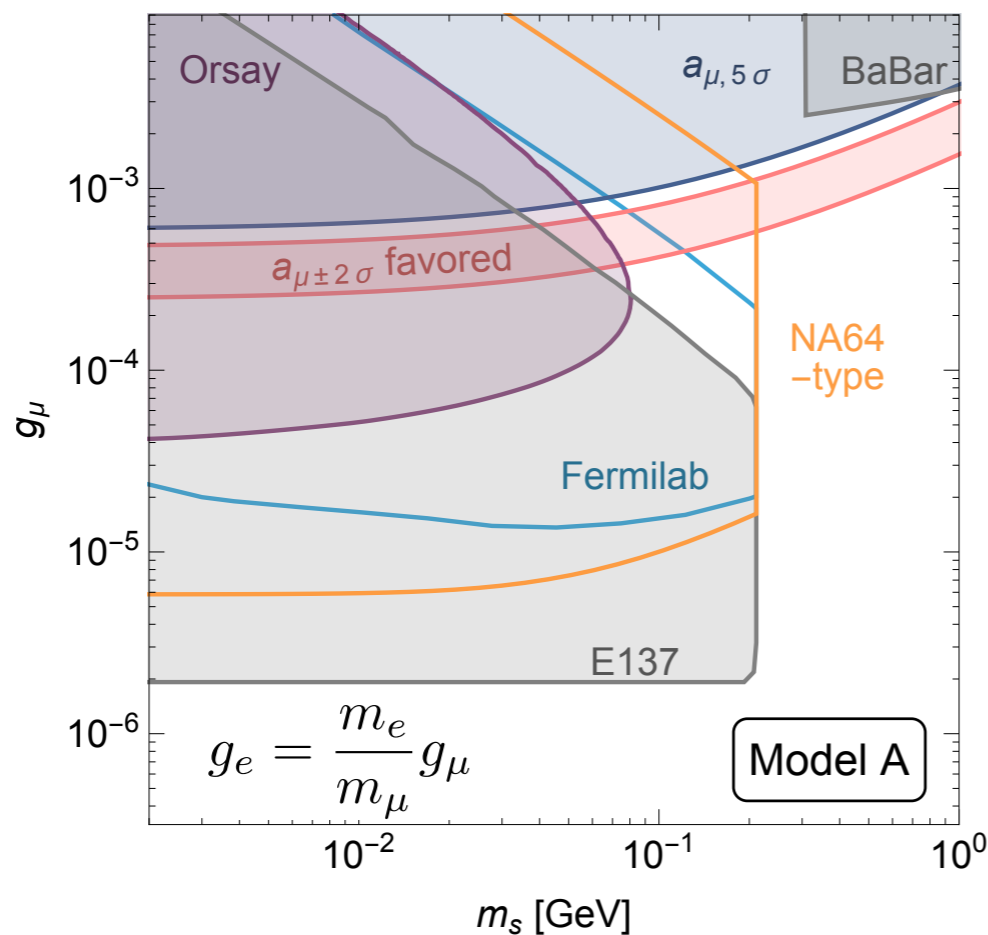
SHiP collab 1504.04855

# Options For Light BSM?

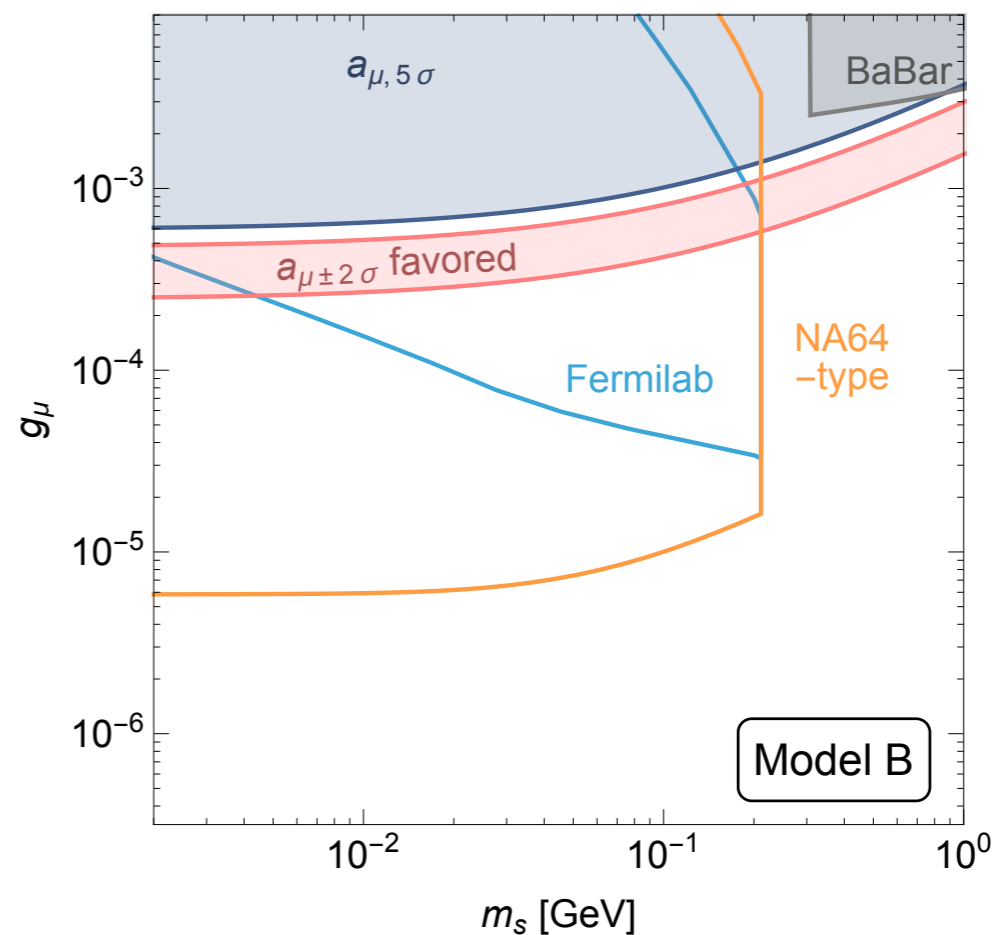
2) Couple S/V to heavy states that mix with the muon

$$\mathcal{L} \supset -\frac{1}{2}m_S^2 - \left( y_\mu H^\dagger L\mu^c + \frac{c_s}{M} S H^\dagger L\mu^c \right) \rightarrow \mathcal{L}_{\text{eff}} = g_\mu S \mu\mu^c$$

$S \rightarrow e^+e^-$  decay allowed



$S \rightarrow \gamma\gamma$  via muon loop

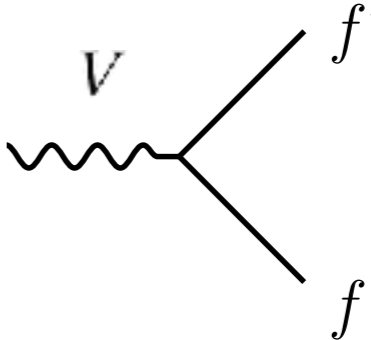


Both viable below  $\sim 200$  MeV

## Options For Light BSM?

3)  $V$  is the gauge boson of a new  $U(1)$  SM extension

SM particles now carry a new gauge quantum number

$$\mathcal{L} \supset g V_\mu J_{\text{SM}}^\mu, \quad J_{\text{SM}}^\mu \equiv \sum_f Q_f \bar{f} \gamma^\mu f$$
A Feynman diagram illustrating the interaction of a gauge boson  $V$  with a fermion  $f$ . The gauge boson is represented by a wavy line on the left, labeled  $V$ . It connects to two fermion lines on the right, both labeled  $f$ . The top fermion line is an incoming line, and the bottom fermion line is an outgoing line.

Only anomaly free possibilities:

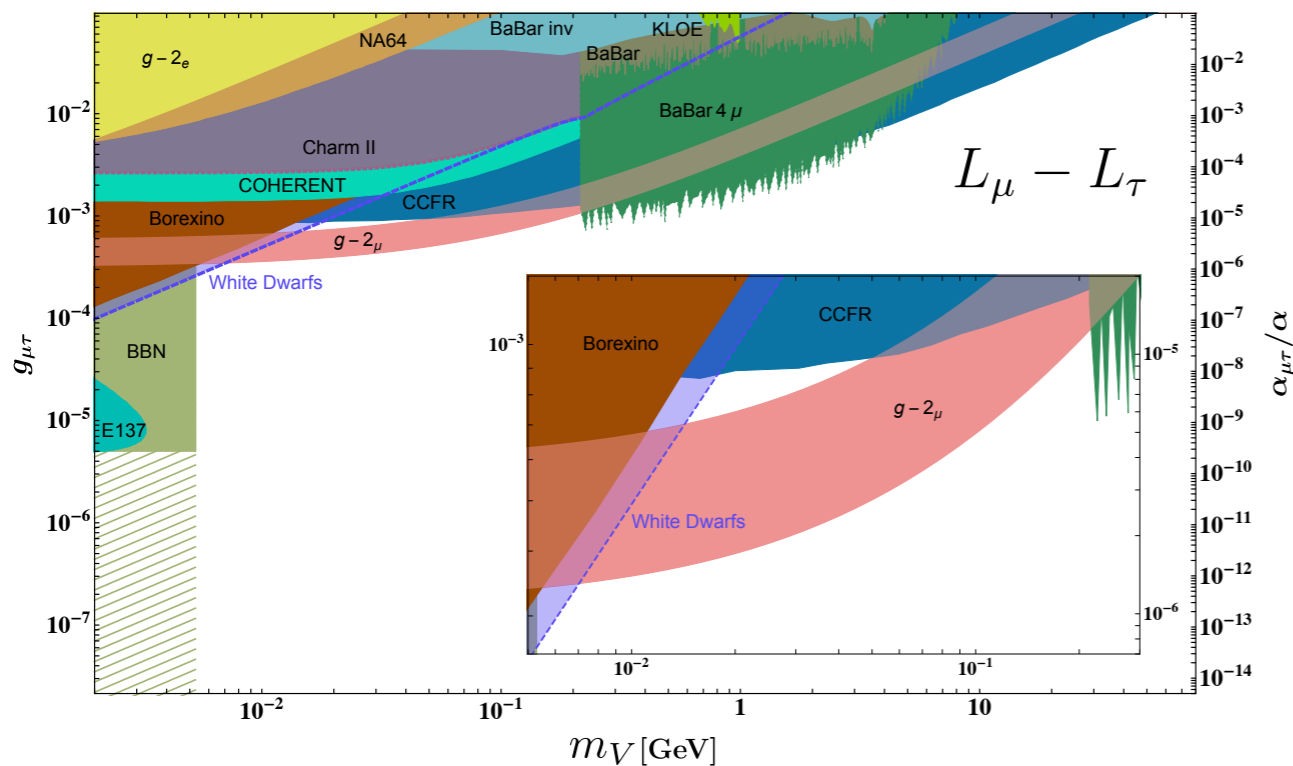
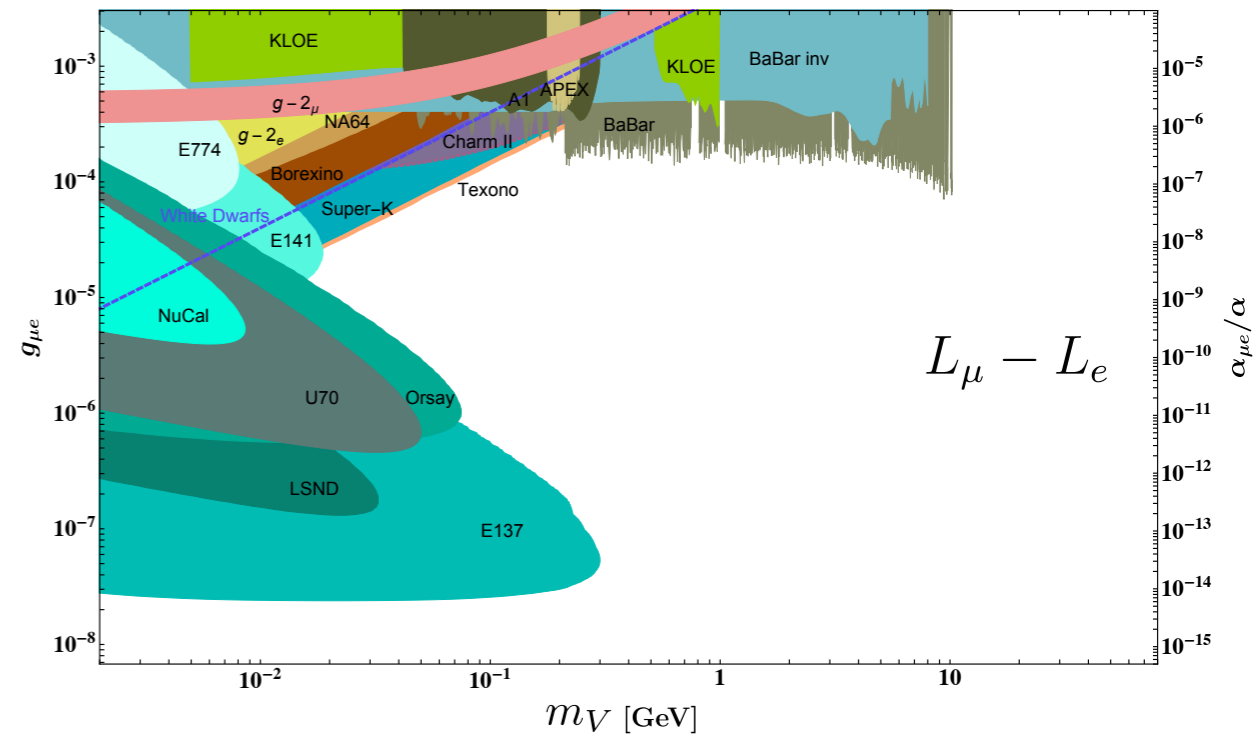
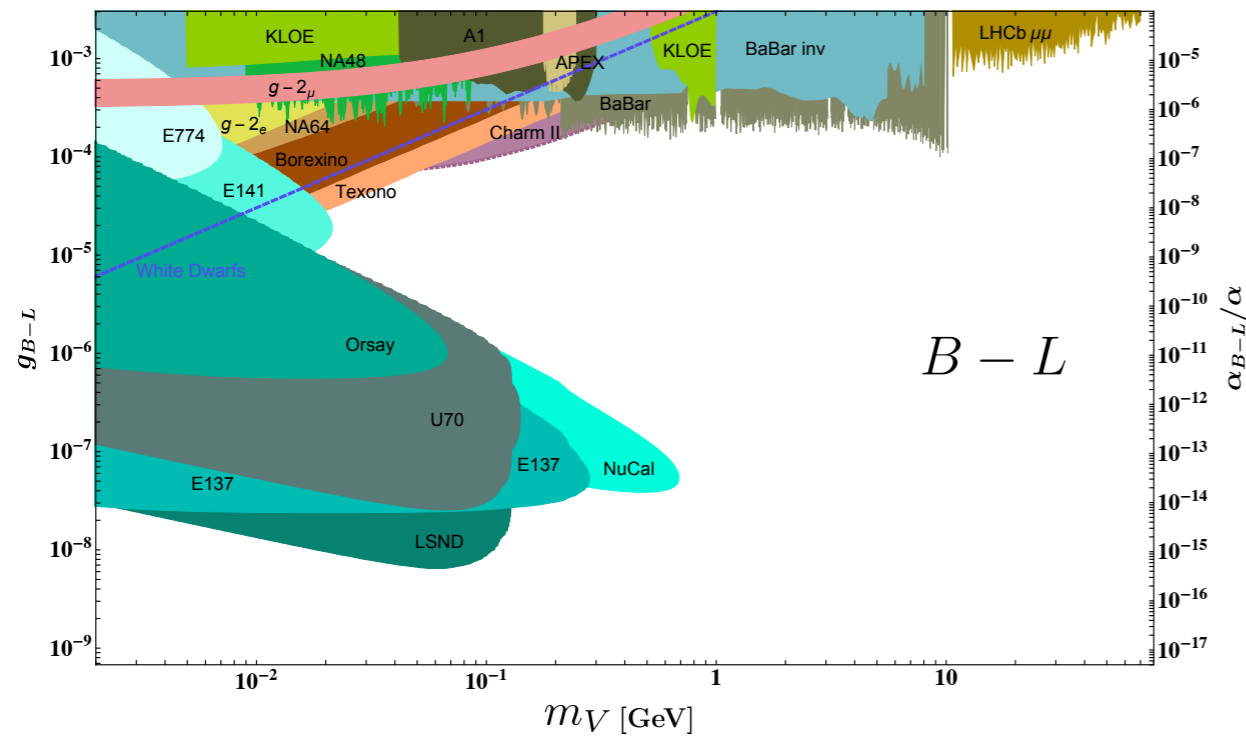
$$U(1)_{B-L}, \quad U(1)_{L_i-L_j}, \quad U(1)_{B-3L_i}$$

Qualitatively similar, but some differences in bounds

Two parameter family of models:  $\{g, m_V\}$

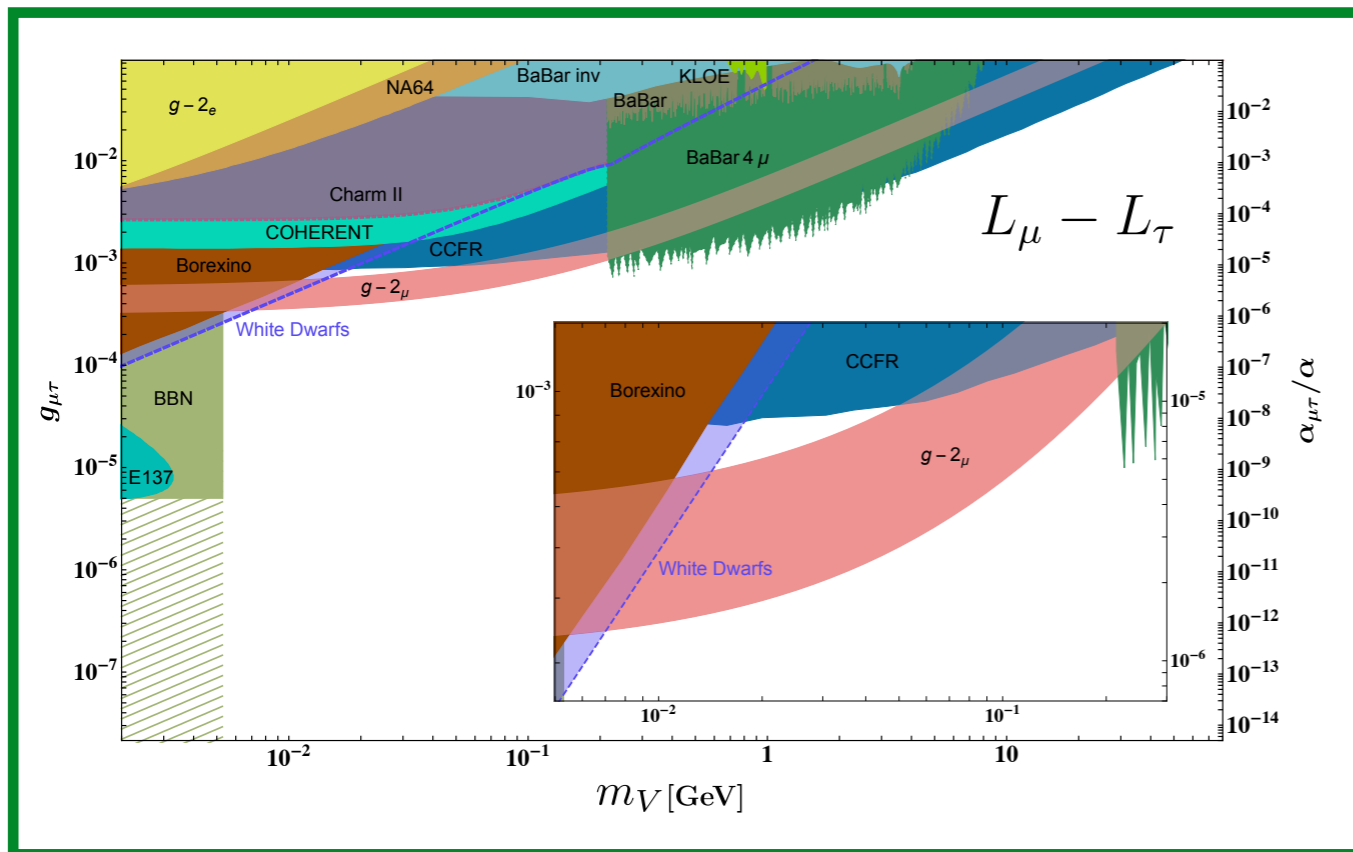
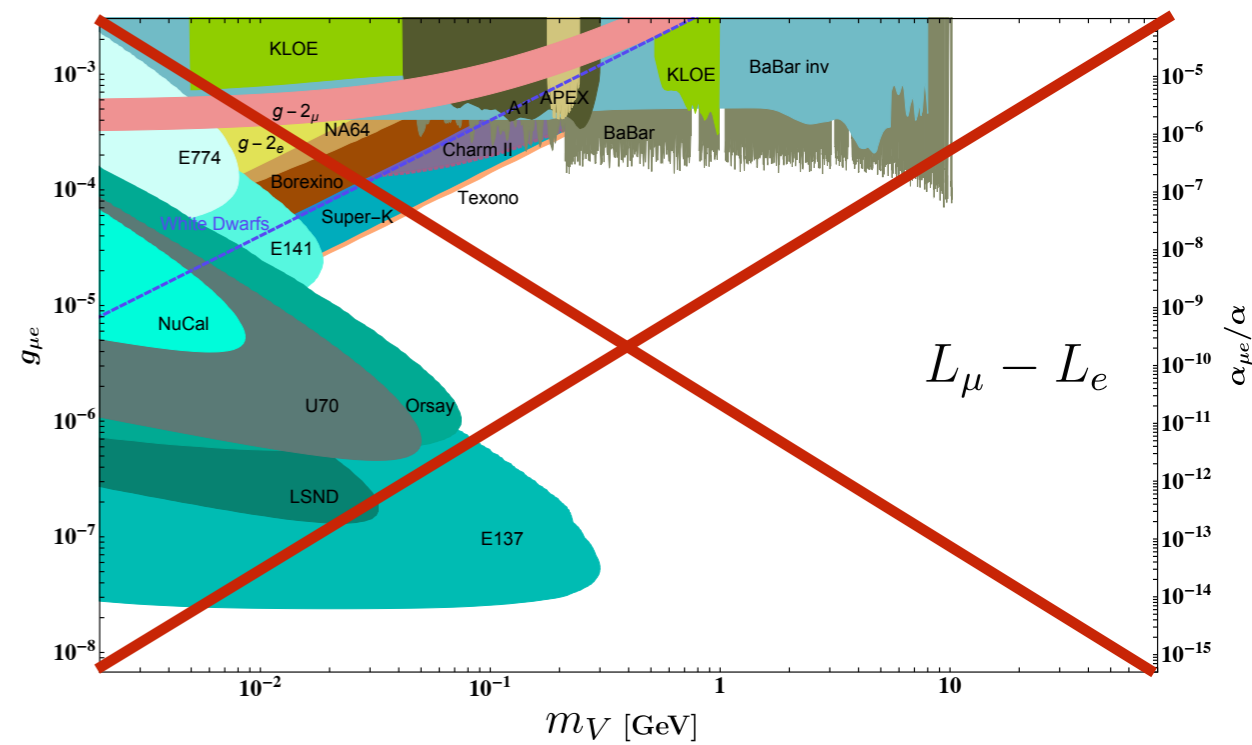
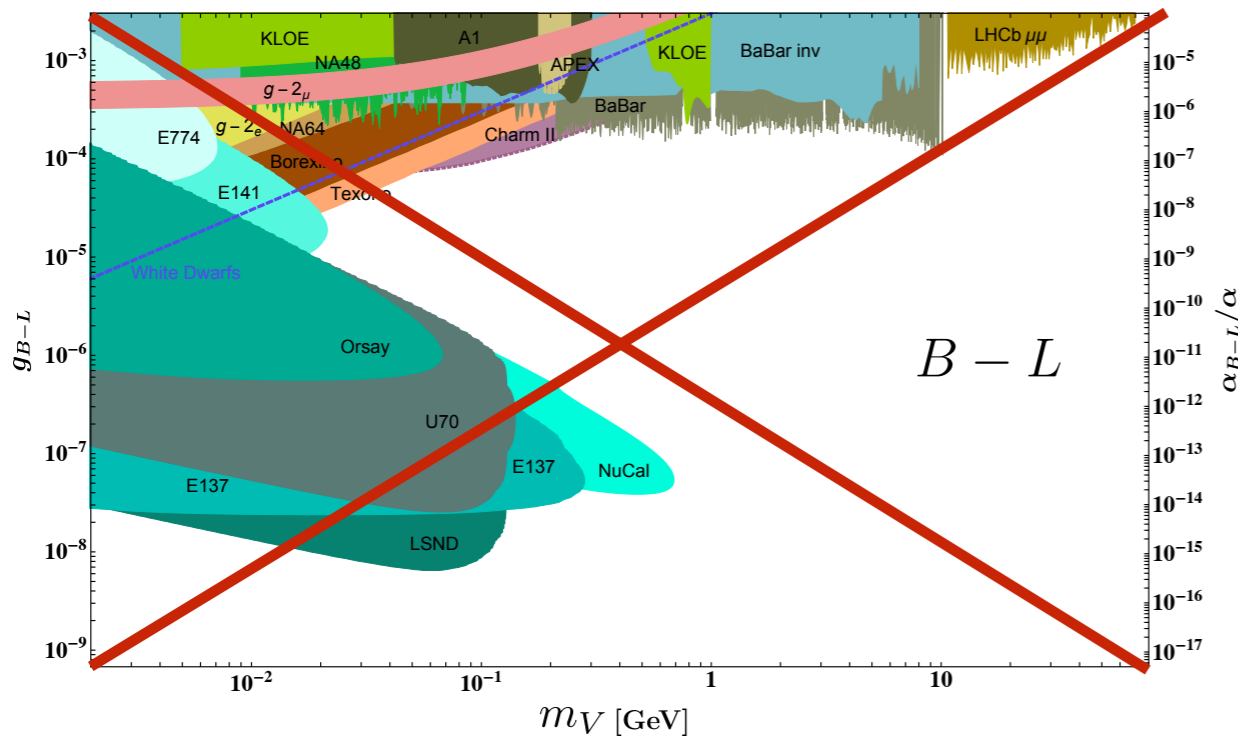
# Options For Light BSM?

3)  $V$  is the gauge boson of a new  $U(1)$



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Only one possibility left  
Viable between  $\text{MeV} - 2m_\mu$

$< \text{MeV}$  spoils BBN

$> 2m_\mu$  decays to muons

## Summary of what's left

Experimental bounds require muon-philic forces for  $< \text{GeV}$  singlets

### Scalar model

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}m_S^2 S^2 - \sum_{\ell=e,\mu,\tau} g_\ell S \bar{\ell}\ell,$$

Generically need  $g_{e,q} \ll g_\mu$

Some model dependence in decays

Need additional SM charged fields in UV

### Vector model

$$U(1)_{\mu-\tau}$$

$$\mathcal{L} \supset \frac{m_V^2}{2} V_\alpha V^\alpha + g_V V_\alpha J_{\mu-\tau}^\alpha$$

$$J_{\mu-\tau}^\alpha \equiv \bar{\mu}\gamma^\alpha\mu + \bar{\nu}_\mu\gamma^\alpha P_L\nu_\mu - (\mu \rightarrow \tau)$$

For viable mass range  $< 200 \text{ MeV}$ ,  $V$  always decays invisibly

See Cristina Mantilla-Suarez's talk

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**Dark Matter Connection**



**Q: Why haven't we discovered dark matter in the lab?**

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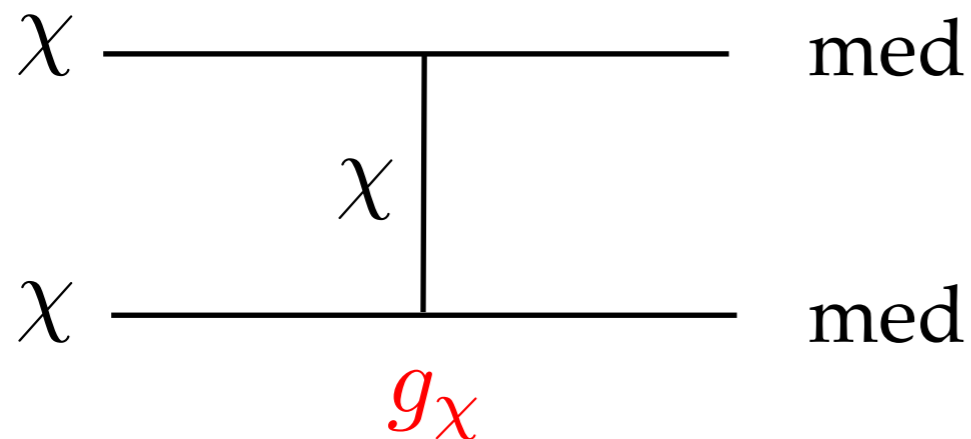
**A: Maybe because it couples more to muons**

The same bosons (S/V) for  $g-2$  can also mediate DM annihilation during “freeze out” in early universe

# Who's heavier: mediator or DM?

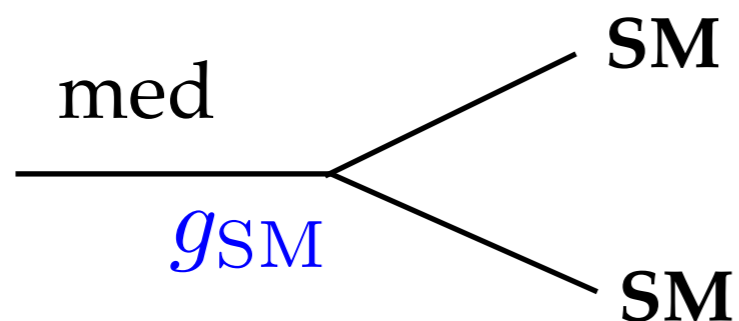
## Secluded Annihilation

$$m_\chi > m_{\text{med}}$$



No clear experimental target

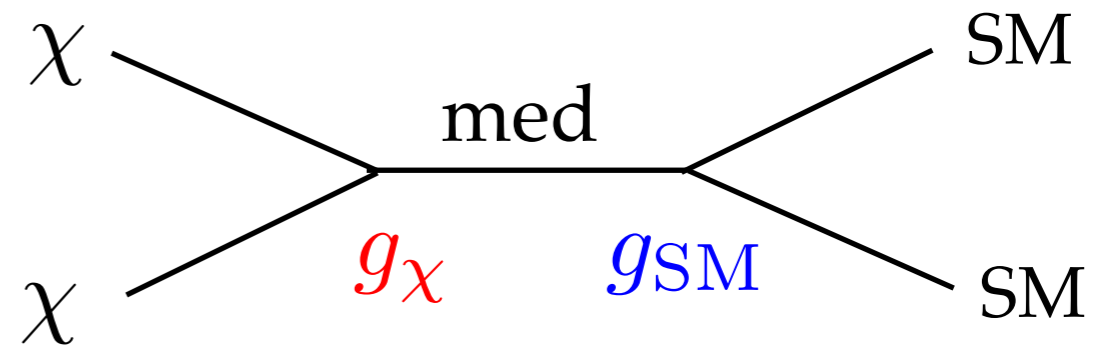
Abundance set by  $g_\chi$



Mediator decays **visibly**

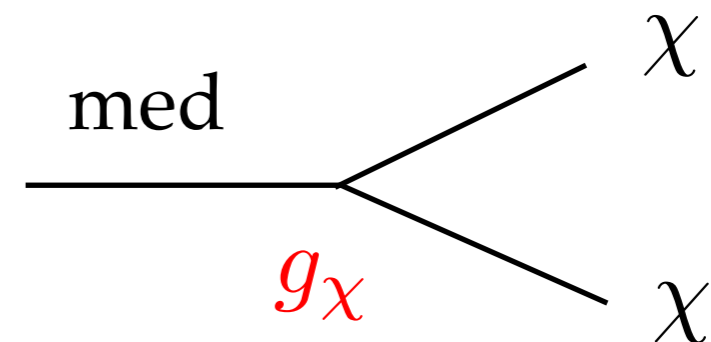
## Direct Annihilation

$$m_\chi < m_{\text{med}}$$



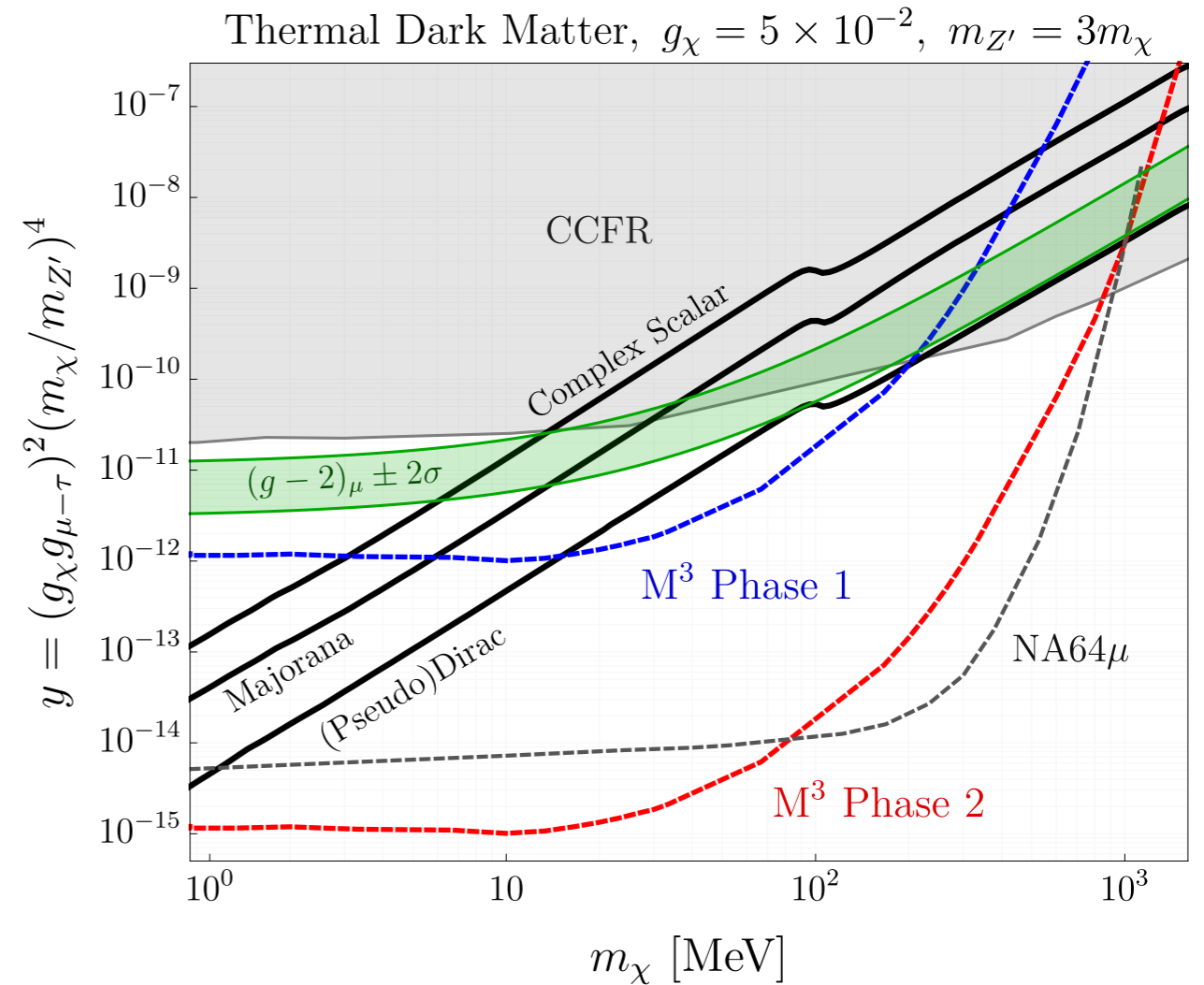
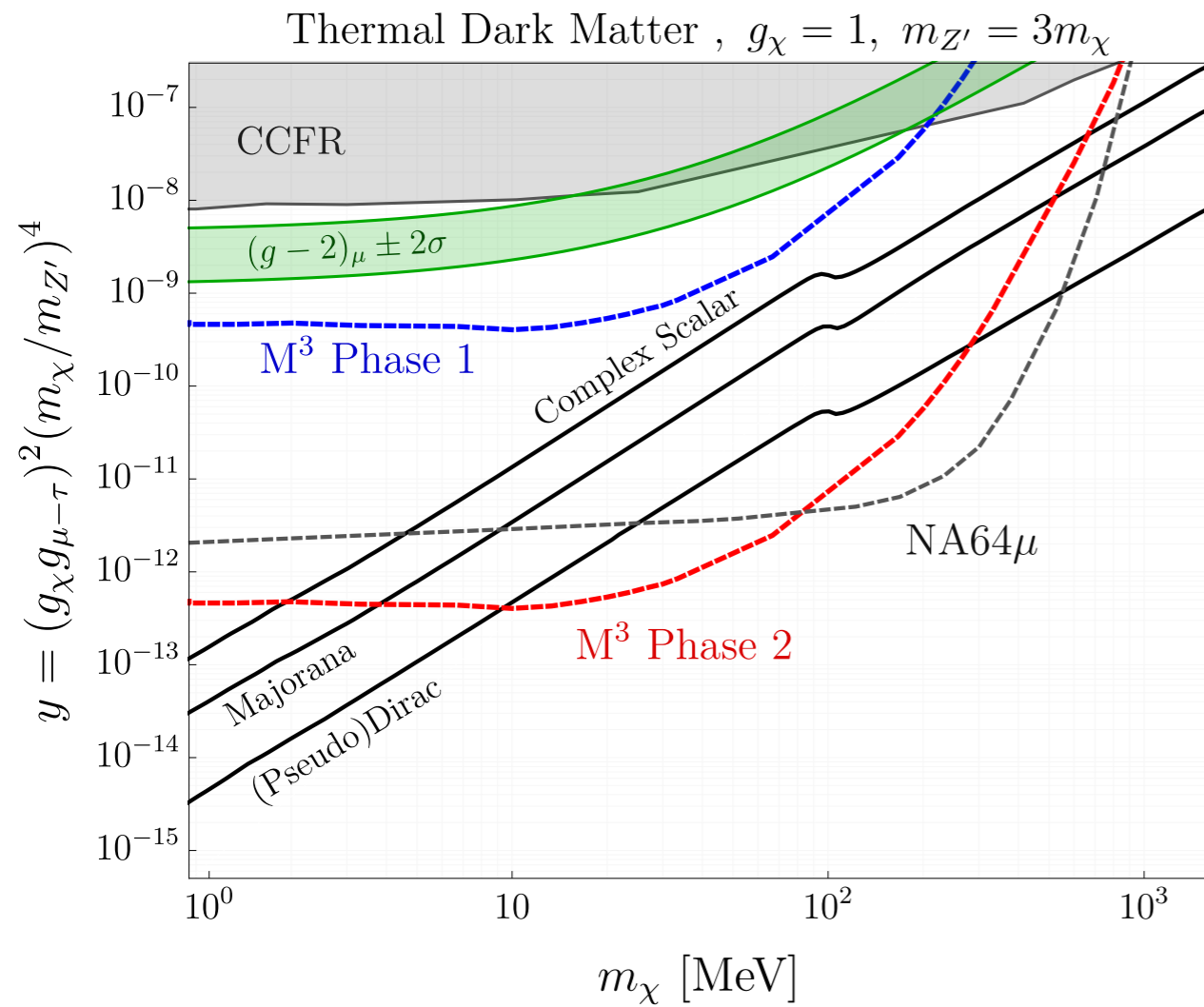
Predictive thermal targets

Abundance depends on  $g_{\text{SM}}$



Mediator decays **invisibly\***

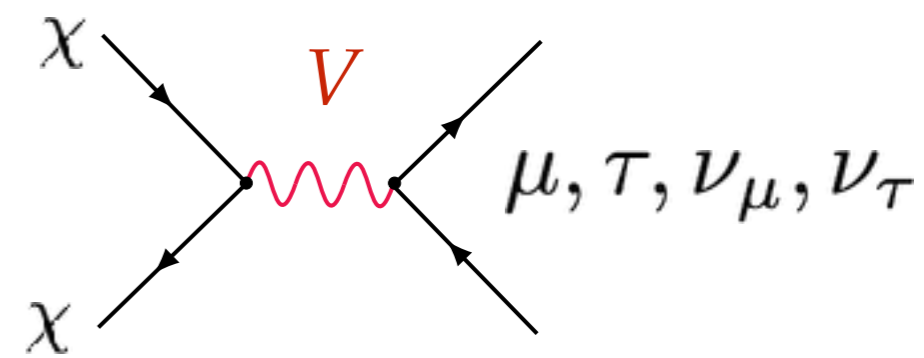
# Comprehensive Coverage



Gauged  $L_\mu - L_\tau$  Interaction

Also resolve muon  $g-2$  with light physics

Compatible parameter space for freeze-out



Phase 1,2:  $1e^{10}$ ,  $1e^{13}$  muons

See Cristina Mantilla-Suarez's talk

## Concluding Remarks

### Exciting time for $g-2$ , new results soon!

Awaiting new SM lattice / R-ratio results  
4.2 sigma anomaly probably not systematic error

### If anomaly is due to light $< \text{GeV}$ BSM physics

Must be muon-philic boson: S/V  
Can decay either visibly or invisibly  
Muon fixed targets can probe nearly all variations

See Cristina, Yiming, and Brian's talks

### Same particles can couple to dark matter

Common parameter space for  $g-2$  + freeze out  
Same fixed target searches give this for free

**Thanks!**

## Constraints: Big Bang Nucleosynthesis

$V$  is in chemical equilibrium with SM in early universe

$$n_V \propto \begin{cases} T^3 & (T \gg m_V) \\ e^{-m/T} & (T \ll m_V) \end{cases}$$

When  $T < m$ , the  $V$  decays transfer entropy to SM particles  
Must happen before neutrinos decouple from photons

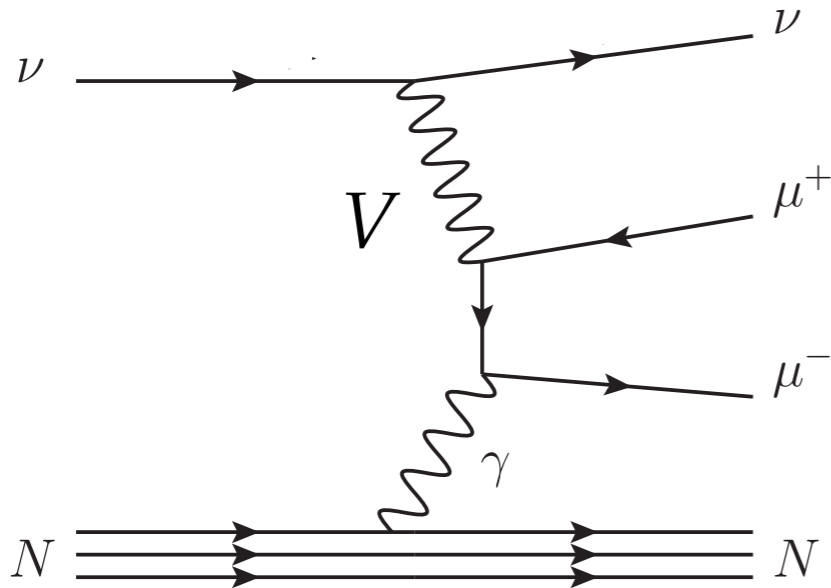
$$m \gtrsim T_{\nu,\text{dec}} \approx 2 \text{ MeV}$$

Otherwise  $V$  decays heat neutrinos not CMB  $\rightarrow \Delta N_{\text{eff}} \gtrsim 0.5$

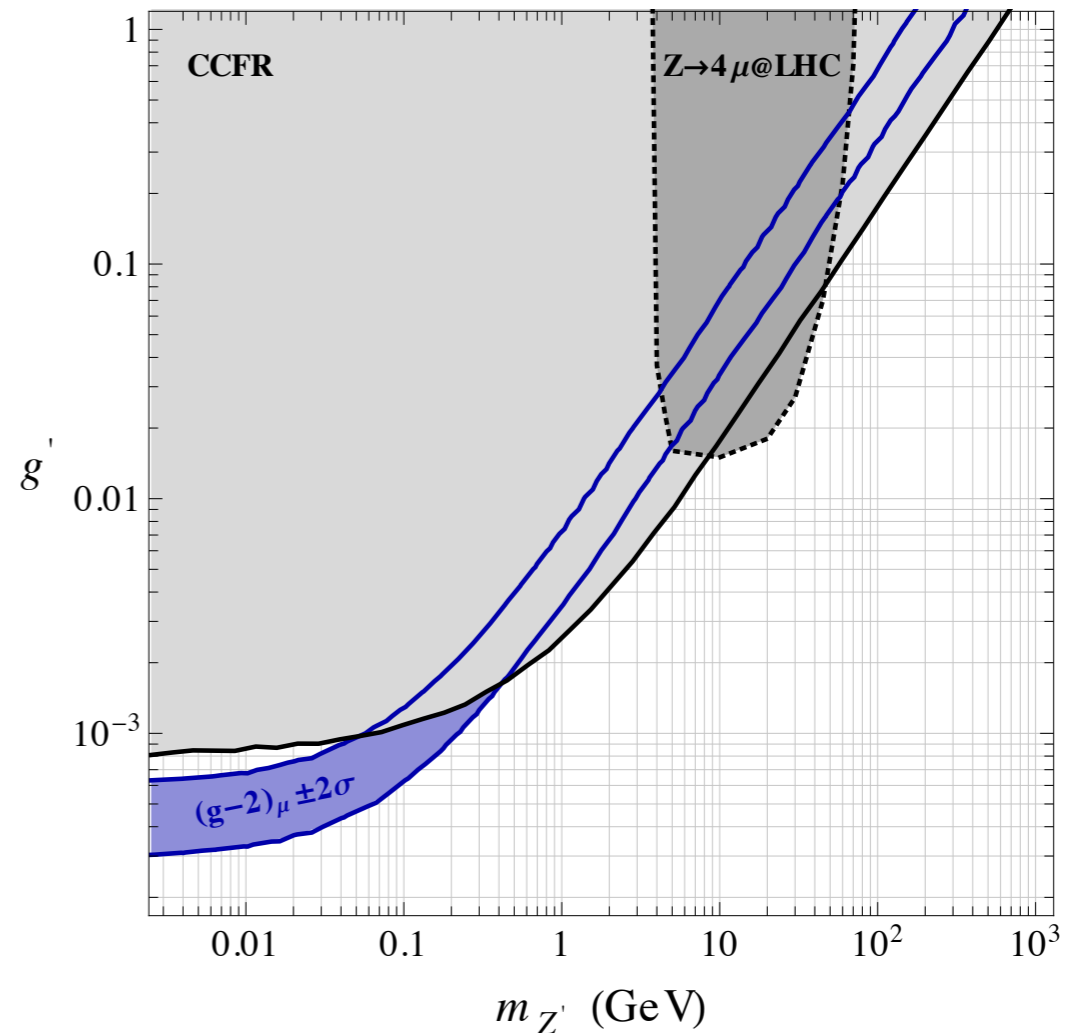
Spoils BBN element yields

\*mild contribution for  $m \sim$  few MeV may reduce Hubble tension

# Constraints: Neutrino Trident, CCFR + CHARM II



$$\frac{\sigma^{\text{CCFR}}}{\sigma^{\text{SM}}} = 0.82 \pm 0.28.$$

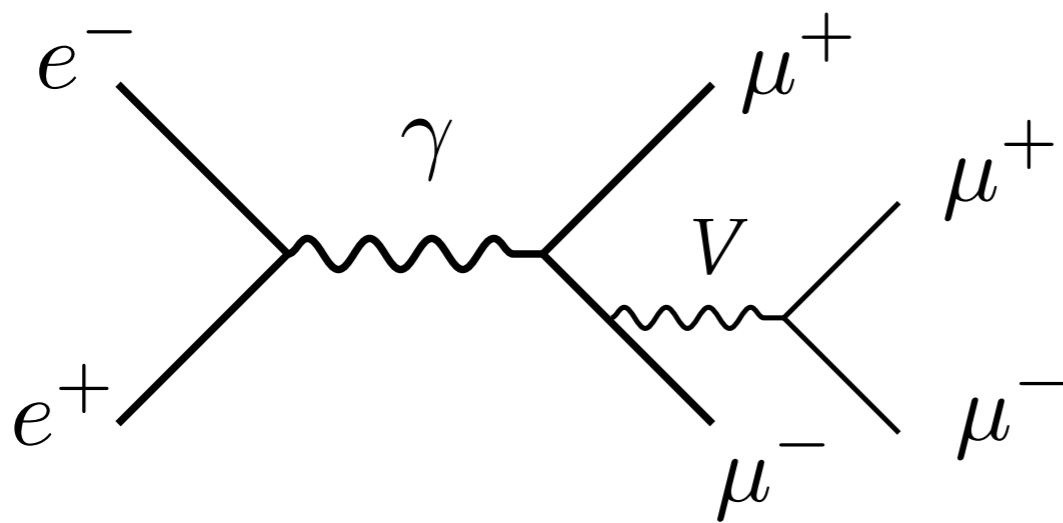
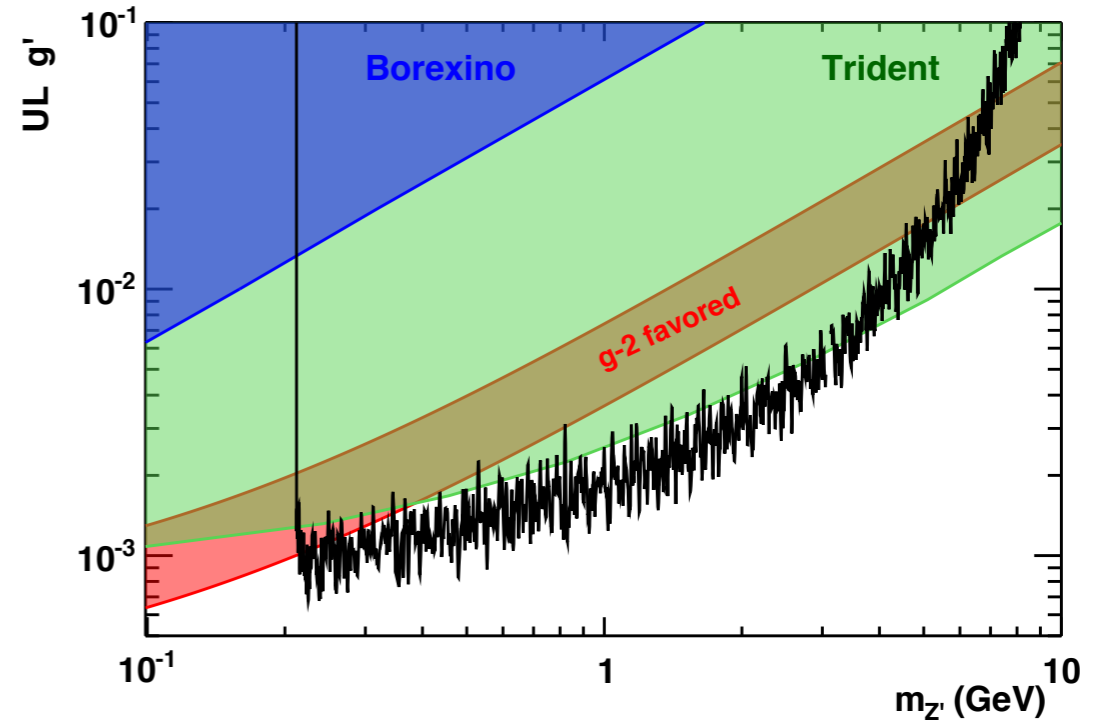
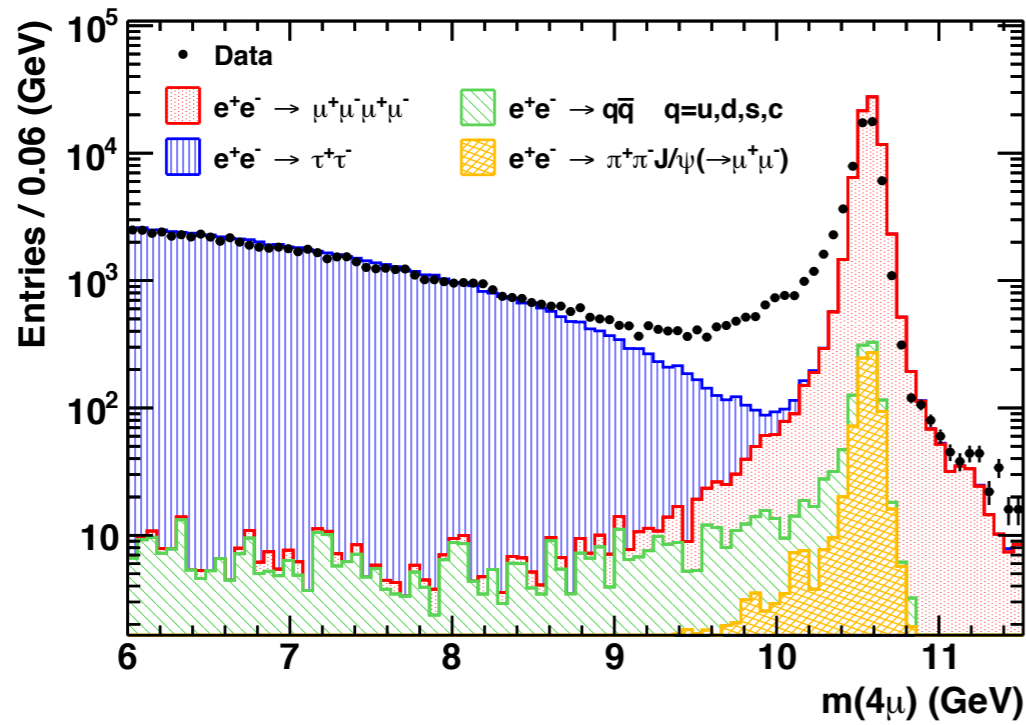


S. Mishra et al. (CCFR Collaboration), Phys.Rev.Lett. 66, 3117 (1991)

Altmanshoffer, Pospelov, Gori, Yavin 1406.2332



# Constraints: BABAR Experiment



Search for 4 muon excess  
Excludes  $g-2$  for  $m > 200$  MeV