



QED ISR/FSR with YFS Resummation



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Overview

1. Motivation
2. Theory
3. Some Results
4. Outlook and Conclusion



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Sherpa has traditionally been more focused on Hadron-Hadron collisions and the improvements needed for e^+e^- have been mostly implemented by one Phd/Post-Doc (me)



Motivation

Observable	Where from	Current (LEP)	FCC (stat.)	FCC (syst.)	$\frac{\text{Now}}{\text{FCC}}$
M_Z [MeV]	Z linesh. [32]	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3
Γ_Z [MeV]	Z linesh. [32]	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$ [33]	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12
σ_{had}^0 [nb]	σ_{had}^0 [32]	$41.541 \pm 0.037\{0.025\}$	$0.1 \cdot 10^{-3}$	$4 \cdot 10^{-3}$	6
N_ν	$\sigma(M_Z)$ [32]	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6
N_ν	$Z\gamma$ [34]	$2.69 \pm 0.15\{0.06\}$	$0.8 \cdot 10^{-3}$	$< 10^{-3}$	60
$\sin^2 \theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$ [33]	$23099 \pm 53\{28\}$	0.3	0.5	55
$\sin^2 \theta_W^{eff} \times 10^5$	$\langle \mathcal{P}_\tau \rangle, A_{FB}^{pol, \tau}$ [32]	$23159 \pm 41\{12\}$	0.6	< 0.6	20
M_W [MeV]	ADLO [35]	$80376 \pm 33\{6\}$	0.5	0.3	12
$A_{FB, \mu}^{M_Z \pm 3.5 \text{ GeV}}$	$\frac{d\sigma}{d\cos\theta}$ [32]	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100

- ❑ Emission of soft/collinear photons lead to large logs $\sim \log(s/m^2)$
- ❑ LEP era calculations will not be sufficient for future e+e- machines

QED corrections needed for FCC-ee, adapted from [\(Jadach et al, Eur. Phys. J. C79\(2019\)\)](#)

How to Treat ISR?

Collinear Resummation

- ❑ Calculate ISR using electron PDF ([Jadach et.al, Z.Phys.C 49 \(1991\) 577-584](#), [Europhys. Lett.17\(1992\) 123–128](#))
- ❑ Recently calculated up to NLL, improvement beyond this very difficult ([Bertone et.al 1911.12040](#))
- ❑ New calculations also include photon pdf for photon initiated processes
- ❑ Needs to be matched to a Parton Shower for no inclusive observables
- ❑ Standard treatment of ISR in e+e- MC tools such as Whizard and Sherpa v1.x/2.x

Soft Resummation

- ❑ Soft photons can be resummed to all orders ([Yennie, Frautshci, Suura, Annals Phys. 13 \(1961\) 379-452](#))
- ❑ Fully differential treatment of the multi-photon phasespace
- ❑ **Can be systematically improved order-by-order**
- ❑ Collinear logs are included in a truncated expression

Inclusive Calculations

- ❑ Inclusive calculation for e+e-> Υ^*/Z up to $\mathcal{O}(\alpha^6 L^6)$ ([J. Blümlein, A. De Freitas, K. Schönwald et al, Nucl. Phys. B955 \(2020\), Phys. Lett. B 816 \(2021\)](#))

How to Treat ISR?

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Resummation à la YFS

Yennie, Frautschi, and Suura showed that in the soft limit the total cross section for a given process with n_V virtual and n_R real soft photons can be expressed as,

$$\sigma = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\Phi_f e^{2\alpha B + 2\alpha \tilde{B}} \prod_{j=1}^n \tilde{S}(k_j) \theta(\Omega; k_j) \left[\tilde{\beta}_0(p_1, p_2; q_1, \dots, q_{n'}) \right. \\ \left. + \sum_{j=1}^n \frac{\tilde{\beta}_1(p_1, p_2; q_1, \dots, q_{n'}, k_j)}{S(k_j)} \right. \\ \left. + \sum_{\substack{j,l=1 \\ j \neq l}}^n \frac{\tilde{\beta}_2(p_1, p_2; q_1, \dots, q_{n'}, k_j, k_l)}{S(k_j)S(k_l)} + \dots \right]$$

YFS Resummation

$$\sigma = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\Phi_f e^{2\alpha B + 2\alpha \tilde{B}} \prod_{j=1}^n \tilde{S}(k_j) \theta(\Omega; k_j) \left[\tilde{\beta}_0(p_1, p_2; q_1, \dots, q_{n'}) \right. \\ \left. + \sum_{j=1}^n \frac{\tilde{\beta}_1(p_1, p_2; q_1, \dots, q_{n'}, k_j)}{S(k_j)} \right. \\ \left. + \sum_{\substack{j,l=1 \\ j \neq l}}^n \frac{\tilde{\beta}_2(p_1, p_2; q_1, \dots, q_{n'}, k_j, k_l)}{S(k_j)S(k_l)} + \dots \right]$$

- ❑ β are the IR finite ME
- ❑ Currently they are hard coded into Sherpa but can be taken from external tools e.g Recola, COMIX

$$\tilde{\beta}_i = \sum_{n_v} \beta_i^{n_v}$$

$$\tilde{\beta}_0^0 = M_0^0 M_0^{0*}$$

$$\tilde{\beta}_0^1 = M_0^1 M_0^{0*} + M_0^{1*} M_0^0$$

$$\tilde{\beta}_1^1 = \frac{1}{2(2\pi)^3} M_0^{\frac{1}{2}} M_0^{\frac{1}{2}*} - \tilde{S}(k) M_0^0 M_0^{0*} = \frac{1}{2(2\pi)^3} M_0^{\frac{1}{2}} M_0^{\frac{1}{2}*} - \tilde{S}(k) \tilde{\beta}_0^0$$

YFS Resummation

$$\sigma = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\Phi_f e^{2\alpha B + 2\alpha \tilde{B}} \prod_{j=1}^n \tilde{S}(k_j) \theta(\Omega; k_j) \left[\tilde{\beta}_0(p_1, p_2; q_1, \dots, q_{n'}) \right. \\ \left. + \sum_{j=1}^n \frac{\tilde{\beta}_1(p_1, p_2; q_1, \dots, q_{n'}, k_j)}{S(k_j)} \right. \\ \left. + \sum_{\substack{j,l=1 \\ j \neq l}}^n \frac{\tilde{\beta}_2(p_1, p_2; q_1, \dots, q_{n'}, k_j, k_l)}{S(k_j)S(k_l)} + \dots \right]$$

- ❑ The explicit form factor is known explicitly
- ❑ Treatment of the full phasespace was detailed in

[Comput.Phys.Commun. 56 \(1990\) 351-384](#)

$$\tilde{B} = -\frac{1}{8\pi^2} \int \frac{d^3k}{k^0} \Theta(\Omega, k) \left(\frac{p_1}{p_1 k} - \frac{p_2}{p_2 k} \right)^2$$

$$B = 2\alpha \Re \int \frac{d^4k}{k^2} \frac{i}{(2\pi)^2} \left(\frac{2p_1 - k}{2kp_1 - k^2} - \frac{2p_2 - k}{2kp_2 - k^2} \right)^2$$

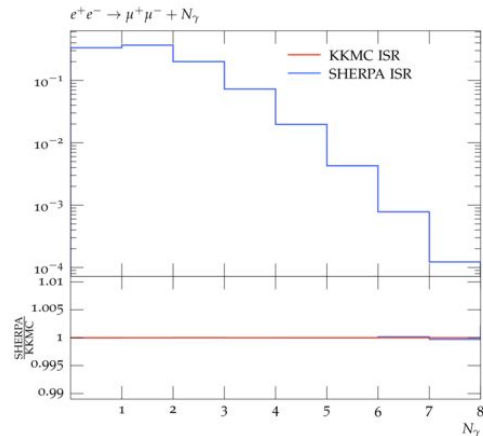
Status in Sherpa

- ❑ For initial state radiation (ISR), the YFS algorithm can be applied to any e^+e^- process
 - ISR includes corrections up to $\alpha^3 L^3$, inclusion of FO calculations trivial
 - Full treatment of the Photon Phasespace, which allows for explicit photon creation
- ❑ Recently final state radiation has been added
 - It was implemented for decays in PHOTONS++ ([JHEP 2008\(12\):018](#))
 - New treatment added to account for FSR in the total XS
 - Well validated for $e^+e^- \rightarrow f\bar{f}$ and testing is ongoing for WW/ZZ/ZH
- ❑ Initial-Final Interference
 - Currently not included
 - For $e^+e^- \rightarrow f\bar{f}$ can be included by “hand” but difficult to automate

$$e^+ e^- \rightarrow f \bar{f}$$

- ❑ State of the art is KKMC
([Comput.Phys.Comm. 130 \(2000\) 260-325](#))
- ❑ KKMC includes initial, final, and initial-final interference
- ❑ Sherpa does not include Initial-final interference

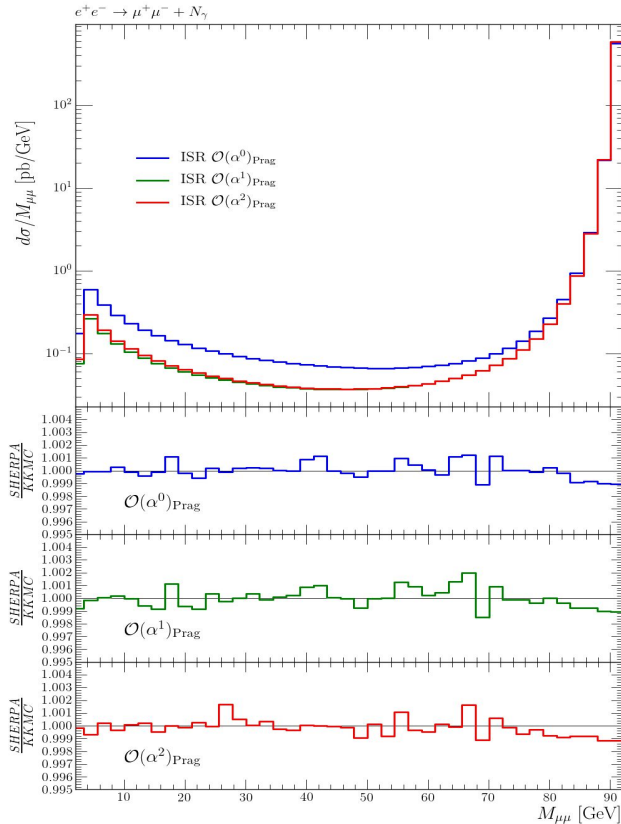
	Born [pb]	FSR [pb]	ISR [pb]	ISR+FSR [pb]
KKMC	1822.60	1863.03 +0.33	1249.53+/- 0.37	1281.611 +0.001
SHERPA	1822.60	1863.62 +/- 0.32	1249.49+/- 0.44	1282.28 +/- 0.4



There is excellent agreement between KKMC and Sherpa. Above is Xs for muon production at 91 GeV

- ❑ For final state leptons QED emission can be resummed in the YFS framework
- ❑ For final state quarks it is better to use Parton Shower with QED splittings (also in Sherpa)

$$e^+e^- \rightarrow f\bar{f}$$

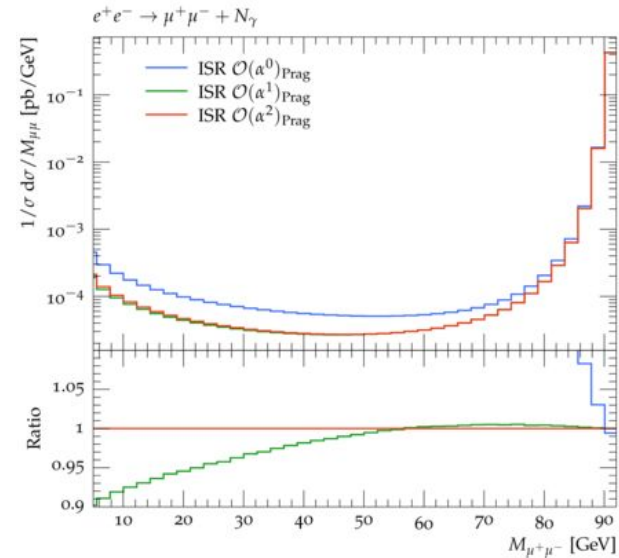


$$\mathcal{O}(\alpha^0)_{\text{Prag}} = \text{Resummation only}$$

$$\mathcal{O}(\alpha^1)_{\text{Prag}} = \alpha, \alpha L$$

$$\mathcal{O}(\alpha^2)_{\text{Prag}} = \alpha, \alpha L, \alpha^2 L^2$$

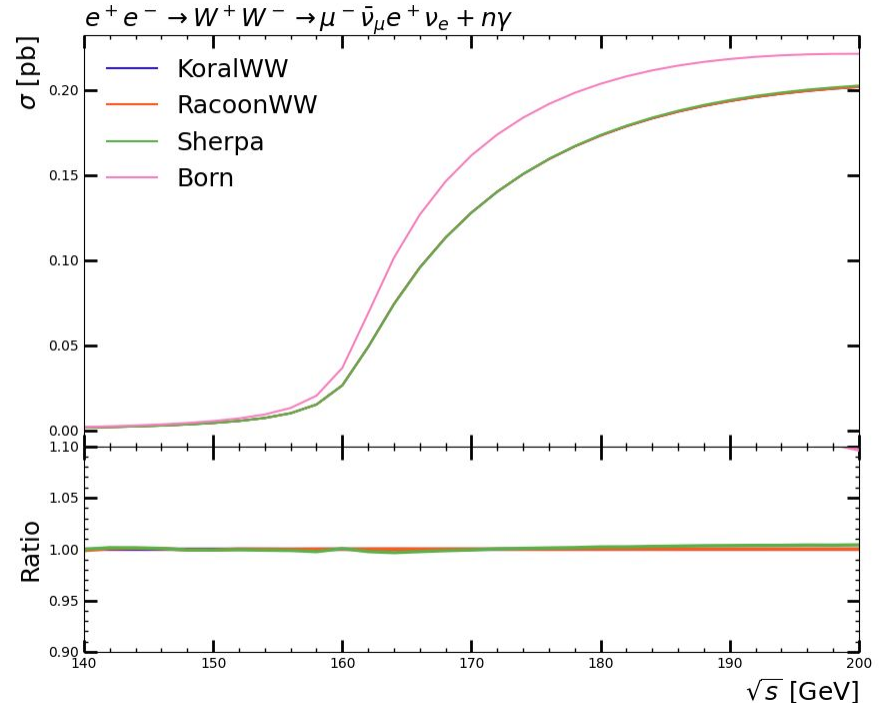
$$\mathcal{O}(\alpha^3)_{\text{Prag}} = \alpha, \alpha L, \alpha^2 L^2, \alpha^2 L, \alpha^3 L^3$$



$$e^+e^- \rightarrow W^+W^-$$

- ❑ Dedicated codes during LEP era:
 - YFSWW/KoralW ([Comput.Phys.Comm. 140 \(2001\) 475-512](#))
 - ❑ ISR Corrections via YFS
 - ❑ Complete $O(\alpha)$ corrections included
 - ❑ Option of FSR via Photos
 - ❑ Coulomb corrections also implemented
 - RacoonWW ([Nucl.Phys.B 587 \(2000\) 67-117](#))
 - ❑ ISR corrections via electron PDF
 - ❑ Complete $O(\alpha)$ corrections included
 - ❑ Coulomb corrections also implemented

- ❑ Sherpa
 - ISR Corrections via YFS
 - FSR corrections via YFS or PS, being tested, with option to combine PS and YFS for semi-leptonic decays
 - Coulomb Corrections included
 - Soft Photons for W also resummed
 - Complete $O(\alpha)$ with EW loops from Recola (under test)



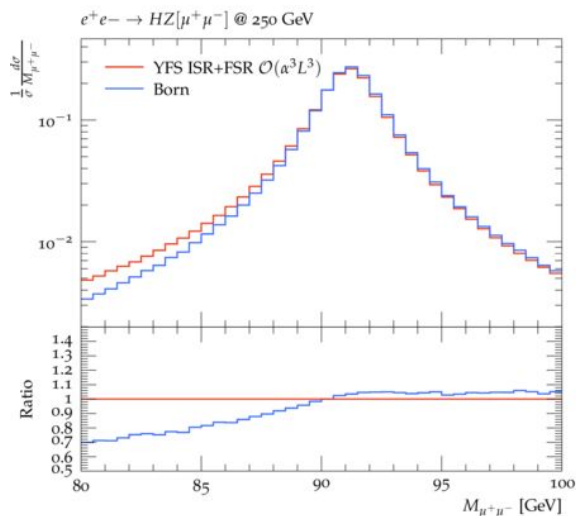
$$e^+ e^- \rightarrow ZH$$

- ❑ ISR has only been modelled via electron PDFs before
- ❑ Sherpa can now use YFS for ISR+FSR

250 GeV	Born	ISR+FSR
SHERPA [fb]	7.05	5.98

Above is example Xs for Z-> mu mu and H -> tau, tau at 250 GeV

- ❑ For final state leptons QED emission can be resummed in the YFS framework
- ❑ For final state quarks it is better to use Parton Shower with QED splittings (also in Sherpa)
- ❑ Ongoing study to investigate effect of FSR on Higgs mass from Z-recoil.



- ❑ Sherpa has traditionally been more focused on Hadron-Hadron collisions and the improvements needed for e^+e^- have been mostly implemented by one Phd/Post-Doc (me)
- ❑ IFI still needs to be implemented
 - Method is known but difficult to automate. Work ongoing with ME generators
- ❑ More loops will be needed
 - Full 1-loop EW corrections can be included via Recola
 - Framework exists to include 2-loops as and when they become available
- ❑ For Linear colliders
 - Interface to LCIO ([eConf C0303241 \(2003\) TUKT001](#)) has been written and is undergoing testing
 - Interface to CIRCE ([Comput.Phys.Comm. 101 \(1997\) 269-288](#)) (for beam spectra) is under development
 - Planned to be added to Sherpa 3.X release

Conclusion

- ❑ ISR corrections have been implemented in a process independent manner and validated against existing calculations.
- ❑ Inclusion of FSR via YFS resummation is possible for some processes, work is ongoing to automate this and include IFI
- ❑ These features will be released in Sherpa 3.X but dedicated samples can be provided
- ❑ Can be used for BSM physics via UFO interface