Fitting NOvA cross-section parameters with Markov Chain Monte Carlo

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Motivation of the MCMC fit

- **NuMI Off-Axis $\nu_e$ Appearance (NOvA) Experiment** observes neutrino interactions to measure oscillation parameters.
- Contains a Near Detector (ND) at Fermilab & Far Detector (FD) 810 km away in Minnesota.
- Primary physics goals: (1) the mass ordering and value of $\Delta m^2_{23}$, (2) CP-violation ($\delta_{CP}$), (3) the mixing angle $\theta_{23}$. 
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- Primary physics goals: (1) the mass ordering and value of $\Delta m^2_{23}$, (2) CP-violation ($\delta_{CP}$), (3) the mixing angle $\theta_{23}$.
- The long-term goal of this MCMC work is to...
  
  ...fit the neutrino oscillation ($\sin^2(\theta_{23}), \Delta m^2_{23}$, and $\delta_{CP}$) and NOvA’s physics model parameters...
  
  ...simultaneously to the NOvA ND + FD data...
  
  ...with MCMC.

- This talk demonstrates the principle of MCMC fitting.
- Fit a set of NOvA physics model parameters to fake data.
What is MCMC?

- MCMC is a Bayesian inference parameter estimation tool.
- In Bayes' theorem, we seek to locate the maximum of the posterior probability distribution.
  - Posterior is complex: $N$ parameters $\rightarrow N$ dimensions ($\dim(\mathbf{q}) = 16$, in this talk).

Bayes Theorem:

$$P(\mathbf{q} | \text{data}) \propto P(\text{data} | \mathbf{q}) \times P(\mathbf{q})$$

posterior $\propto$ likelihood $\times$ prior

2D example
posterior distribution

maximum

$q_1$

probability

$q_2$
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- Explore space by a Markov transition (i.e. a conditional prob.) $- T(\vec{q}^\prime | \vec{q})$.
- Use Hamiltonian Monte Carlo (HMC) to explore posterior Efficiently & Repeatedly:
  - Include conjugate variable: $\vec{p}$, so $(\vec{q}) \rightarrow (\vec{q}, \vec{p})$.
  - $pdf(\vec{q}, \vec{p}) \equiv H(\vec{q}, \vec{p}) = KE(\vec{q}, \vec{p}) + V(\vec{q})$.
- In 2D, ball in a gravitational field analogy:
  - Efficiently: if $V(\vec{q}) \downarrow$, then probability $\uparrow$.
  - Repeatedly: “Drop” ball from different locations $-$ each “drop” is known as a sample.
Divide the ND predictions into topologies

- Split $\nu$ & $\bar{\nu}$ ND interactions into five topologies in two variables: $E_{\text{vis}}$ & $Reco|\vec{q}_3|$. 
  - $E_{\text{vis}}$ — energy deposited in the detector. 
  - Will focus on $\nu$ and $E_{\text{had}}$. 

Muon

Muon + Pion + other

Muon + Proton

Muon + Proton + other

Remaining
Divide the ND predictions into topologies: $
u$ in $Reco\ E_{had,\ vis}$

- GENIE v3 simulation of the five ND topologies in $E^{vis}_{had}$.
  - $Reco\ |\vec{q}_3|$ in backup slide.
  - Next is to create ND fake data in these topologies...
Creating Fake Data

- Identify the parameters from NOvA’s systematic uncertainties to fit.
- Apply a random shift to each parameter (from a Gaussian).
- Apply these shifts to the nominal simulation...
- ...to get a new, distorted distribution: the fake data.
- Repeat for each topology.
In a perfect fit, many MCMC samples will identically match the true pull values...
Fake Data Fitting

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- RES & FSI parameters: a perfect fit – nearly all (dark red) samples overlap the true pull (black).
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- RES & FSI parameters: a perfect fit – nearly all (dark red) samples overlap the true pull (black).
- QE: samples produce a wide spread of pull values (pink).
- MCMC tells us about parameters that are not well constrained.
Fitted ND topological distributions:

\( \nu \) in \( E_{\text{had, vis}} \)

- MCMC fit is strong:
  - ‘Rep Sample’ (min \( \chi^2 \)) from posterior agrees with the fake data (black) for all topologies.
Assessing the Fake Data fit

- Use a quantitative metric to evaluate MCMC, the \( \chi^2 \) — plotted for each topology.
- \( \chi^2 \approx 0 \) for all topologies:
  - MCMC fit agrees almost identically to the fake data.
- MCMC can successfully fit NOvA physics model parameters.
Conclusions

- Markov Chain Monte Carlo is a Bayesian inference parameter fitting tool.

- This talk is a demonstration of the fitting procedure:
  - MCMC successfully fits NOvA physics model parameters to fake data.

- Future directions:
  - fit all physics model parameters to ND data.
  - simultaneously fit ND & FD data to all NOvA systematic uncertainties.

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http://novaexperiment.fnal.gov
Back up
Fitting Fake Data

![Graph showing pull (σ) vs fraction of MCMC samples for various systematic errors. The graph includes MCMC samples and MCMC (mean+RMS) distributions, with systematic errors represented by labels such as Z-Expansion Axial Form Factor Eigen Value 1, Z-Expansion Axial Form Factor Eigen Value 2, and so on.]
We believe any pull beyond $\pm 3\sigma$ does not have a solid physical meaning — “extreme region”.

We use a narrow, Gaussian-like prior to prevent pulls in the “extreme region”.

**Custom Prior:** $p(x) = e^{-e^{0.3025x^2}}$
ND Topological distributions:
ν in Reco | $\vec{q}_3$ |
Fitted ND topological distributions:

\( \nu \text{ in } \text{Reco } |\vec{q}_3| \)
Examining marginal distributions

One advantage of Bayesian inference is the ability to marginalize over undesired parameters to examine the probability distributions for desired parameters.
ND Topological distributions:

$\bar{\nu}$ in $Reco \ E_{had,\ vis}$
ND Topological distributions:

$$\bar{\nu} \text{ in } \text{Reco } |\vec{q}_3|$$

- Antineutrino Beam
  - $\mu$ only
  - $\mu + \pi^\pm + X$
  - $\mu + X (0\pi)$

- Antineutrino Beam remaining

Graphs show distributions of events in the Reco $|\vec{q}_3|$ for different final states. The graphs are color-coded to represent different categories of events.