Theory working group

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UNIVERSITY of VIRGINIA

Theory working group

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  - Yuichi Uesaka (Kyushu Sangyo University)
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Comments, questions, and members welcome!

Muon decay in orbit (DIO)

- Near endpoint, improved expression

$$\frac{1}{\Gamma_{\text{free}\,\mu}} \frac{\mathrm{d}\Gamma_{\text{DIO}}}{\mathrm{d}E_e}\Big|_{E_e \sim E'_{\text{end}}} = B\, E'^5_{\text{end}} \left(1 - \frac{E_e}{E'_{\text{end}}}\right)^{5.023}$$

[Czarnecki, Tormo, Marciano, 1106.4756; Szafron & Czarnecki, 1505.05237]

with corrected endpoint

$$E'_{\text{end}} \equiv E_{\text{end}} + \frac{\alpha}{\pi} (Z\alpha)^2\, m_\mu \left(\frac{11}{9} - \frac{2}{3} \log\left[\frac{2 m_\mu Z\alpha}{m_e}\right]\right)$$

[Szafron & Czarnecki, 1608.05447]

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[Szafron & Czarnecki, 1608.05447]

- Coefficient B and endpoint E_{end} obtained by solving Dirac equation for muon and electron in electric field of nucleus.

Dirac equation

- Assuming *spherical symmetry*, solve

$$\frac{d}{dr}\begin{pmatrix} g^{\kappa}(r) \\ f^{\kappa}(r) \end{pmatrix} = \begin{pmatrix} -(1+\kappa)/r & E_e + m_e + e\phi(r) \\ -E_e + m_e - e\phi(r) & -(1-\kappa)/r \end{pmatrix} \begin{pmatrix} g^{\kappa}(r) \\ f^{\kappa}(r) \end{pmatrix}$$

  with nuclear Coulomb potential

$$\phi(r) = \int_0^{\infty} dr' r'^2 \left[\theta(r-r')\frac{1}{r} + \theta(r'-r)\frac{1}{r'}\right]\rho(r')$$

  for a given nuclear charge distribution ρ(r).

- Solved numerically by Yuichi.
- Matches known results (Al, Ti, Cu, Se, Sb, Au). [Czarnecki, Tormo, Marciano, '11]

How well do we know these charge distributions?

Charge distributions

Via electron scattering or muonic atom spectroscopy.

1) Charge radii: take Fermi function

$$\rho \propto \left[1 + \exp\left(\frac{r-c}{a}\right)\right]^{-1}$$

with fixed a = 0.5 fm and fit c (or radius) to data.
Up-to-date values for all isotopes of interest. [Angeli & Marinova '13]
Ignores all substructure in ρ.

2) Substructure: take ρ ansatz with more parameters and fit to data.
Probably more realistic shapes.
Not clear which parametrization is best.
Does not exist for all isotopes. [de Vries et al. '87, Fricke et al. '95]

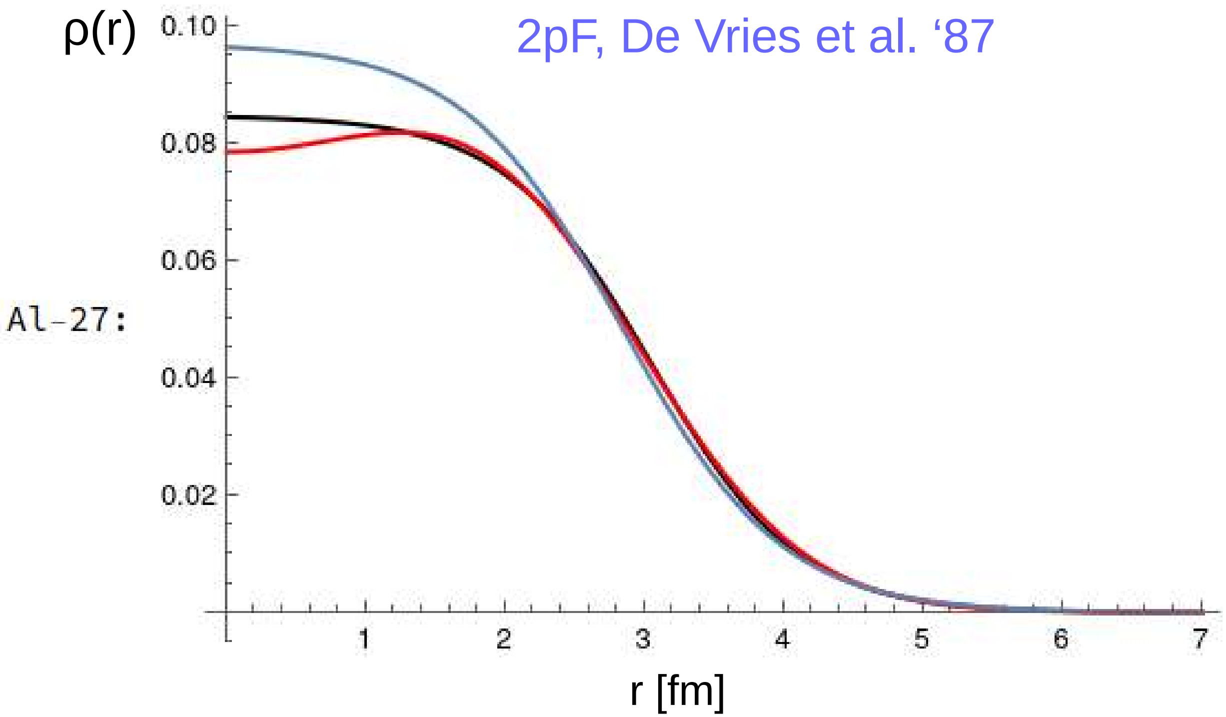
Charge distributions

DIO

Fermi, Angeli et al. '13

Fourier-Bessel, De Vries et al. '87

2pF, De Vries et al. '87



$E_{\rm end} = 104.973\,\mathrm{MeV}$

$(E'_{\rm end} = 104.970\,\mathrm{MeV})$

$B = 8.81 \times 10^{-17}\,\mathrm{MeV}^{-6}$

$B = 8.91 \times 10^{-17}\,\mathrm{MeV}^{-6}$

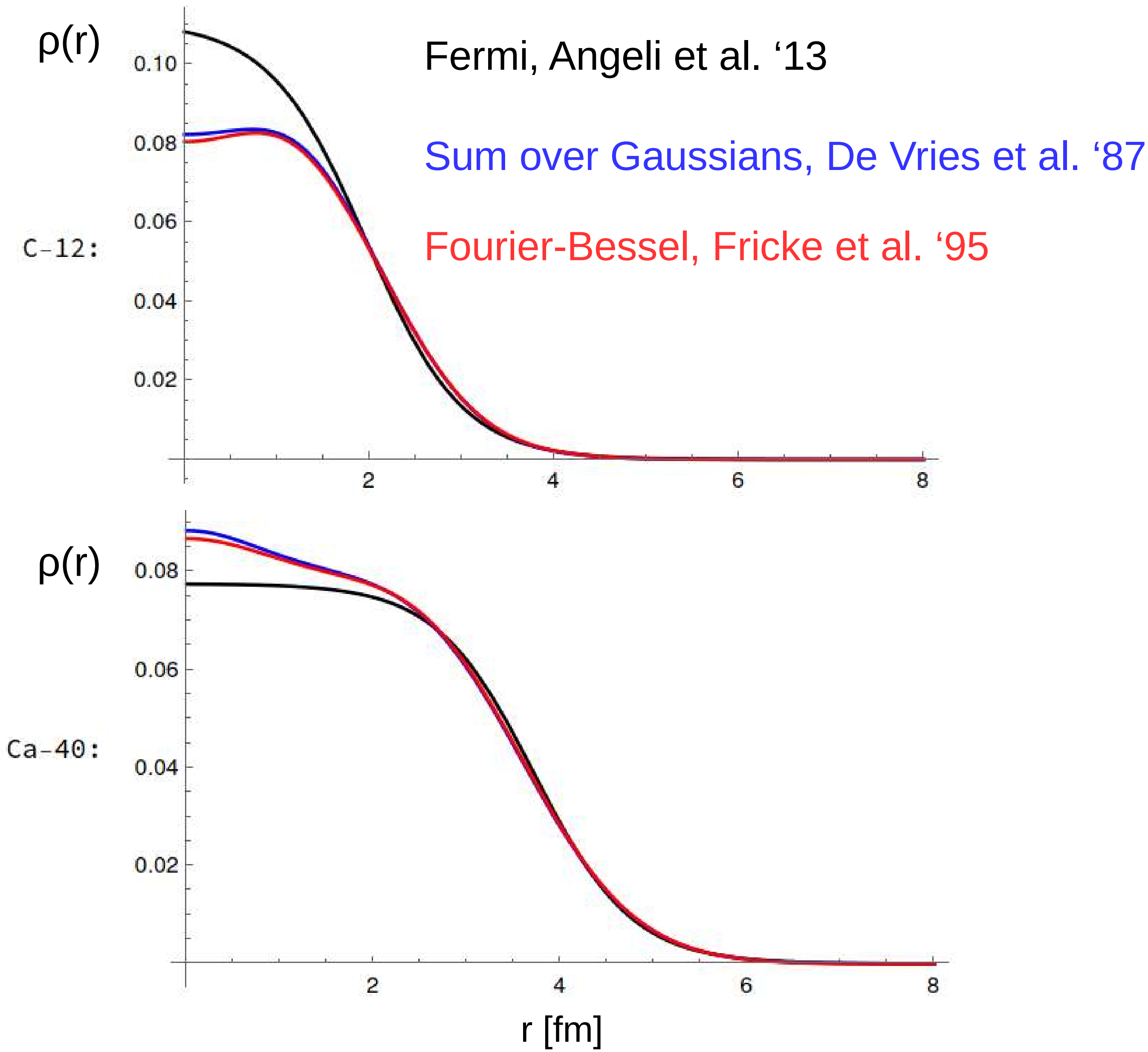
$B = 8.96 \times 10^{-17}\,\mathrm{MeV}^{-6}$

$B = 8.98 \times 10^{-17}\,\mathrm{MeV}^{-6}$

[Czarnecki et al., 1106.4756]

< 2% difference

Charge distributions DIO



$E_{\text{end}} = 105.059\,\text{MeV}$

$B = 3.55 \times 10^{-18}\,\text{MeV}^{-6}$

$B = 3.55 \times 10^{-18}\,\text{MeV}^{-6}$

$B = 3.53 \times 10^{-18}\,\text{MeV}^{-6}$

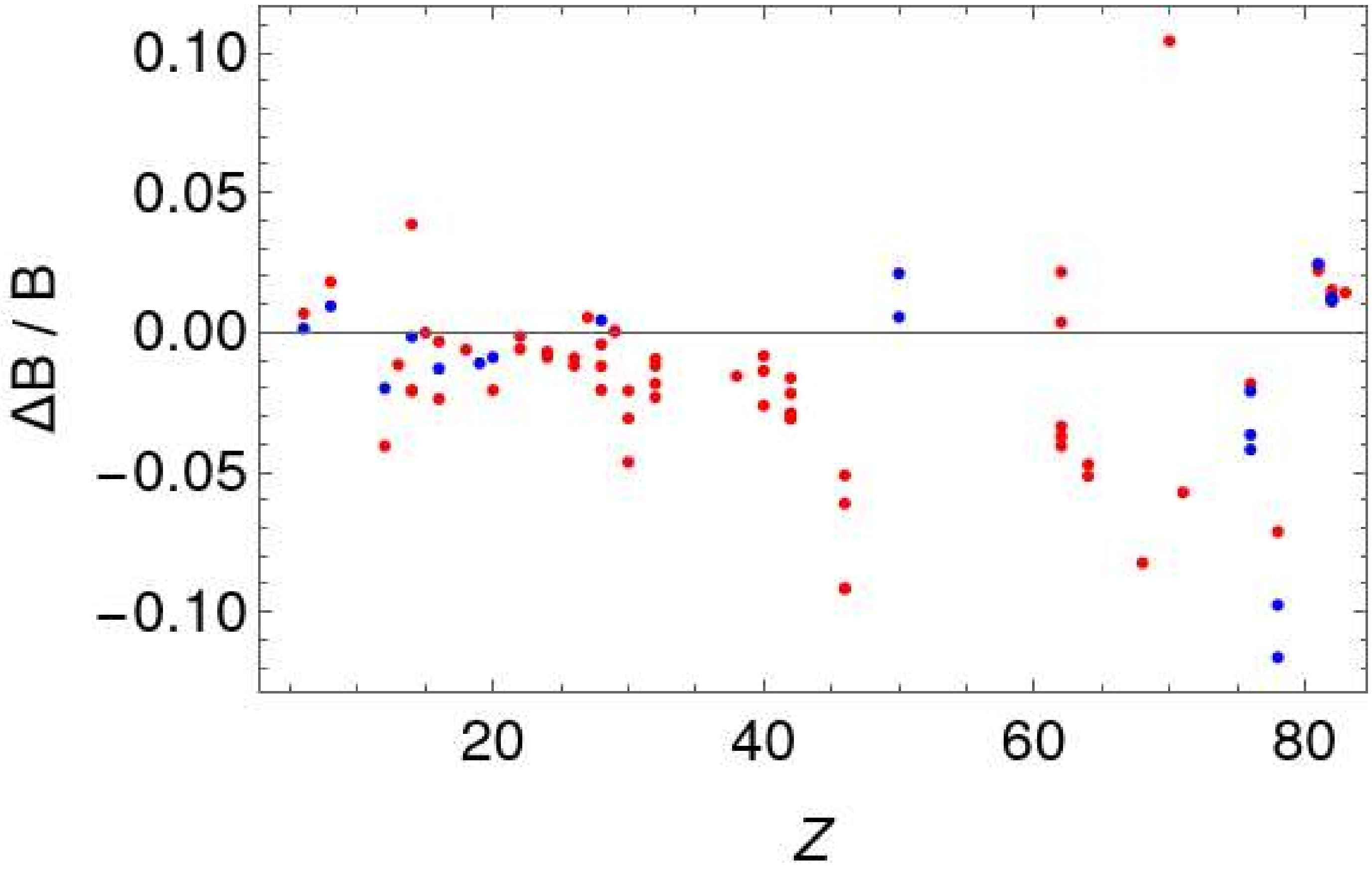
$E_{\text{end}} = 104.450\,\text{MeV}$

$B = 4.01 \times 10^{-16}\,\text{MeV}^{-6}$

$B = 4.04 \times 10^{-16}\,\text{MeV}^{-6}$

$B = 4.09 \times 10^{-16}\,\text{MeV}^{-6}$

Parametrization dependence



Sum over Gaussians

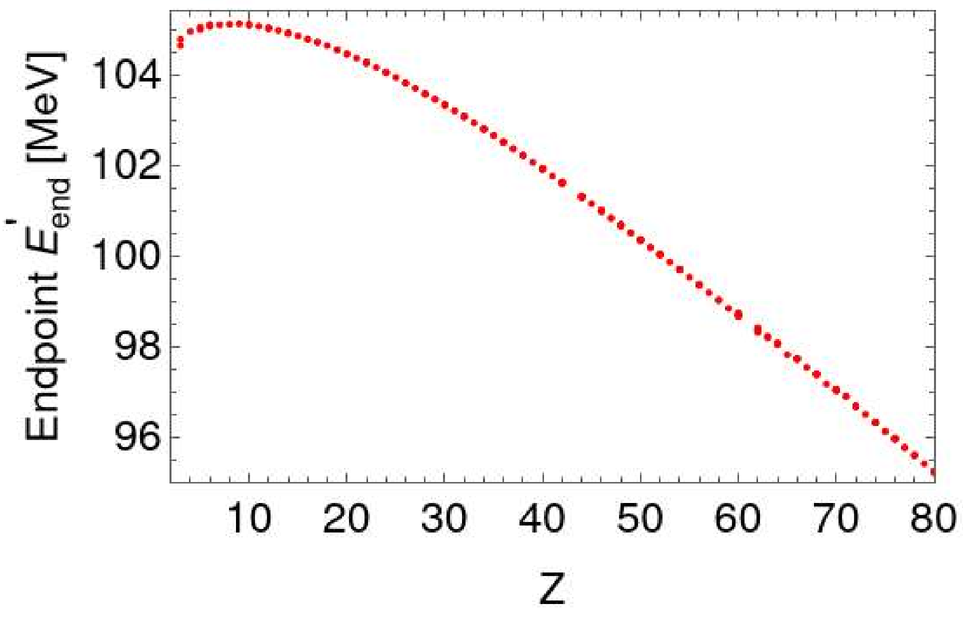
Fourier-Bessel

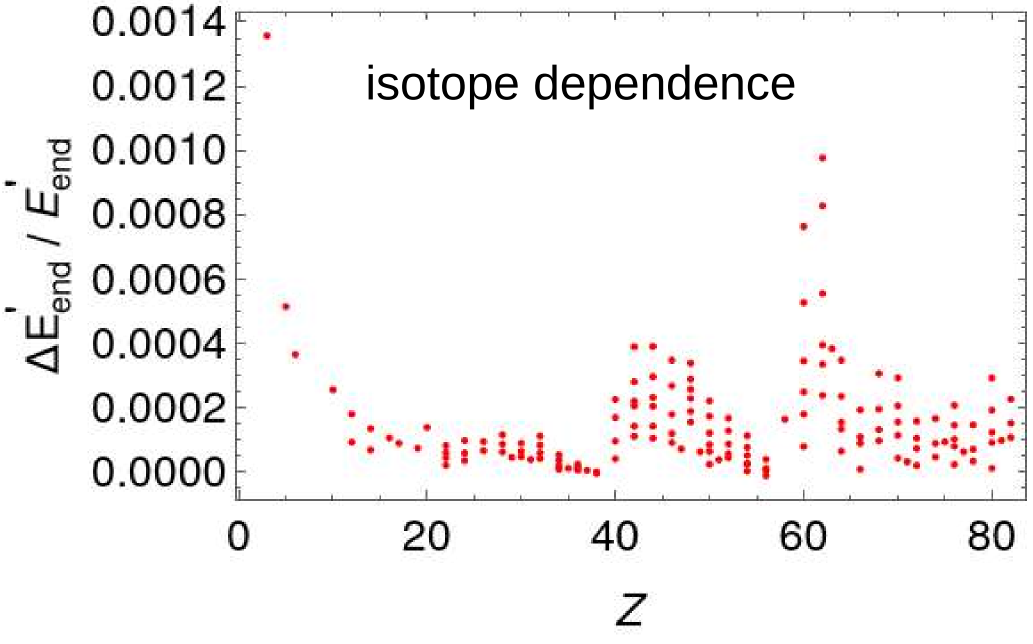
relative to simple
Fermi function

- Parametrization dependence of B: ~10%, smaller for Z < 30.
- Fermi function surprisingly good!

Endpoint energy

- Using Fermi function for all 236 isotopes:



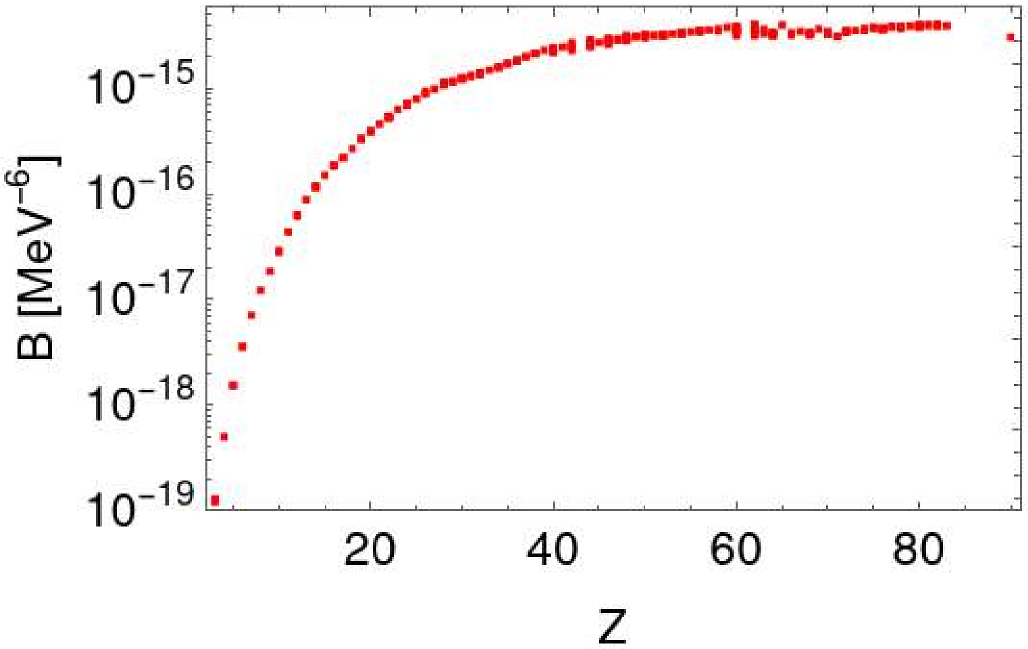


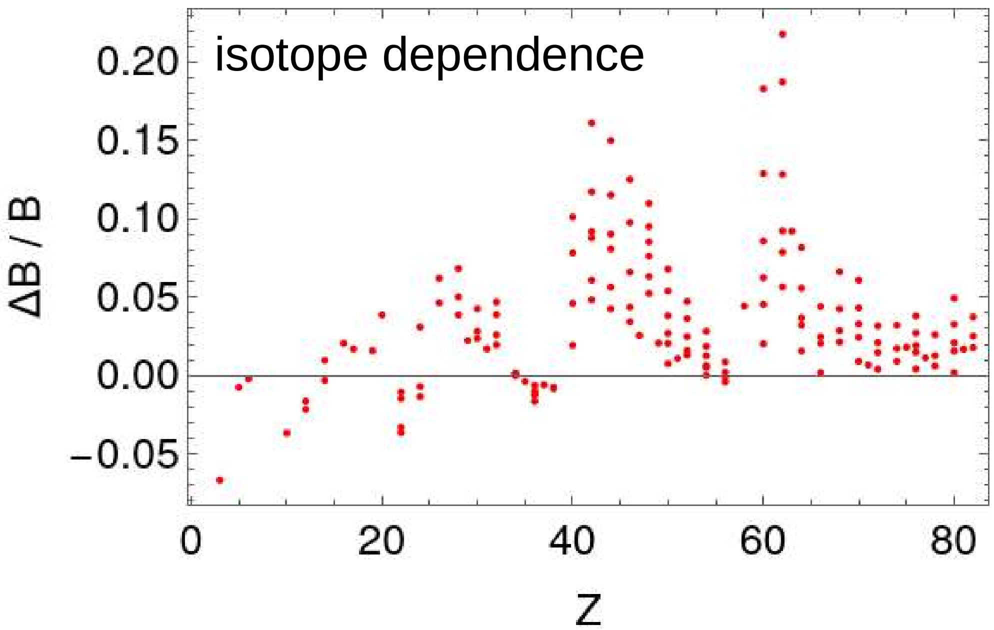
- Isotope dependence small, e.g. ΔE_{end}(Li) = 0.14 MeV, but often bigger than parametrization dependence.
- At large Z:

$$E'_{\mathrm{end}} = 108.75\,\mathrm{MeV} - 0.17\,\mathrm{MeV}\,Z$$

B coefficient

- Using Fermi function for all 236 isotopes:



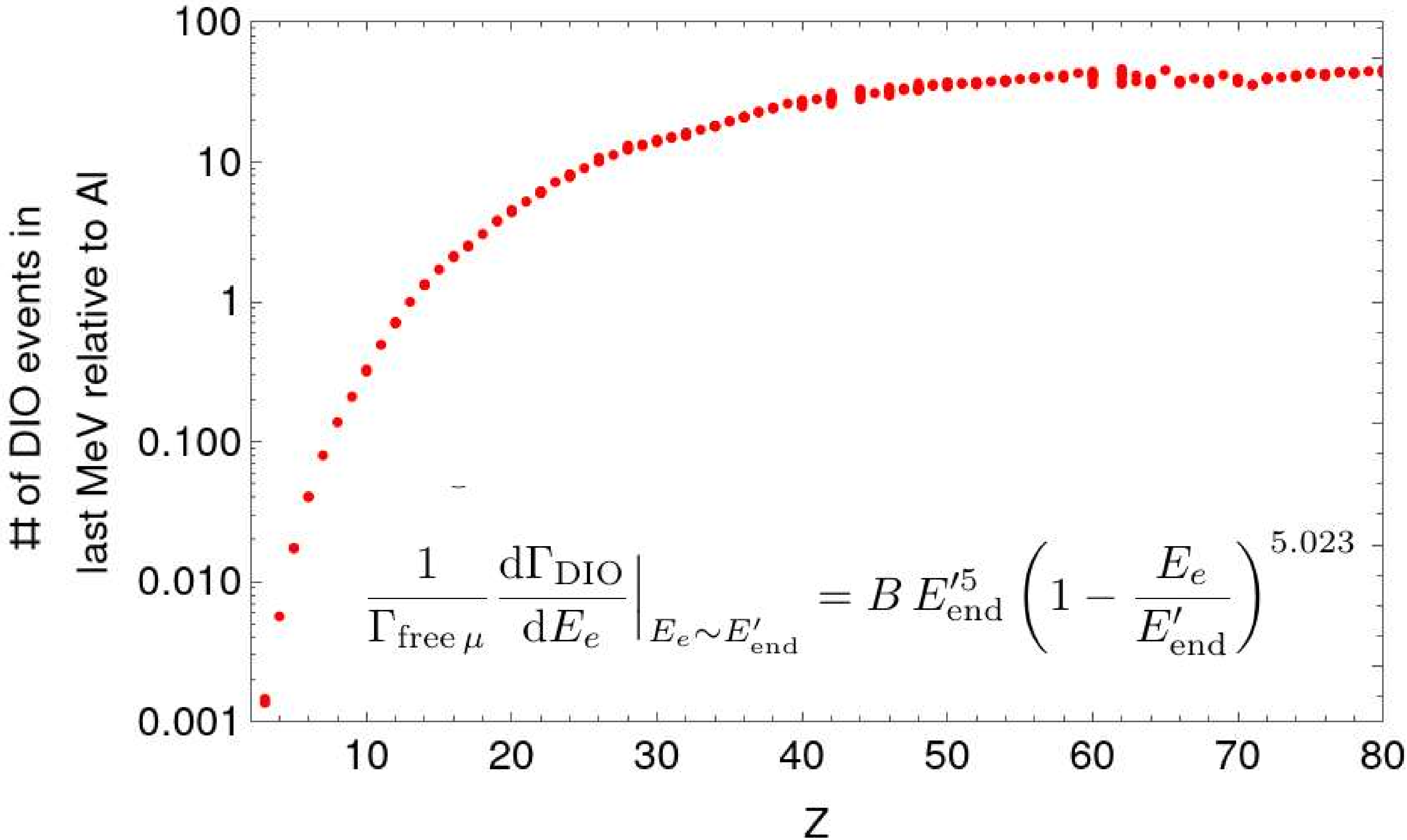


- Isotope dependence small, e.g. ΔB/B (Li) = 7%, but often bigger than parametrization dependence.
- At large Z:

$$B = 3 - 4 \times 10^{-15}\,\text{MeV}^{-6}$$

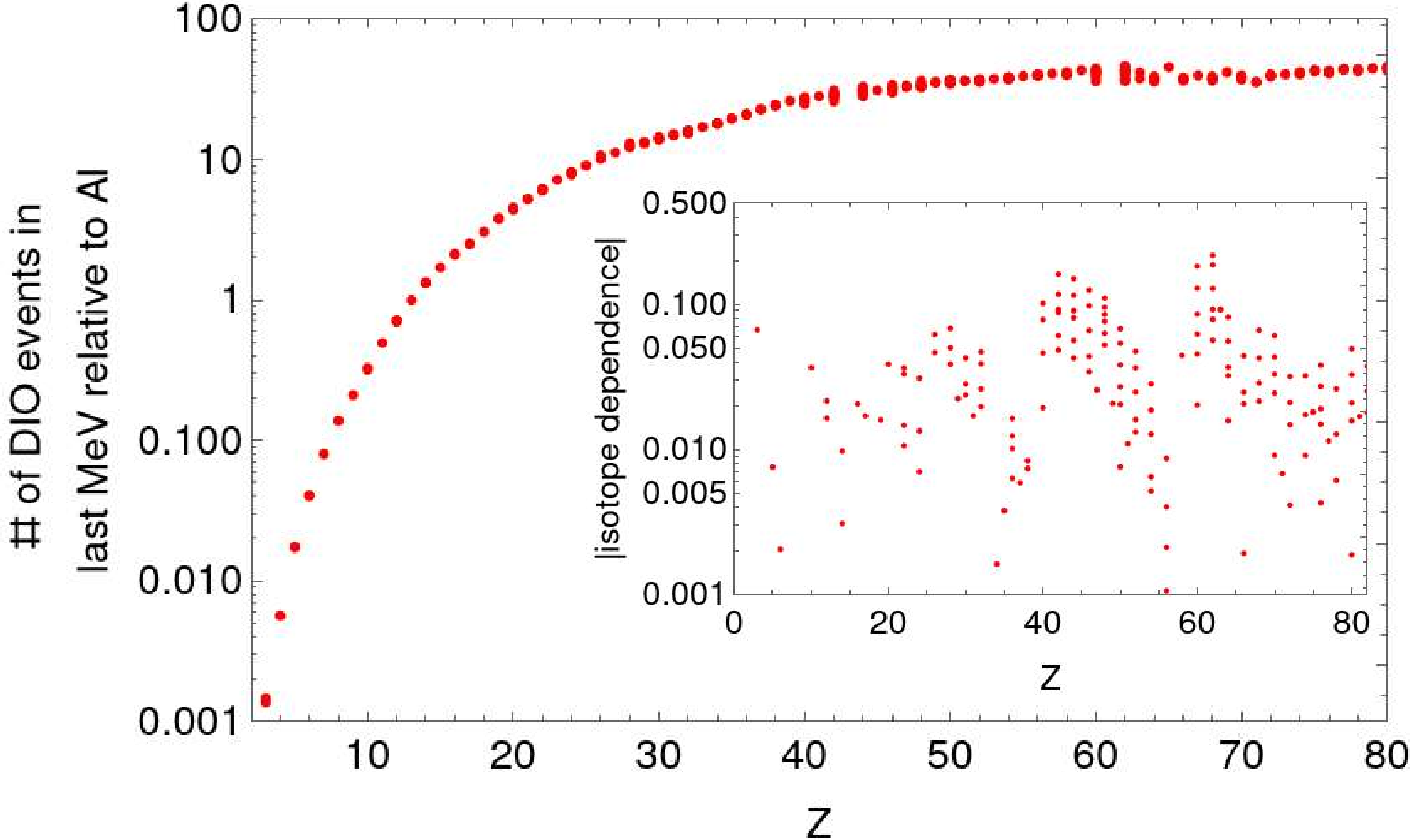
#DIO events

- Using Fermi function for all 236 isotopes:



#DIO events

- Using Fermi function for all 236 isotopes:



Summary

- Calculated DIO spectrum near endpoint for 236 nuclei.
- Uncertainty due to charge distribution below ~10%, often smaller than isotope dependence.
- Good enough for global overview.
- Next steps:
  - Finish up DIO analysis, add last parametrizations.
  - Move on to LFV overlap integrals, which also require much more uncertain *neutron* density.
  - Maybe improve precision for select isotopes (full DIO spectrum, higher orders, deformed nucleus,...)