Understanding Initial Emittance Growth in IOTA

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Outline

- Part 1: Initial emittance growth
  - TM report recap
  - Sources of emittance growth
  - Mismatch theory
  - KV vs Gaussian distributions
  - How to minimize early loss
- Part 2: Octupoles and space charge
  - TM report recap
  - Low intensity results
  - Truncated distribution results
  - Full intensity results
  - Conclusions/future work

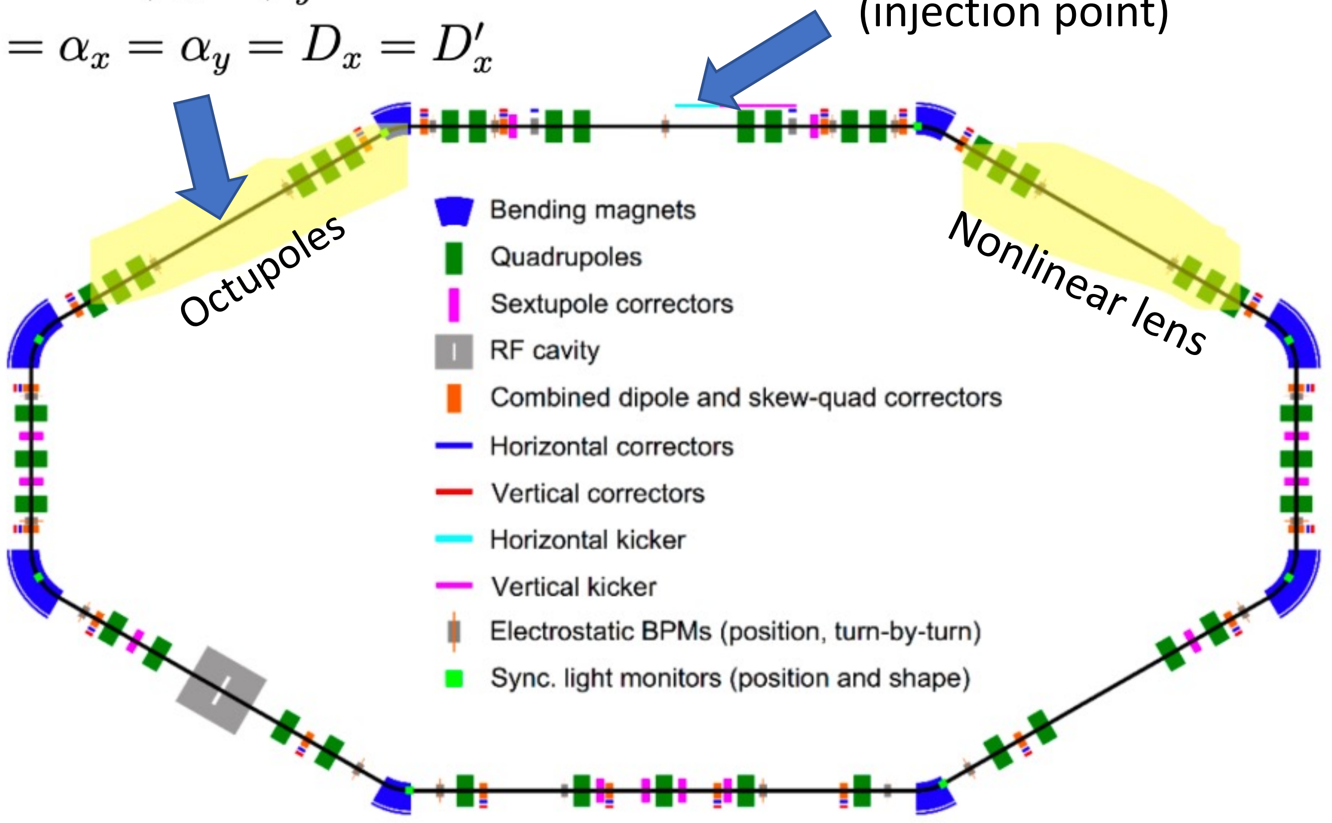
Project Goals

- Original
  - Investigate if octupoles and/or a specially profiled lens can mitigate space charge driven resonances in IOTA
    - Retain the benefits of integrability
    - Keep loss at an acceptable level
- Revised
  - Understand sources of initial emittance growth
  - Minimize initial loss

IOTA Proton Beam Parameters

$$\beta_x = \beta_y$$
$$0 = \alpha_x = \alpha_y = D_x = D'_x$$

Initial start of lattice (injection point)

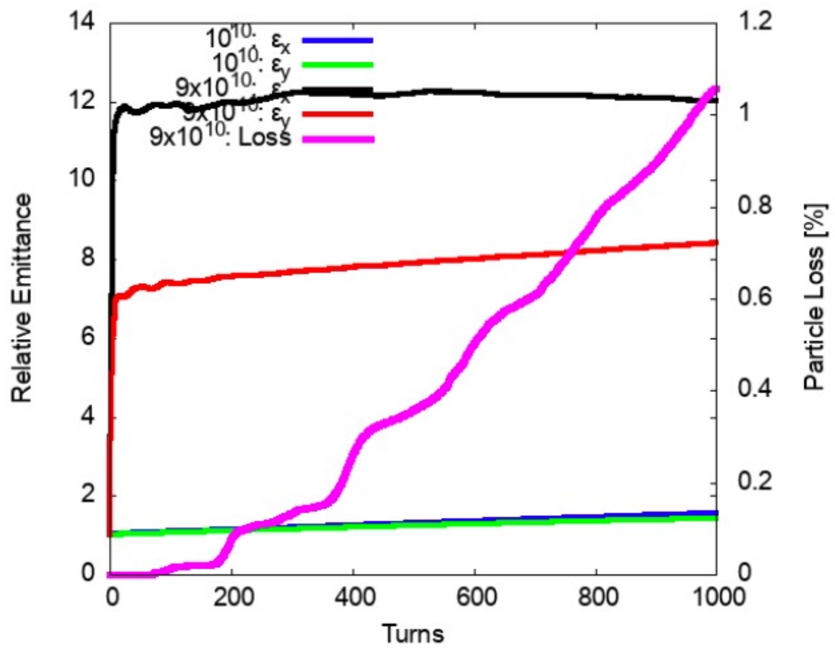


IOTA proton parameters	
Circumference	39.97 [m]
Kinetic Energy	2.5 [MeV]
Maximum bunch intensity /current	9×10^{10} / 8 [mA]
Transverse normalized rms emittance	(0.3, 0.3) [mm-mrad]
Betatron tunes	(5.3, 5.3)
Natural chromaticities	(-8.2, -8.1)
Average transverse beam sizes (rms)	(2.22, 2.22) [mm]
Kinematic γ / Transition γ_T	1.003 / 3.75
Rf voltage	400 [V]
Rf frequency / harmonic number	2.2 [MHz] / 4
Bucket wavelength	$\sim$ 10 [m]
Bucket half height in $\Delta p/p$	3.72×10^{-3}
rms bunch length	1.7 [m]
rms energy /momentum spread	1.05×10^{-5} / 1.99×10^{-3}
Beam pipe radius	25 [mm]
Bunch density	6.9×10^{14} [m^{-3}]
Plasma period τ_p	0.18 [μ-sec] / 0.1 [turns]
Average Debye length λ_D	559 [μm]

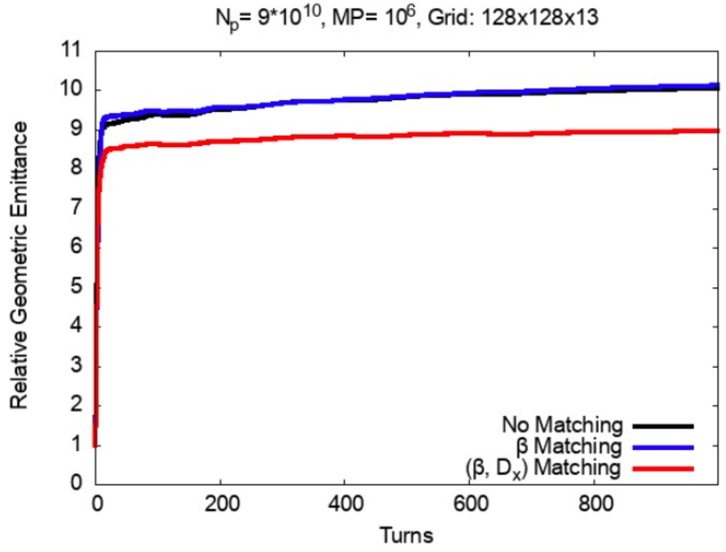
Emittance Growth/Mismatch

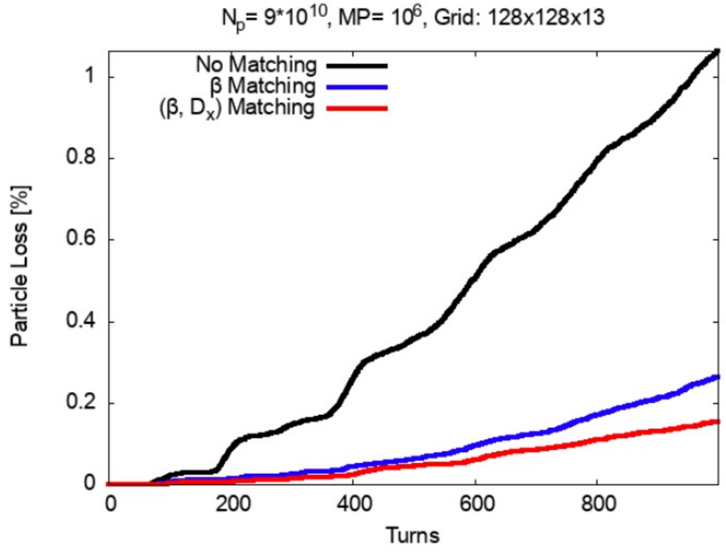
Fermilab TM Report Recap*

- At full intensity, RMS emittance grows 10-fold and loss exceeds 1% after 1000 turns (top right)
  - Most growth is in the first few turns
    - Time scale set by plasma oscillation period
      - 0.1 turns for a 3D Gaussian Bunch



- RMS matching at injection point (where $\beta_x \neq \beta_y$) does not reduce emittance growth much but reduces loss to about 0.2% over 1000 turns
  - RMS matching emittance growth (bottom left) and particle loss (bottom right)

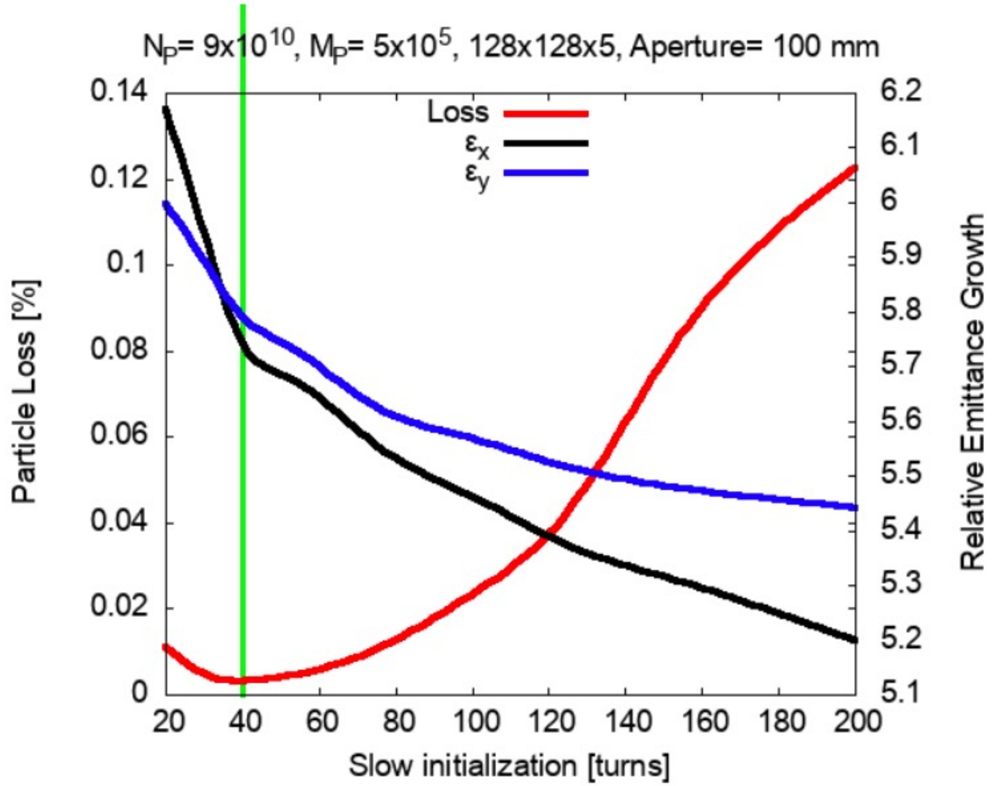




*Results from FERMILAB-TM-2753-AD

Fermilab TM Report Recap cont.*

- Slow initialization (w/o RMS matching) reduces emittance growth by a factor of 2 (10-fold increase to 6-fold increase) and particle loss to ~0.01%
  - Shown on the right, where emittance growth and loss are plotted as a function of slow initialization turns
- Important takeaways:
  - Most growth is over the first few turns, therefore not due to betatron resonance effects
  - RMS matching has minimal impact on emittance growth, but significantly affects loss, suggesting the halo population is sensitive to the RMS mismatch



*Results from FERMILAB-TM-2753-AD

Relevant Sources of Emittance Growth

- Nonstationary beam distribution
  - Stationary distribution retains it shape as it moves about the accelerator
  - Determined by forces acting on beam and self forces
  - Stationary if distribution is any function of beam Hamiltonian $\rho = f(H(\mathbf{x}, \mathbf{p}))$
  - Ex: KV distribution $\rho \propto \delta(E - H(\mathbf{x}, \mathbf{p}))$; uniform density in any plane
  - Gaussian distribution is nonstationary in a linear lattice, so emittance will grow
- Mismatched beam size
  - RMS beam size needs to be matched to lattice for beam size to remain stationary
    - Determined by solutions to envelope equation
  - Both a mismatched KV or Gaussian beam will experience emittance growth
    - If both have same RMS sizes and experience similar growth, this would suggest that mismatch is dominant source of growth
- Coupling from longitudinal to transverse plane
  - Dispersion and chromaticity are coupling sources

Mismatch theory

- A beam is matched if its emittance is stationary. This is a perfect balance between the external focusing force, the space charge force, and the emittance term, shown in the envelope equation
- If the beam is mismatched, there will be increased field energy, and the emittance will evolve. If the space charge (second) term dominates (greater than the third emittance term), the beam can grow without bound.

$$a = 2x_{rms} \qquad k_0 = Q/R$$

$$K_{sc} = \frac{e\lambda_L}{2\pi\epsilon_0\beta\gamma^2 pc}$$

$$\lambda_L = \frac{N}{2\sqrt{2\pi}\sigma_z}, \quad \text{Bunched beam}$$

$$\lambda_L = \frac{N}{C}, \quad \text{Coasting beam}$$

$$k_0^2 a - \frac{K_{sc}}{a} - \frac{\epsilon^2}{a^3} = 0$$

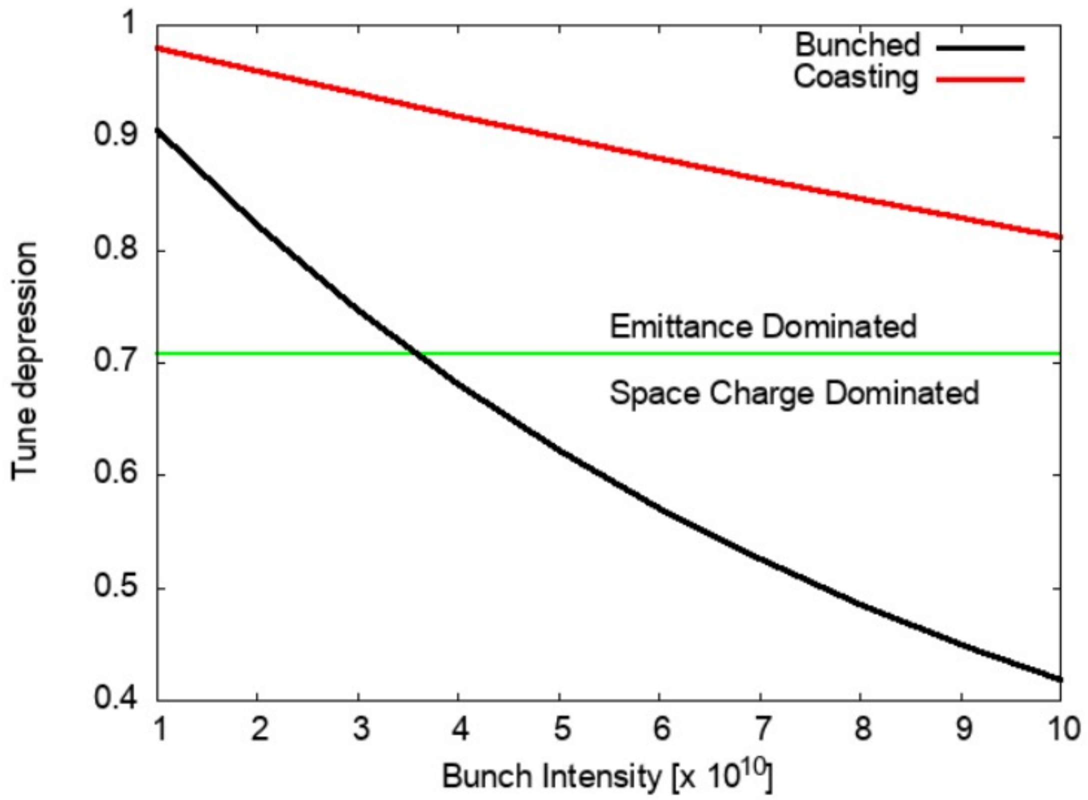
SC dominated: $K_{sc}a^2 > \epsilon^2$

Emittance dominated: $K_{sc}a^2 < \epsilon^2$

Mismatch theory cont.

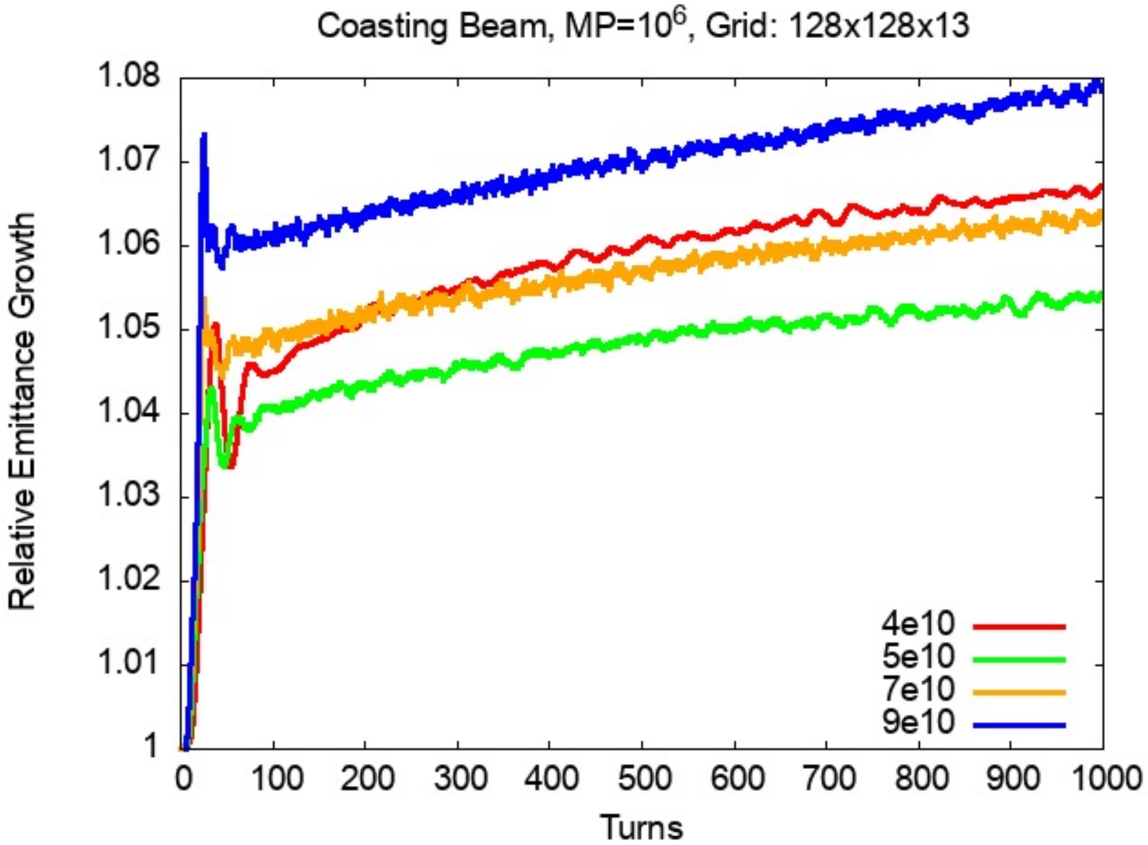
$$K_{sc} = k_i^2 a^2 \Rightarrow a^2(k_0^2 - k_i^2) = k_i^2 a^2$$
$$\frac{k_i}{k_0} = \sqrt{\frac{1}{2}} \approx 0.707$$

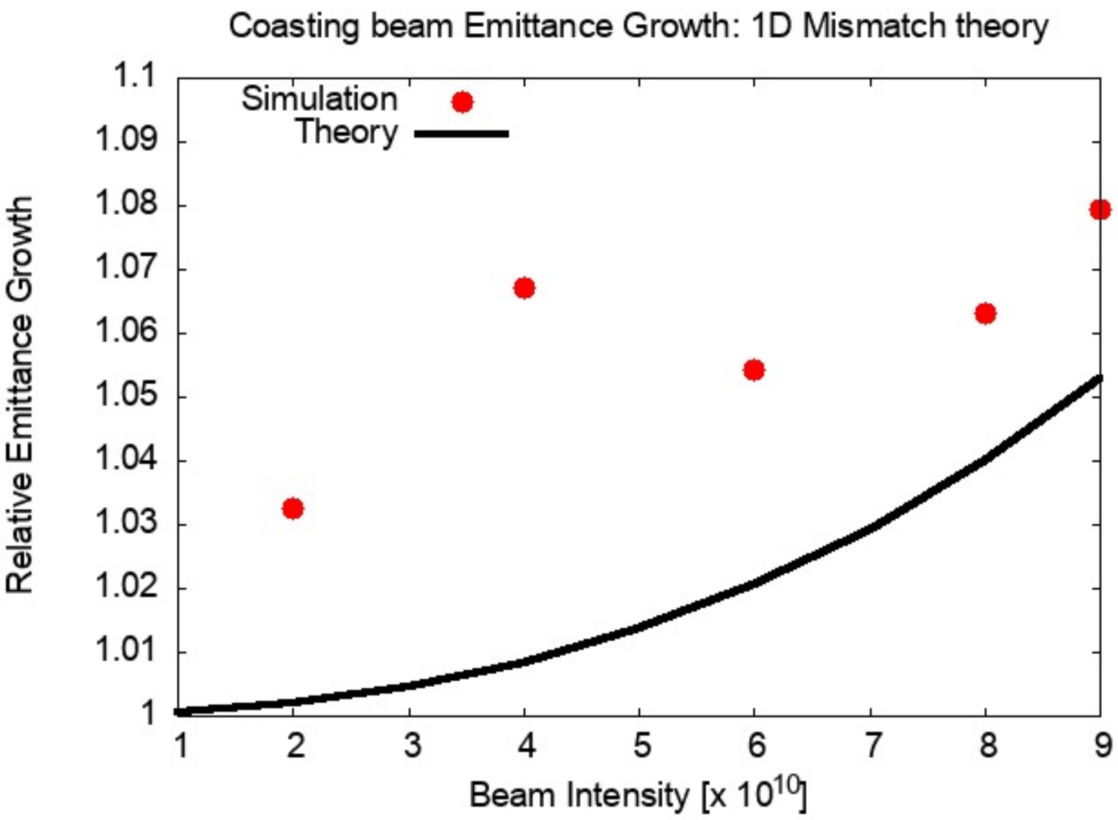
- This transition occurs at a tune depression (tune with space charge/tune without space charge) of ~ 0.7
- Coasting beams are not space charge dominated in IOTA even at full intensity
- Bunched beams become space charge dominated at intensities > 4 x 10^{10}



Coasting Beam Emittance Growth

- Emittance growth for a coasting beam, where perveance is smaller (left)
  - No loss and low growth even at full intensity
- Good agreement between theory and simulations (right)
  - Discrepancy is ~2%

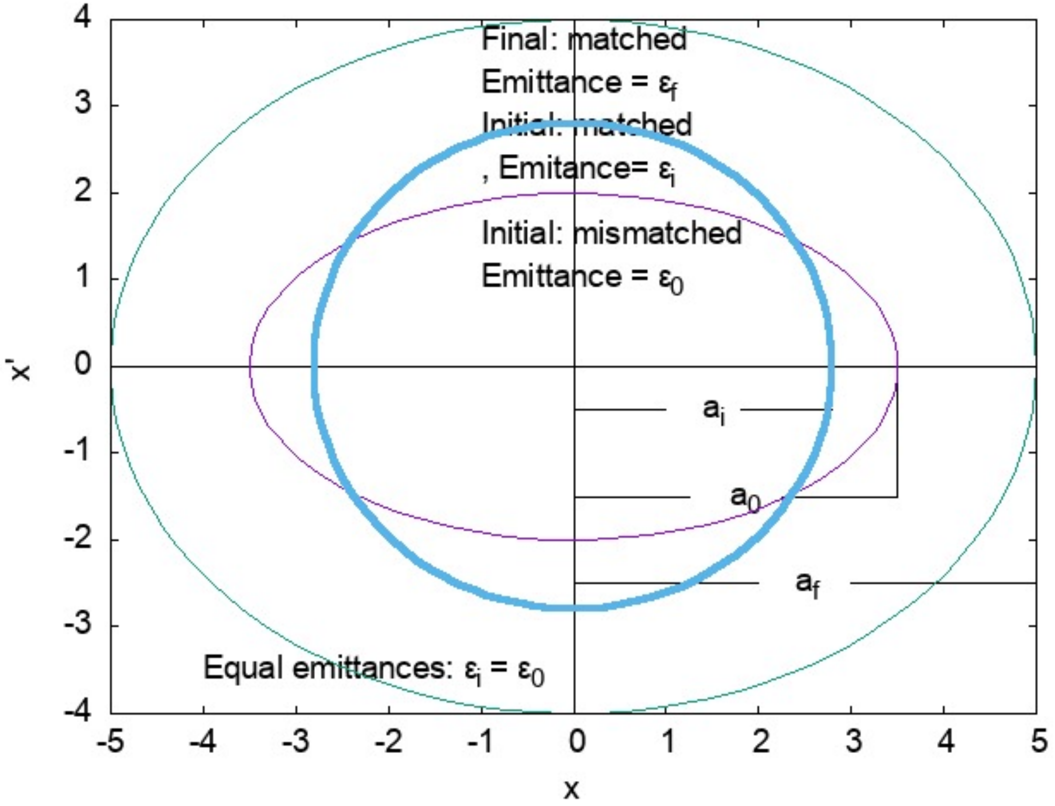




Mismatch theory cont.*

- Key assumptions: Smooth focusing and perfect axial symmetry in the x-y plane
- If the beam has an RMS size a_0 that is different from the matched beam size a_i for the same emittance, the excess energy due to the mismatch can be thermalized
- The beam will then relax to a matched final state with an increased emittance
- Using the conservation of energy in the transverse planes, we can attain a relation for the initial to final beam radius, and use this to find the change in emittance (equation in top right)

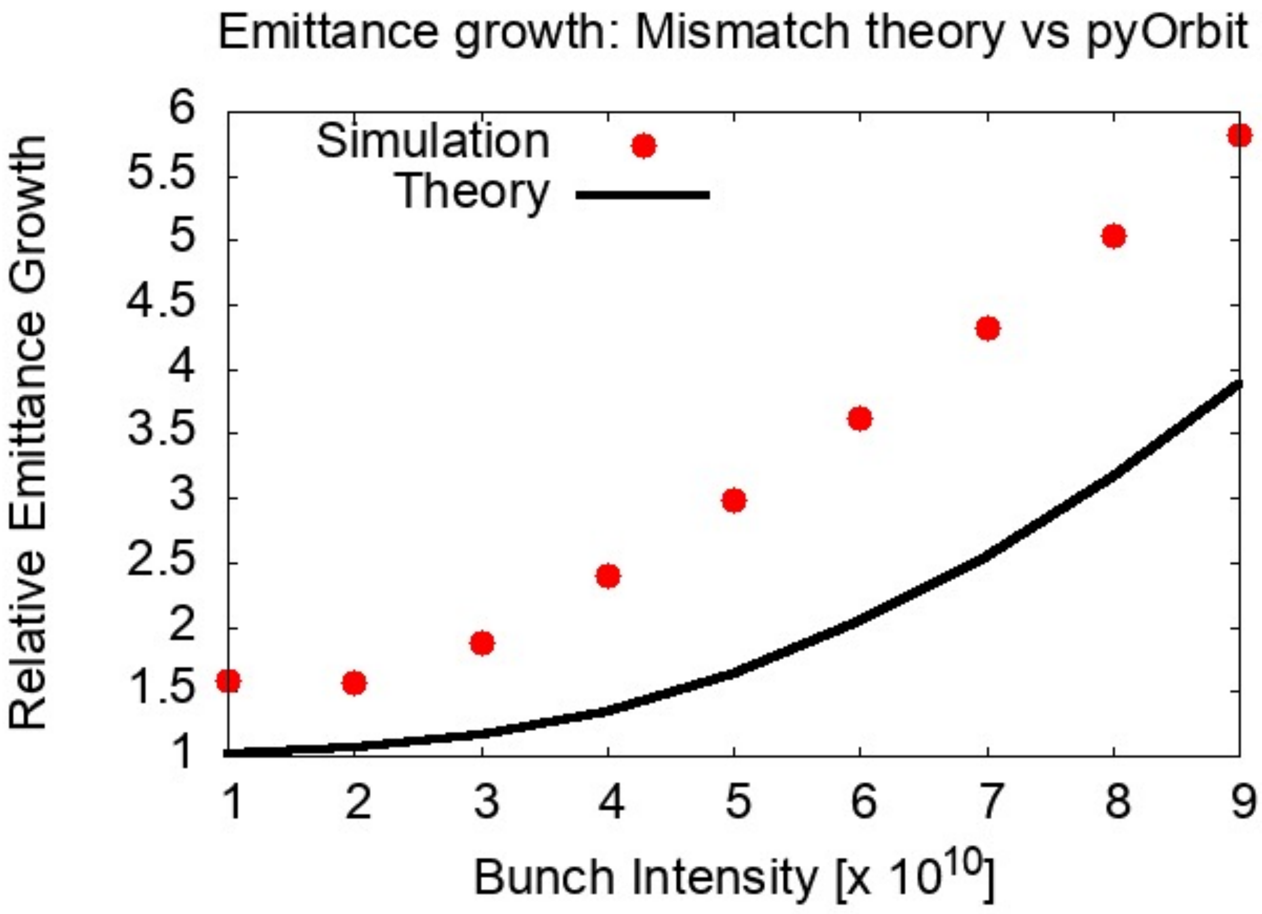
$$\frac{\epsilon_f}{\epsilon_i} = \frac{a_f}{a_i}\left[1+\frac{k_0^2}{k_i^2}\{(\frac{a_f}{a_i})^2-1\}\right]^{1/2}$$



*M. Reiser, *J. App. Phys.*, 70, 1919 (1991)

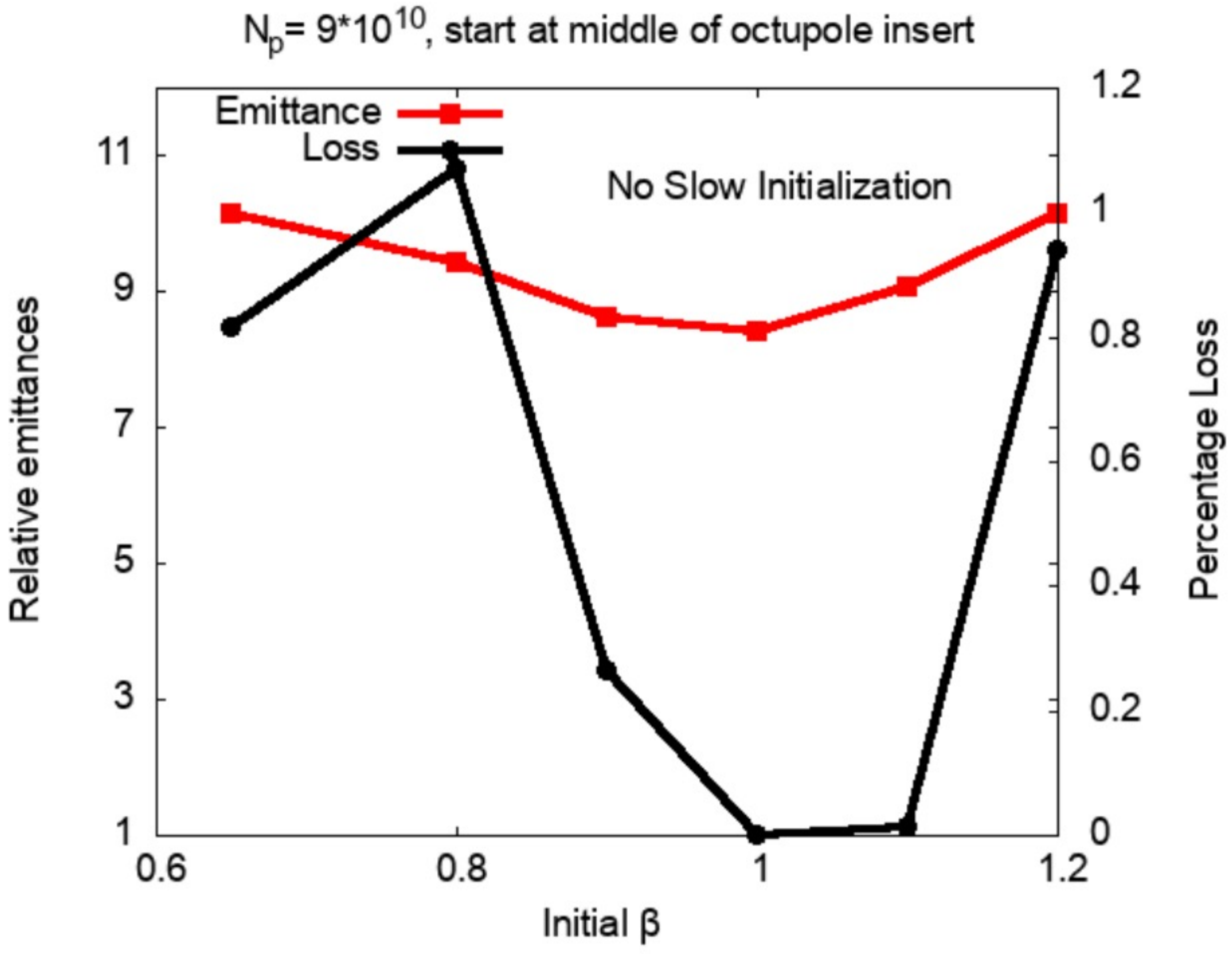
Bunched Beam Growth Theory vs Simulation

- Simplified 1D theory accounts for more than half of the growth at full intensity
  - Simple theory
  - Noisy simulations
- Improvements to theory will include:
  - Dispersion
  - Transverse coupling
  - Drop the smooth focusing approximation

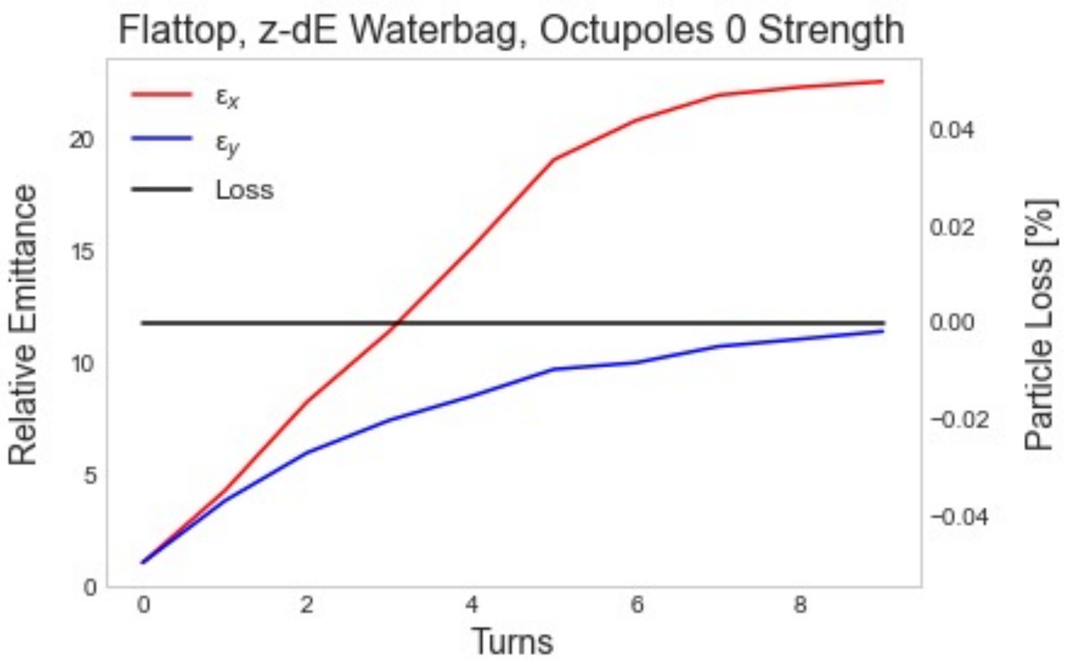


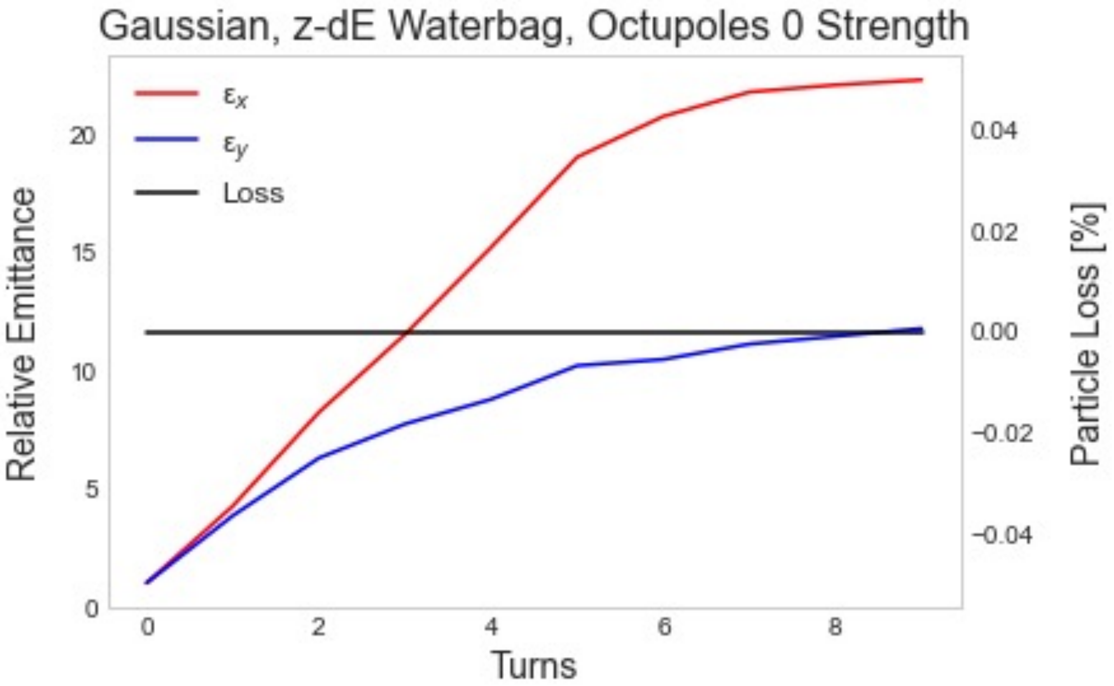
RMS Matching with New Injection Point $\beta_x = \beta_y$

- RMS matching can minimize beam loss
  - To 0% over 1000 turns
- Emittance growth not very sensitive to RMS matching
- Same result was seen in the original lattice



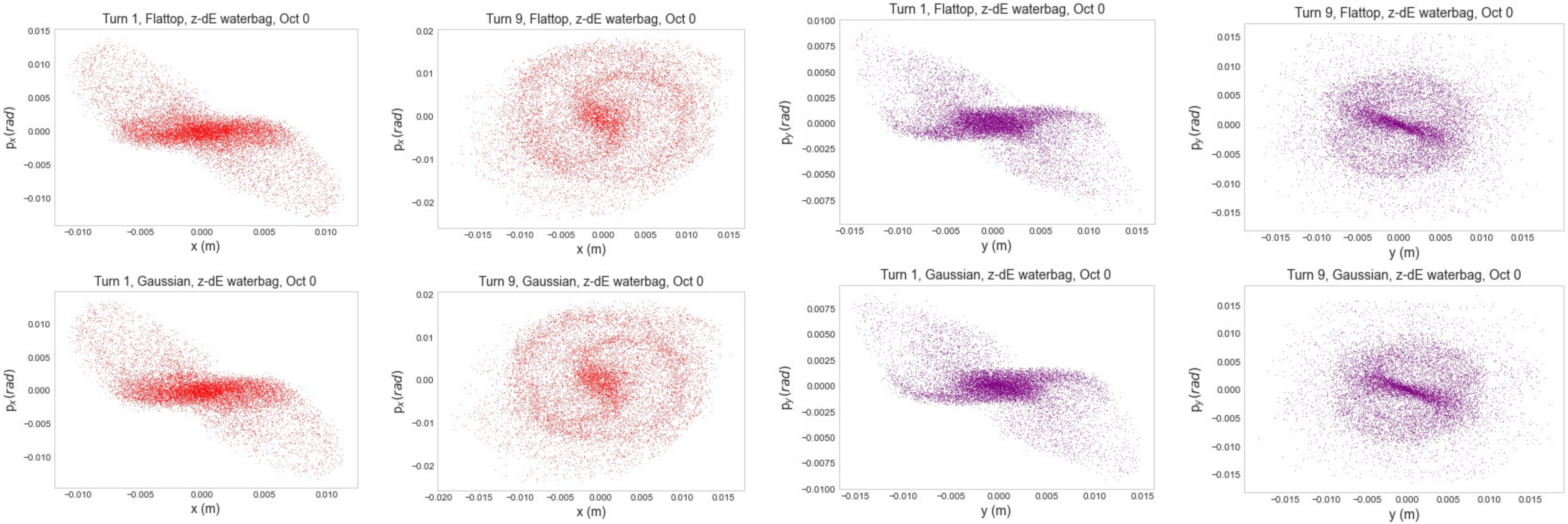
KV vs Gaussian Initial Emittance - bunched





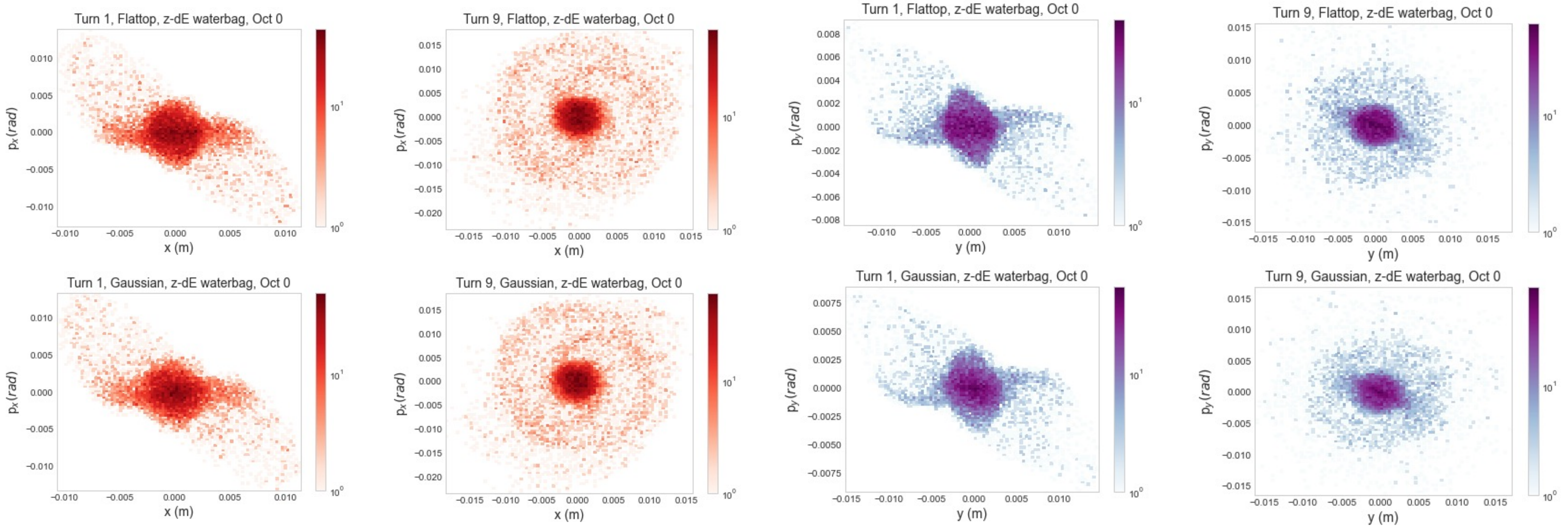
- Full intensity (9e10)
- No slow initialization, trunc distributions, matched emittance, old lattice
- Growth is very similar between distributions, and greater initial growth in x suggests a more rapidly populated halo
- No loss in either case

KV vs Gaussian Phase Space - Scatterplots



- Bunched, no slow init, trunc distributions, matched emittance, old lattice, 9e10
- Significantly more growth in x in both cases, with x’ almost doubling after 10 turns and radius increasing from ~0.1 to ~0.15
- Y develops a halo, but it is less populated, and far fewer particles double in y’ and even fewer increase in radius

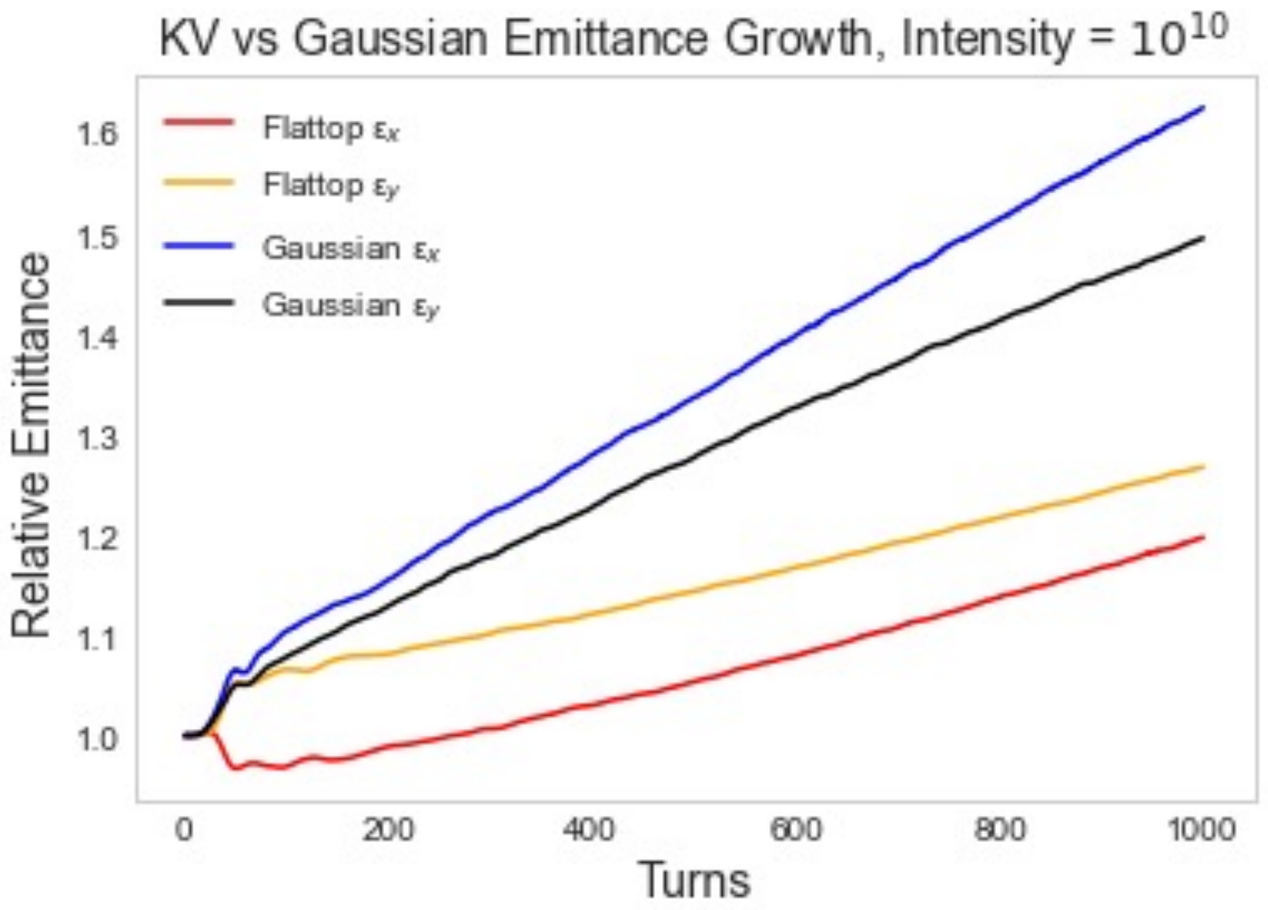
KV vs Gaussian Phase Space - Histograms



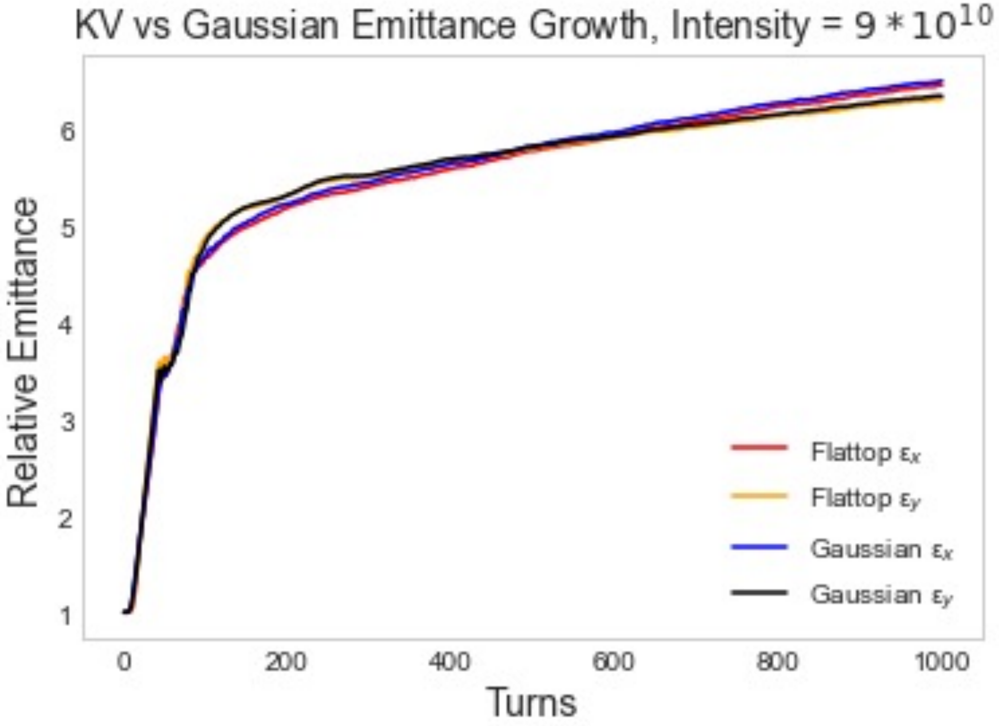
- Bunched, no slow init, trunc distributions, matched emittance, old lattice, 9e10
- Again, there is little disparity in behavior between distributions
- We can see even more clearly the densely populated halo that develops in x for than y, likely a result of dispersion

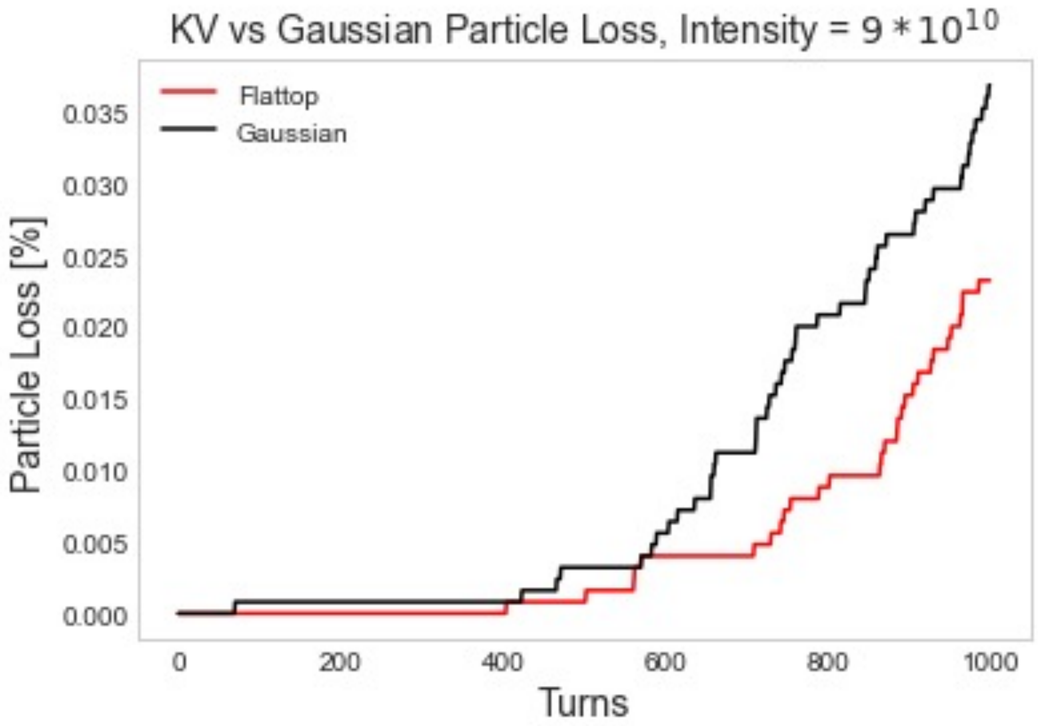
KV vs Gaussian Emittance - bunched

- Slow initialization, full distributions, matched emittance, 1e10
- New RMS matched lattice at $\beta_x = \beta_y$
- KV growth is significantly lower than Gaussian, reaffirming that nonequilibrium beam distribution is a significant source of growth
- No loss in either case



KV vs Gaussian Emittance - bunched





- Slow initialization, full distributions, matched emittance, new lattice, 9e10
- Growth is very similar in both cases, suggesting mismatch, rather than the nonequilibrium distribution, is a stronger source of growth at 9e10
- Loss are greater for Gaussian at ~0.35% vs ~0.2% after 1000 turns

Ways to Minimize Early Beam Loss

- Coasting beams have significantly lower loss
- Bunched beams
  - Operate in emittance (not space charge) dominated intensities, < 4×10^{10}
  - If higher intensities are needed, use slow initialization
  - Proper RMS matching, reduces loss to 0 under ideal conditions
    - Requires confirmation under non-ideal conditions and experimentally
  - When testing quasi-integrability with octupoles or integrability with the nonlinear lens
    - Maintain a phase advance of 2pi through the rest of the ring
    - Currently not tested

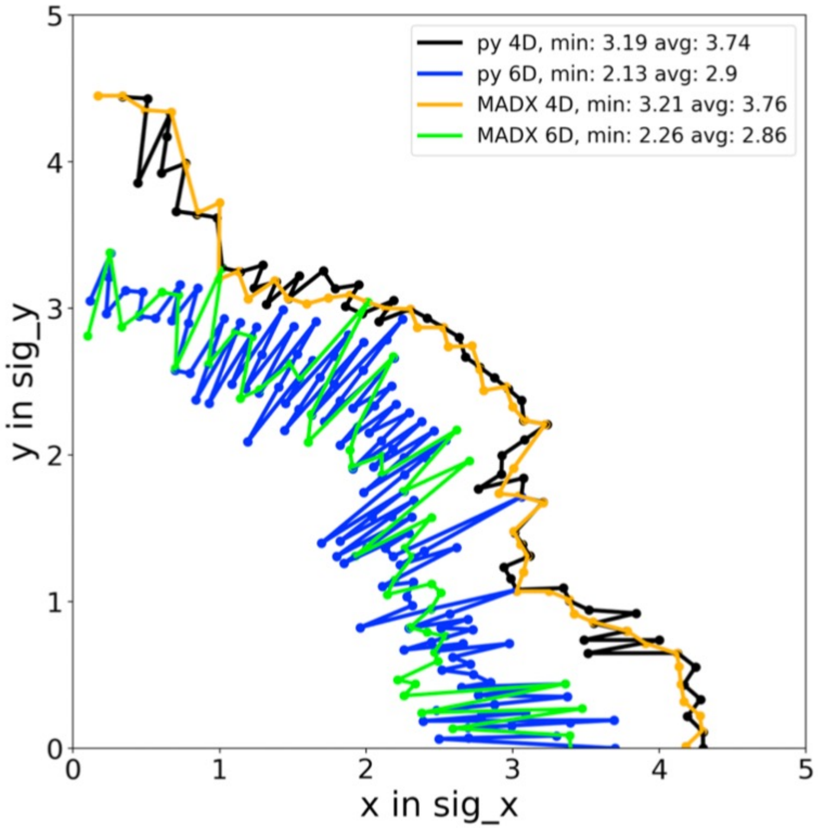
Space Charge + Octupoles

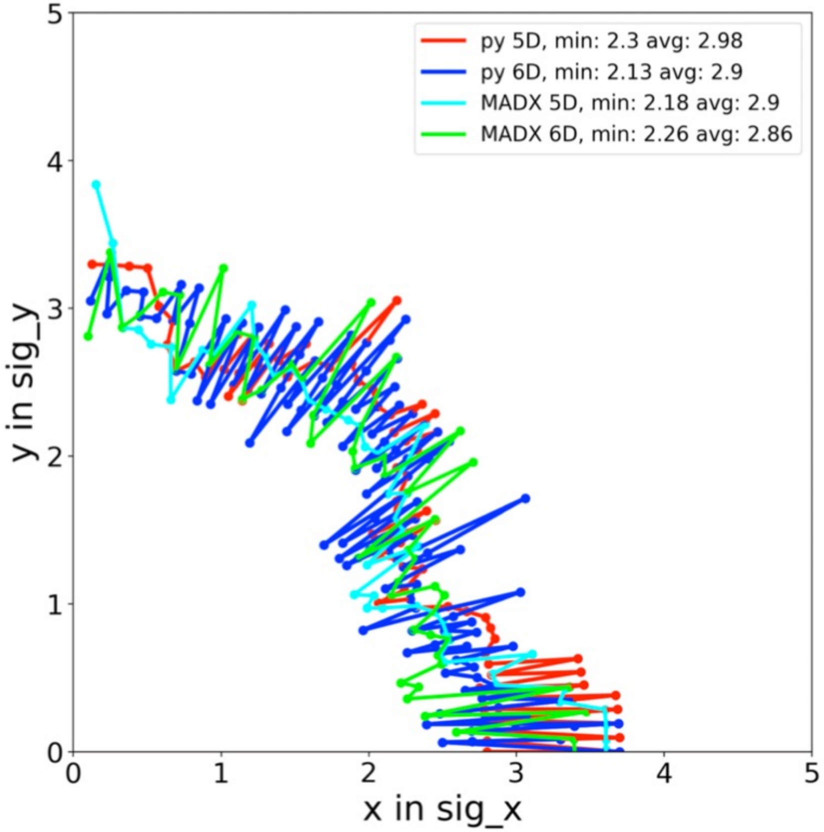
Dynamic Aperture with Octupoles*

- Physical aperture set to 25mm, 5000 test particles (50 amplitudes, 100 angles from 0-90°), initialized as:

1. 4D: RF Cavity is turned off, particles are initialized at (x_i, 0, y_i, 0, 0, 0)
2. 5D: RF Cavity is turned off, particles are initialized (x_i, 0, y_i, 0, 0, σ_p) Chromaticity effect included
3. 6D: RF Cavity is turned on, particles are initialized (x_i, 0, y_i, 0, 0, σ_p), so synchrotron motion is included

- Tracked for 10,000 turns, largest displacement of remaining particles is an upper bound for the DA





- Good agreement between MADX and PyORBIT

Aperture type	Minimum DA		Average DA	
	pyORBIT	MADX	pyORBIT	MADX
4D	3.19	3.21	3.74	3.76
5D	2.3	2.18	2.98	2.9
6 D	2.13	2.26	2.9	2.86

4D, 5D (left) and 6D (right) dynamic apertures and their minimum and average values from MADX and pyORBIT

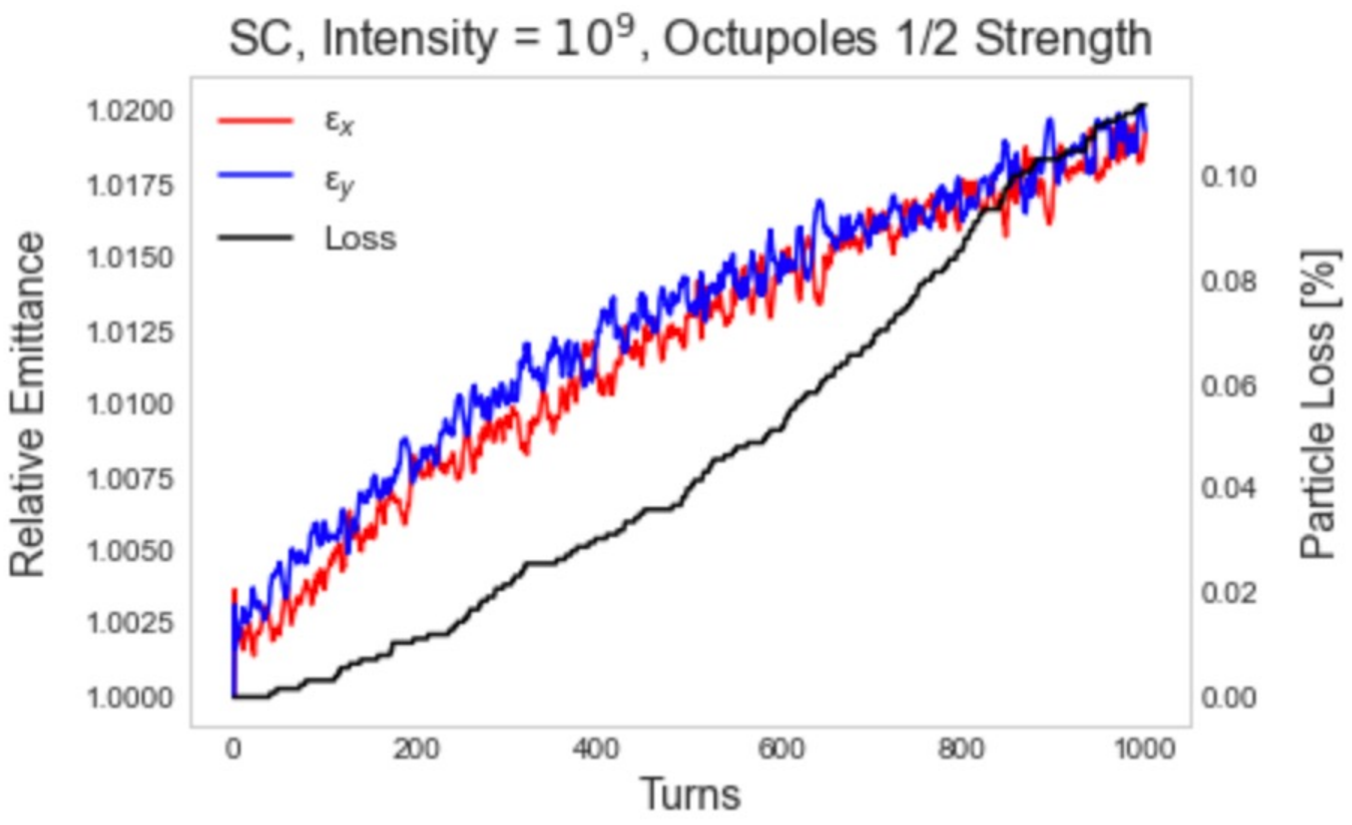
*Results from FERMILAB-TM-2753-AD

Methods

- Particles are tracked for 1000 turns in PyORBIT
- Physical aperture is set to 10cm
  - Allow beam to relax to any possible steady state
- Simulations have examined a bunched beam with:
  - Octupoles at various strengths
  - Reduced and full beam intensities
  - Varied initial distributions
    - Thereby changing charge distribution

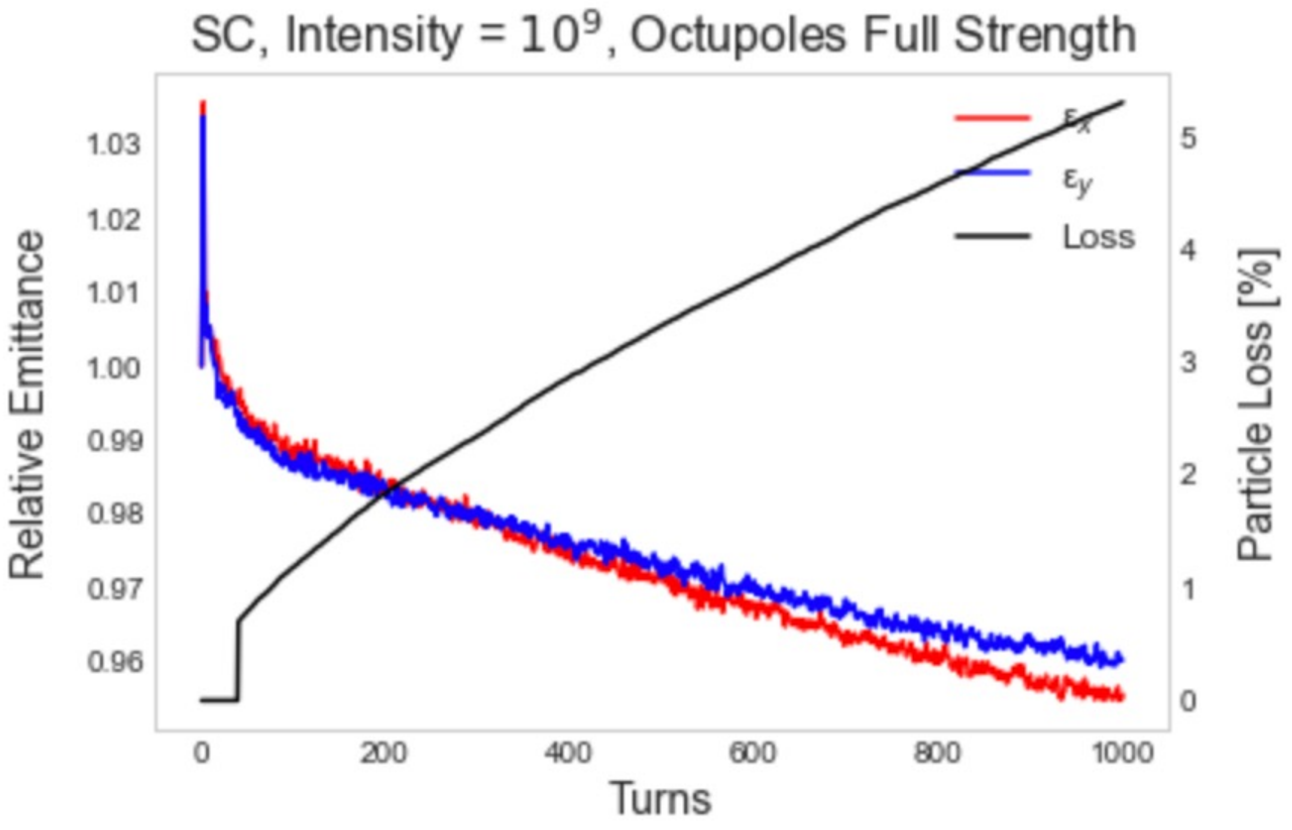
Low Intensity Results (1e9)

- Full Gaussian
- No loss for 0-1/4 strength octupoles
- Emittance growth increases from 0.2% to 2%
- No DA scraping

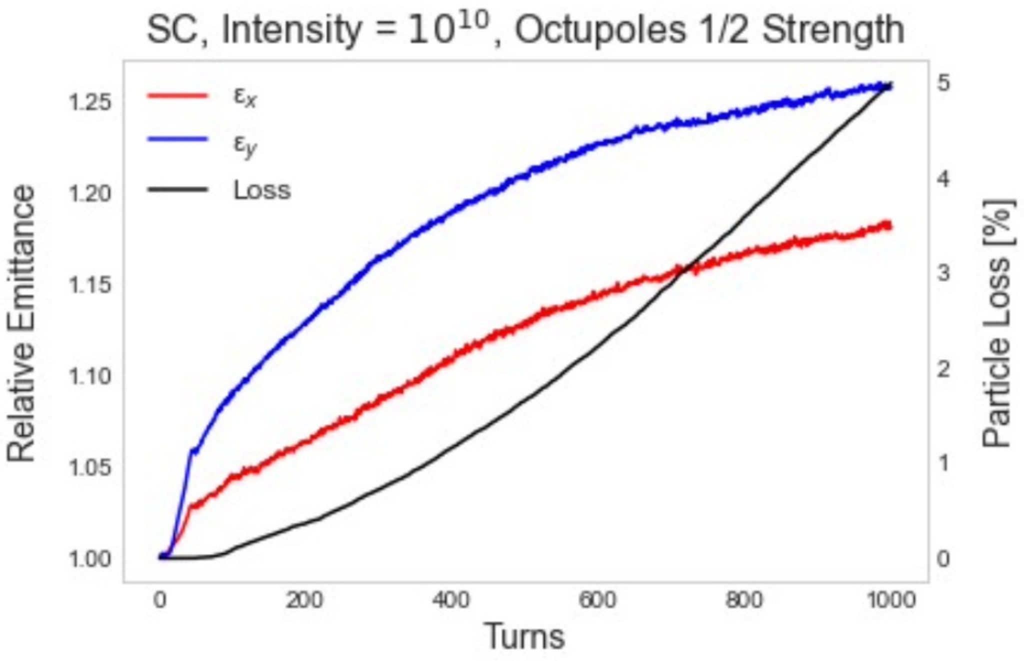


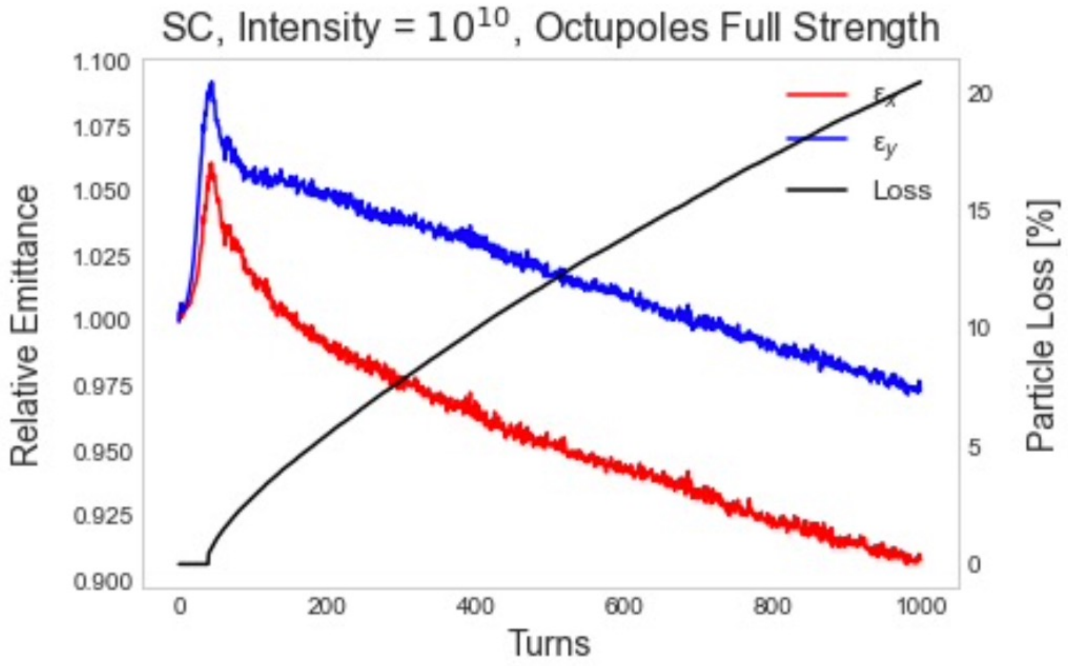
Low Intensity Results (1e9)

- Full Gaussian
- Loss increase to >5%
- Particles hit DA quickly, resulting in the increased loss



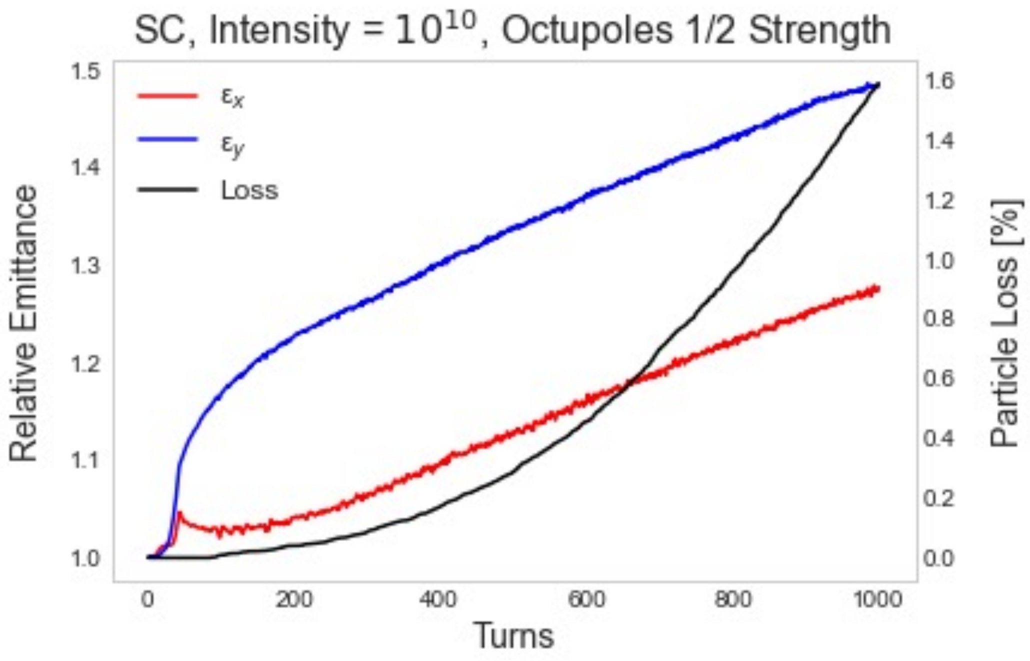
Low Intensity Results (1e10) – Full Gaussian

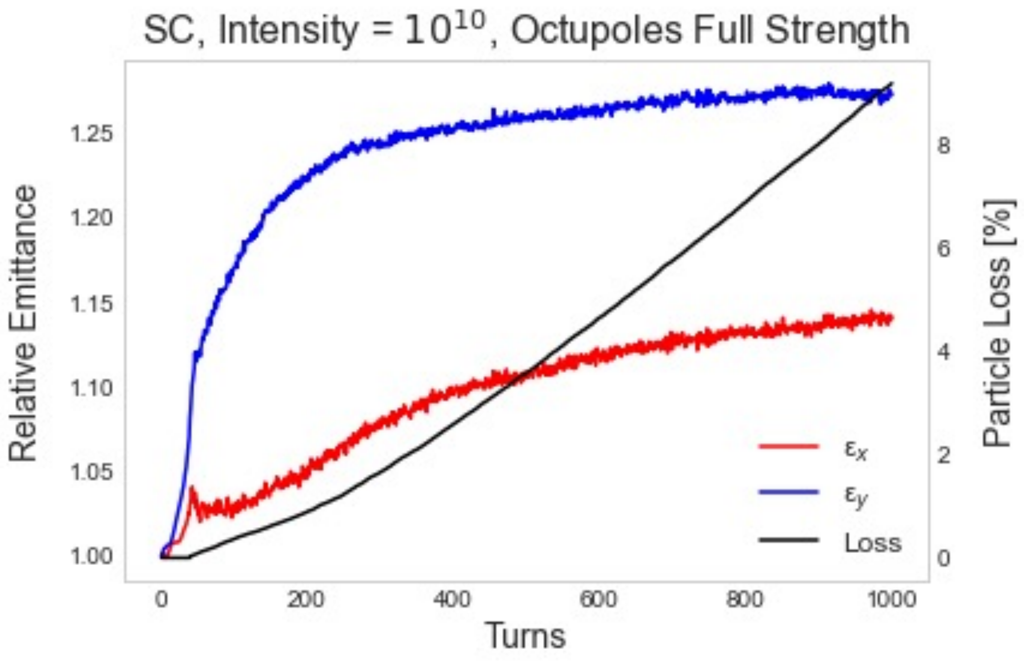




- Oct 1/2: No immediate DA induced loss, more growth in y than x (~25% vs ~15%), loss reaches 5%
- Oct Full: DA scraping, induces loss of ~20%
- Dispersion increases horizontal beam size, reducing horizontal space charge effects, resulting in less horizontal emittance growth

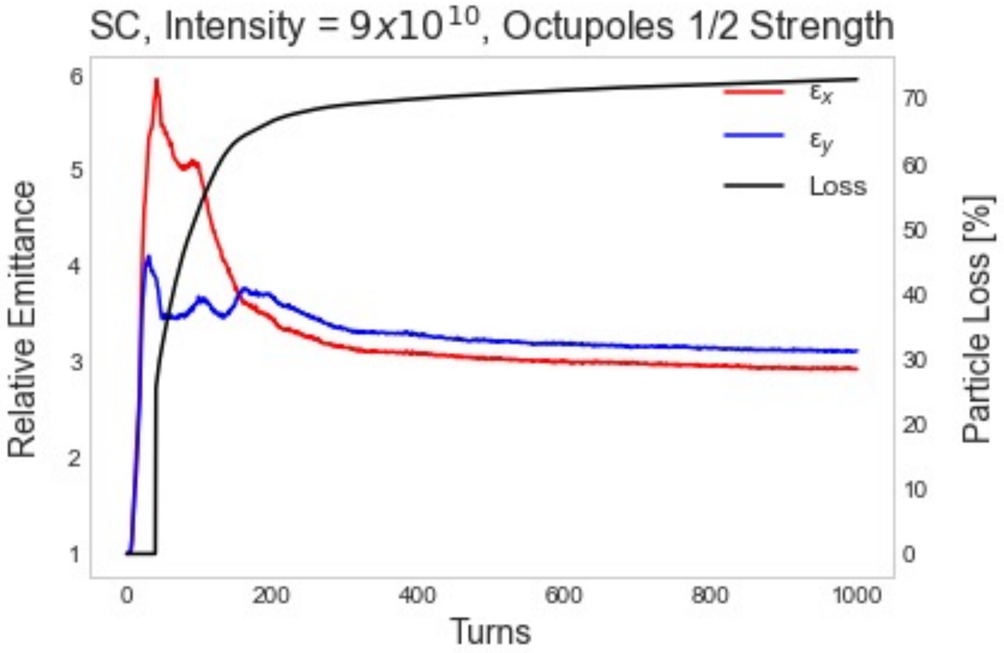
Low Intensity Results (1e10) – Trunc Gaussian

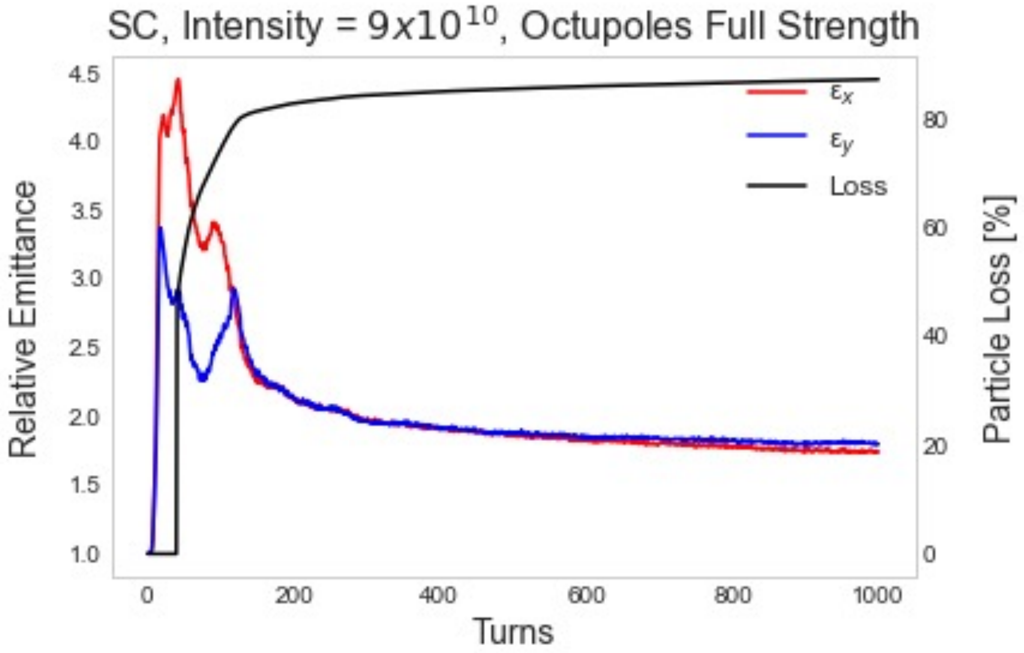




- More growth in y than x, same dispersion effect
- Growth increases, but no DA scraping and loss decreases from 5% to 1.6% and from 20% to ~10%

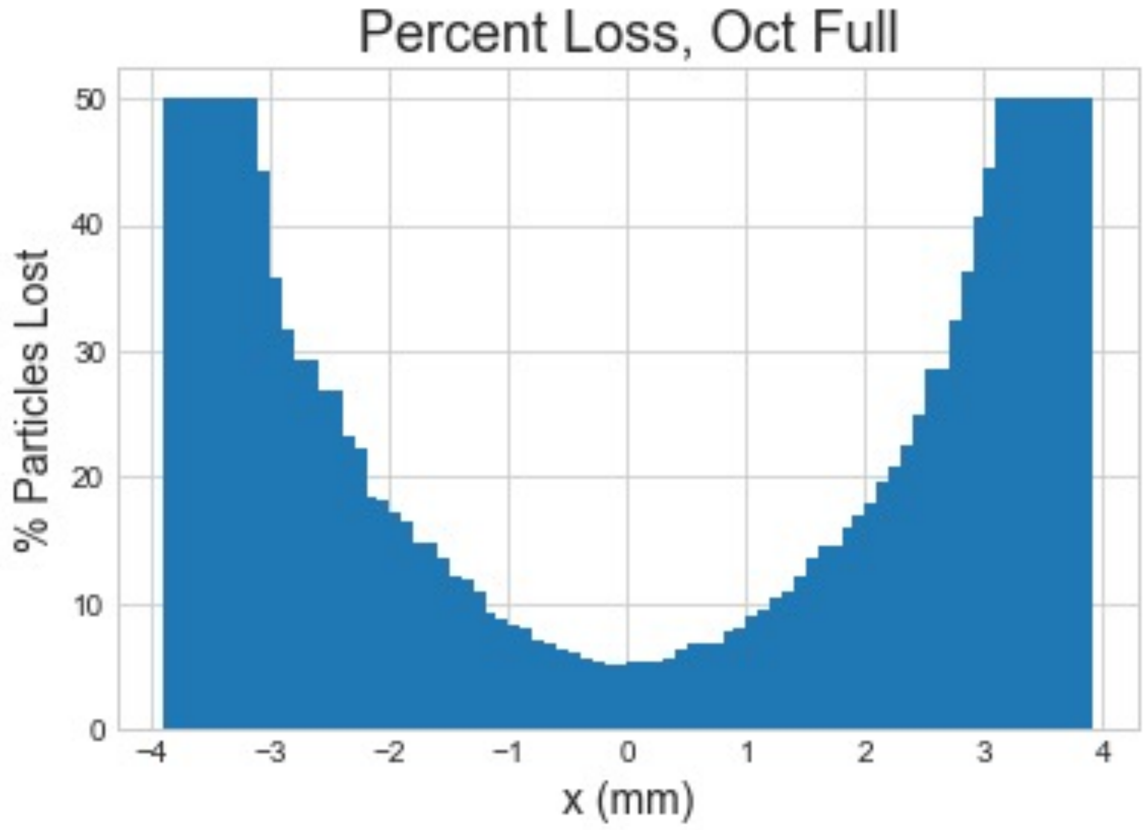
Full Intensity Results (9e10) – Trunc Gaussian

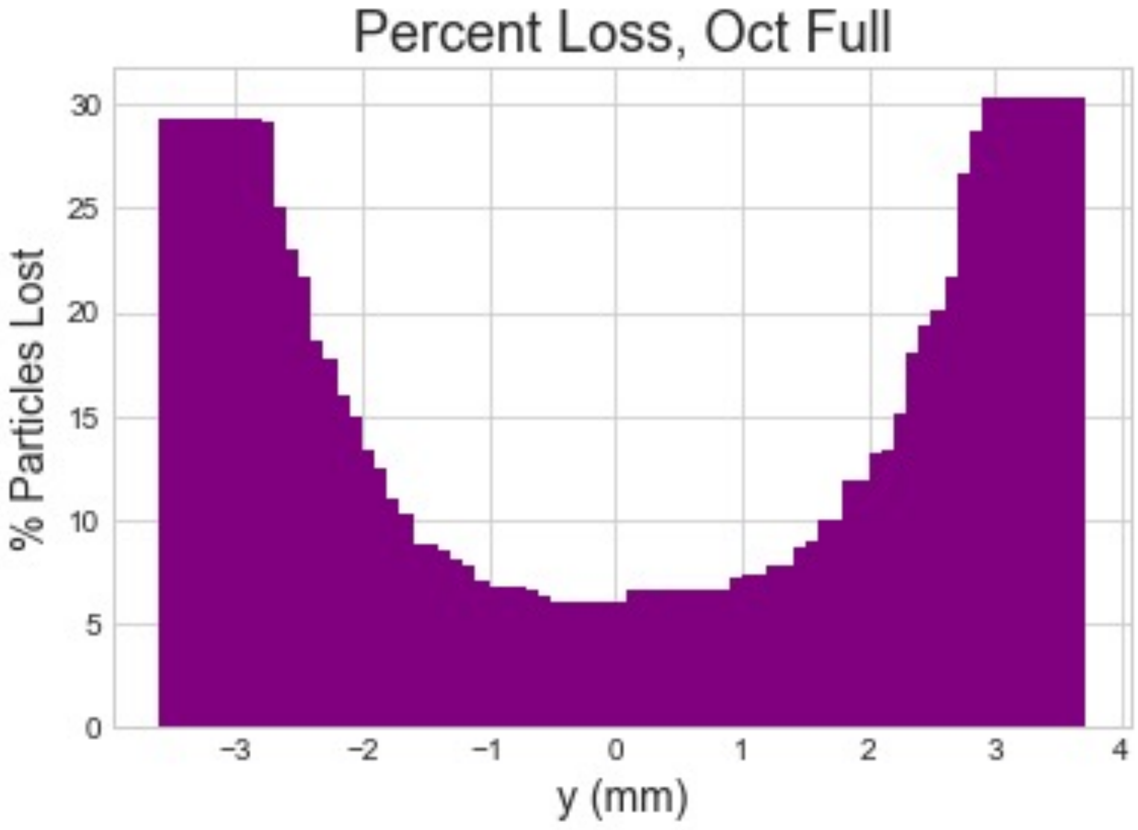




- Particles hit DA even at half strength, loss skyrockets to ~70%/80%
- Emittance growth still on the order of 2-4x increase
- Dispersion increases individual horizontal amplitudes, increasing horizontal loss relative to vertical loss

Loss Example Distributions (1e10 Trunc Gaussian)





- As we expect, particles are more likely to get lost the further they are initialized from the beam center
- Losses in x greater than in y

Conclusions

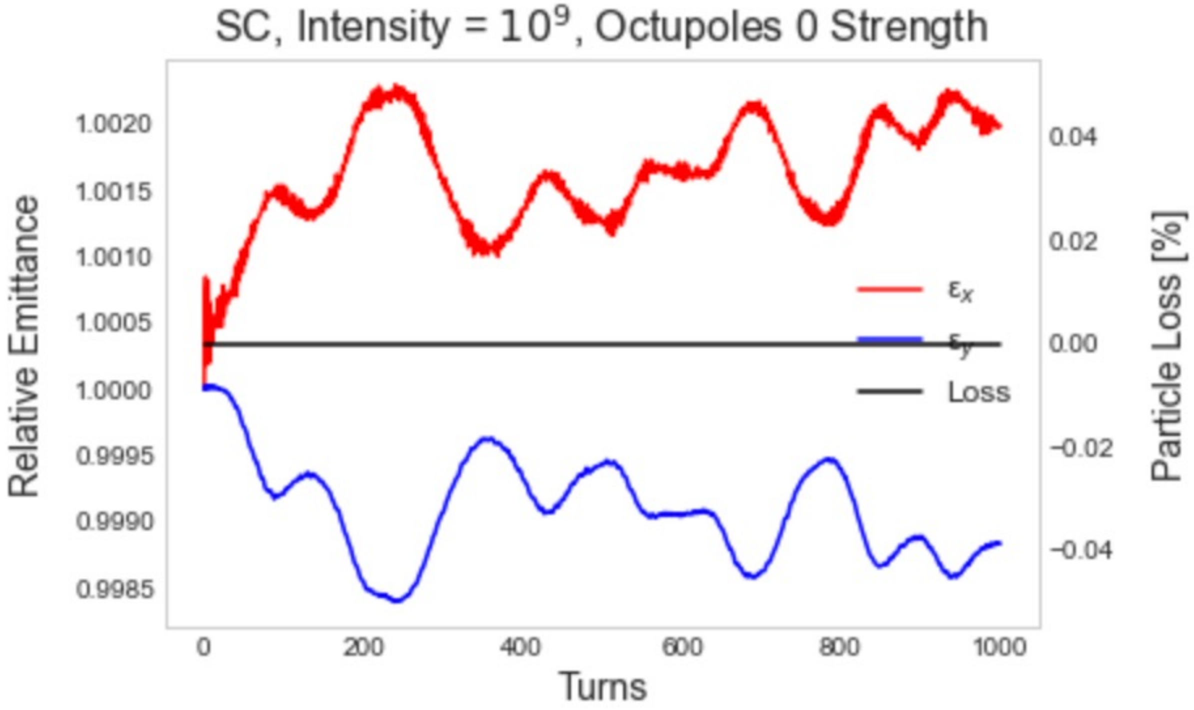
- Theory suggests bunched beams are space charge dominated at intensity above 4e10
- Bunched beams w/transverse Gaussian distributions
  - Space charge without octupoles leads to large emittance growth + loss at low intensity, occurring over the first few turns, not due to betatron resonances
  - A simplified theory of rms mismatch in 1D accounts for more than half of the growth. Work to improve theory is ongoing
- Octupoles greatly reduce the dynamic aperture ($\sim 3\sigma$)
  - Necessitates a truncated initial distribution to avoid large immediate loss
  - Space charge with a Gaussian distribution moves particles beyond the DA and increases loss
- RMS matching is effective at reducing beam loss
- The impact of dispersion is evident in differences in emittance growth and loss in the x-y planes
- At full intensity (9e10), mismatch is a stronger source of emittance growth than nonequilibrium distribution

Next Steps

- RMS matching with slow initialization: aim to minimize emittance growth and loss
- Improve the mismatch theory: Include 3D envelope equation with dispersion, transverse coupling, drop smooth focusing approximation, etc.
- Maintain integrability with space charge by matching phase advance to 2π through the rest of the ring
- Minimize losses and emittance growth with octupoles
- Open question:
  - Why does RMS matching help reduce the beam halo (and loss) but has a small impact on emittance growth?

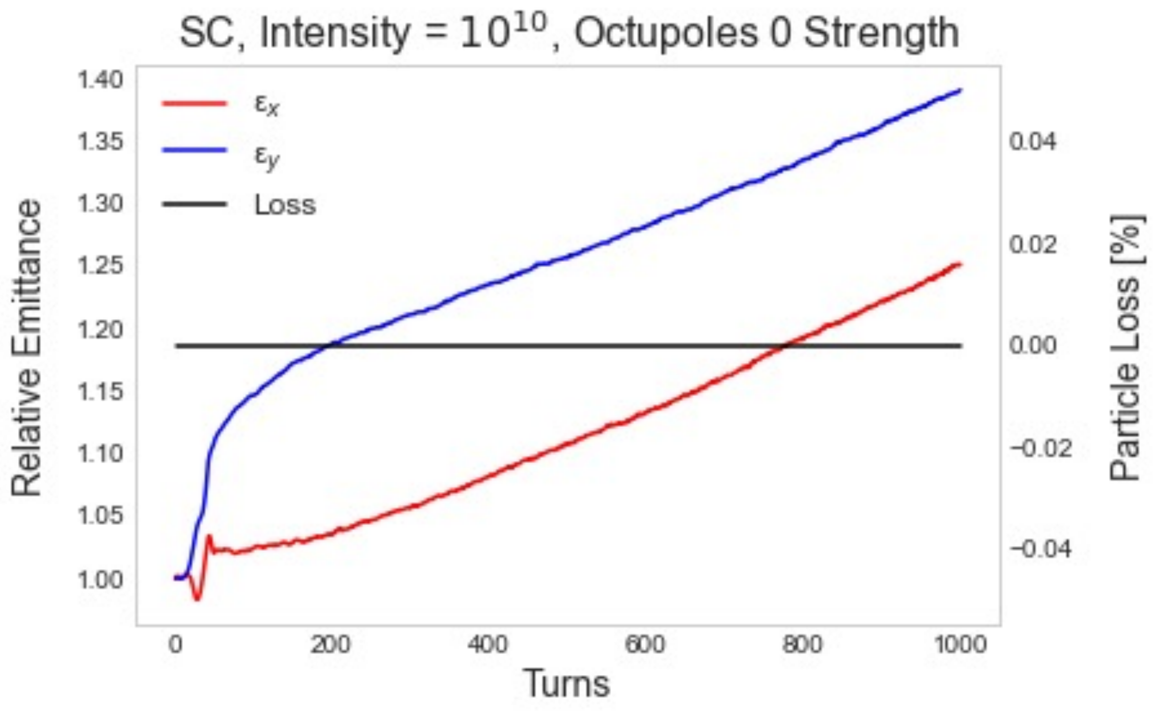
QUESTIONS?

Backup – 1e9 no octupoles, full gaussian distribution, bunched beam



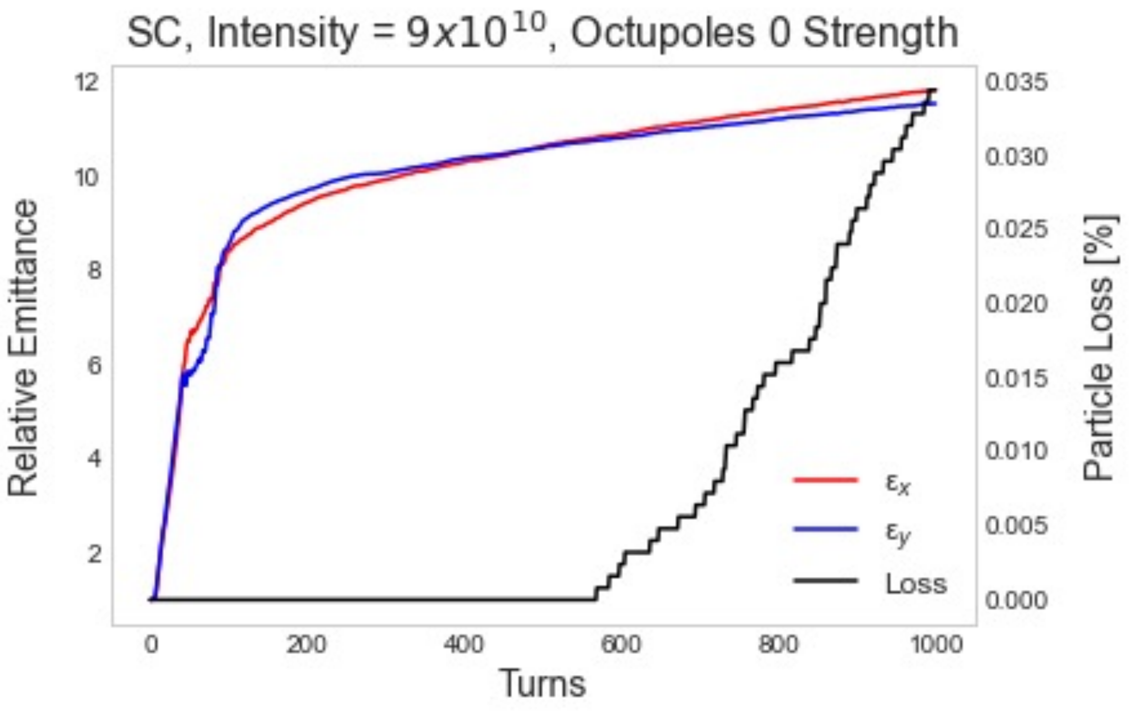
- No loss
- 0.2% emittance growth in x, decrease in y

Backup – 1e10 no octupoles, truncated gaussian distribution, bunched beam



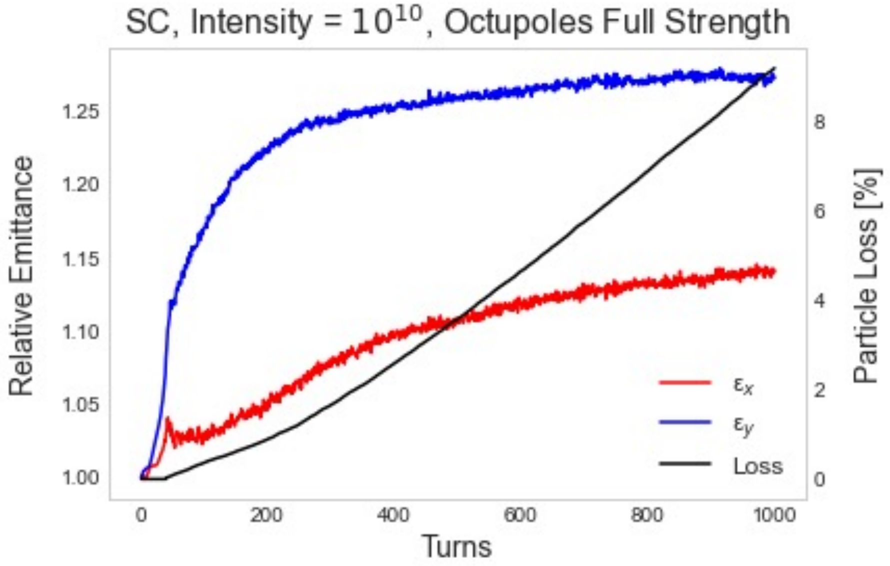
- No loss
- 40% emittance growth in y, 25% growth in x

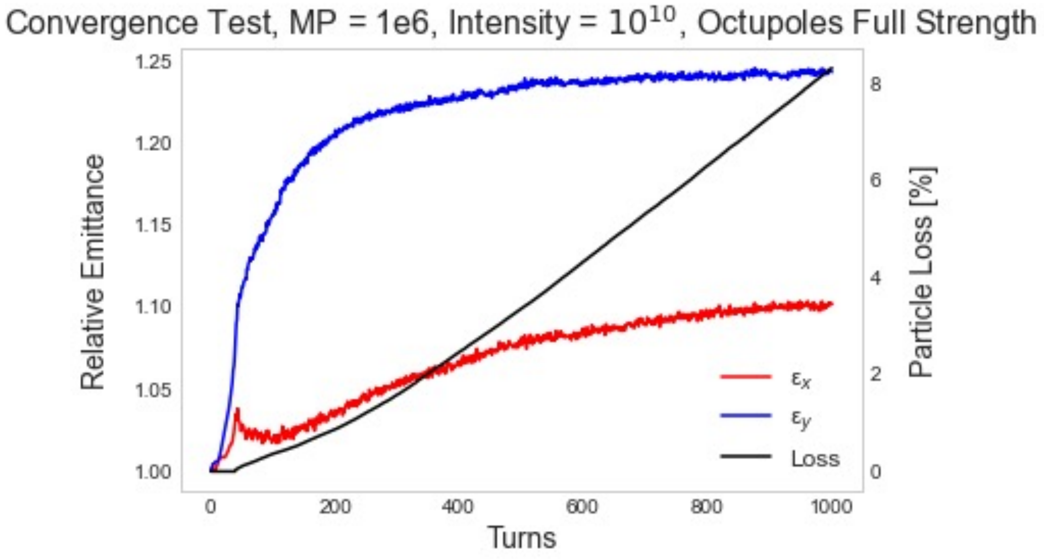
Backup – 9e10 no octupoles, truncated gaussian distribution, bunched beam

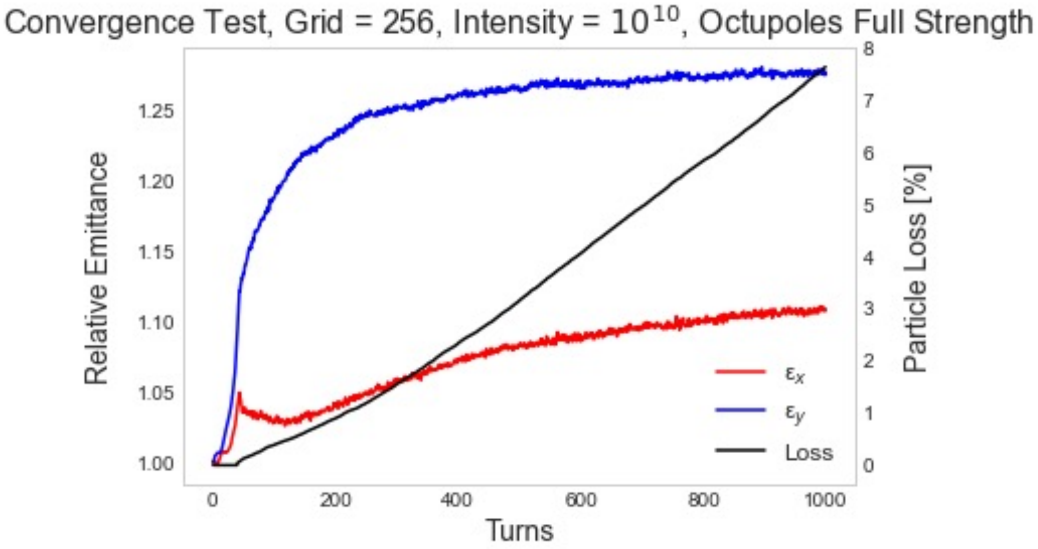


- 0.035% loss, starting around turn 600
- Emittance growth still about 12-fold, rapid in first few turns

Backup - Convergence Tests (bunched)







Backup – Mismatch theory complete equations

$$k_i^2 = k_0^2 - \frac{K_{sc}}{a_i^2} \quad (209)$$

$$\chi = 1 - \frac{k_i^2}{k_0^2} \quad (210)$$

$$h = \frac{1}{2}\frac{k_i^2}{k_0^2}\left[\frac{a_i^2}{a_0^2} - 1 + \frac{k_0^2}{k_i^2}\left(\frac{a_0^2}{a_i^2} - 1\right) + 2\frac{K_{sc}}{k_i^2 a_i^2}\ln\left(\frac{a_i}{a_0}\right)\right] \quad (211)$$

$$h = \frac{a_f^2}{a_i^2} - 1 - \chi \ln\left(\frac{a_f}{a_i}\right) \quad (212)$$

$$\frac{\epsilon_f}{\epsilon_i} = \frac{a_f}{a_i}\left[1 + \frac{k_0^2}{k_i^2}\left\{\left(\frac{a_f}{a_i}\right)^2 - 1\right\}\right]^{1/2} \quad (213)$$

Procedure

1. a_0: initial unmatched beam size, $k_0 = Q/R$
2. Given the tune, initial emittance and bunch intensity, solve the envelope equations to find the initial matched bean size a_i.
3. Use Eq.(211) to find h.
4. Use Eq.(212) to find the ratio of final to initial matched beam sizes a_f/a_i
5. Use Eq.(213) to find the ratio of final to initial emttances ϵ_f/ϵ_i

Backup - Reiser sources of emittance growth

6.1
Causes of Emittance Change

In the self-consistent theory of Chapter 5 we limited our analysis for the most part to stationary or quasistationary beams where the applied focusing forces are linear and the emittances associated with each direction are constant. These beams are best described by a Maxwell–Boltzmann distribution with different transverse and longitudinal temperatures. The forces arising from the space charge of such stationary beams are in general nonlinear except at very low temperatures, where the perveance dominates over the emittance and where the transverse density profile tends to be uniform. However, in the equilibrium state the nonlinear space-charge forces do not, by definition, cause any changes in temperature and emittance.

Real laboratory beams are usually not in perfect equilibrium, and there are a large number of effects that can cause the temperature and emittance to increase.

The most important causes of emittance growth are the following:

- Nonlinearities in the applied forces
- Chromatic aberrations
- Nonlinear forces arising from nonstationary beam density profiles
- Beam mismatch causing oscillations of the rms radius
- Beam off-centering causing coherent oscillations around the optical axis or central orbit
- Misalignments of the focusing and accelerating elements
- Collisions between the beam particles (Coulomb scattering) and between the beam and a background gas or a foil
- Instabilities, including unstable interactions with applied or beam-generated electromagnetic fields
- Nonlinear single-particle resonances and nonlinear coupling between longitudinal and transverse motion (especially important in circular accelerators)
- Beam–beam effects in the interaction regions of high-energy colliders

Taken from “Theory and Design of Charged Particle Beams”, second edition, 2008

Backup - Reiser emittance growth theory

Taken from “Emittance Growth in Mismatched Charged Particle Beams”, 1991

Emittance Growth in Mismatched Charged Particle Beams*

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Abstract

A new theoretical model of emittance growth in nonstationary charged particle beams has been developed which generalizes the previous theory of nonuniform charge distributions to include rms mismatched and off-centered beams. First the behavior of mismatched uniform beams in linear focusing channels, where no emittance growth occurs, is discussed. Then the results of the new theory are presented and compared with numerical simulation studies for rms mismatched, nonuniform beams.

I. Introduction

Past theoretical studies of mismatched beams used the uniform beam model to calculate the frequencies of the envelope oscillations[1]. Since all forces acting on the particles are linear in this model, the emittance remains constant. In more realistic nonuniform beams, however, the nonlinear space charge forces may cause emittance growth, as has been shown for rms-matched particle distributions in several theoretical[2-5] and experimental investigations[6,7].

Recently, the author developed a new model of emittance growth in nonstationary distributions[8] that extended and generalized the previous theory to include rms mismatched and off-centered beams. The generally nonuniform distribution is modelled by the equivalent uniform beam having the same perveance, rms radius $\tilde{x}$, and rms emittance $\tilde{\epsilon}_x$ according to Lapostolle[9] and Sacherer[10]. In nonstationary beams the total energy per particle is higher than in the equivalent stationary beam by an amount ΔE which constitutes “free energy” that can be thermalized via collisions or nonlinear space charge forces. By assuming that the beam relaxes into a final stationary state at the higher energy and comparing it with the initial stationary state one obtains analytical relations for the increase of the beam radius and for the emittance growth.

In the following we will first describe the behavior of a mismatched uniform beam. Then we will present the results of the new theory for the increase of beam radius and emittance resulting from the thermalization of the free (mismatch) energy and compare them with simulation.

II. Behavior of a Mismatched Uniform Beam

Let us consider a beam with a uniform density profile (K-V distribution) in a linear “smooth” transport channel. The stationary state is characterized by a constant average beam radius a and perfect balance between the external focusing force, $k_0^2 a$, the space charge force, K/a, and the emittance term, ϵ^2/a^3, according to the envelope equation[11]

$$k_0^2 a - \frac{K}{a} - \frac{\epsilon^2}{a^3} = 0. \quad (1)$$

Here, $k_0 = 2\pi/\lambda_0 = \sigma_0/S$ represents the external focusing force, λ_0 the betatron oscillation wavelength without space charge, σ_0 the phase advance per period without space charge and S the length of one focusing period. $K = (I/I_0)(2/\beta^3\gamma^3)$ is the generalized perveance, $I_0 = 120\pi\ mc^2/q$ is the characteristic current, m the particle mass, q the particle charge, c the speed of light, $\beta = v/c$, $\gamma = (1-\beta^2)^{-1/2}$, and v the particle velocity.

Defining the above parameters with space charge by k, λ, σ, respectively, and using the relation

$$k^2 = k_0^2 - \frac{K}{a^2}, \quad (2)$$

one may rewrite Eq. (1) in the form

$$k^2 a - \frac{\epsilon^2}{a^3} = 0, \text{ or } \epsilon = ka^2. \quad (3)$$

The total energy E per particle (transverse kinetic energy E_k + potential energy of the applied field E_p + self-field energy E_s) for the stationary beam is found to be[8]

$$E = \frac{\gamma m v^2}{4}\left[k^2a^2 + k_0^2a^2 + \frac{1}{2}\left(k_0^2 - k^2\right)a^2\left(1 + 4\ell n\frac{b}{a}\right)\right], \quad (4)$$

where b is the radius of the beam tube.

If the beam is mismatched, the beam envelope will perform oscillations about the equilibrium radius a with wave constant[1]

$$k_e = (2k_0^2 + 2k^2)^{1/2} \quad (5)$$

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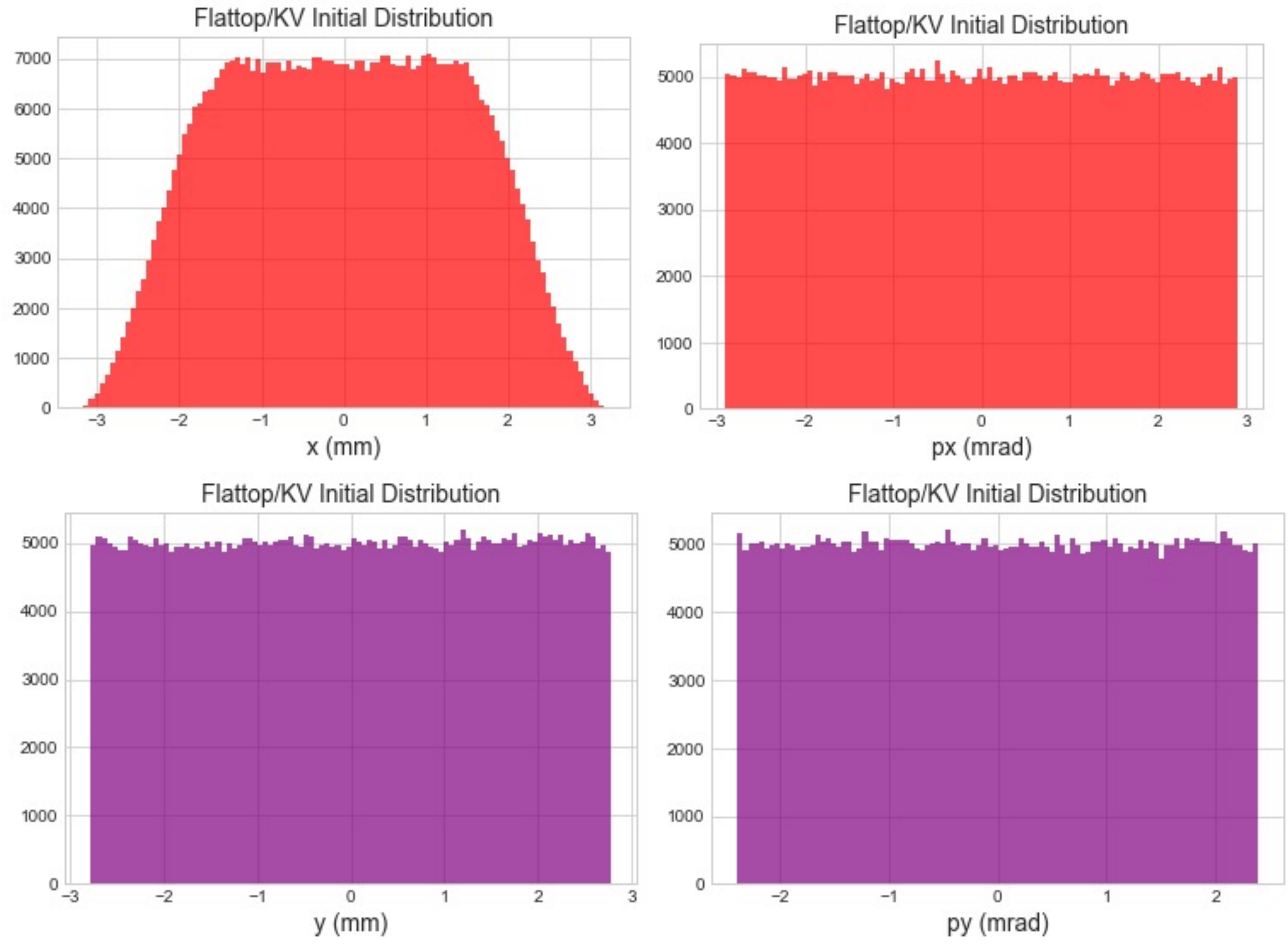
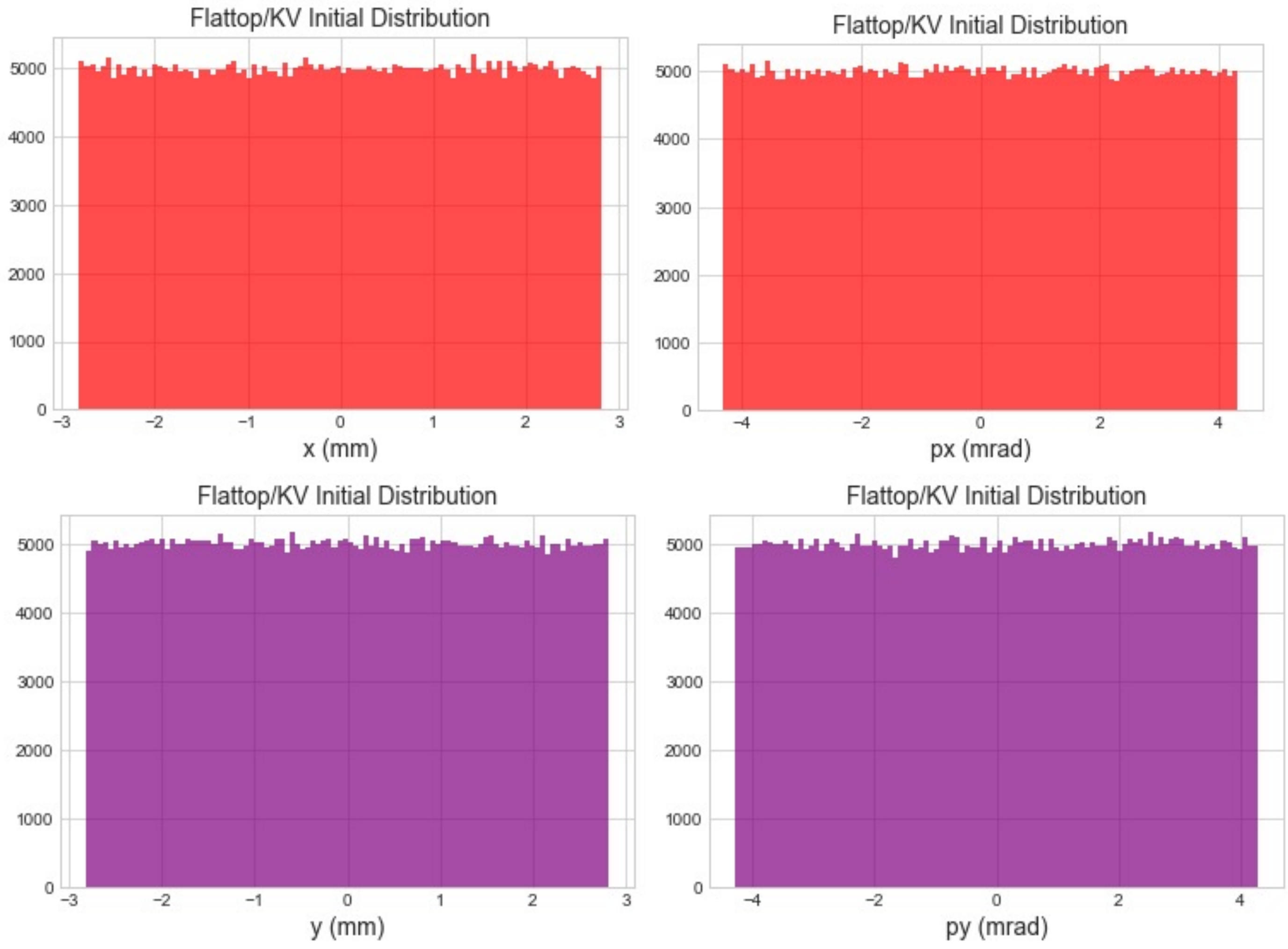
Backup - KV initial distributions with original lattice (RMS matched at injection point)



Backup - KV initial distributions with new lattice (RMS matched at octupole center where $\beta_x = \beta_y$)



Backup - Gaussian initial distributions