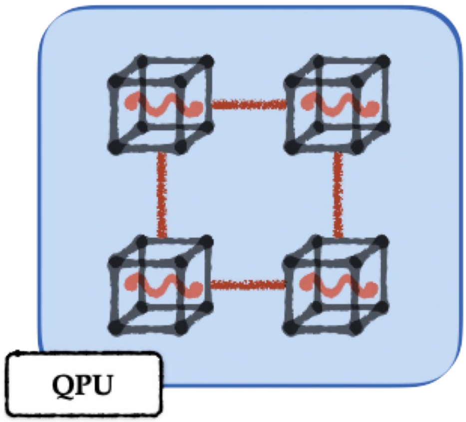
Quantum Bits



chughes
@fnal.gov

Fermilab

Lecture Overview

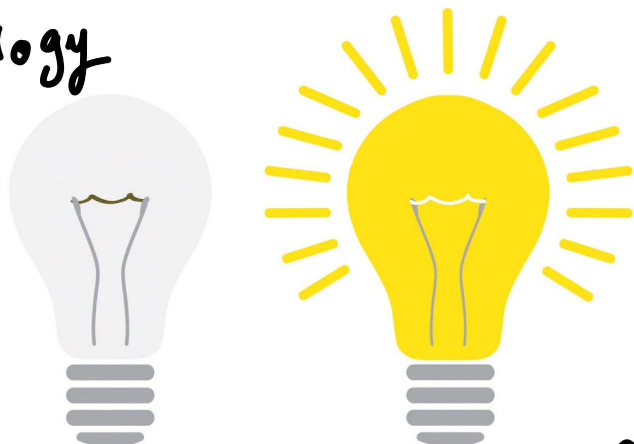
1. Classical bits
2. Quantum bits
3. Measuring, manipulate qubit
4. Bloch Sphere

Classical bits

classical are either
0 or 1

physically the wire either
if current exists $\equiv 1$
if current doesn't exist $\equiv 0$

classical
bit analogy



off $\equiv 0$

on $\equiv 1$

Quantum Bits

Qubits have two Measurable basis state, $|0\rangle, |1\rangle$

$\cancel{|H\rangle}$ $\cancel{|T\rangle}$

$\cancel{|alive\rangle}$ $\cancel{|dead\rangle}$

Special because a general qubit state in a superposition:

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

qubit

$$|\psi\rangle = a \, \square + b \, \boxed{\sim}$$

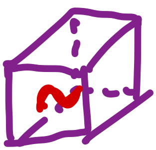
??

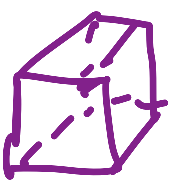
Conceptual ways to picture a qubit

Later: Spin-$\frac{1}{2}$ particle (electron)

$|0\rangle \equiv |\uparrow\rangle$, $|1\rangle \equiv |\downarrow\rangle$

Here: whether a photon exists in a box or not

 $\equiv |1\rangle$

 $\equiv |0\rangle$

Qubit Amplitudes

$$|\psi\rangle = a|0\rangle + b|1\rangle$$

'a' and 'b' are called amplitudes
Think of them as real numbers $a = \sqrt{\frac{1}{2}}$

AMPLITUDES ALLOW US TO REPRESENT all possible superpositions

IMPORTANT because tells us the probability of measuring $|0\rangle$, $|1\rangle$

MEASUREMENT RULE

$$|\psi\rangle \xrightarrow{M} |\psi\rangle_M \begin{cases} |0\rangle \text{ with prob } a^2 \\ |1\rangle \text{ with prob } b^2 \end{cases}$$

why squared: Nature is probabilistic

if $a = 0.3 \Rightarrow a^2$ is prob

Normalisation Rule

If you measure all outcomes
then you measure all probs,
then the probs need to
add to one

$$\boxed{a^2 + b^2 = \underline{1}}$$

Common Misconception

Measurement of a single qubit will result in a weighted average of both $|0\rangle$ and $|1\rangle$

WRONG

3 MINS

Breakout Rooms

Your friend gives you 100 instances of a qubit $|\psi\rangle = a|0\rangle + b|1\rangle$.
They won't tell you a, b.
Given 100 qubits, you need to find a, b!!

$q_1 \boxed{\psi} \xrightarrow{M} \boxed{} \equiv |0\rangle$ ← measurement of single qubit doesn't you anything

$q_2 \boxed{\psi} \xrightarrow{M} \boxed{} \equiv |0\rangle$

$q_3 \boxed{\psi} \xrightarrow{M} \boxed{\sim} \equiv |1\rangle$

$\vdots$

$q_{99} \boxed{\psi} \xrightarrow{M} \boxed{} \equiv |0\rangle$

$q_{100} \boxed{\psi} \xrightarrow{M} \boxed{\sim} \equiv |1\rangle$

$60 \boxed{} \equiv |0\rangle$

$40 \boxed{\sim} \equiv |1\rangle$

$|\psi\rangle \xrightarrow{M} |\psi\rangle_m \quad |0\rangle$ prob $a^2 = \frac{60}{100} = \frac{3}{5}$

$\rightarrow \quad |1\rangle$ with $b^2 = \frac{40}{100} = \frac{2}{5}$

$$|\psi\rangle = \pm\sqrt{\frac{3}{5}}\,|0\rangle \pm \sqrt{\frac{2}{5}}\,|1\rangle$$

Matrix Representation of qubits

EQUIVALENT TO KET NOTATION

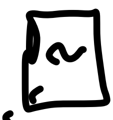
$$|\psi\rangle = a|0\rangle + b|1\rangle \equiv \begin{pmatrix} a \\ b \end{pmatrix}$$

$$|0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\psi\rangle = a\begin{pmatrix} 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$

How DO YOU make qubits do anything

Real world $\longrightarrow$ Maths



$\longrightarrow$ $|\psi\rangle \equiv \begin{pmatrix} a \\ b \end{pmatrix}$

laser

change $\longrightarrow$ $U|\psi\rangle$

unitary

Unitary Transformations

Untary are special transformations the perserve probability $a^2+b^2=1$

Unitary matrices 'U' obey $\dagger = *^T$

$$UU^\dagger = U^\dagger U = \mathbb{1} \; ; \; U^\dagger = U^{-1}$$

Ex $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

X-matrix/gate flip $|0\rangle \leftrightarrow |1\rangle$

$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$X|0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0\cdot 1 + 1\cdot 0 \\ 1\cdot 1 + 0\cdot 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

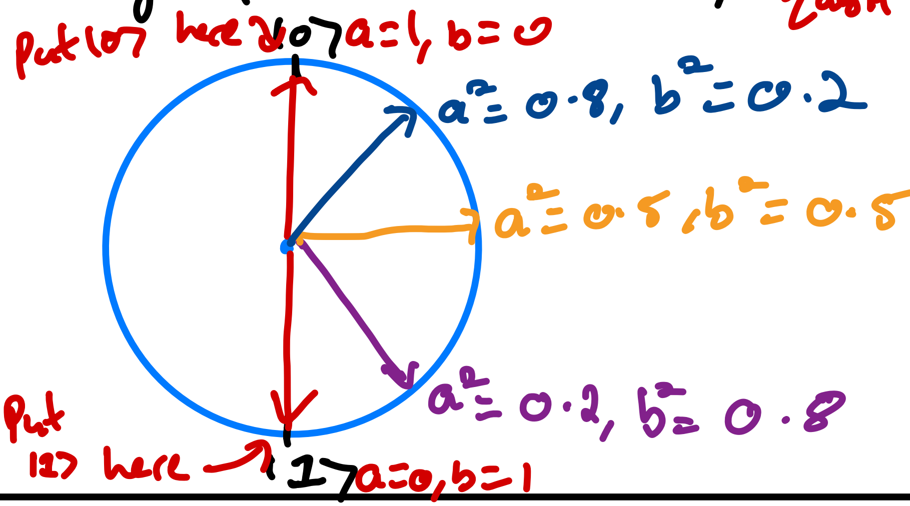
$$= \begin{pmatrix} \\ \end{pmatrix}$$

$$= \begin{pmatrix} a \cdot 1 + b \cdot 2 \\ c \cdot 1 + d \cdot 2 \end{pmatrix}$$

Bloch Sphere

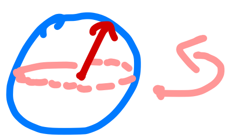
way to visualise a single qubit: $|\psi\rangle = a|0\rangle + b|1\rangle$ with $a^2+b^2=1$

if $a,b\in\mathbb{R}$, then $a^2+b^2=1$ is a circle (only sphere if $a,b\in\mathbb{C}$): arrow represent qubit

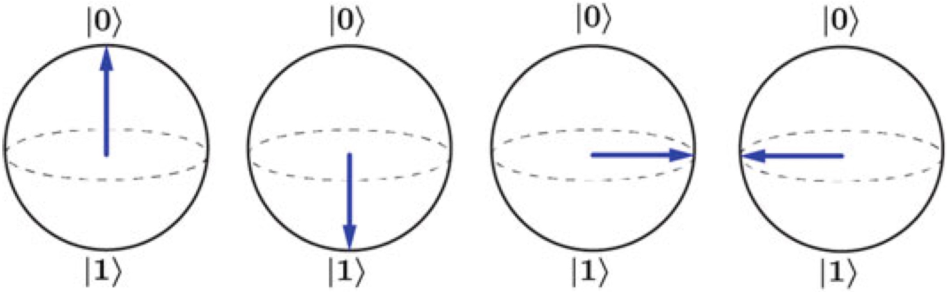


Extend a,b from $\mathbb{R}$ to $\mathbb{C}$, then $a^2+b^2=1$

$\Rightarrow$ Rotate circle into 3D to make sphere



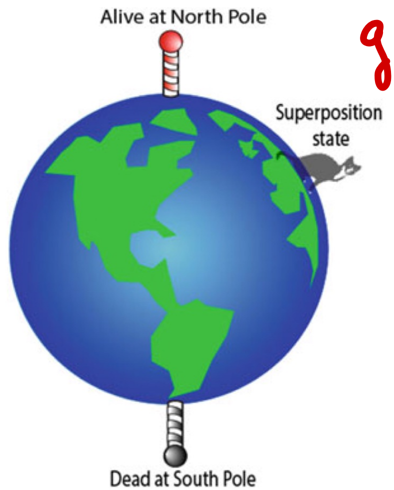
Now qubit live on sphere. Anything not on circle has complex phase



$|\Psi\rangle = |0\rangle$ $\quad$ $|\Psi\rangle = |1\rangle$ $\quad$ $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\quad$ $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

Schrödinger's Cat

As quantum practioners, the goal is to make the catwalk to do interesting things



Poll

Schrodinger's cat is measured, and found to be alive. Where on earth could the cat be before measurement

A) North Pole
B) South pole
C) ANYWHERE Except SOUTH POLE

SOLUTION

$$|cat\rangle_{before} = a|alive\rangle + b|dead\rangle$$

$$\xrightarrow{M} |alive\rangle \text{ with prob } a^2$$

$$|dead\rangle \text{ with prob } b^2$$

- If $|alive\rangle \Rightarrow$ measured $a^2 \neq 0$

Since we have only done one single measurement, we only have a single piece of info. $a \neq 0$. We do not know anything about b.

$\Rightarrow |cat\rangle_{before}$ = anywhere except south pole; since south pole has $a = 0$

Big Ideas

Qubits are written as kets matrices in terms of amplitudes, which obey normalisation rule

AMPLITUDES give prob of measuring qubit $|0\rangle$ or $|1\rangle$

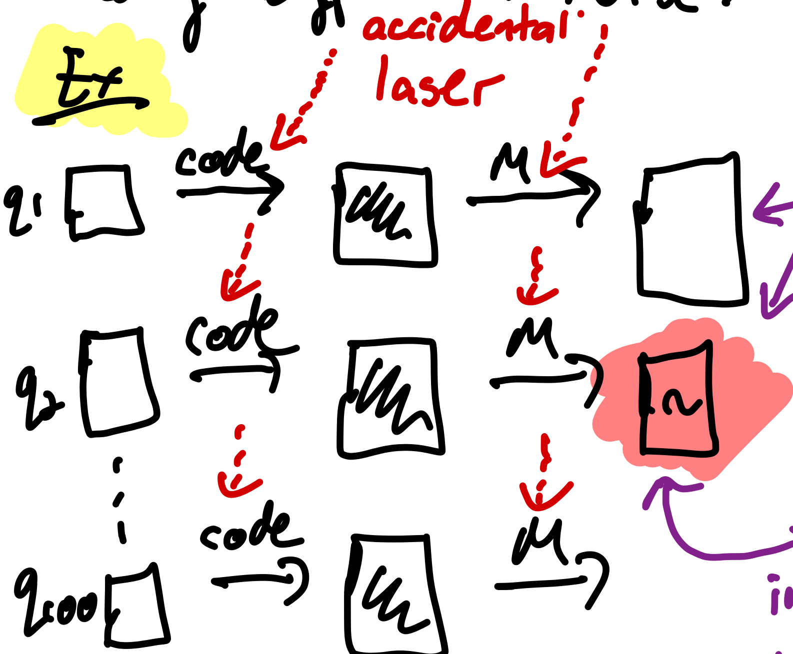
A physical change to a qubit, mathematically, correspond to unitary matrices acting on qubit.

gates.

Aside: Noisey measurement

Imagine you unknowingly shoot a laser at your box. It changes the box in some way. You don't know you did it. This is an example of noise in your calculation. There are many types of noise.

Ex



Imagine result should be $|0\rangle$

Incorrect Result due to not perfect implementation of tech in code or measurement

$\Rightarrow$ noise

Aside: Finite stats estimate (not specific to Quantum)

Ex Imagine you flip 100 classical coins. How many come up heads?

A) 50; B) 0; C) SOMETHING NEAR 50

STATS ⇒ Every estimate of an expectation/average value will give an error on the estimate from finite stats.

Answer: C). Flipping 100 coins does NOT give exactly 50 heads. Try it yourself.

• Expectation = true value $\pm \underbrace{\frac{\sigma}{\sqrt{N}}}_{}$

→ error from using N data points.

If $N \to \infty$, error $\to 0$

- Same with measuring qubits' amplitudes.

Only find 'a' and 'b' exactly with ∞ stats/measurements.

At finite measurements, get

$$|\psi\rangle \xrightarrow{M} \begin{cases} |0\rangle \text{ with prob } a^2 \pm \frac{\sigma}{\sqrt{N}} \\ |1\rangle \text{ with prob } b^2 \pm \frac{\sigma}{\sqrt{N}} \end{cases}$$