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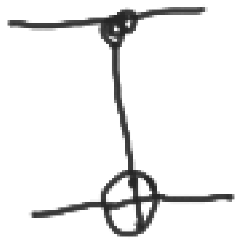
$CX = CNOT$



$|00\rangle \to |00\rangle$

$|01\rangle \to |01\rangle$

$|10\rangle \to |11\rangle$

$|11\rangle \to |10\rangle$

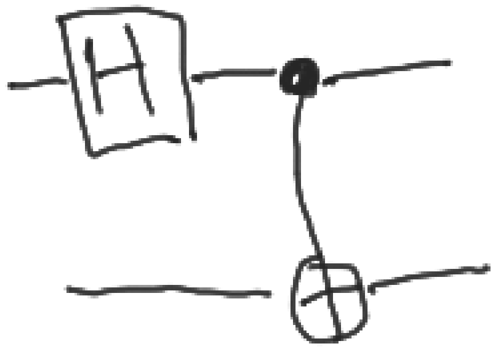
Ordered Basis

$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$|c\ t\rangle \to |c, c+t \bmod 2\rangle$

CX used to create entanglement



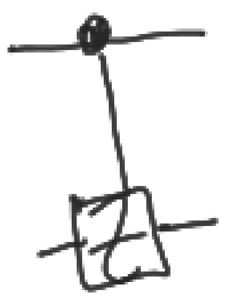

$$|00\rangle = |0\rangle \otimes |0\rangle$$

$$\xrightarrow{H} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$$

$$= \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

$$\xrightarrow{CX} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

CZ

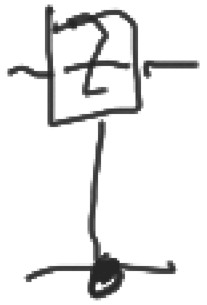



$|00\rangle \to |00\rangle$

$|01\rangle \to |01\rangle$

$|10\rangle \to +|10\rangle$

$|11\rangle \to -|11\rangle$

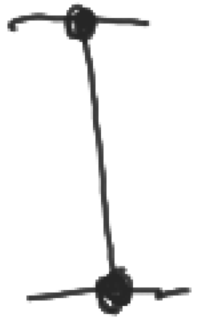



$|00\rangle \to |00\rangle$

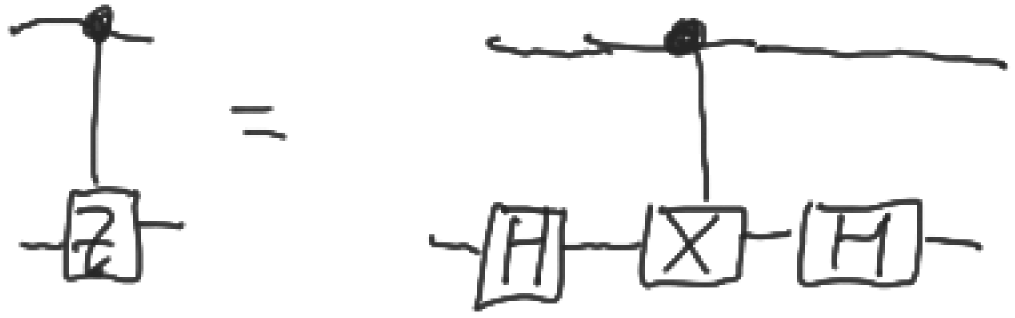
$|01\rangle \to +|01\rangle$

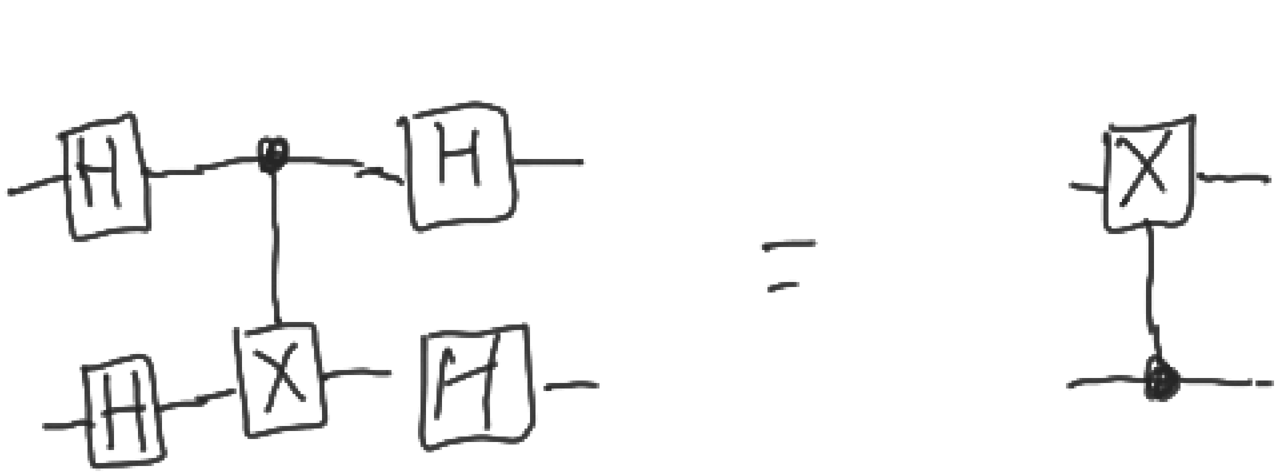
$|10\rangle \to |10\rangle$

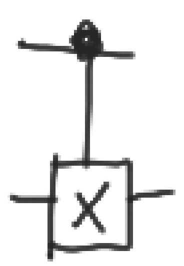
$|11\rangle \to -|10\rangle$



$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$$







00 → 00

01 → 01

10 → $|1\rangle \otimes X|0\rangle$

$= |1\rangle \otimes |1\rangle$

$= |11\rangle$

what if we want to swap then?

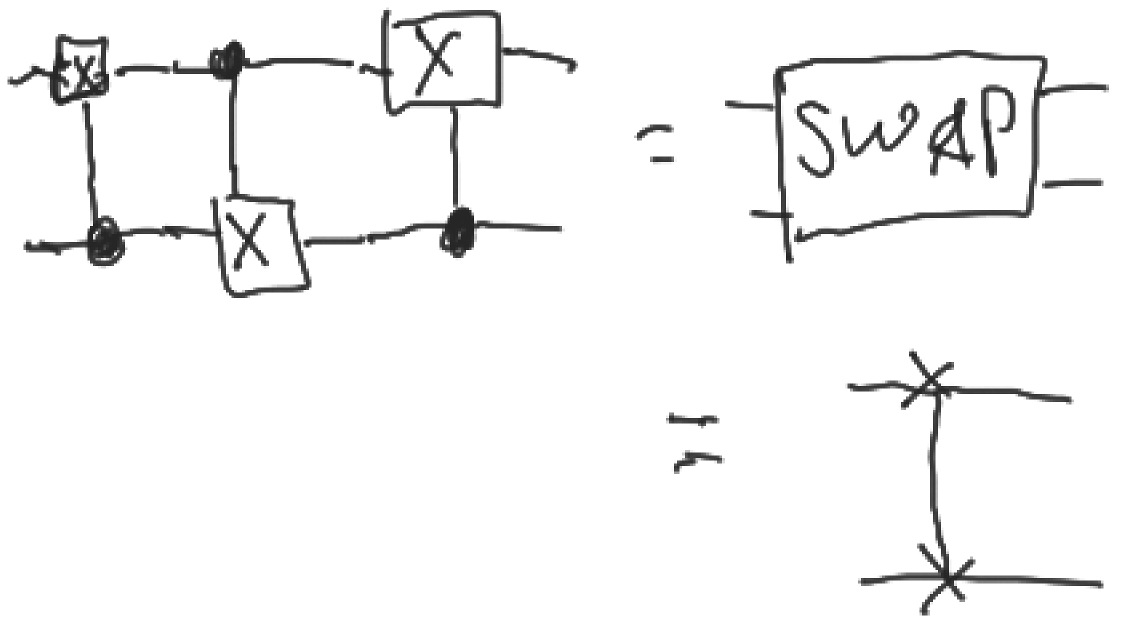

00 → 00

01 → 10

10 → 01

11 → 11

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



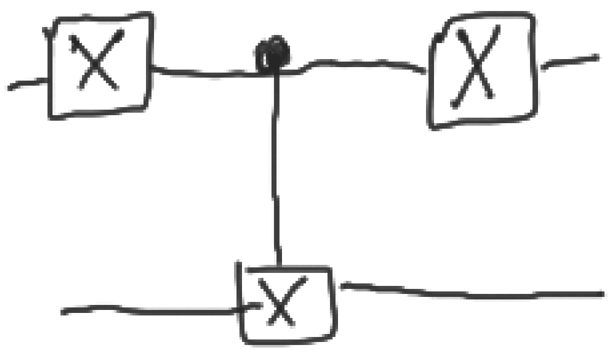

00 → 01

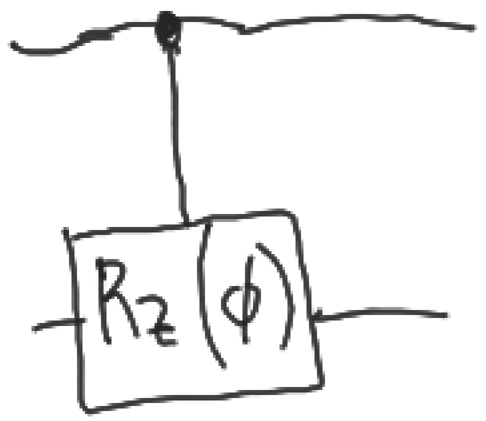
01 → 00

10 → 10

11 → 11

=







$$R_z(\phi) = e^{-\frac{i\phi}{2} Z} \quad \checkmark$$

Typical gate in quantum simulation

$$e^{-i\theta Z_1 Z_2}$$

$$Z_1 Z_2 = Z \otimes Z$$

$$(Z|0\rangle) \otimes (Z|0\rangle)$$

$$Z \otimes Z |00\rangle = (+1)(+1)|0\rangle \otimes |0\rangle = |00\rangle$$

$$|01\rangle = (+1)(-1)|0\rangle \otimes |1\rangle = -|01\rangle$$

$$|10\rangle = \qquad = -|10\rangle$$

$$|11\rangle = (-1)(-1)|1\rangle \otimes |1\rangle = |11\rangle$$

$0+0 \bmod 2 = 0$

$0+1 \bmod 2 = 1$

$\ldots = 1$

$1+1 \bmod 2 = 0$

$0 \bmod 2 = 0$

$1 \bmod 2 = 1$

$2 \bmod 2 = 0$

$3 \bmod 2 = 1$

11:00 + 5hrs → 16:00 → 4:00

$$e^{-i\theta Z\otimes Z}:$$

$$|00\rangle \to e^{-i\theta(+1)(+1)}|00\rangle = e^{-i\theta}|00\rangle$$

$$|01\rangle \to e^{-i\theta(+1)(-1)}|01\rangle = e^{+i\theta}|01\rangle$$

$$|10\rangle \to e^{-i\theta(-1)(+1)}|10\rangle = e^{+i\theta}|10\rangle$$

$$|11\rangle \to e^{-i\theta(-1)(-1)}|11\rangle = e^{-i\theta}|11\rangle$$

$$e^{x} = \sum_{k=0}^{\infty} \frac{1}{k!} x^{k}$$

$$e^{A} = \sum_{k=0}^{\infty} \frac{1}{k!} A^{k}$$

Action is dictated by sum modulo 2:

if $0 \longrightarrow$ give $e^{-i\theta}$

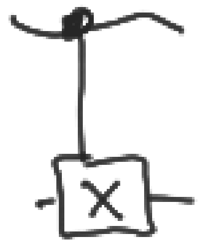
if $1 \longrightarrow$ give $e^{+i\theta}$

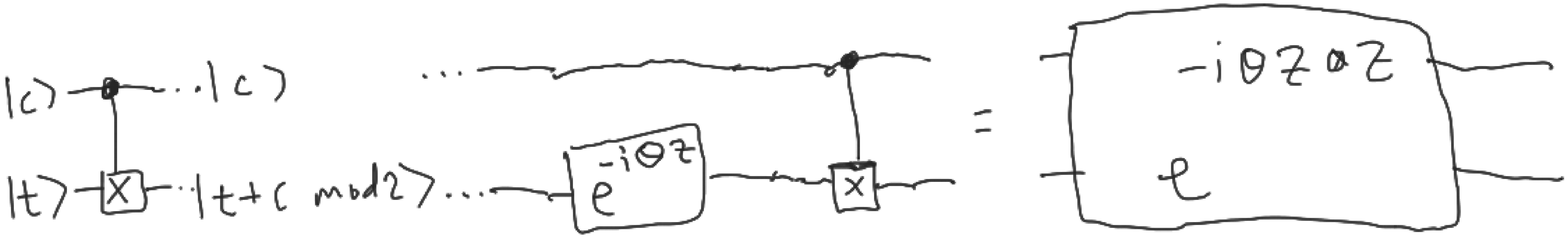
$e^{-i\theta Z\otimes Z}$

Apply $e^{-i\theta}$ for sum $= 0$

$e^{+i\theta}$ for sum $= 1$

↳ $e^{-iZ}$

 $= |c,t\rangle \to |c, t+c \text{ modulo } 2\rangle$



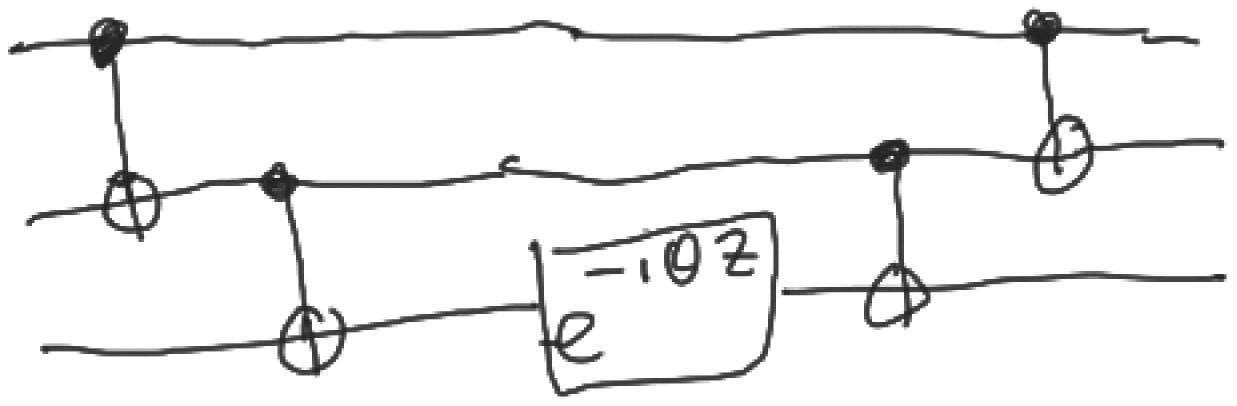


$HZH = X$

$HXH = Z$

$e^{-i\theta Z\otimes Z}$ generalizes to 3 or $n$ qubits

$$e^{-i\theta Z\otimes Z\otimes Z} =$$




Turned a 3-qubit operator into a sequence of CX's and a single-qubit rotation.