

Simultaneous extraction of m_t and α_s from differential $t\bar{t}$ distributions

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Simultaneous extraction of m_t and α_s from differential $t\bar{t}$ distributions

- ▶ 8 TeV data from ATLAS and CMS collected in Run 1
- ▶ Differential distributions of tops reconstructed from lepton+jets analyses, common binning
- ▶ Transverse momentum p_t^T , invariant mass $M_{t\bar{t}}$, single and pair rapidities $y_t, y_{t\bar{t}}$
- ▶ Absolute and normalised distributions—separate data sets from ATLAS, only normalised from CMS (absolute inferred)

Fit methodology

Least squares extraction for normalised and absolute distributions

$$\zeta_i = \zeta_i^{\text{data}} - \zeta_i^{\text{theory}}$$

$$\chi_{\text{norm}}^2 = \frac{1}{N_{\text{data}}} \left[\sum_{i,j=1}^{N_{\text{data}}-1} \zeta_i C_{ij}^{-1} \zeta_j + \frac{(\sigma_{\text{NNLO}} - \sigma_{\text{data}})^2}{\delta\sigma_{\text{data}}^2} \right]$$

$$\chi_{\text{abs}}^2 = \frac{1}{N_{\text{data}}} \sum_{i,j=1}^{N_{\text{data}}} \zeta_i C_{ij}^{-1} \zeta_j$$

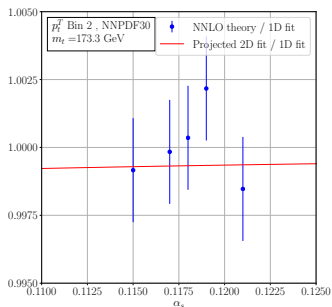
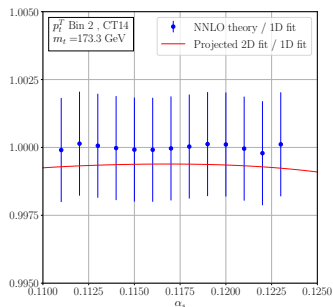
- ▶ Measured values of $\sigma_{t\bar{t}}$ taken from separate 8 TeV ATLAS/CMS measurements ¹
- ▶ Theory values of $\sigma_{t\bar{t}}$ calculated using top++2.0 at NNLO with NNLL resummation of soft gluons

¹1406.5375,1603.02303

Theory input

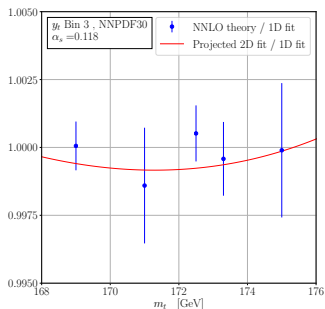
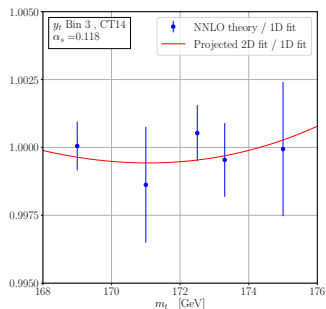
- ▶ For each distribution, need differential cross section in each bin as a function of α_s , m_t
- ▶ Precompute each bin weight on a grid of α_s , m_t and interpolate parameter dependence
- ▶ α_s dependence determined by PDF set, so procedure needs to be done for each choice
- ▶ Possible through use of FastNLO tables for values of $m_t = \{169.0, 171.0, 172.5, 173.3, 175.0\}$ GeV
- ▶ 3 sets chosen, CT14, NNPDF3.0, NNPDF3.1
- ▶ Different parametrisations chosen for each distribution, PDF choice

Assessing fit quality



- ▶ Factorised form taken for 2D parametrisation,
 $f(\alpha_s, m_t) = g(\alpha_s)h(m_t)$
- ▶ Points removed and fit redone to ensure no overfitting

Assessing fit quality



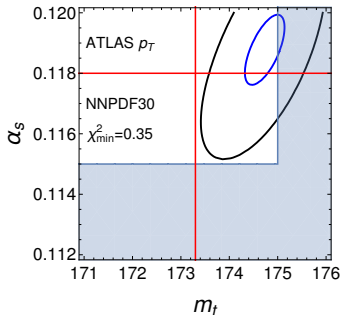
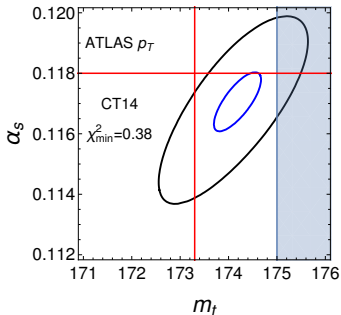
- ▶ Factorised form taken for 2D parametrisation,
 $f(\alpha_s, m_t) = g(\alpha_s)h(m_t)$
- ▶ Points removed and fit redone to ensure no overfitting

Results: p_t^T

ATLAS, normalised results

White region: interpolated, blue region: extrapolated

Blue line: $\Delta\chi^2 = 0.1$, black line: $\Delta\chi^2 = 1.0$



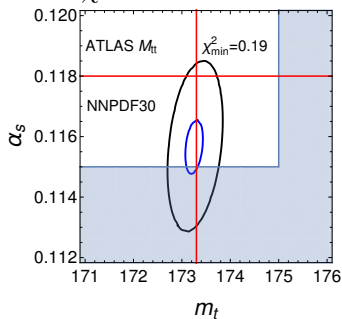
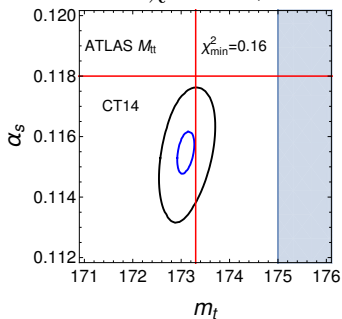
	CT14			NNPDF30		
	α_s	m_t	χ^2_{\min}	α_s	m_t	χ^2_{\min}
p_T^t	0.1171	174.2	0.38	0.1188	174.7	0.35

Results: $M_{t\bar{t}}$

ATLAS, normalised results

White region: interpolated, blue region: extrapolated

Blue line: $\Delta\chi^2 = 0.1$, black line: $\Delta\chi^2 = 1.0$



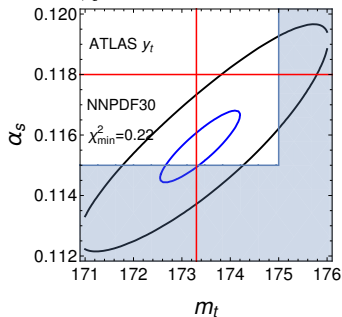
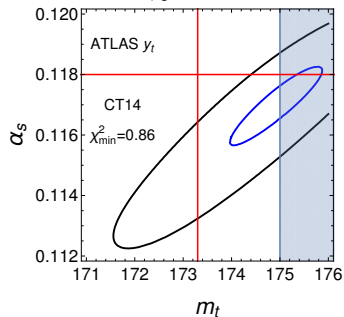
	CT14			NNPDF30		
	α_s	m_t	χ^2_{\min}	α_s	m_t	χ^2_{\min}
$M_{t\bar{t}}$	0.1155	173.1	0.16	0.1157	173.2	0.19

Results: y_t

ATLAS, normalised results

White region: interpolated, blue region: extrapolated

Blue line: $\Delta\chi^2 = 0.1$, black line: $\Delta\chi^2 = 1.0$



	CT14			NNPDF30		
	α_s	m_t	χ^2_{\min}	α_s	m_t	χ^2_{\min}
y_t	0.1171	175.0	0.86	0.1156	173.4	0.22

Combining distributions and experiments

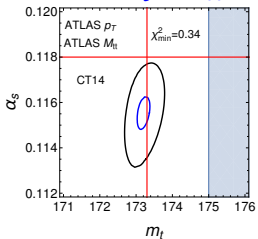
- ▶ Combine distributions from ATLAS making use of available correlations between distributions
- ▶ Combine distributions from ATLAS and CMS assuming no correlations (luminosity?)

$$\chi_{\text{norm}}^2 = \frac{1}{(N_{\text{ATLAS}} + N_{\text{CMS}})} \left(\sum_{i,j=1}^{N_{\text{ATLAS}}-1} \zeta_{i,\text{ATLAS}} C_{ij,\text{ATLAS}}^{-1} \zeta_{j,\text{ATLAS}} + \sum_{i,j=1}^{N_{\text{CMS}}-1} \zeta_{i,\text{CMS}} C_{ij,\text{CMS}}^{-1} \zeta_{j,\text{CMS}} \right) + \frac{(\sigma_{\text{NNLO}} - \sigma_{\text{ATLAS}})^2}{\delta\sigma_{\text{ATLAS}}^2} + \frac{(\sigma_{\text{NNLO}} - \sigma_{\text{CMS}})^2}{\delta\sigma_{\text{CMS}}^2}$$

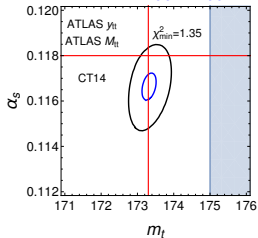
$$\chi_{\text{abs}}^2 = \frac{1}{(N_{\text{ATLAS}} + N_{\text{CMS}})} \left(\sum_{i,j=1}^{N_{\text{ATLAS}}} \zeta_{i,\text{ATLAS}} C_{ij,\text{ATLAS}}^{-1} \zeta_{j,\text{ATLAS}} + \sum_{i,j=1}^{N_{\text{CMS}}} \zeta_{i,\text{CMS}} C_{ij,\text{CMS}}^{-1} \zeta_{j,\text{CMS}} \right)$$

CT14

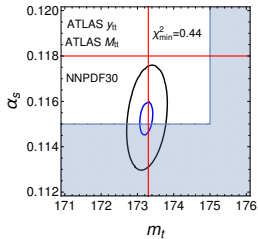
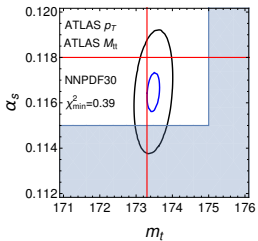
ATLAS $p_t^T, M_{t\bar{t}}$



ATLAS $M_{t\bar{t}}, y_{t\bar{t}}$

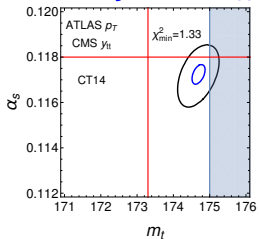


NNPDF3.0

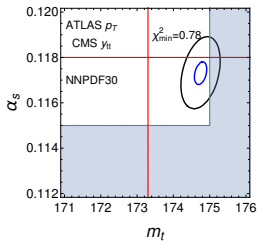


CT14

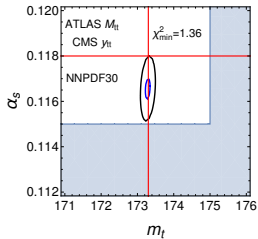
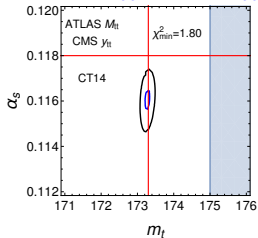
ATLAS p_t^T , CMS $y_{t\bar{t}}$



NNPDF3.0



ATLAS $M_{t\bar{t}}$, CMS $y_{t\bar{t}}$



Best fit values via averaging

- ▶ In light of differences, obtain best quality results via a weighted average.
- ▶ Consider only extractions returning $0.115 \leq \alpha_s \leq 0.121$ and $170.0 \leq m_t \leq 176.0$ GeV ($\pm \sim 3\sigma$ around world average).
- ▶ Exclude cases where χ_{\min}^2 is uniformly bad.

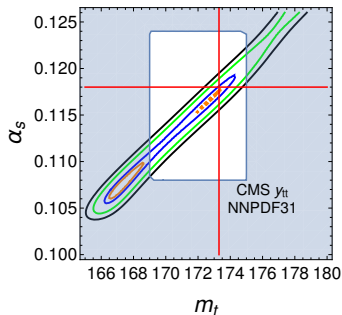
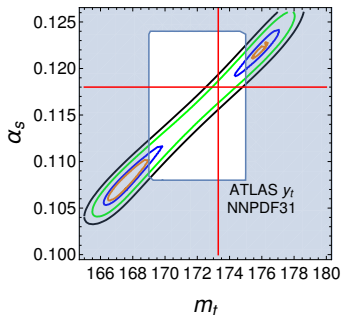
$$\bar{\beta} = \frac{\sum_i u_i \beta_i}{\sum_i u_i}, \quad u_i = \int_{\chi_{\min,i}^2}^{\infty} f(z; n_d) dz \quad (1)$$

$$\delta\beta_{\text{stat}}^{\text{up/down}} = \frac{\sqrt{\sum_i u_i^2 (\delta\beta_i^{\text{up/down}})^2}}{\sum_i u_i}, \quad \delta\beta_{\text{sys}} = \frac{\sum_i u_i |\beta_i - \bar{\beta}|}{\sum_i u_i} \quad (2)$$

Extraction	CT14		NNPDF3.0	
	α_s	m_t	α_s	m_t
Single ATLAS	$0.1163^{+0.0015}_{-0.0016}$	$173.8^{+0.8}_{-0.8}$	$0.1164^{+0.0019}_{-0.0019}$	$173.6^{+0.8}_{-0.8}$
ATLAS combination	$0.1159^{+0.0013}_{-0.0014}$	$173.8^{+0.8}_{-0.8}$	$0.1161^{+0.0011}_{-0.0010}$	$173.7^{+0.6}_{-0.6}$

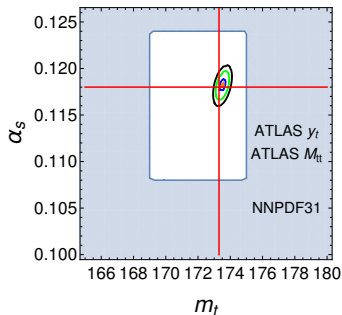
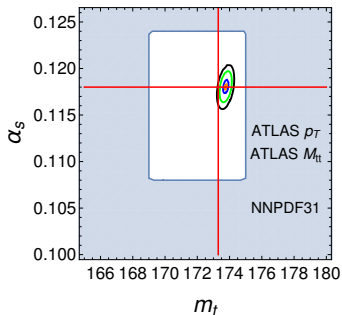
Extractions using NNPDF3.1

- ▶ ATLAS y_t and CMS $y_{t\bar{t}}$ used in this work also included in NNPDF3.1 fit
- ▶ What effect does this have on extractions from these distributions? Are others affected?



Extractions using NNPDF3.1

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Theory systematic uncertainties

- ▶ Given the differences seen between experiments/PDFs/distributions, we neglect all potential sources of systematic uncertainty in the theory predictions.
- ▶ Most pragmatic way to include effects due to MHO is *alla NNPDF*.
 - ▶ Assume Gaussian uncertainties, construct a theory covariance matrix.
- ▶ More sophisticated approach (see Tackmann, Les Houches 2019):
 - ▶ Regard missing higher order terms as nuisance parameters.
 - ▶ $\sigma = c_0 + \alpha_s(\mu)[c_1 + \alpha_s(\mu)c_2 + \dots]$
 - ▶ In the simplest case c_2 is a number, more generally a function.
 - ▶ Correct correlations obtained, when multiple parameters involved CLT implies total theory uncertainty is Gaussian.

Conclusions

- ▶ 8 TeV ATLAS, CMS data compared to NNLO theory to extract α_s , m_t simultaneously.
- ▶ Find noticeable differences between
 - ▶ ATLAS and CMS
 - ▶ Different PDF choices
 - ▶ Different distributions

indicating large sensitivity to all factors—data, PDF and kinematics.

- ▶ In order to reconcile differences/arrive at best values,
 - ▶ Restrict $0.115 \leq \alpha_s \leq 0.120$ and $170.0 \leq m_t \leq 175.0$ GeV ($\pm \sim 3\sigma$ around world average);
 - ▶ Perform a weighted average over various extractions.

Conclusions

- ▶ Averaging procedure performed on different types of extraction and different PDFs results in consistent values.
- ▶ Absolute distributions give larger errors in general.
- ▶ 'Best' value from combining two ATLAS distributions and averaging CT14 results.
- ▶ $\alpha_s = 0.1159_{-0.0014}^{+0.0013}$, $m_t = 173.8_{-0.8}^{+0.8}$ GeV.
- ▶ Prospects for future inclusion of theory uncertainties due to MHO.
- ▶ Results available at <http://www.precision.hep.phy.cam.ac.uk/results/ttbar-fastnlo/>