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Lattice Renormalization of Quantum Simulations

arXiv:2107.01166

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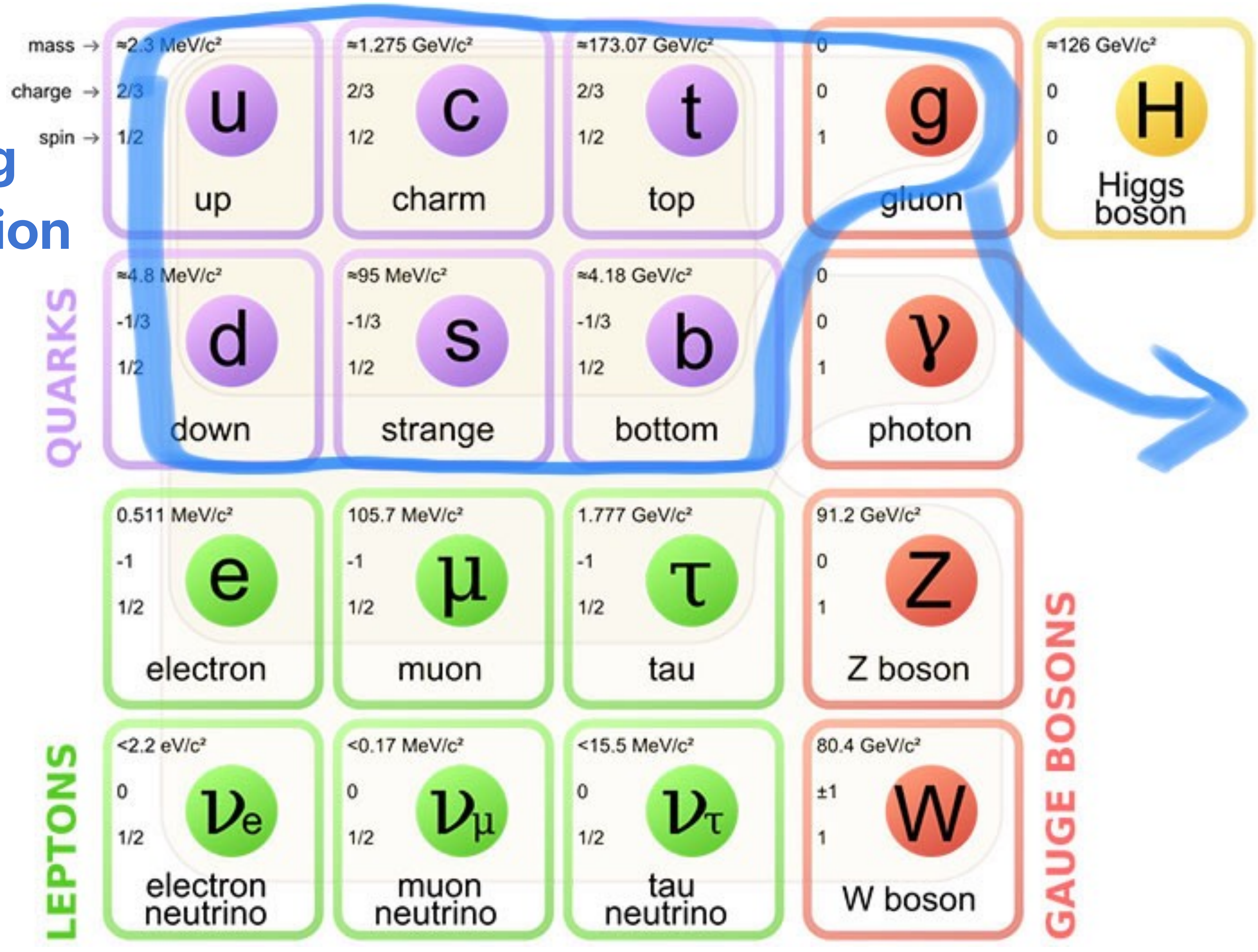
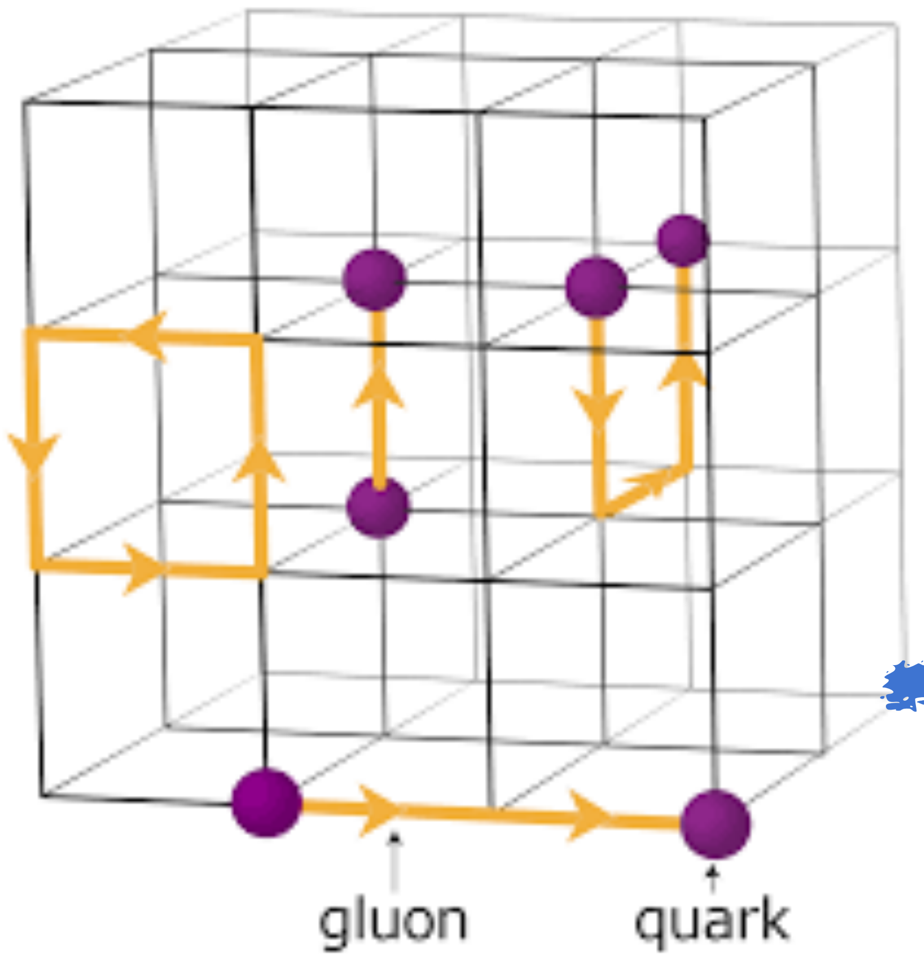
Lattice simulations:
the non-perturbative tool for Quantum Field Theories (QFT)

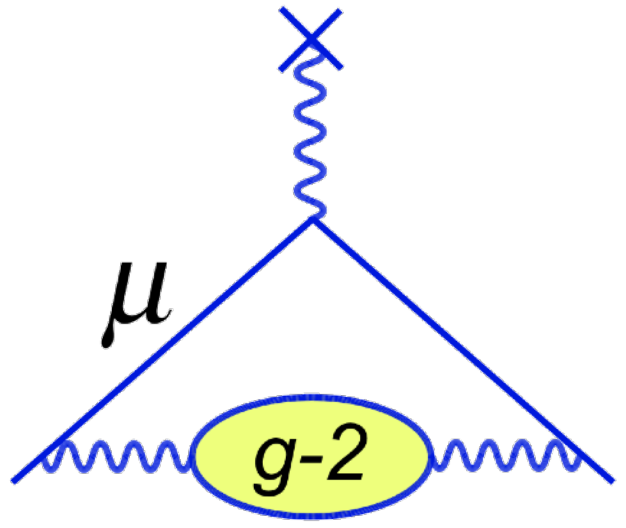
Strong interaction



The standard model of particle physics



Lattice QCD



Neutron stars

...

Lattice QFT on classical computers: the method, the strengths and the weaknesses

- Imaginary time QFT (or equilibrium thermodynamics) : $e^{i\hat{H}t} \to e^{-\beta\hat{H}}$

$$Z = \mathrm{Tr} e^{-\beta\hat{H}} = \sum_{G_{luons}, Q_{uarks}} e^{-S(G_{luons}, Q_{uarks})} \Longleftrightarrow \text{Monte Carlo simulations}$$

- Difficult problems for classical lattice QFT:
  - Finite fermion density (S is complex)
  - Viscosity (needs real time evolution)
  - ...

Easier on a (future) QC!

$$e^{itH_{QFT}} \xrightarrow[\text{Regularization}]{} e^{itH_{QC}}$$

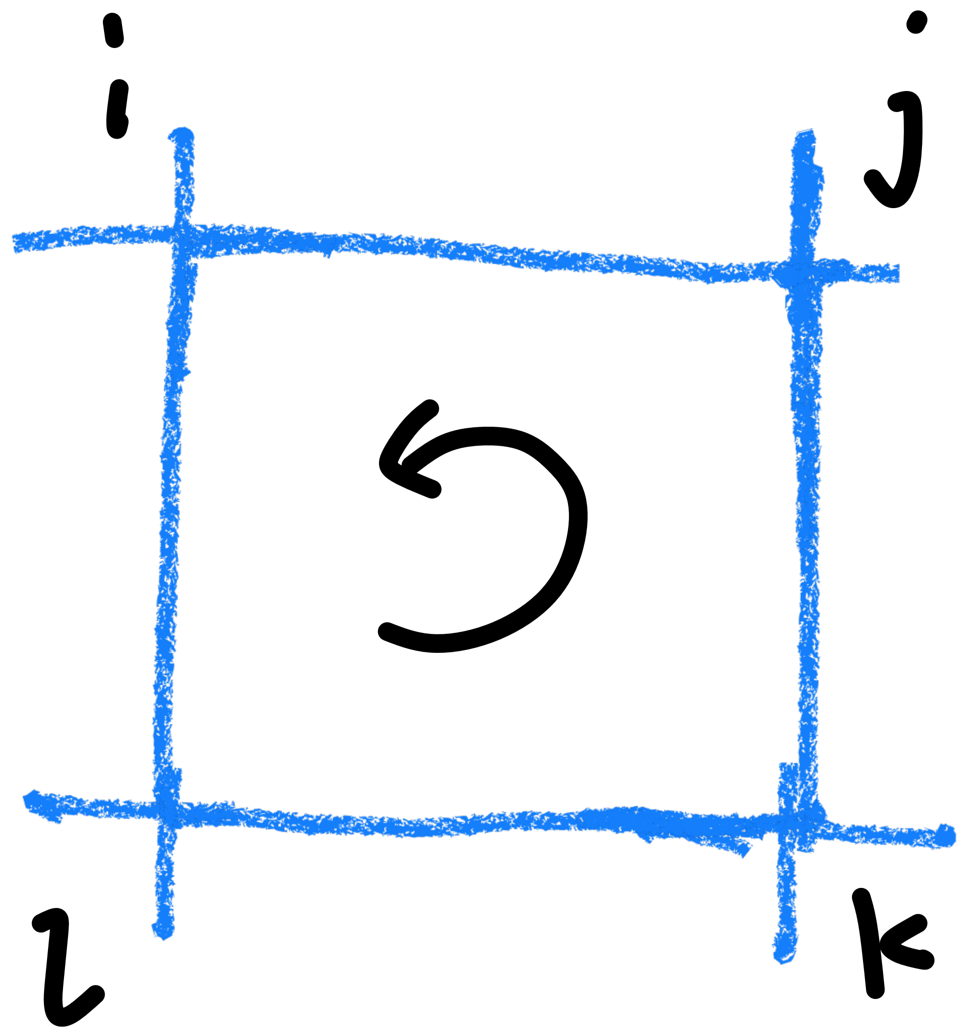
Simulating a lattice gauge theory on a QC

- Kogut-Susskind Hamiltonian

EM field: energy density $= \frac{1}{2}(E^2 + B^2)$

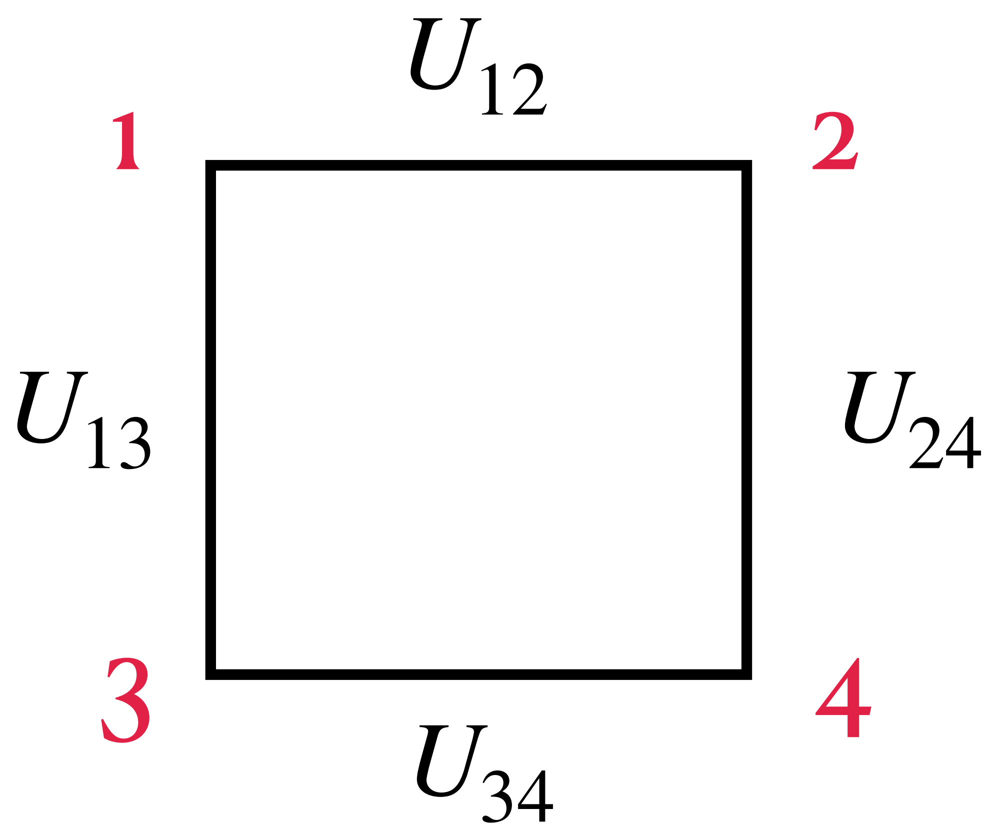
$$H_{KS}(a) = \frac{c(a)}{a}\left(g_H^2(a)\sum_{\{ij\}} l_{ij}^2 - \frac{1}{g_H^2(a)}\sum_s \mathrm{Re}\,\mathrm{Tr}\,U_s\right) \equiv H_K + H_V$$

Electric   Magnetic



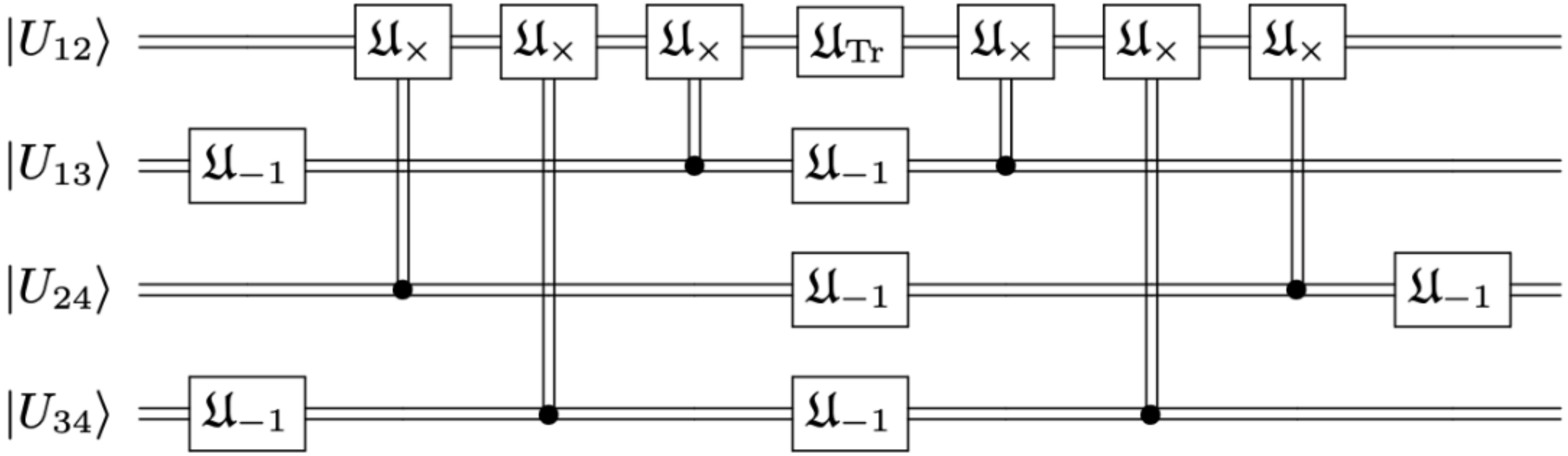
- Digitization:
  - A high- l^2 cut-off;
  - A discrete subgroup
  - …

A quantum circuit for simulating lattice gauge theories

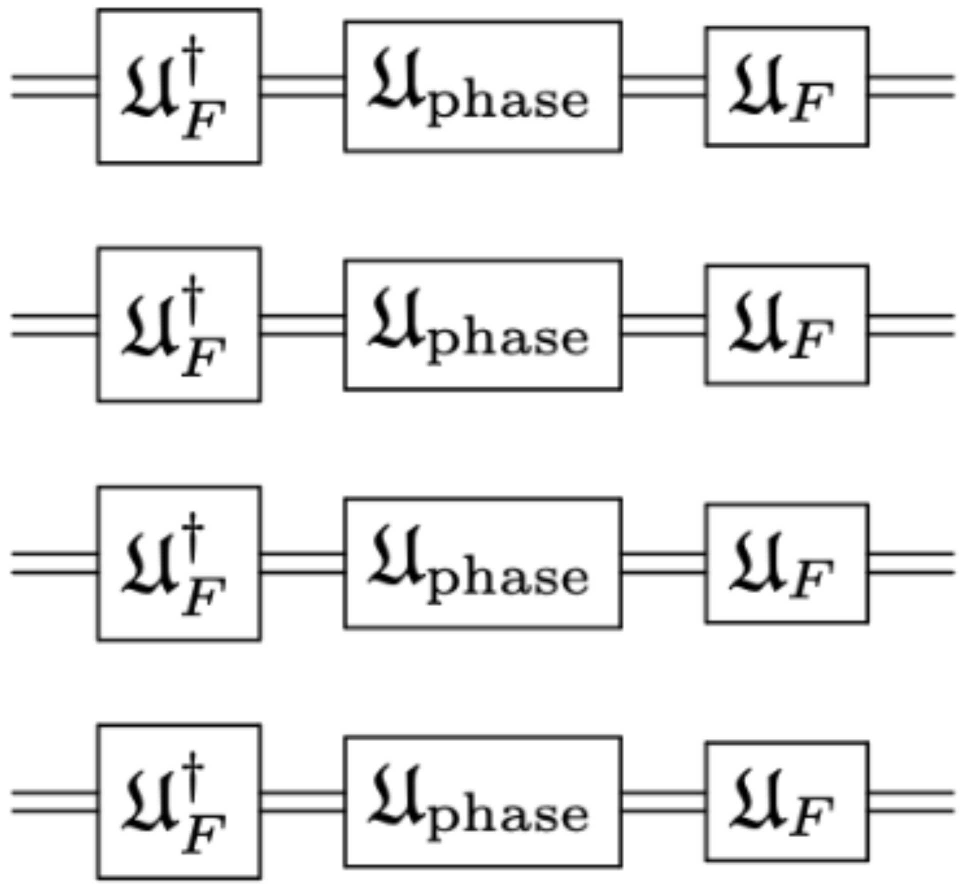


$$U(\delta t) = e^{i(H_K+H_V)a_t} \approx e^{iH_V a_t} e^{iH_K a_t}$$

Trotter approximation



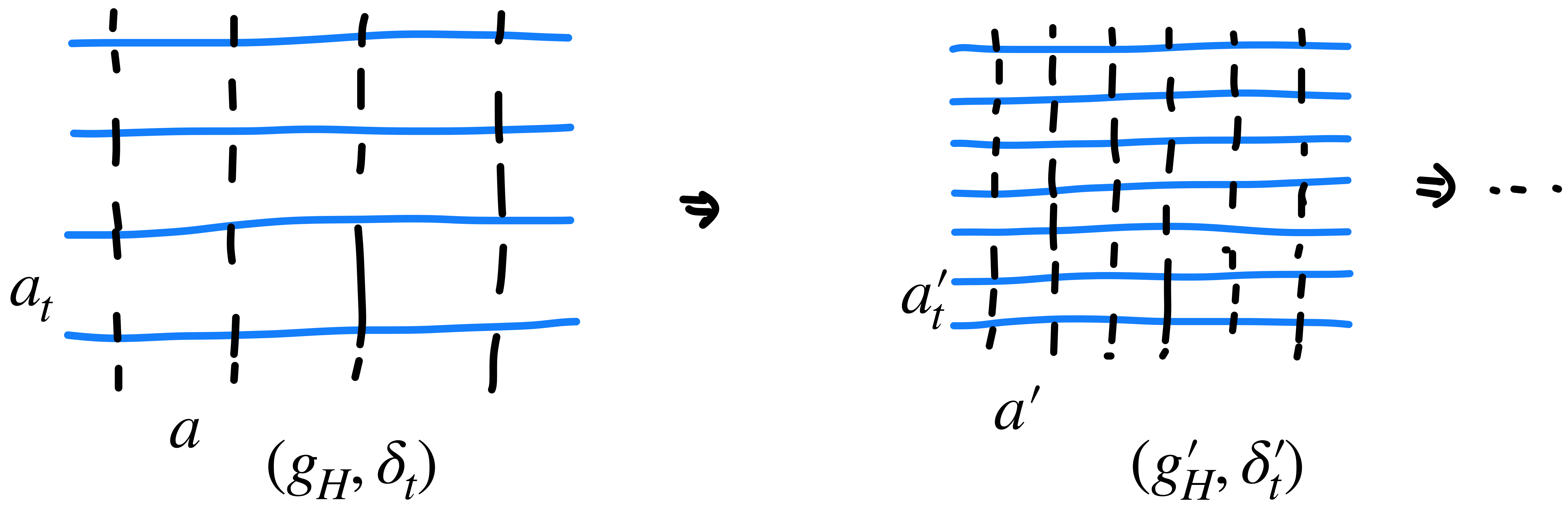
$$e^{iH_V a_t}$$



$$e^{iH_K a_t}$$

H. Lamm, et al, arXiv:1903.08807

The need of scale-setting (renormalization)



Continuum limit

$a \to 0, \quad a_t \to 0$

- Scale-setting: determine physical separations a_t, a from input parameters g_H, δ_t
- Computation resources $\propto \left(\frac{L}{a(g_H, \delta_t)}\right)^d \frac{T}{a_t(g_H, \delta_t)}$

Physics exists at the continuum limit

- Example: particle mass

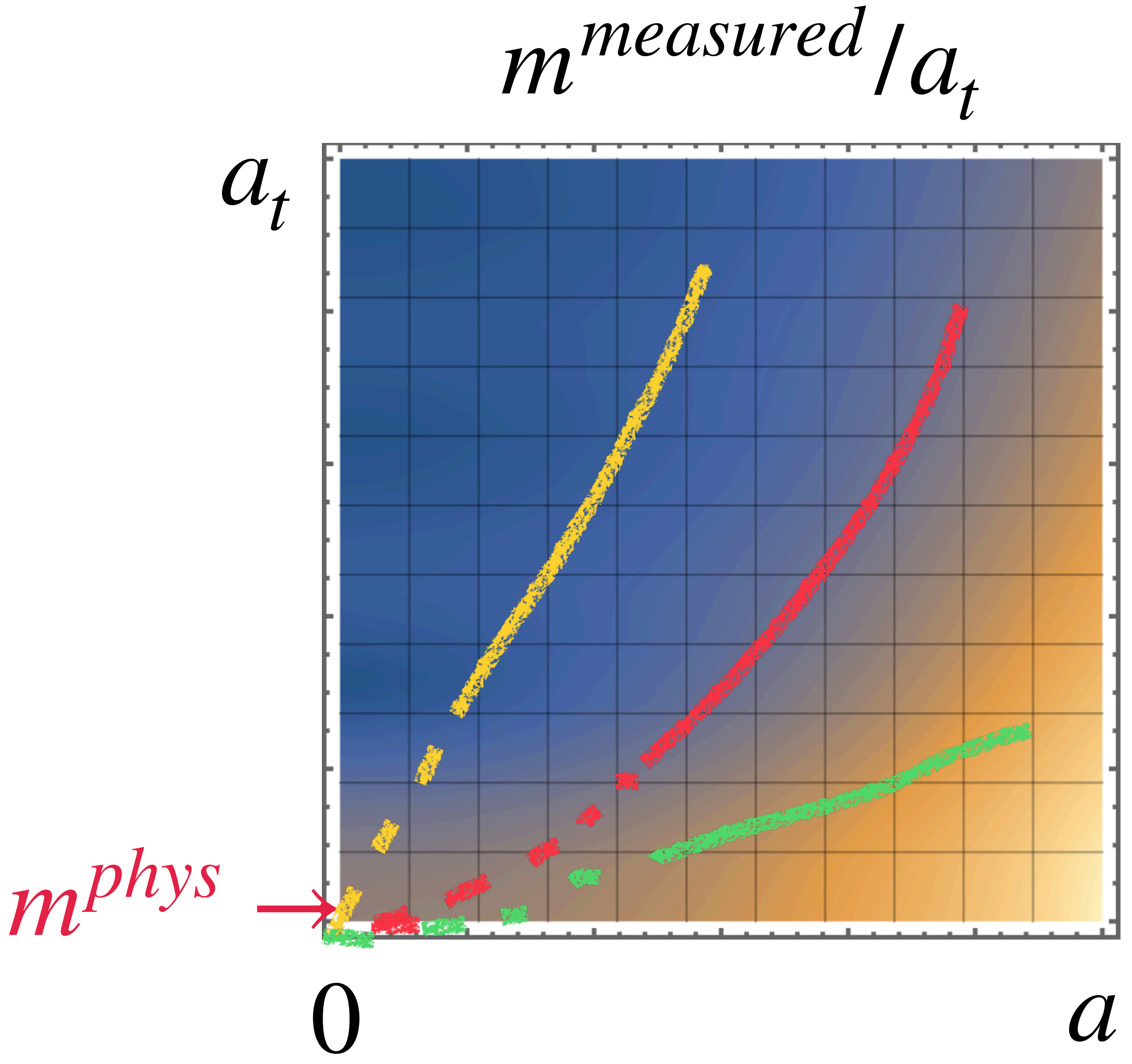
$$m_i^{measured}(g_H, \delta_t) = m_i^{phys} a_t(g_H, \delta_t) + \mathcal{O}(a^2(g_H, \delta_t), a_t^2(g_H, \delta_t))$$

Lattice artifacts

- Choose scale-setting observables $m_{t,s}$; let them **define** the scales:

$$m_{t,s}^{phys} a_{t,s}(g_H, \delta_t) \equiv m_{t,s}^{measured}(g_H, \delta_t)$$

$a_{t,s}(g_H, \delta_t)$ absorbs the lattice artifacts of $m_{t,s}^{measured}$



Computational resources depend on the path to the continuum limit!

Trotter errors as renormalization effects

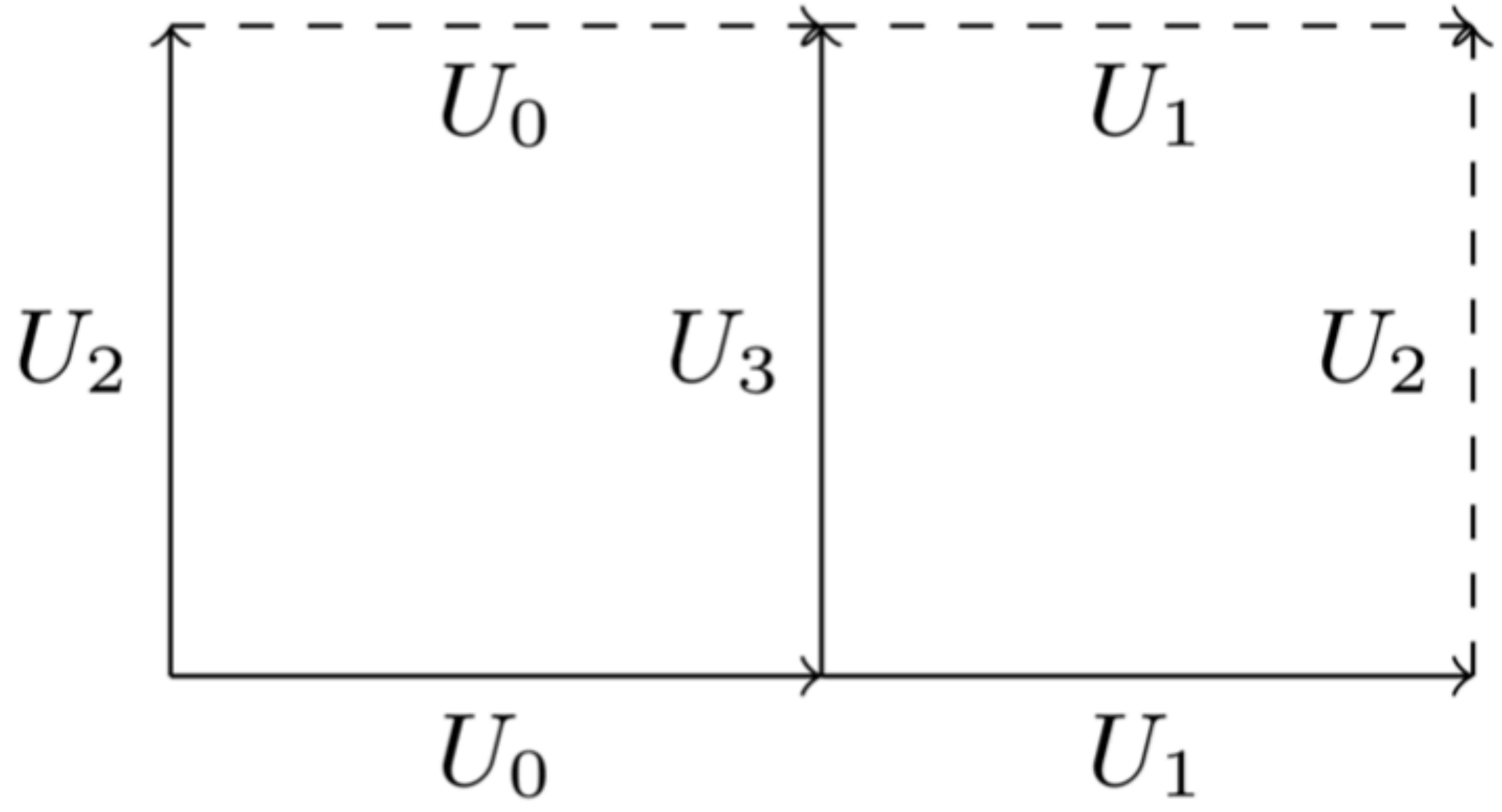
- Second-order Trotterization: $e^{it(H_K+H_V)} \approx (e^{i\frac{t}{2N}H_V}e^{i\frac{t}{N}H_K}e^{i\frac{t}{2N}H_V})^N \equiv e^{it\tilde{H}}$
- Trotter errors come from $[H_K, H_V] \neq 0$

$$e^{ia_tH_V/2}e^{ia_tH_K}e^{ia_tH_V/2} = e^{ia_t(H_K+H_V+\frac{a_t^2}{24}[H_V,[H_V,H_K]]+\frac{a_t^2}{12}[H_K,[H_V,H_K]]+\ldots)}$$

- As renormalization effects: a_t depends non-trivially on δ_t.

($\delta_t \equiv \dfrac{a_t c}{a}$, a dimensionless parameter controlling the size of a step in the circuit)

A numerical study : D4 gauge theory



$$H = \mathrm{Re\,Tr}\left[U_2^\dagger(t)U_0^\dagger(t)U_3(t)U_0(t)\right]$$
$$+\mathrm{Re\,Tr}\left[U_3^\dagger(t)U_1^\dagger(t)U_2(t)U_1(t)\right]$$
$$-\sum_{i=0..3}\log T_K^{(1)}(i)$$

H. Lamm, et al, arXiv:1903.08807

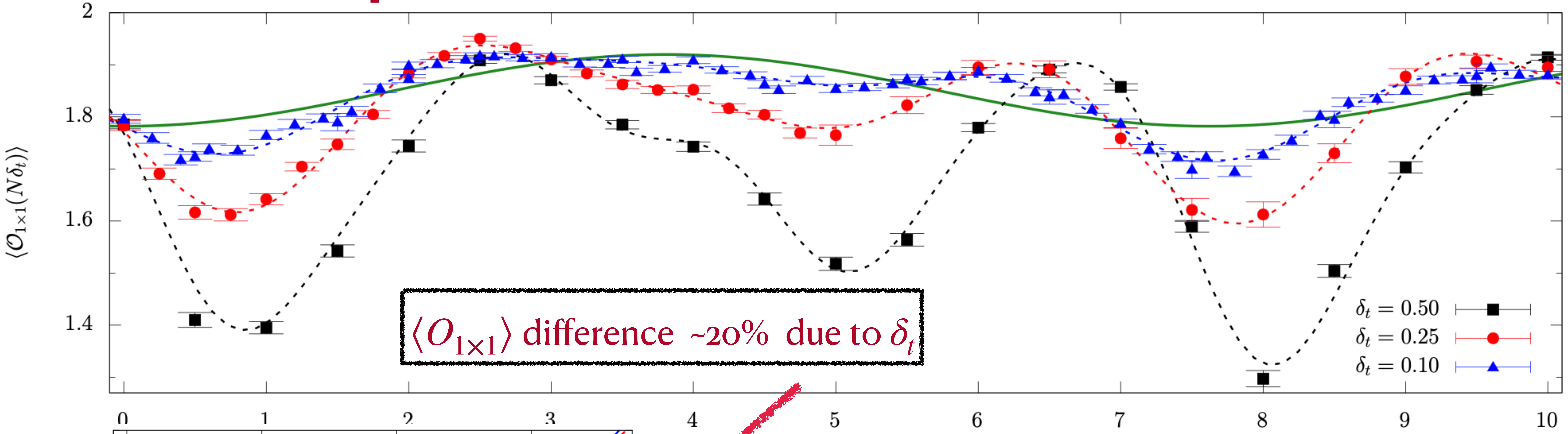
17 qubits:
- 12 for physical degrees of freedom
- 3 for an ancillary group register
- 2 ancillary qubits

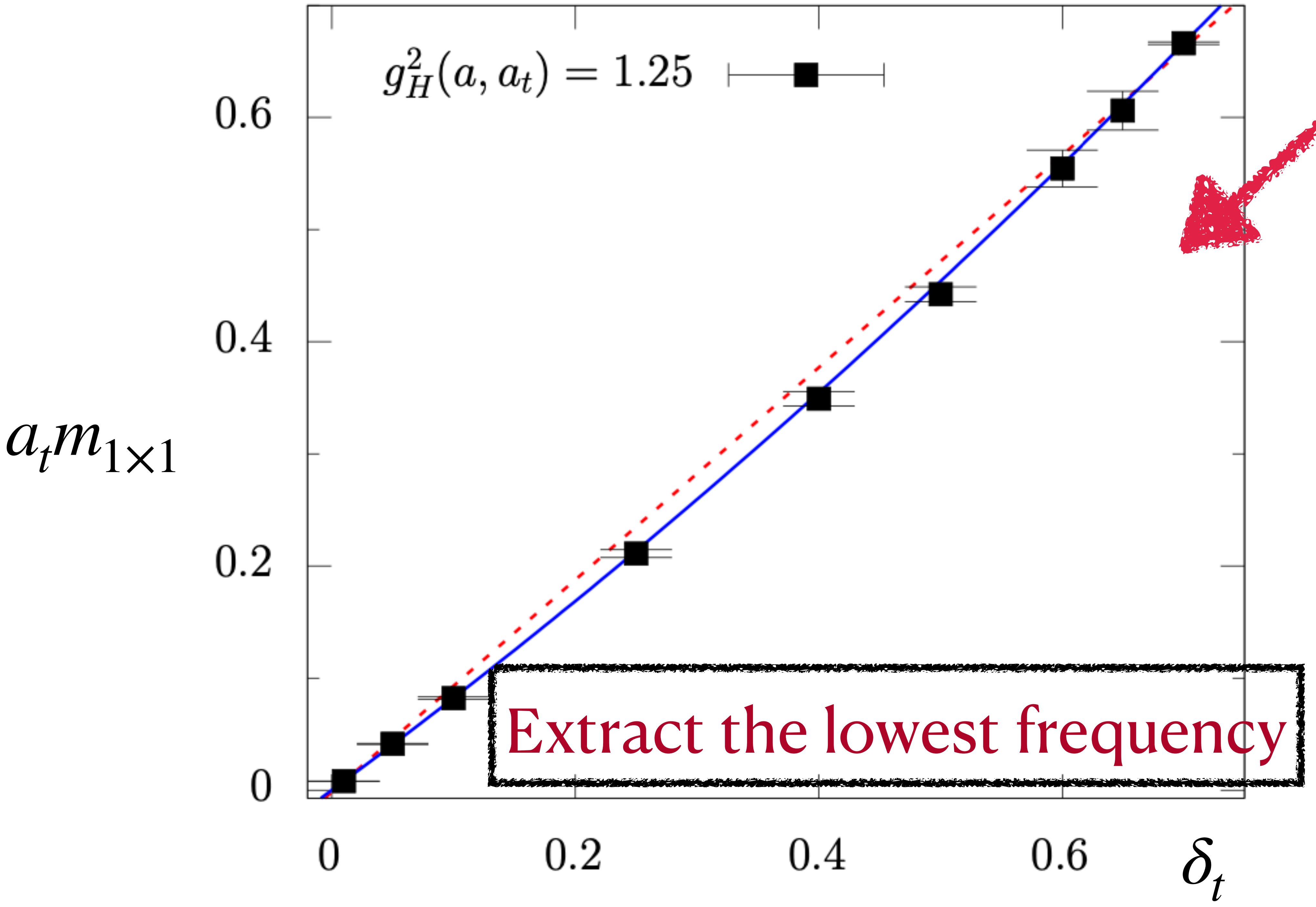
Qiskit@IBM:
Simulating a noiseless quantum circuit on a classical computer.

Initial state: $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\psi_0\rangle + |\psi_2\rangle)$

Observable: $O_{1\times 1} = \mathrm{Re\ Tr}(U_0 U_3 U_0^\dagger U_2^\dagger)$

Trotter & temporal renormalization





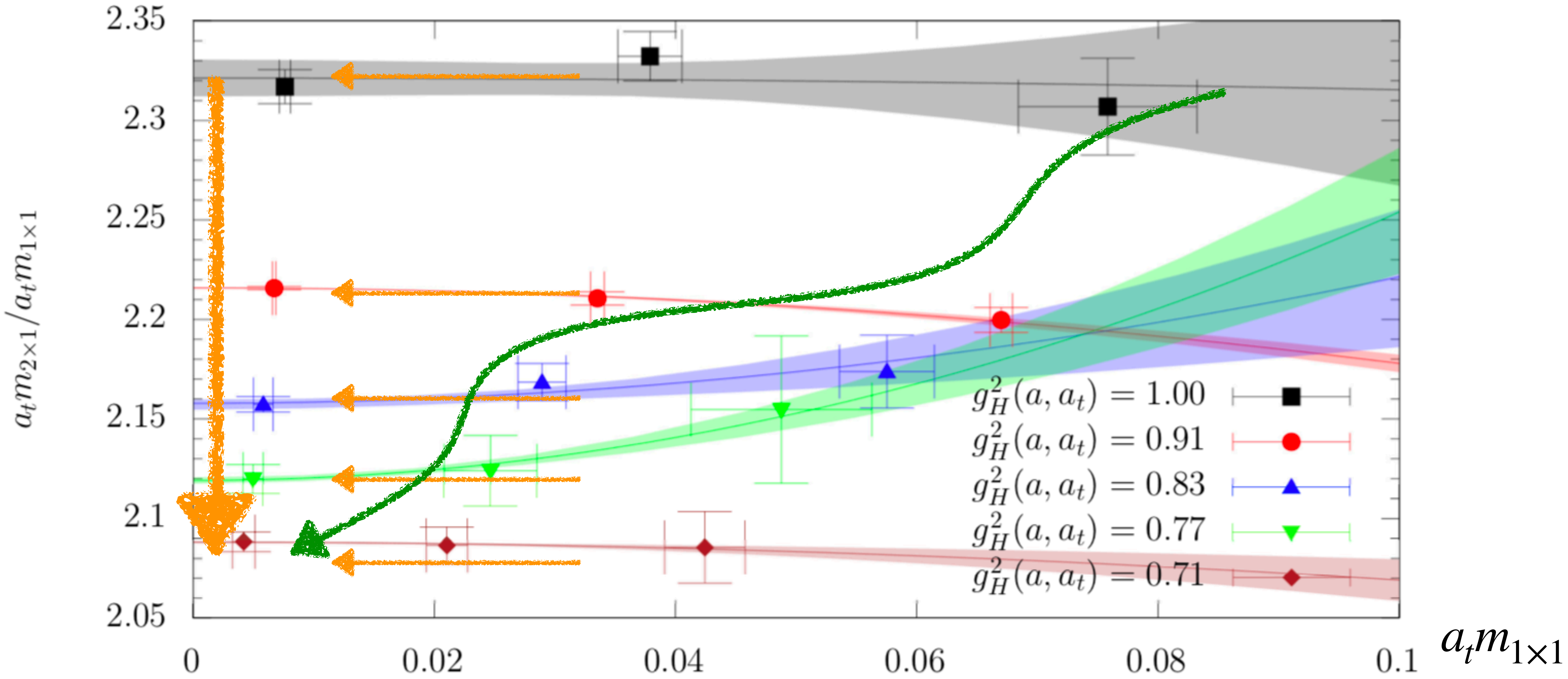
Fit w/. $c_0 + \sum_{i\neq j}^{N\delta_t} \left[c_{i,j} \cos\left(a_t\left(E_j - E_i\right)k\right) + s_{i,j} \sin\left(a_t\left(E_j - E_i\right)k\right)\right]$

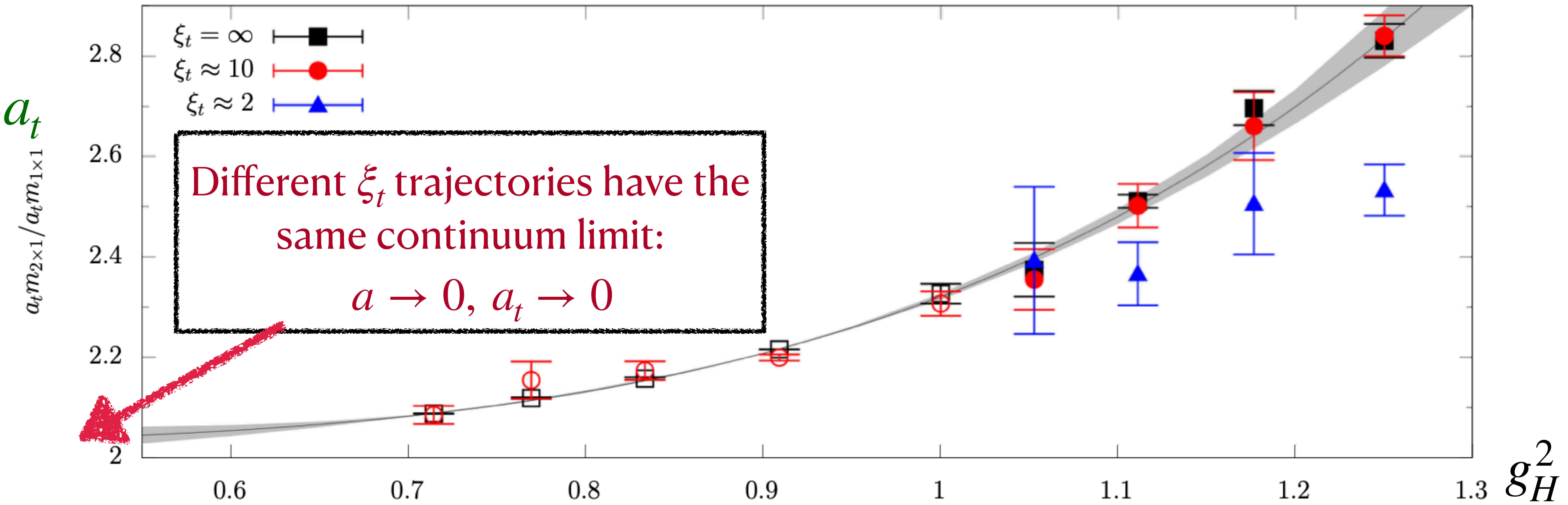
- $\dfrac{a_t m_{1\times1}}{\delta_t}$ is not a constant—δ_t should be renormalized
- This deviation from linear relation is not large— the effect of δ_t renormalization is smaller than state-mixing.

Marcela Carena, Henry Lamm, Ying-Ying Li, and W.L. arXiv:2107.01166

Trajectories to the continuum limit

- *Hamiltonian limit $\delta_t \to 0$ at finite a is expensive and unnecessary*
- *A trajectory to the continuum limit along a finite anisotropy* ☑
- *Requires the knowledge of a, a_t*





Connecting Euclidean and Minkowski scale-settings

- Scale-setting is easier in classical simulations than on a QC.

Can we do scale setting on a classical computer, and use it for a QC?

- Yes— we proposed connecting them with analytic continuation

$$a^E_{t,s}(g_H, \delta_\tau \to i\delta_t) \to a^M_{t,s}(g_H, \delta_t)$$

- The upper bounds of errors for a^M are derived.

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Summary

- QCs are important for future non-perturbative QFT.
- The Hamiltonian formalism of lattice gauge theories and a quantum circuit to implement it.
- Renormalization (scale-setting) is crucial to obtaining any physical results from QC simulation of QFT.
- We demonstrated trotter errors as renormalization effects.
- We demonstrated that that Euclidean (classical) simulations can help with scale-setting on a QC and thus save QC resources.