

# The Higgs Inverse Problem

Based mostly on  
`arXiv:2007.01296`, `arXiv:2102.02823`

**Samuel Homiller**

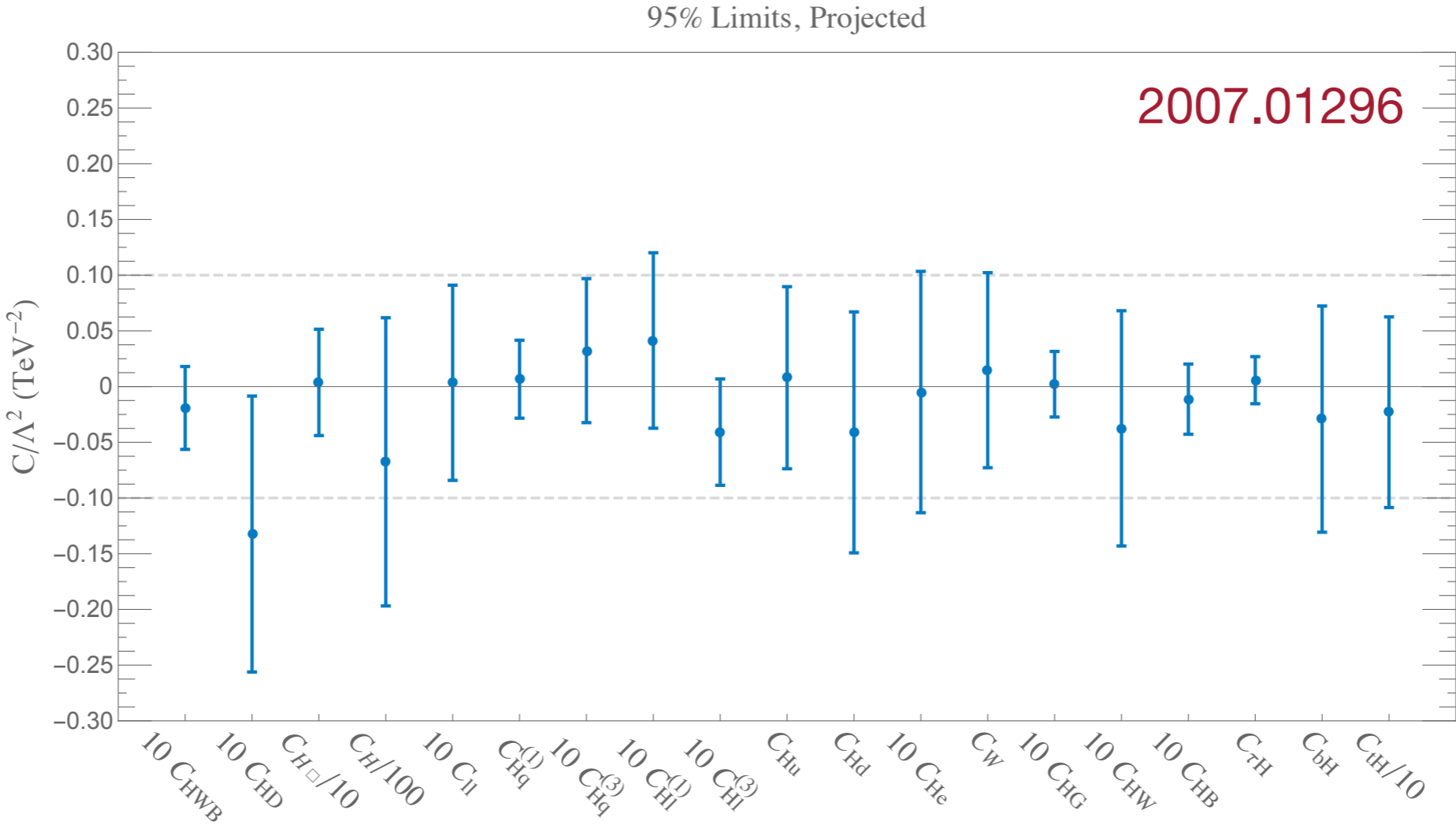
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In collaboration with  
Sally Dawson, Pier Paolo Giardino, and Samuel Lane

**EF01 Meeting — August 18, 2021**

# The Higgs Inverse Problem:

What can we learn from precise measurements of the Higgs?



⇒ assuming we observe some deviation in Higgs couplings, how can we make sense of what UV physics underlies it?

# The SMEFT Framework

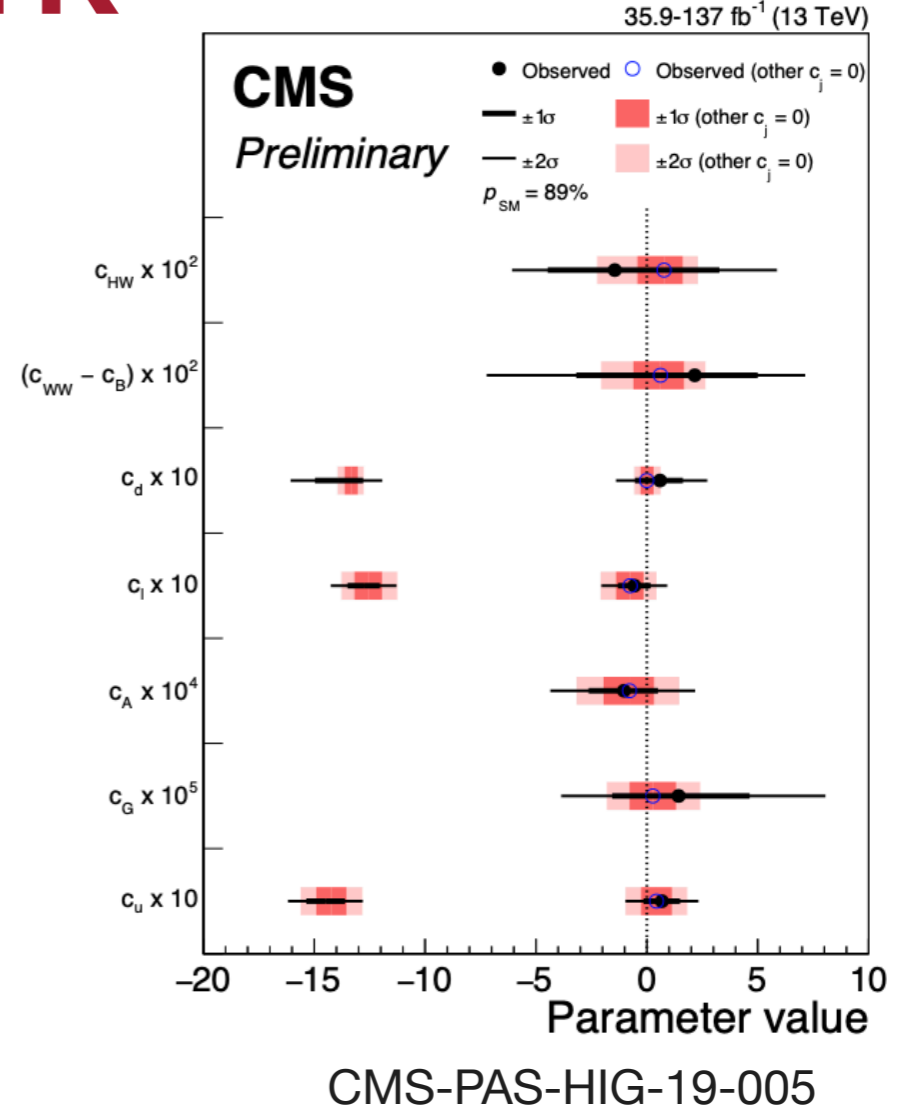
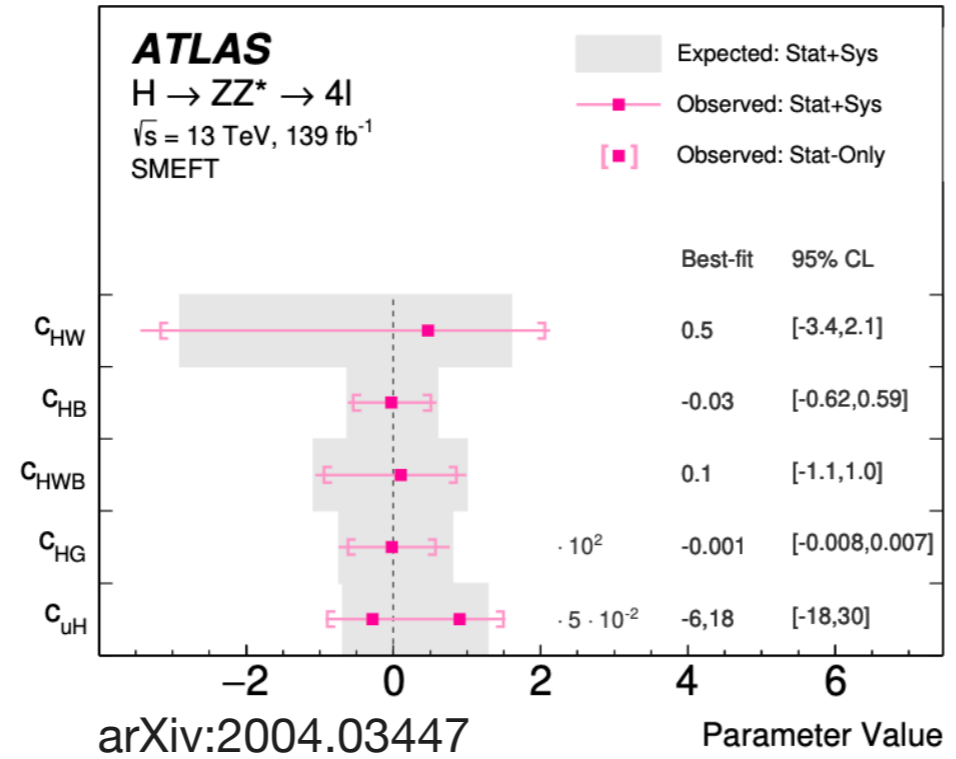
Complete (non-redundant) basis of effective operators that let us search for deviations

$\mathcal{O}_{ll}$	$(\bar{l}_L \gamma_\mu l_L)(\bar{l}_L \gamma^\mu l)_L$	$\mathcal{O}_{HWB}$	$(H^\dagger \tau^a H) W_{\mu\nu}^a B^{\mu\nu}$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^* (H^\dagger D_\mu H)$
$\mathcal{O}_{He}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_R \gamma^\mu e_R)$	$\mathcal{O}_{Hu}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_R \gamma^\mu u_R)$	$\mathcal{O}_{Hd}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_R \gamma^\mu d_R)$
$\mathcal{O}_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^a H)(\bar{q}_L \tau^a \gamma^\mu q_L)$	$\mathcal{O}_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_L \gamma^\mu q_L)$	$\mathcal{O}_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^a H)(\bar{l}_L \tau^a \gamma^\mu l_L)$
$\mathcal{O}_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_L \gamma^\mu l_L)$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	$\mathcal{O}_{eH}$	$(H^\dagger H)\bar{l}_L \tilde{H} e_R$
$\mathcal{O}_{HG}$	$(H^\dagger H)G_{\mu\nu}^A G^{\mu\nu,A}$	$\mathcal{O}_{uH}$	$(H^\dagger H)(\bar{q}_L \tilde{H} u_R)$	$\mathcal{O}_{dH}$	$(H^\dagger H)(\bar{q}_L H d_R)$
$\mathcal{O}_{HB}$	$(H^\dagger H)B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{HW}$	$(H^\dagger H)W_{\mu\nu}^a W^{\mu\nu,a}$	$\mathcal{O}_W$	$\epsilon_{abc} W_\mu^{\nu,a} W_\nu^{\rho,b} W_\rho^{\mu,c}$
$\mathcal{O}_H$	$(H^\dagger H)^3$				

(Note: not the full set here — lots of flavor / model-based assumptions to limit the ~3000 operators in the full EFT!)

# The SMEFT Framework

Experiments starting to present limits in this framework:



Well-known procedures for matching theories onto SMEFT, and getting coefficients in terms of theory parameters

(see e.g., Henning, Lu, and Murayama, arXiv:1412.1837)

Tree-Level Dictionary: de Blas, Criado, Perez-Victoria, Santiago, arXiv:1711.10391

# Beyond Tree Level Matching:

Lots of “higher-order” effects to consider:

- RG Evolution of Wilson Coefficients
- Linear vs. Quadratic Effects in  $(1/\Lambda^2)$
- One-Loop Matching Effects
- Importance of Dimension-8 Operators
- Higher Order QCD / EW Corrections in the EFT

See, e.g, Baglio, Dawson, SH, [arXiv:1909.11576](https://arxiv.org/abs/1909.11576),  
Baglio, Dawson, SH, Lane, Lewis, [arXiv:2003.07862](https://arxiv.org/abs/2003.07862), for importance in  $VV$ ,  $VH$

**Focus on the  
impacts of  
these today**

# Strategy:

**Example:**  $T$  Vector-like Quark

Start with Lagrangian for new states at high scale, ( $M \sim \text{few TeV}$ )

$$\mathcal{L} \supset \lambda_3 \bar{Q}_L \tilde{H} T_R$$

Integrate out new states, generating a subset of SMEFT Coefficients

$$\left(C_{Hq}^{(1)}\right)_{33}, \left(C_{Hq}^{(3)}\right)_{33}, C_{tH}, C_{HG}$$

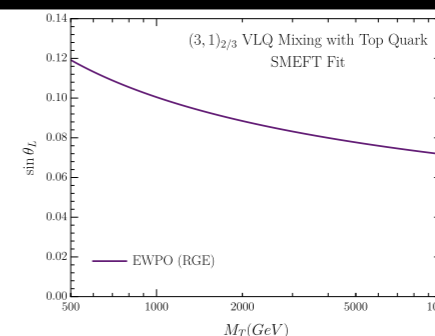
**Evolve Coefficients down to EW scale**

(Using Anomalous Dimensions from Trott et. al, 1308.2627+)

$$C_{HD}, C_{H\Box} \dots$$

Fit to Higgs + Diboson + EWPO Data

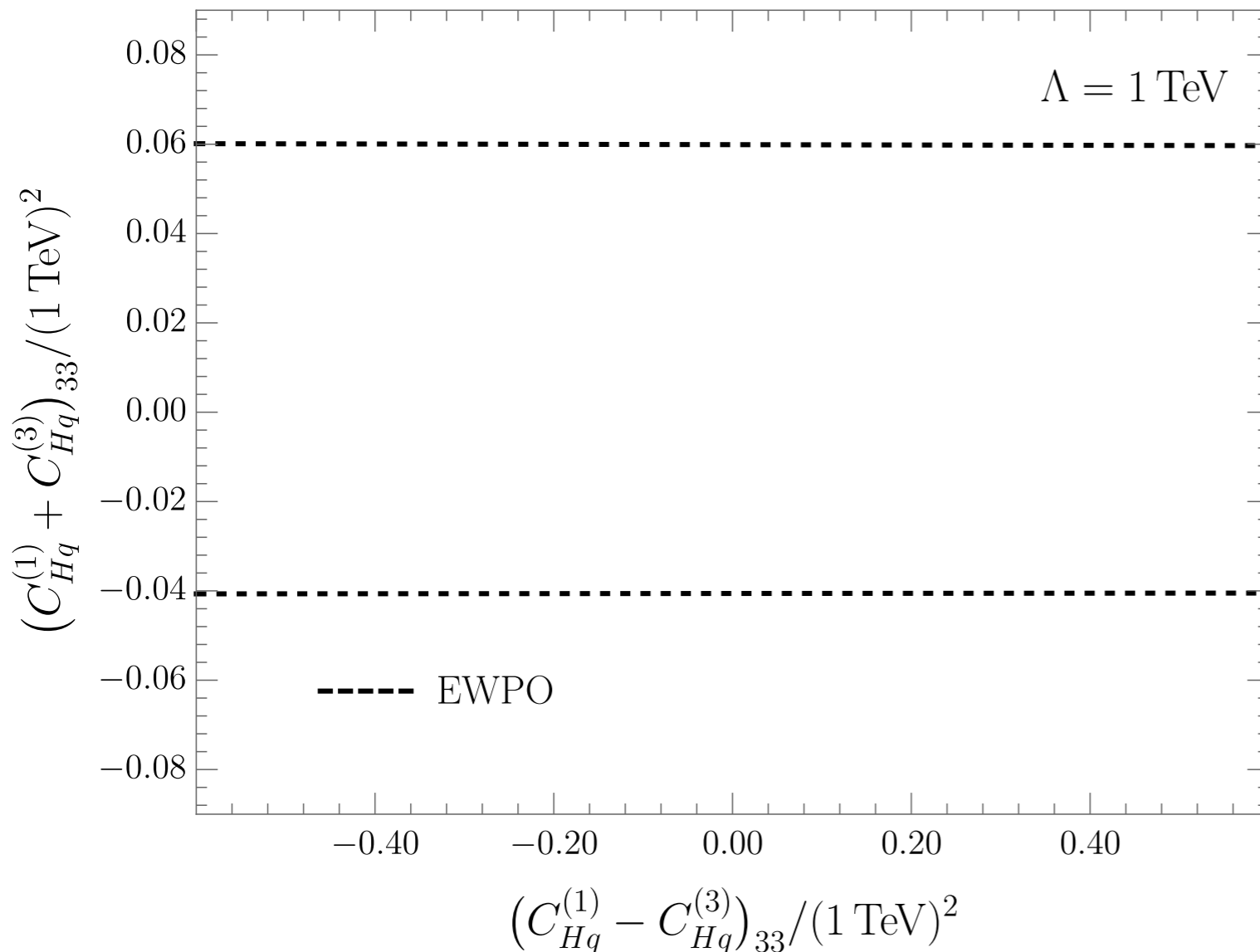
➔ Limits on physical parameters!



# SM + VLQ Singlet Mixing with Top

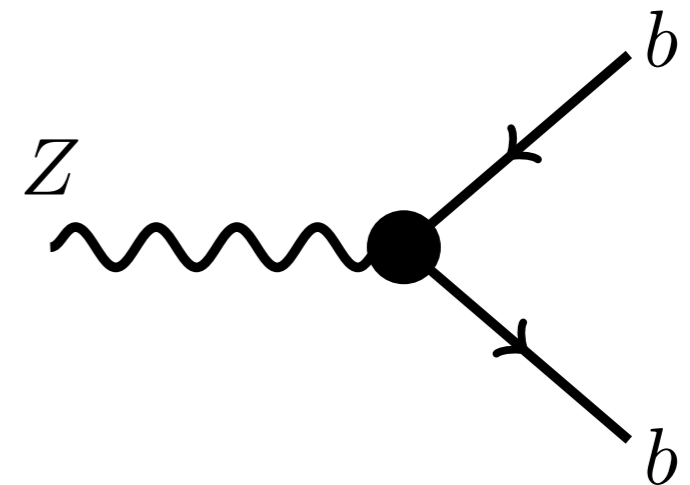
Generates  $C_{tH}$ ,  $(C_{Hq}^{(1)})_{33}$ ,  $(C_{Hq}^{(3)})_{33}$ ,  $C_{HG}$  at the matching scale

RG Effects on the LEP Bounds:



At tree level, only one way to measure operators

$$C_{Hq,33}^{(1)}, C_{Hq,33}^{(3)}$$

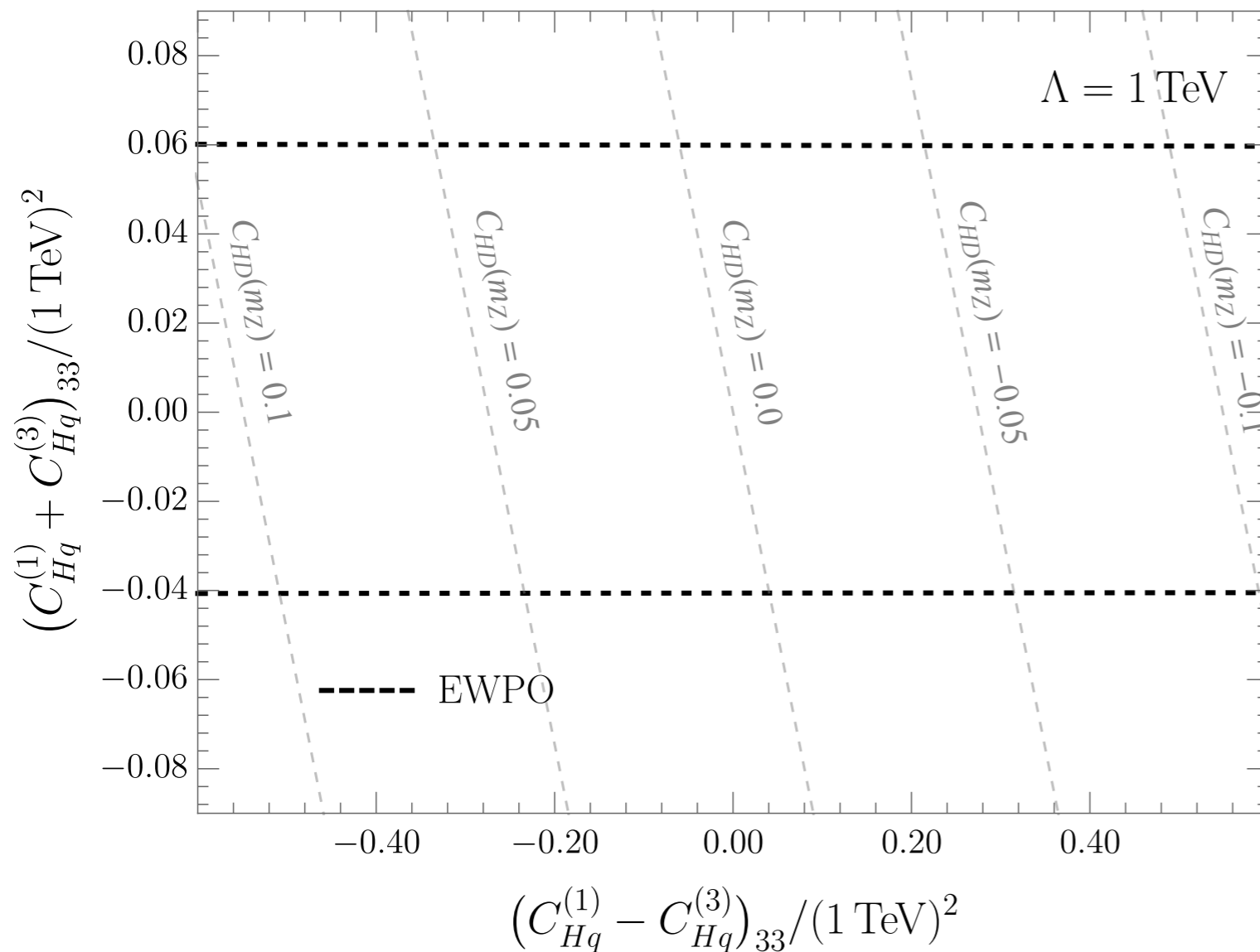


$Z \rightarrow b\bar{b}$  branching ratio constrains **one** combination of operators.

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But if we evolve the operators down to the weak scale...

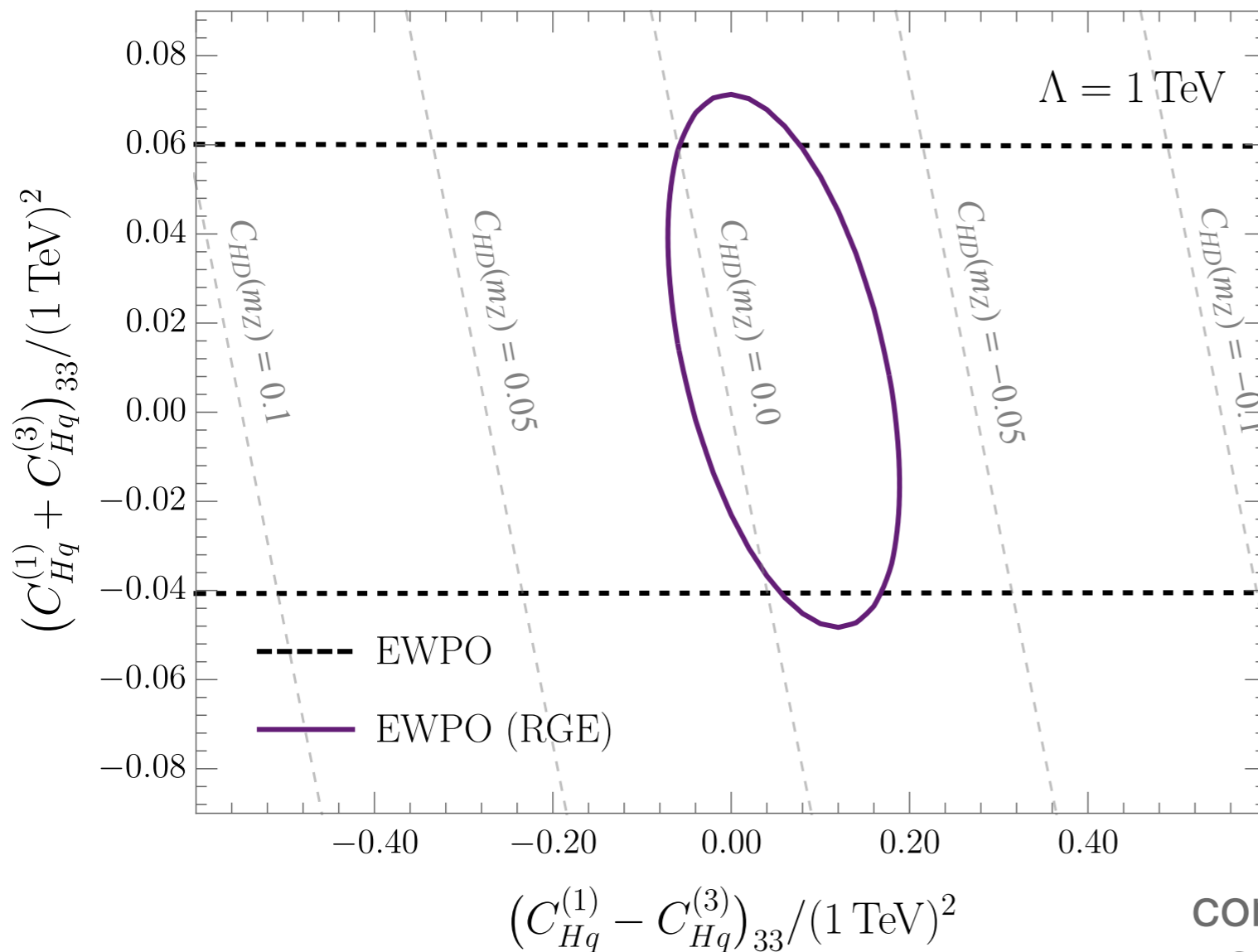
... we generate  $C_{HD}$  — strongly constrained by EW precision data!



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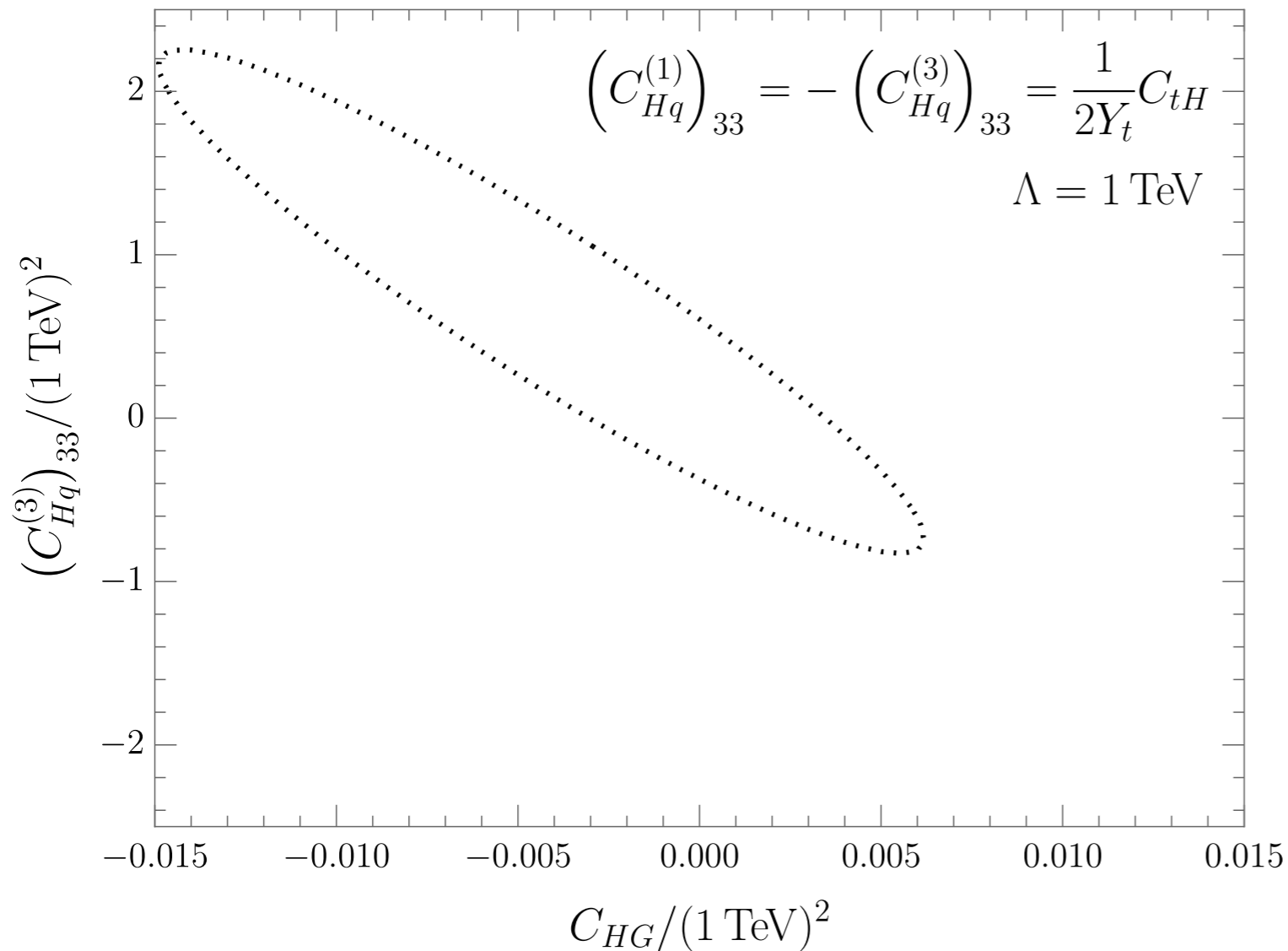
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(Well understood in UV theory — constraints from oblique parameters, e.g., Chen, Dawson, Furlan, [arXiv:1406.3349](https://arxiv.org/abs/1406.3349))

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Similar lessons apply to LHC bounds:

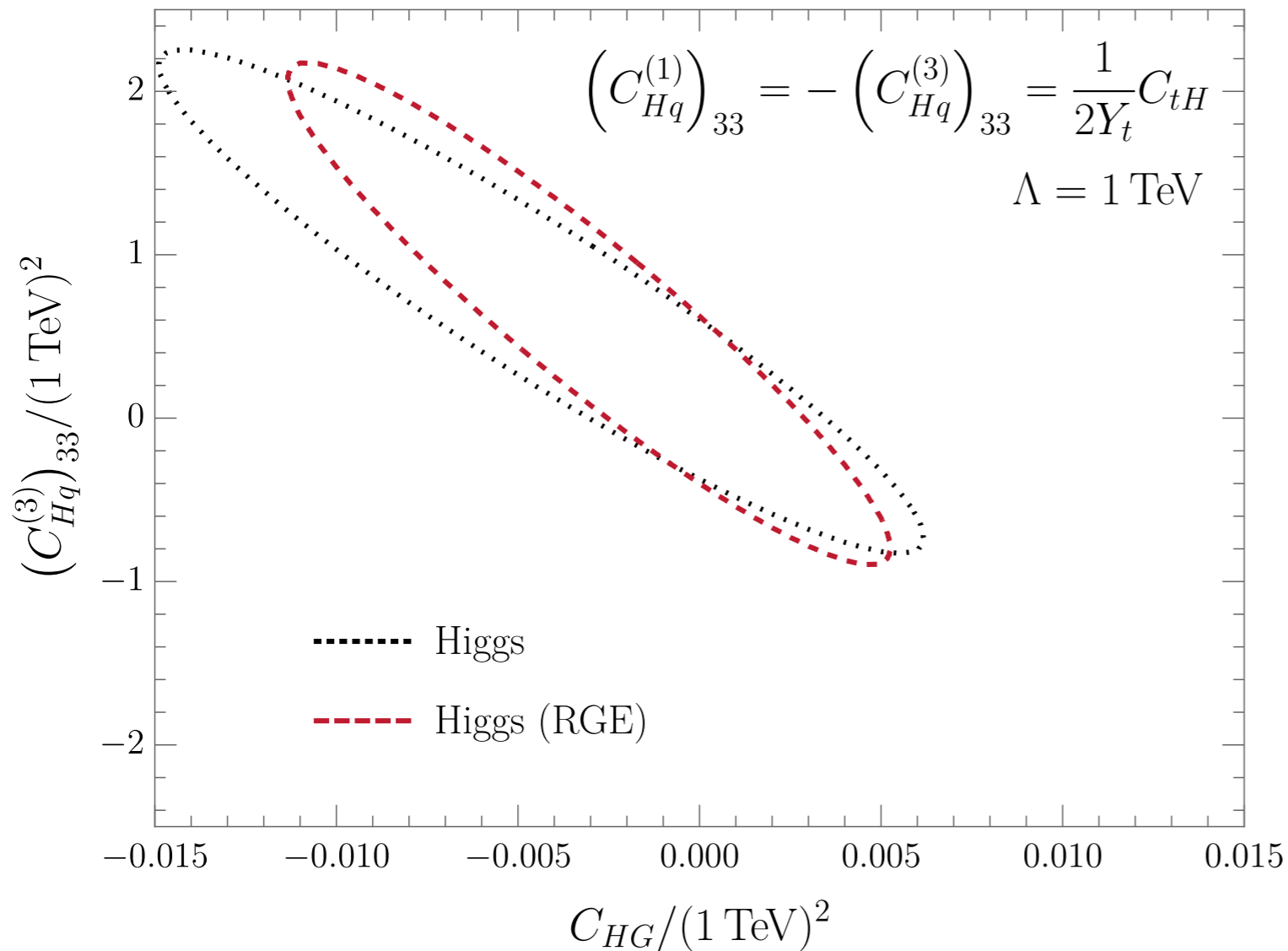


Operators that arise at tree level constrained by Higgs coupling fits

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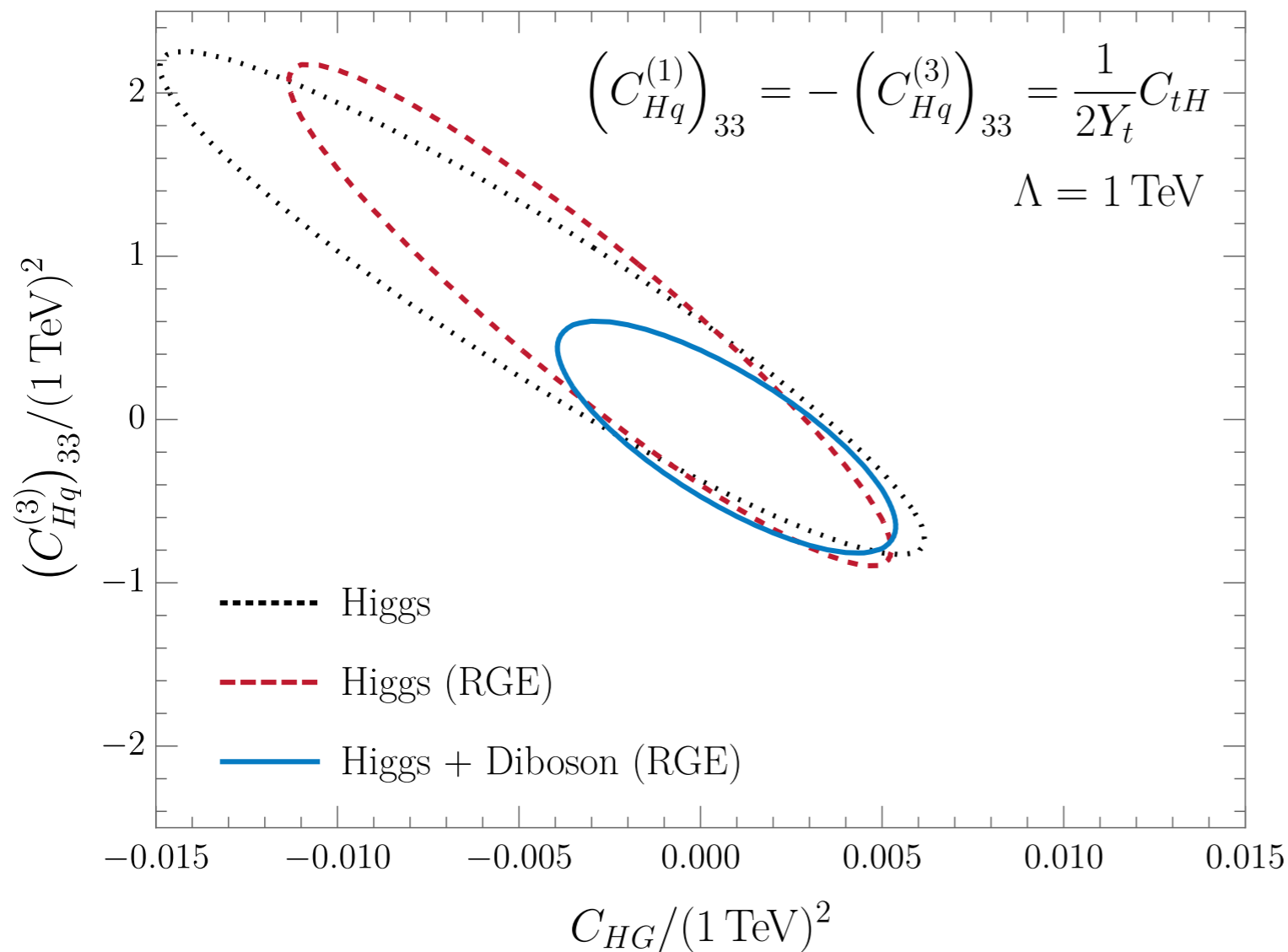
Operators that arise at tree level constrained by Higgs coupling fits

RG evolution changes the values of these operators at the weak scale, so the inferred bounds are different!

# SM + VLQ Singlet Mixing with Top

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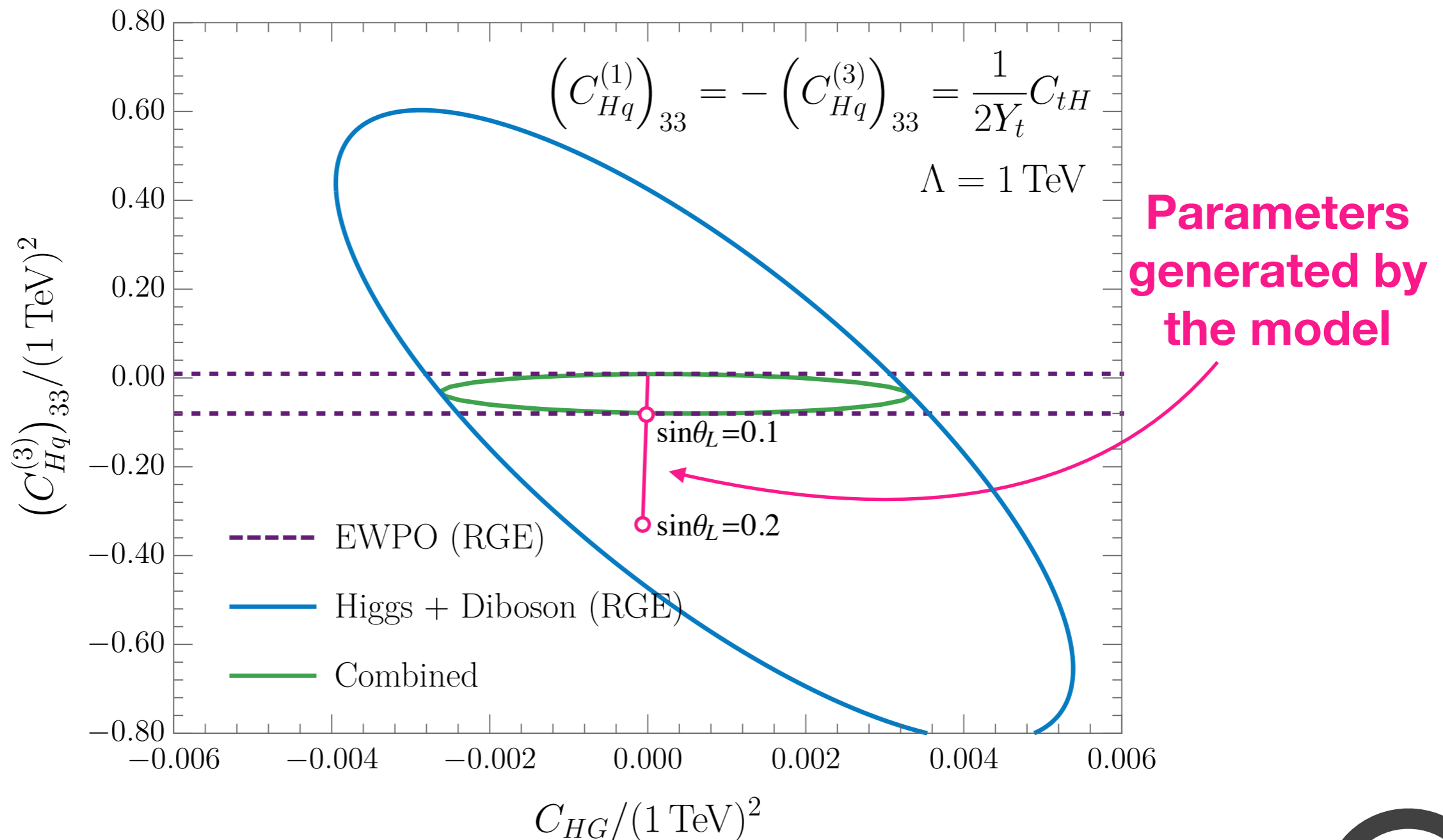
But RG evolution also generates *new* operators measured at the LHC in  $WW$ ,  $WZ$ ,  $WH$ ,  $ZH$  production!

Strongest constraint from RGEs!

Note: NLO-QCD effects are *very* important for diboson limits (see [arXiv:1909.11576](https://arxiv.org/abs/1909.11576), [arXiv:2003.07862](https://arxiv.org/abs/2003.07862))

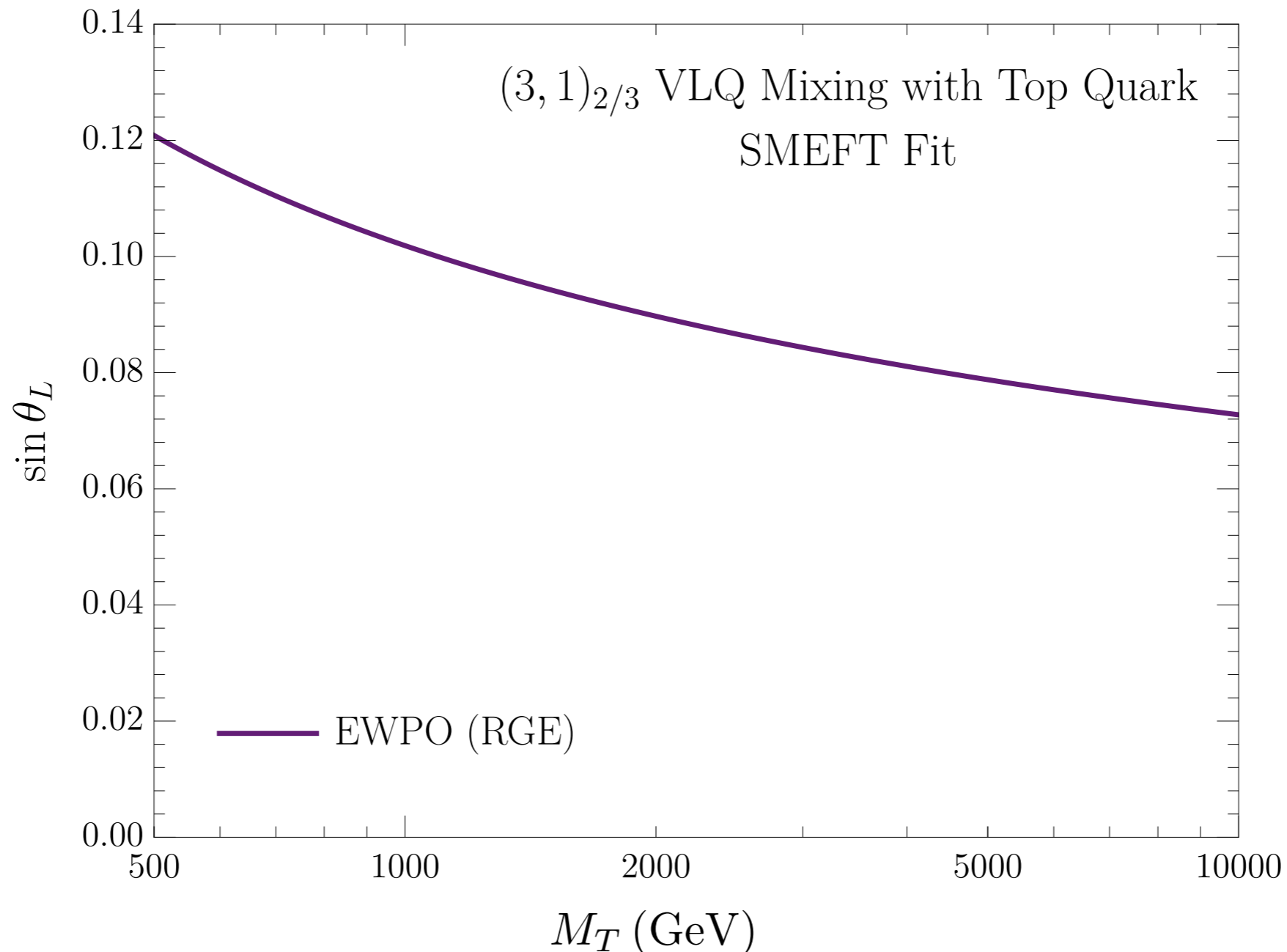
# SM + VLQ Singlet Mixing with Top

Back to the inverse problem: the T VLQ is a 1-parameter model — sweeps out only 1D curve in operator space



# SM + VLQ Singlet Mixing with Top

Now we can re-interpret the bounds on operators as bounds on the physical parameters of the model:



# Example 2: The Singlet Model

arXiv:2102.02823

Simplest extension to the SM — only one additional state

Ideal test case for investigating details of matching procedure

- theoretical constraints well understood
- one-loop matching results are known  
(Jiang et al., 1811.08878, Haisch et al., 2003.05936)

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$$C_i(\mu_R) = c_i(M) + \frac{1}{16\pi^2} d_i(M) + \frac{1}{32\pi^2} \gamma_{ij} c_j(M) \log \left( \frac{\mu_R^2}{M^2} \right)$$

Goal: understand numerical importance of 1-loop matching effects in the context of the singlet model



# The Singlet Model

$$V(\Phi, S) = -\mu_H^2 \Phi^\dagger \Phi + \lambda_H (\Phi^\dagger \Phi)^2 + \frac{1}{2} m_\xi \Phi^\dagger \Phi S + \frac{1}{2} \kappa \Phi^\dagger \Phi S^2 \\ + t_S S + \frac{1}{2} M^2 S^2 + \frac{1}{3} m_\zeta S^3 + \frac{1}{4} \lambda_S S^4$$

In  $Z_2$  non-symmetric case, use shift symmetry to set  $v_S \rightarrow 0$

Physical states:

Masses  $m_h = 125$  GeV,  $M_H$

$$h = \cos \theta \Phi_0 + \sin \theta S$$

Other physical parameters:

$$H = -\sin \theta \Phi_0 + \cos \theta S$$

$$\sin \theta, \kappa, m_\zeta, \lambda_S$$

Higgs couplings universally suppressed by  $\cos \theta$

# Unitarity and Vacuum Stability

The physical parameters are not entirely arbitrary!

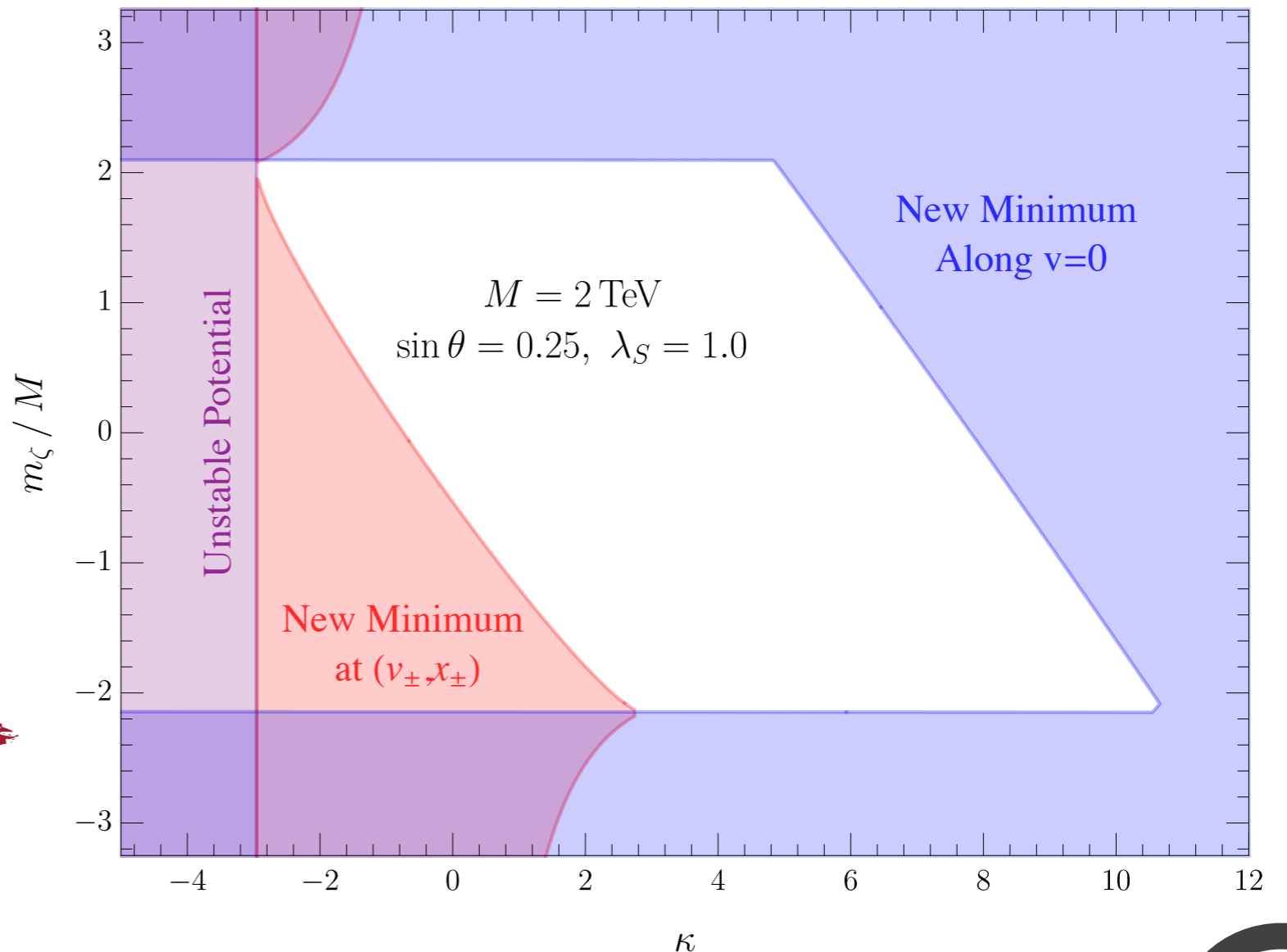
Unitarity of the  $hh$ ,  $hH$ ,  $HH$  amplitudes requires:

$$M_H^2 \sin^2 \theta \lesssim \frac{16\pi}{3} v^2 - m_h^2 \cos^2 \theta$$

$$\lambda_S, \lambda_H \lesssim 8\pi/3$$

$$|\kappa| \lesssim 8\pi$$

Furthermore, have to demand that the EWSB minimum be the global minimum of the potential



# Singlet Matching to SMEFT

Two coefficients are generated at tree-level:

$$C_{H\Box} = -\frac{m_\xi^2}{8M^2}$$

$$C_H = \frac{m_\xi^2}{8M^2} \left( \frac{m_\xi m_\zeta}{3M^2} - \kappa \right)$$

Perform matching at the scale  $M$ ,  
related to the physical mass via

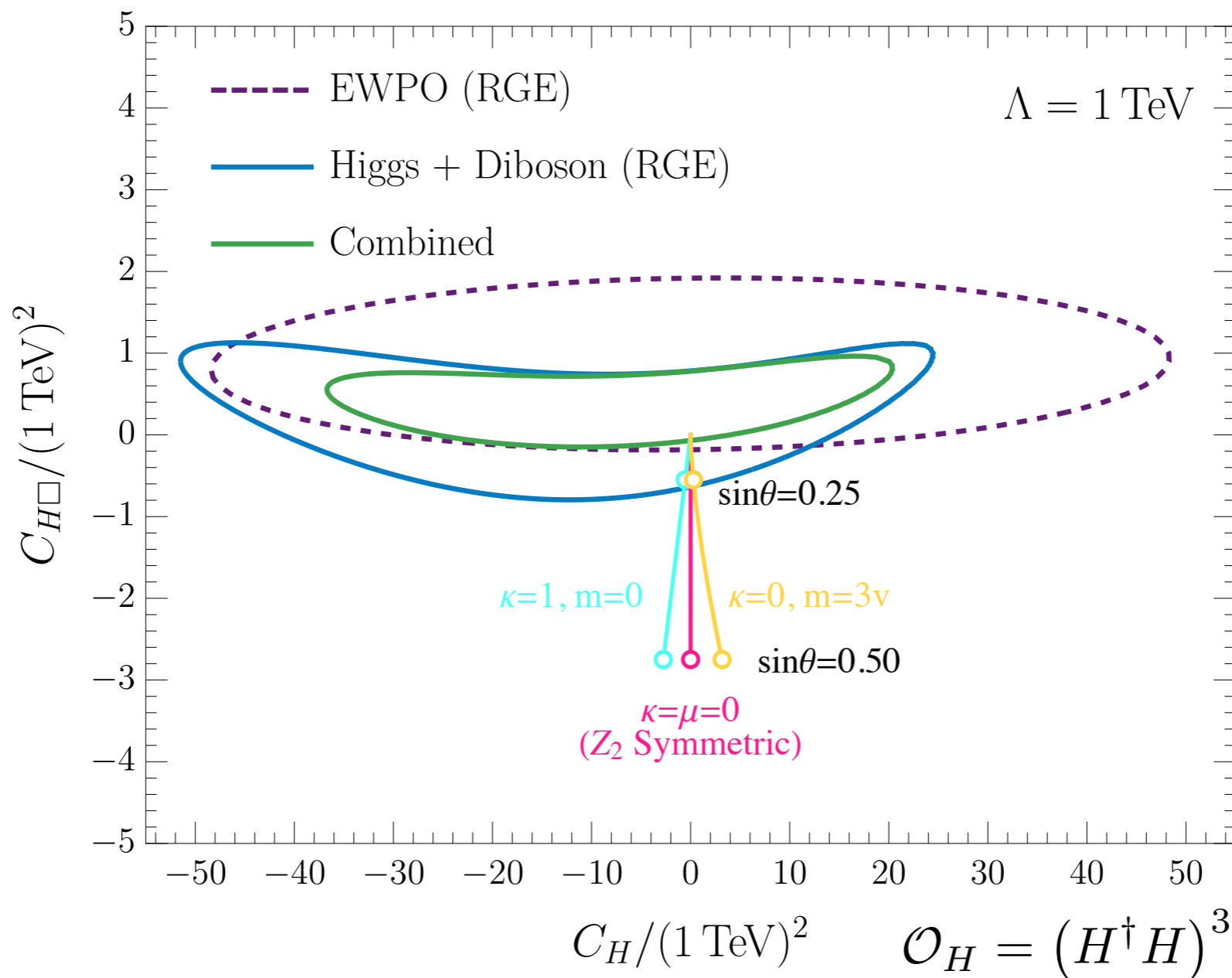
$$M^2 = m_h^2 \sin^2 \theta + M_H^2 \cos^2 \theta - \frac{\kappa}{2} v^2$$

These operators introduce

$$C_{HD}, C_{tH}, C_{bH}, C_{\tau H}, C_{Hl}^{(3)}, C_{Hq}^{(3)}, C_{Htb}$$

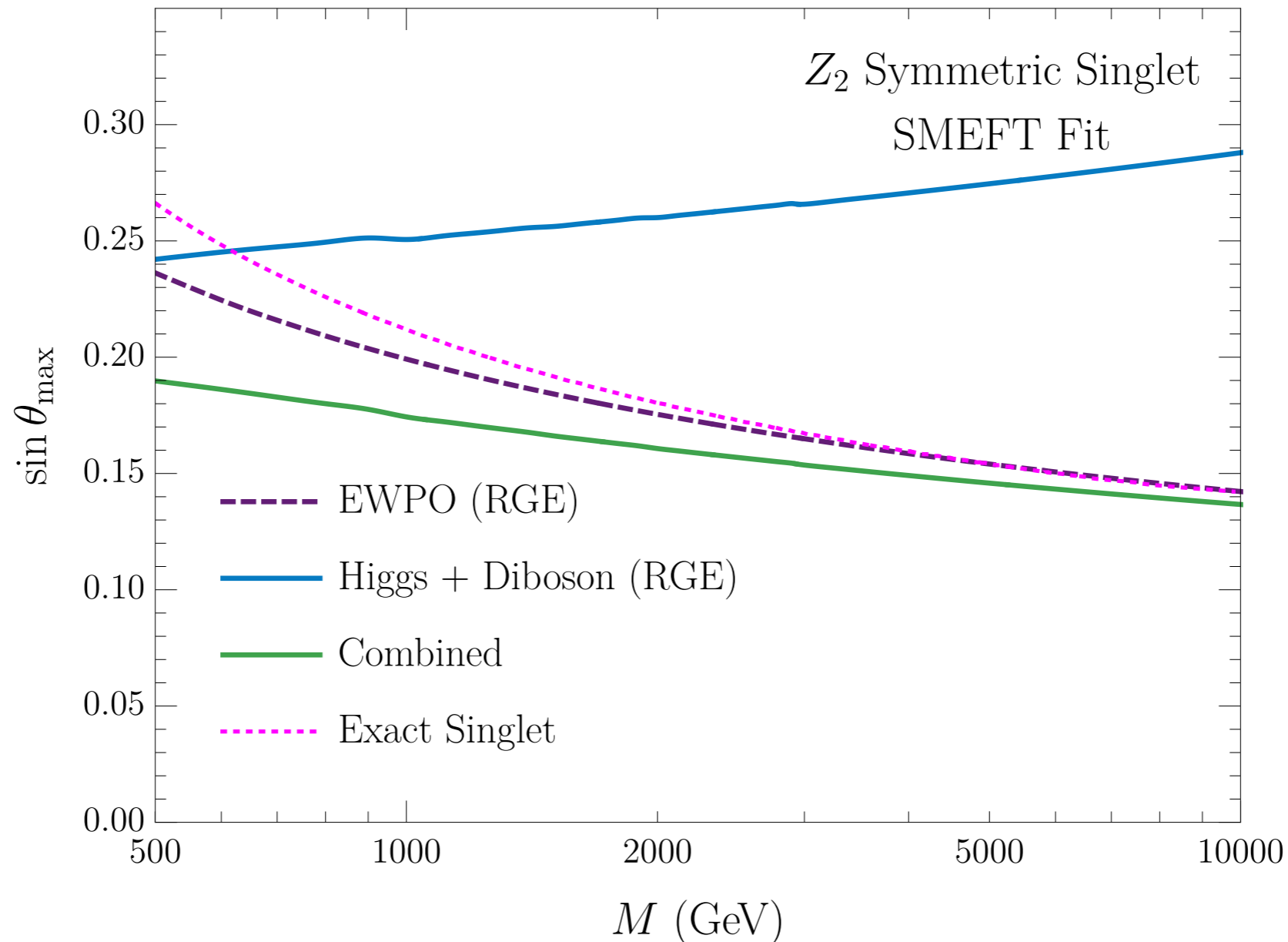
at the weak scale from RG running

# Tree Level (+RGE) Results



Limits on the singlet from EWPO and LHC competitive — but most allowed coefficients cannot be generated in the model

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# One-Loop Matching

Jiang, Craig, Li, Sutherland [1811.08878],

Haisch, Ruhdorfer, Salvioni, Venturini, Weiler [2003.05936]

New contributions to  $C_H, C_{H\Box}$  at the matching scale...

$$d_{H\Box} = -\frac{9}{2}\lambda c_{H\Box} + \frac{31}{36}(3g^2 + g'^2)c_{H\Box} + \frac{3}{2}c_H + \delta d_H + \delta d_{H\Box}^{\text{shift}}$$

$$d_H = \lambda \left[ \frac{1}{9}(62g^2 - 336\lambda)c_{H\Box} + 6c_H \right] + \delta d_H + \delta d_H^{\text{shift}}$$

...as well as many operators that don't appear at tree-level

$C_{HD}, C_{HW}, C_{HB}, C_{HWB}, C_{Hu}, C_{Hd},$

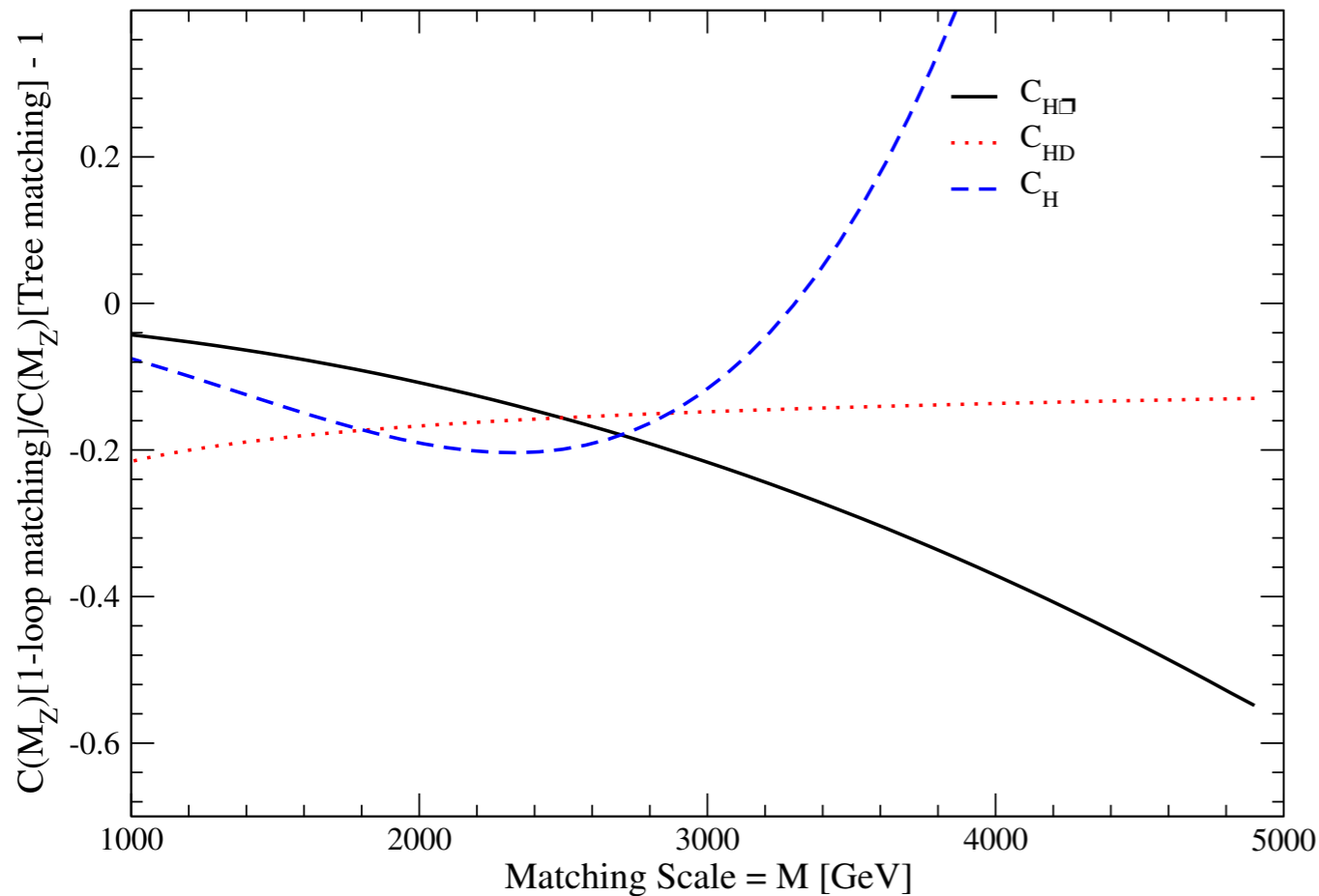
$C_{Hq}^{(1)}, C_{Hq}^{(3)}, C_{Hl}^{(3)}, C_{tH}$

**In principle of comparable size to RGE-induced contribution!**

# One-Loop Matching

SMEFT Limit of Singlet Model

$\cos \theta = .98, \kappa = .5, m_\zeta = M/4, \lambda_s = .03$



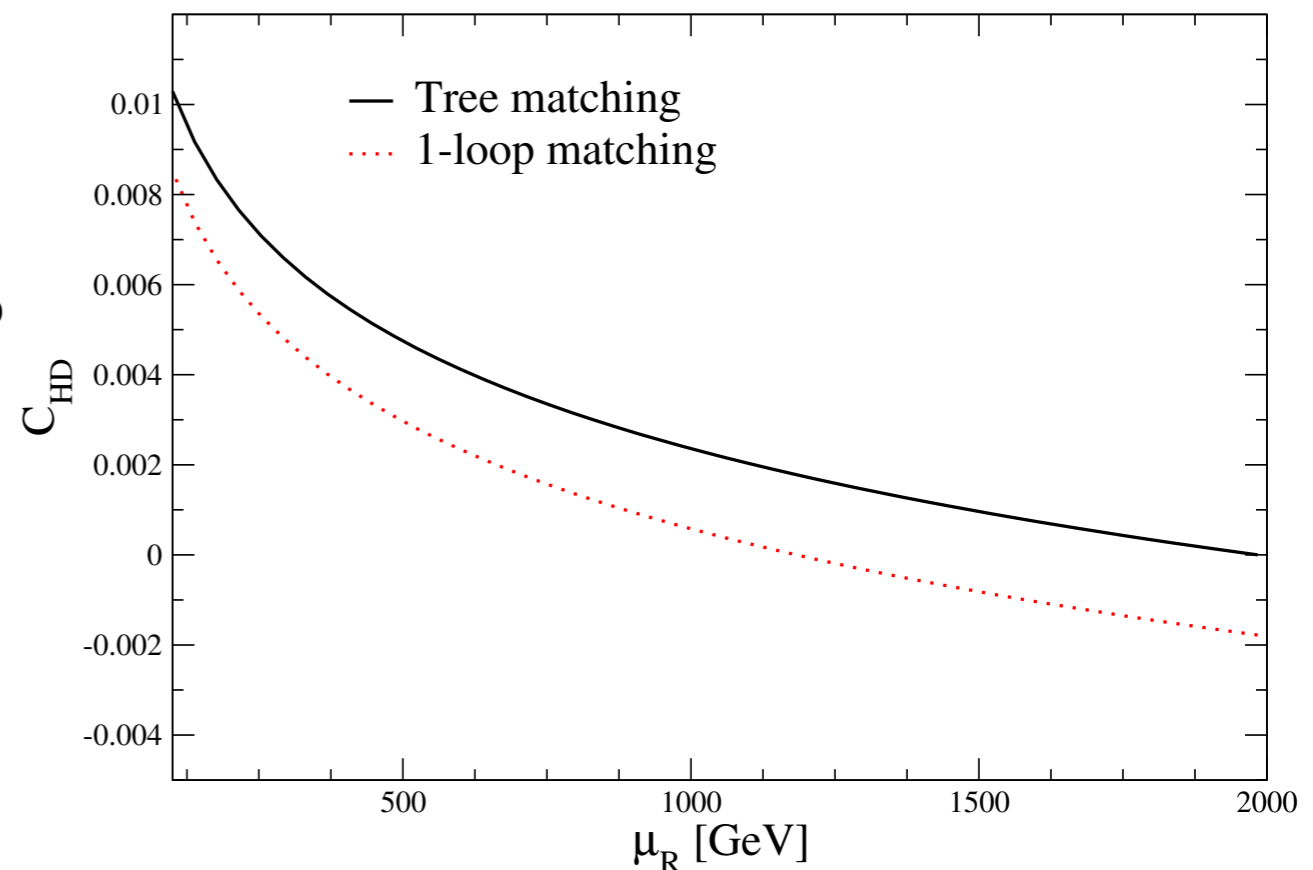
One-loop matching changes operators by  $\sim 10\text{-}20\%$  as measured at the weak scale

Include only one-loop RGEs

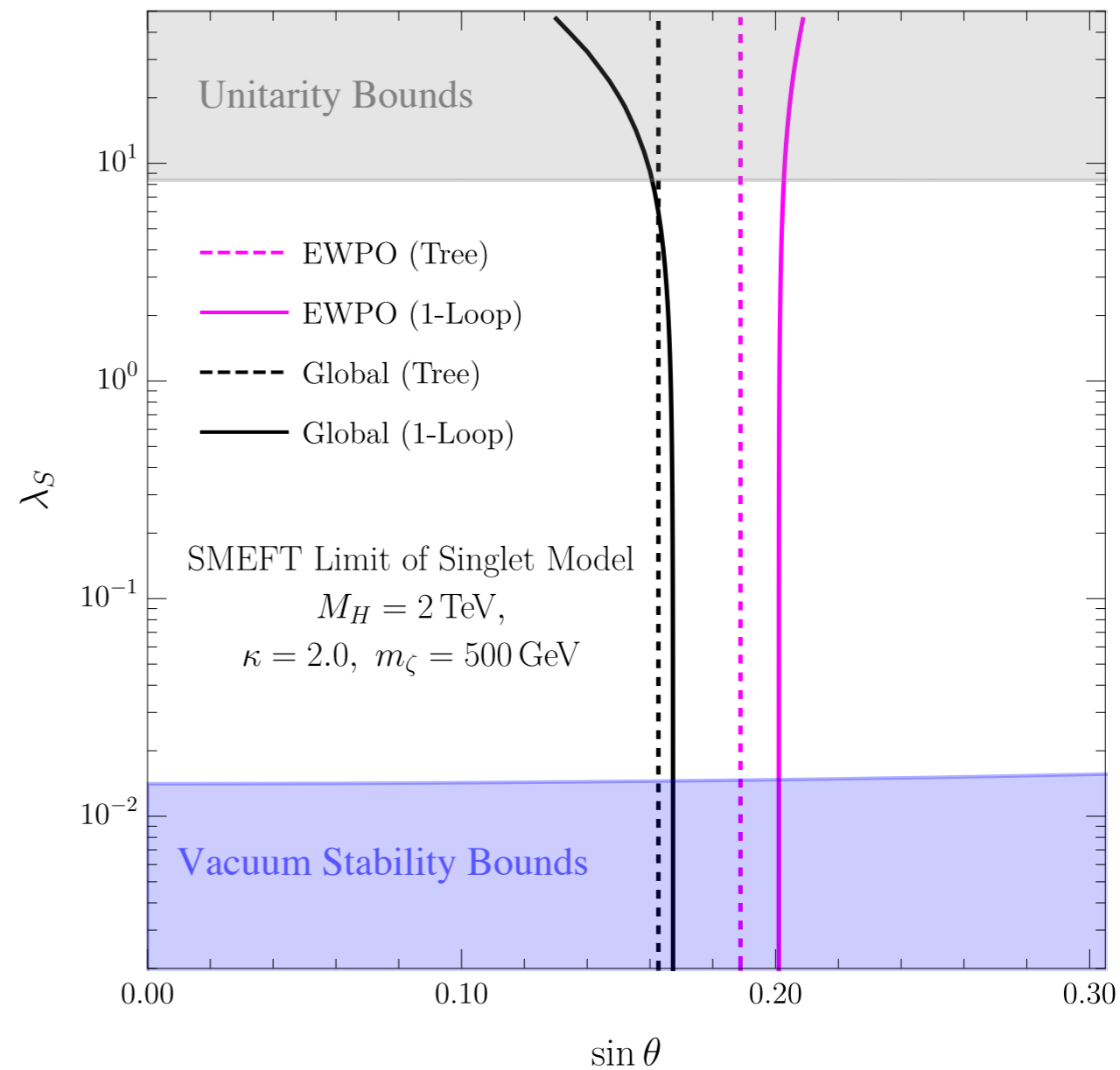
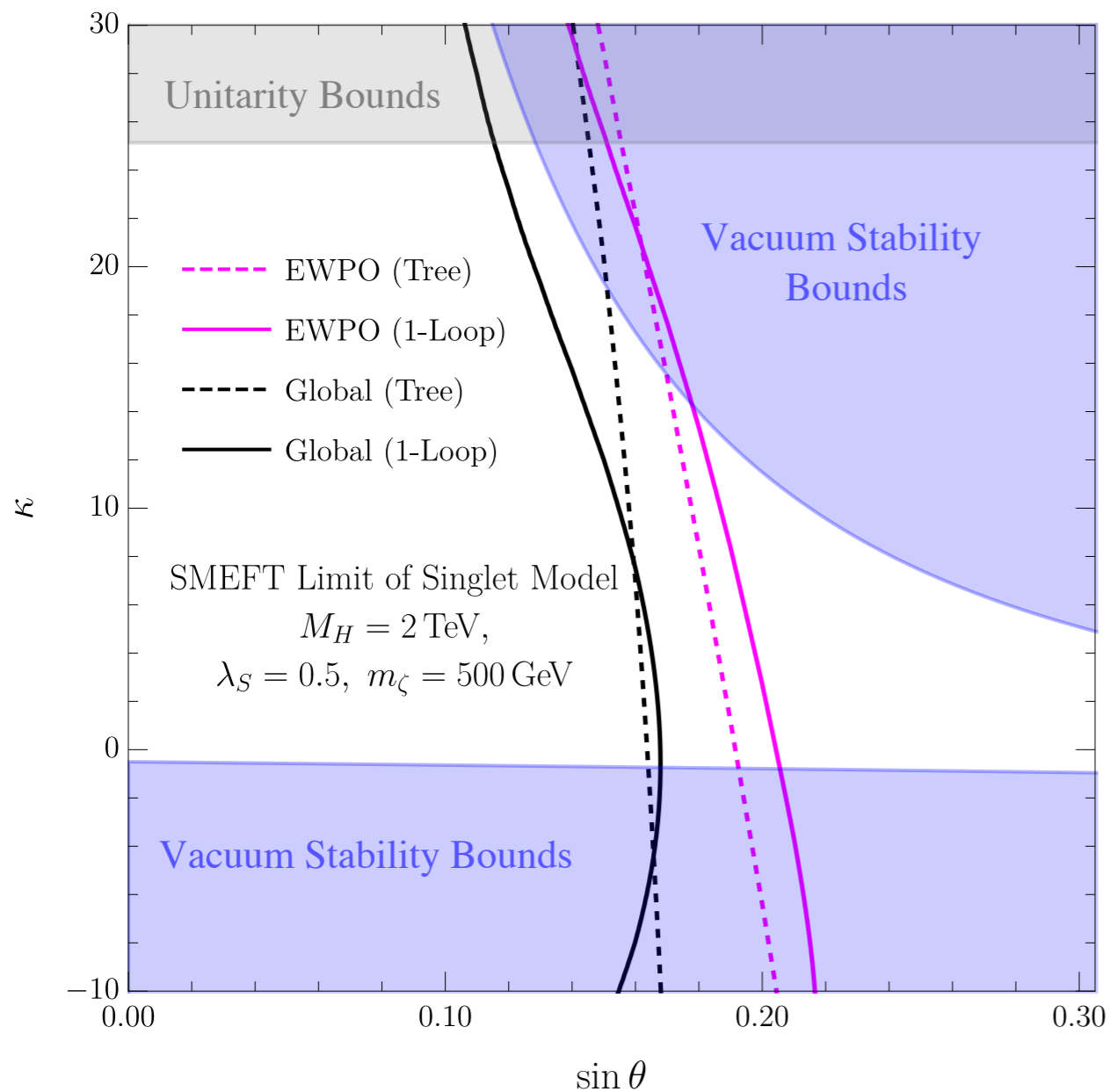
(two loops unavailable, but necessary to run one-loop induced operators)

SMEFT Limit of Singlet Model

$M_H = 2 \text{ TeV}, \cos \theta = .99, \kappa = -.5, m_\zeta = 500 \text{ GeV}, \lambda_s = .03$



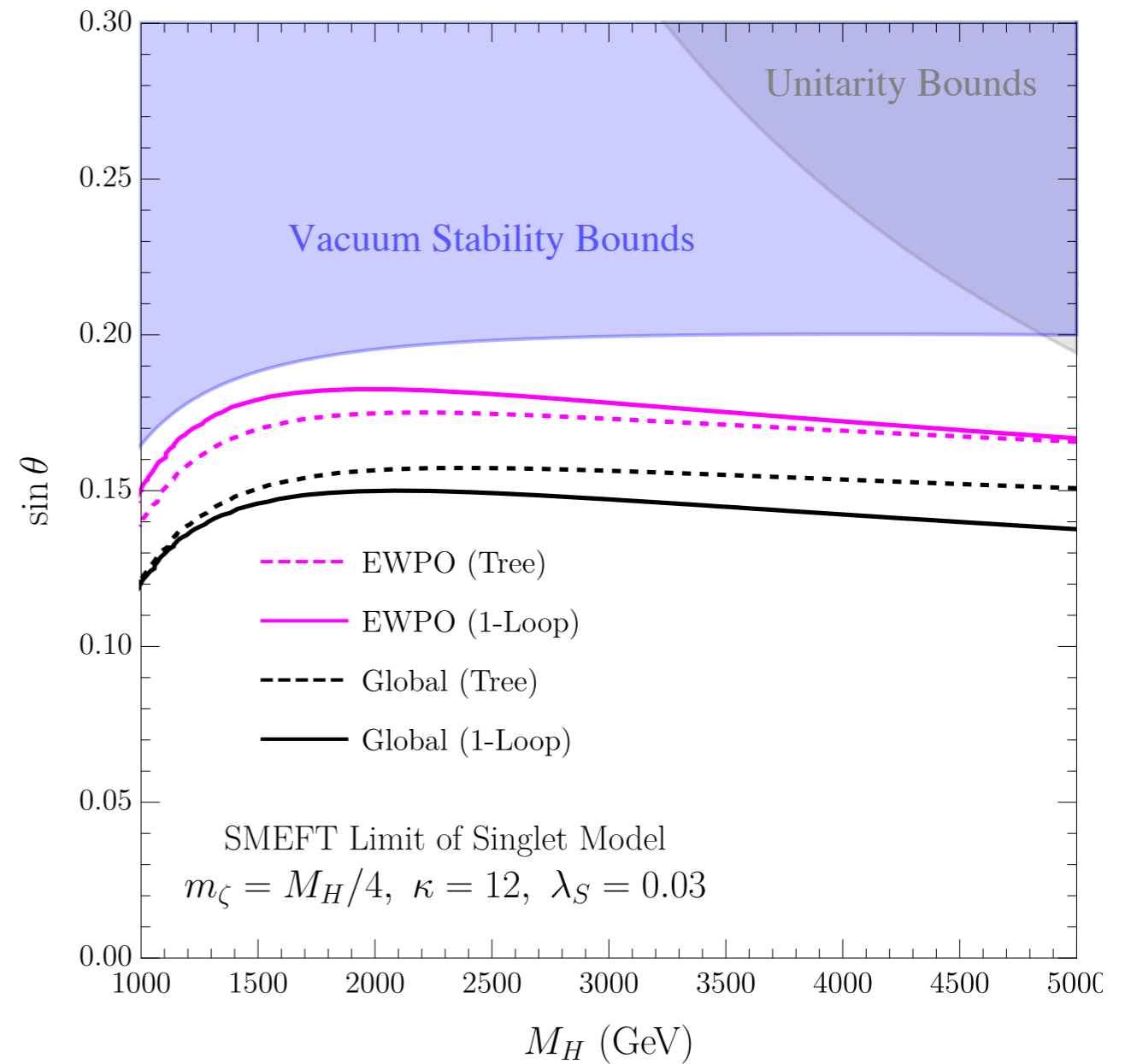
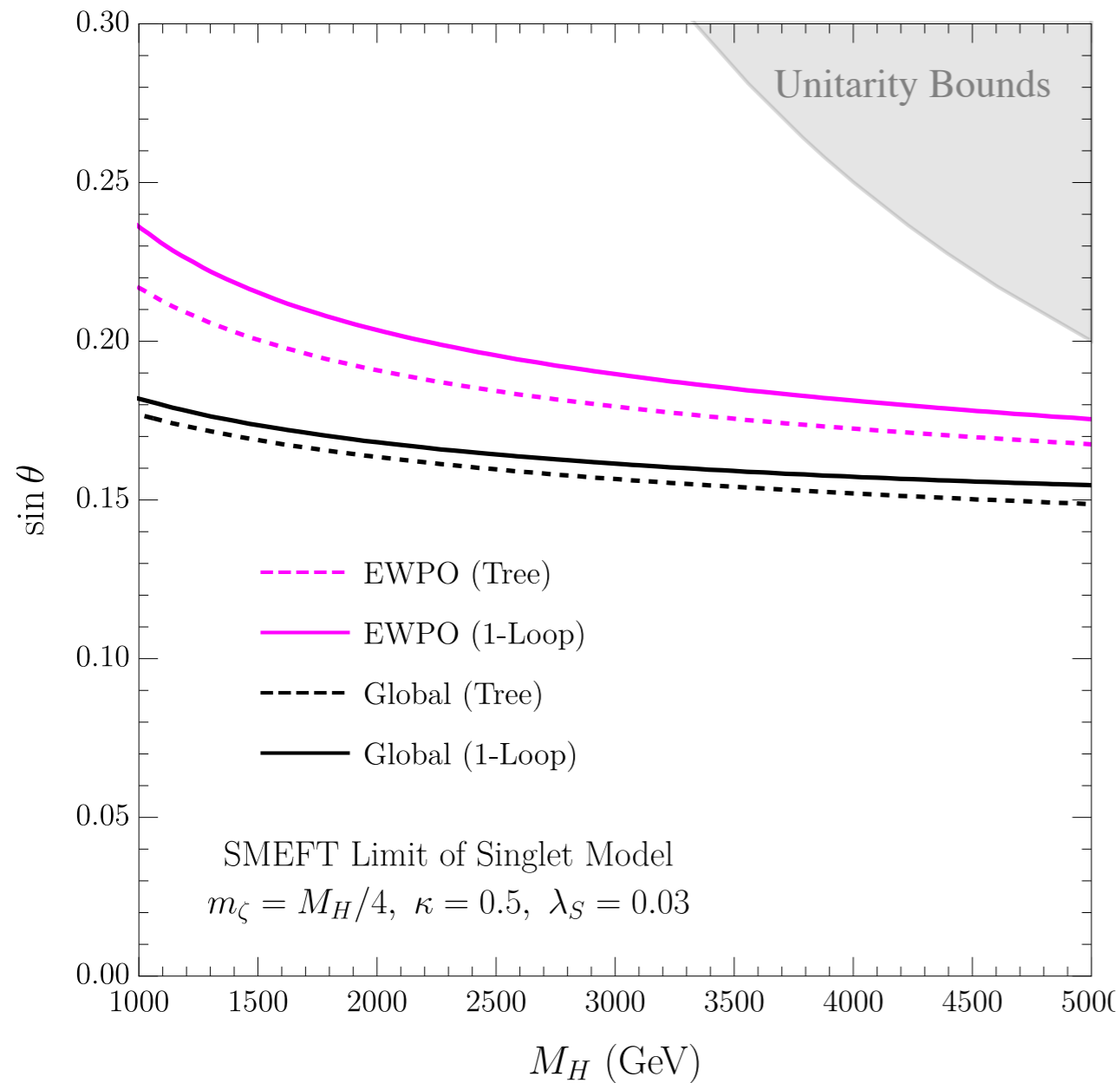
# Effects on the Fit



Effects mostly  $O(10\%)$ , except for large values of portal coupling



# Effects on the Fit



Effects mostly O(10%), except for large values of portal coupling

# Conclusions:

- The Higgs Inverse Problem is the next phase of LHC Higgs Physics
- Tree level interpretations of SMEFT Fits aren't the whole story!  
*RG evolution of coefficients is extremely important*

## Lots more work to do:

- More robust understanding what coefficients can be generated
- Understand linear vs. quadratic approximation in fits in context of models?
- Include complete one-loop matching for other models, more NLO effects in fits, and more distributions
- Importance of effects from dimension-8 operators?

Lots of other recent work on this topic!

See:

- Ellis, et al., [2012.02779]
- Das Bakshi, et al., [2012.03839]
- Marzocca, et al., [2009.01249]
- Brivio et al., [2108.01094]
- Almeida et al., [2108.04828],
- ... and others!