Automatic Leptonic Tensor Generation for BSM Models

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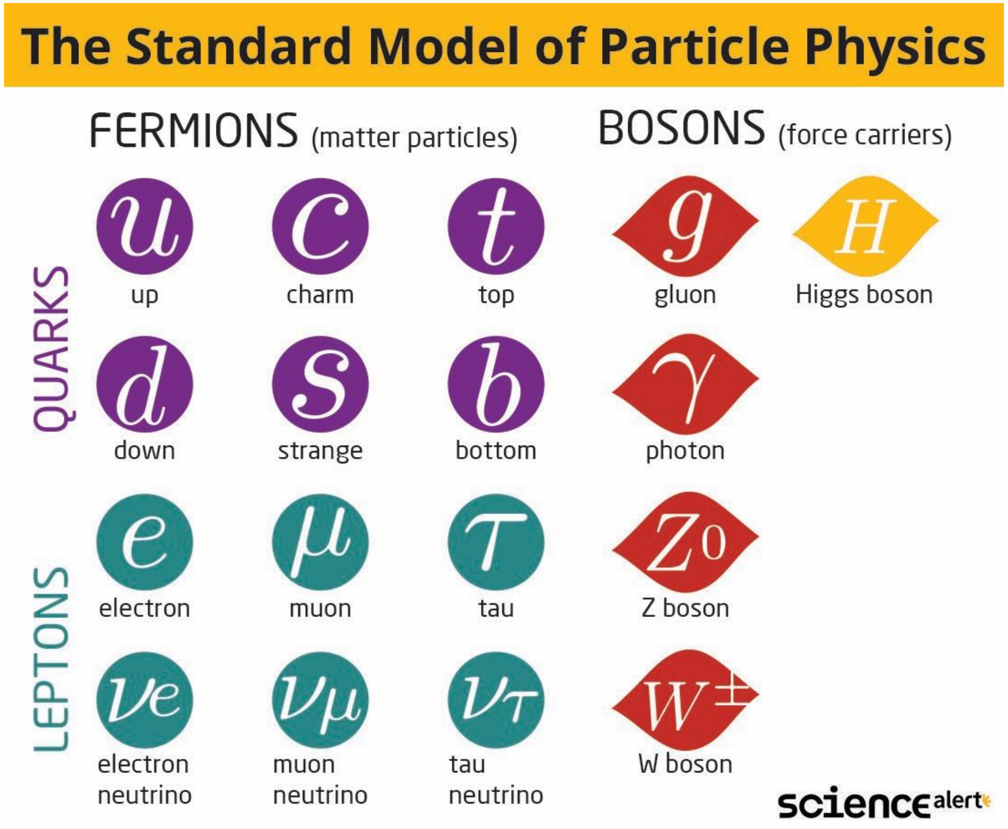
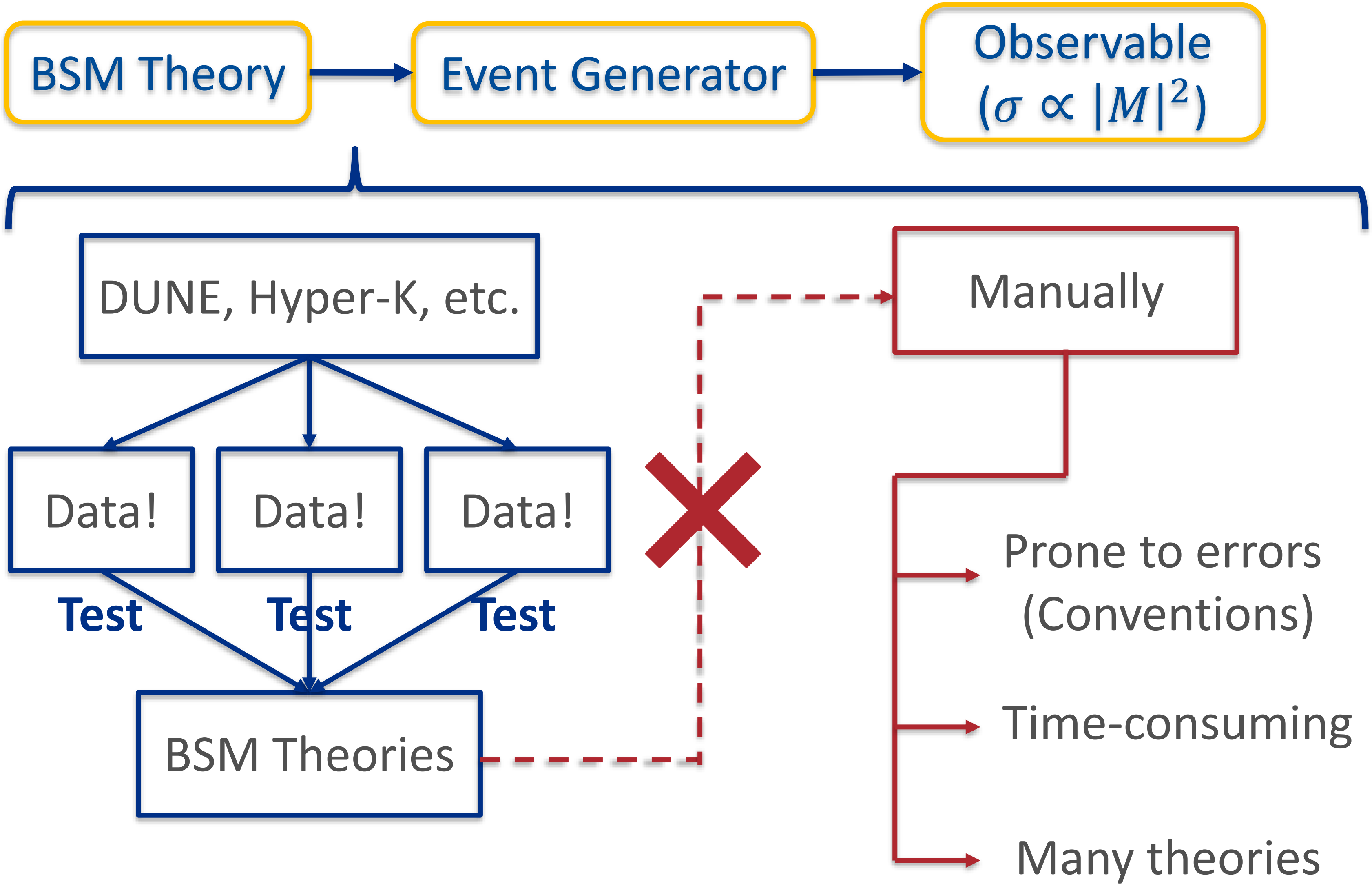
The Standard Model of Particle Physics

- Theory of the EM, weak and strong interactions.
- Describes most phenomena in nature to high accuracy.
- Is *not* complete.
  - Gravity.
  - Dark matter and dark energy.
  - Matter-antimatter asymmetry in the universe.
  - Neutrino oscillations.
  - And many more.

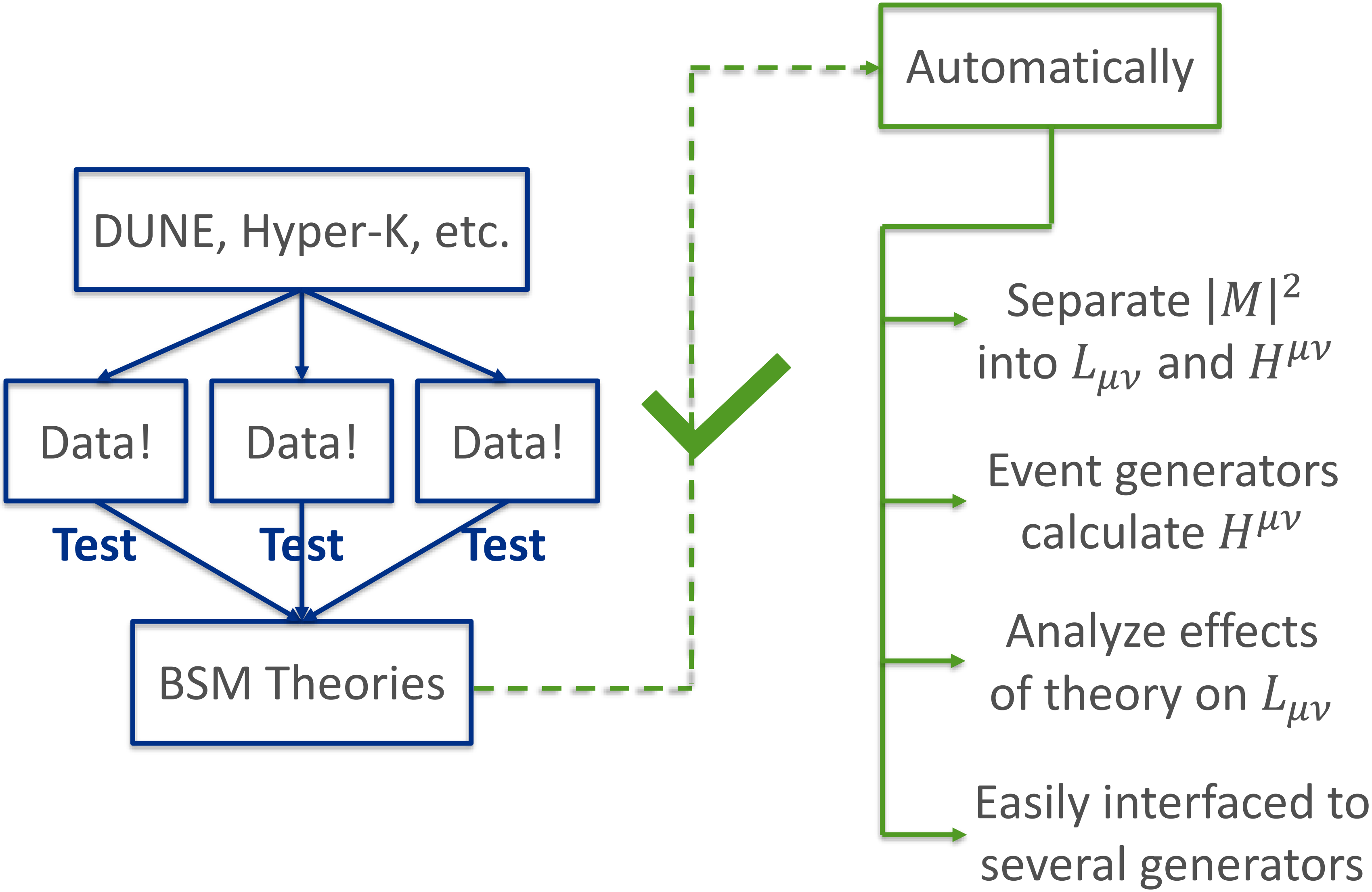


Beyond the Standard Model (BSM) Theories



Fermilab

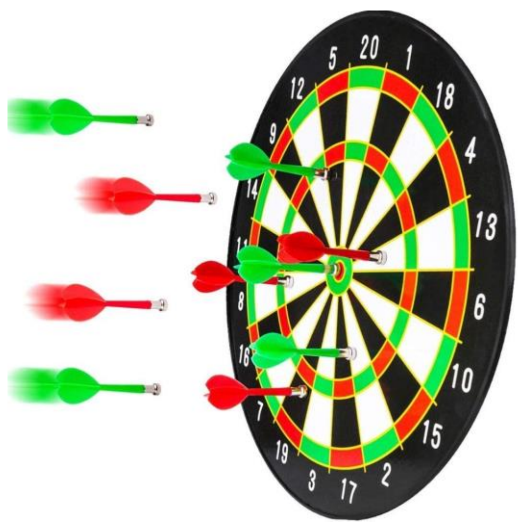
Beyond the Standard Model (BSM) Theories



Cross Section

- Experimental observable.
- Effective area that particle B presents to A when colliding.
- Let A_1, A_2 collide and produce $B_1, B_2, \cdots, B_n$. Then,
$$\sigma(A_1, A_2 \to B_1, B_2, \cdots, B_n) \propto \int \mathrm{d}\Pi_n |M|^2.$$
- $M \to$ Amplitude of process, related to Feynman diagrams.
- $\mathrm{d}\Pi_n \to$ Phase space, allowed states depending on $\overrightarrow{p_i}$ and E_i.

Darts ~ Particles



Area of Dartboard ~ Cross Section (σ)

Lagrangian Formalism and QED

- Equivalent to Newtonian Formalism from intro Physics classes.
- Fundamental quantity: Lagrangian L, contains all information about a theory.
- Quantum Electrodynamics (QED): quantum field theory that governs all electromagnetic interactions.

Free fermions propagate in space.

$$L_{\text{QED}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \sum_{\text{fermions}} \bar{\psi}_f \left(i\gamma^\mu \partial_\mu - m_f \right) \psi_f - Q_f e \bar{\psi} \gamma^\mu \psi A_\mu$$

Free photon propagates in space.

Photon interacts with fermions/antifermions (EM)

Feynman Diagrams

- Pictorial representation of spacetime processes.
- Related to interaction term L_{int} in L.
- In QED,

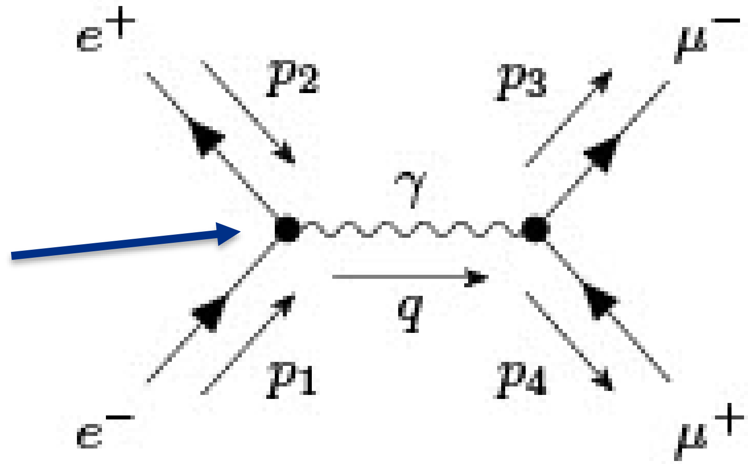
Photon field

$$L_{\text{int}} = \sum_{\text{fermions}} Q_f e \underbrace{\bar{\psi}_f} \gamma^\mu \underbrace{\psi_f} \overbrace{A_\mu}.$$

Antifermion/fermion fields

- Example: $e^- e^+ \rightarrow \gamma \rightarrow \mu^- \mu^+$

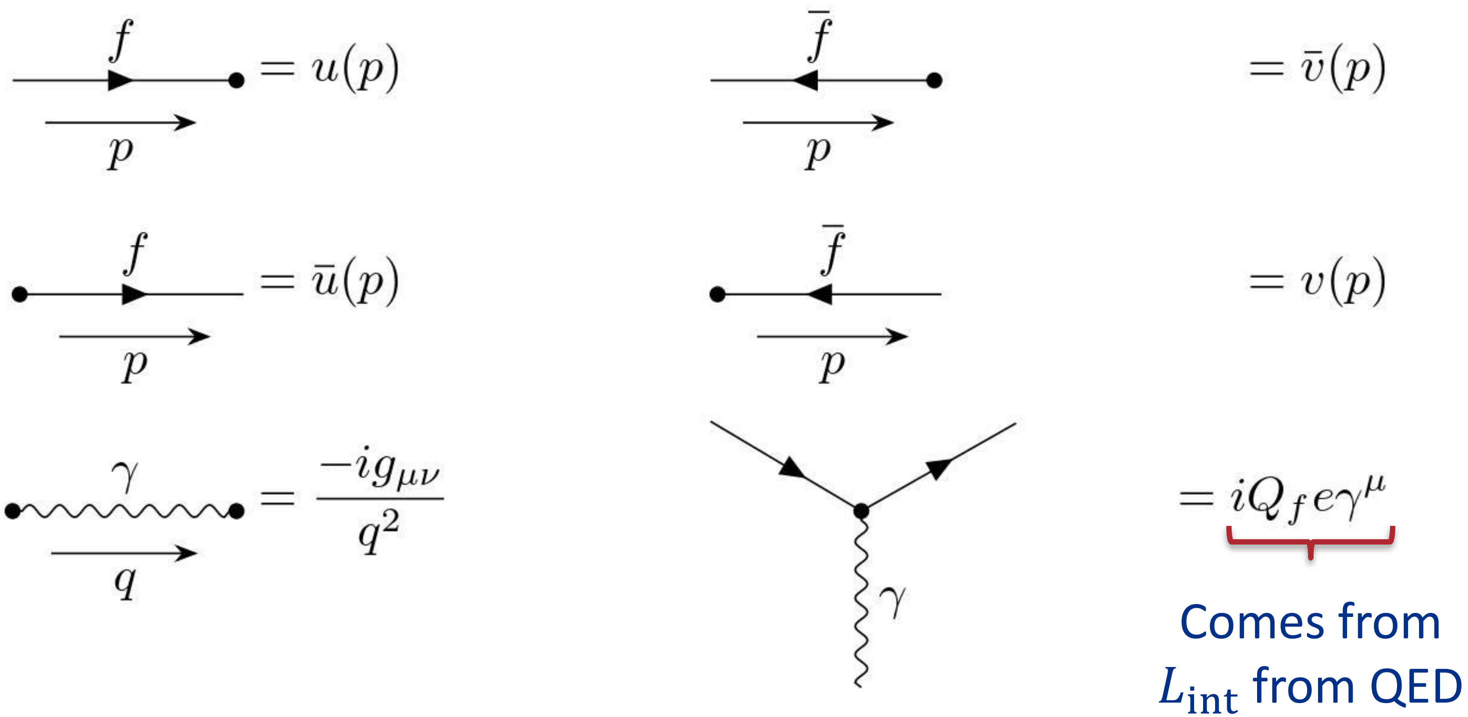
$\propto Q_f e \subset L_{\text{int}}$



Fermilab

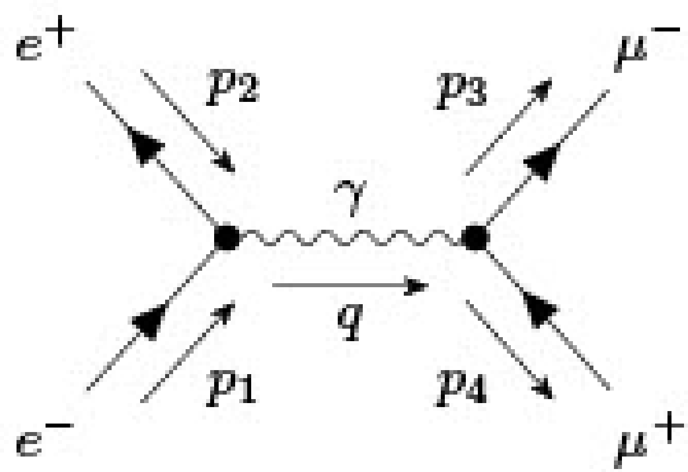
QED Feynman Rules

- Can get M from Feynman diagrams.
- Feynman Rules: express diagrams mathematically.



QED Feynman Rules

- Getting M from Feynman diagram: $e^-e^+ \rightarrow \mu^-\mu^+$



$$M = \underbrace{\bar{v}(p_2)}_{e^+}\overbrace{(-ie\gamma^\mu)}^{e^+e^-\gamma \text{ vertex}}\underbrace{u(p_1)}_{e^-}\overbrace{\left(\frac{-ig_{\mu\nu}}{q^2}\right)}^{\gamma}\underbrace{\bar{u}(p_3)}_{\mu^-}\overbrace{(-ie\gamma^\nu)}^{\mu^+\mu^-\gamma \text{ vertex}}\underbrace{v(p_4)}_{\mu^+}$$

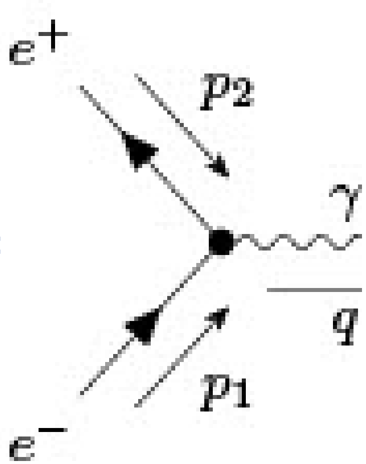
Hadronic and Leptonic Tensor

- $M \rightarrow |M|^2$
- In general,

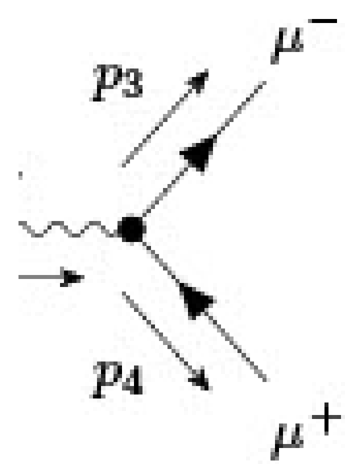
$$|M|^2 = \underbrace{L_{\mu\nu}}_{\text{Leptonic tensor}} \overbrace{H^{\mu\nu}}^{\text{Hadronic tensor}}$$

- For $e^-e^+ \rightarrow \mu^-\mu^+$,

$L_{e^-,\mu\nu} =$

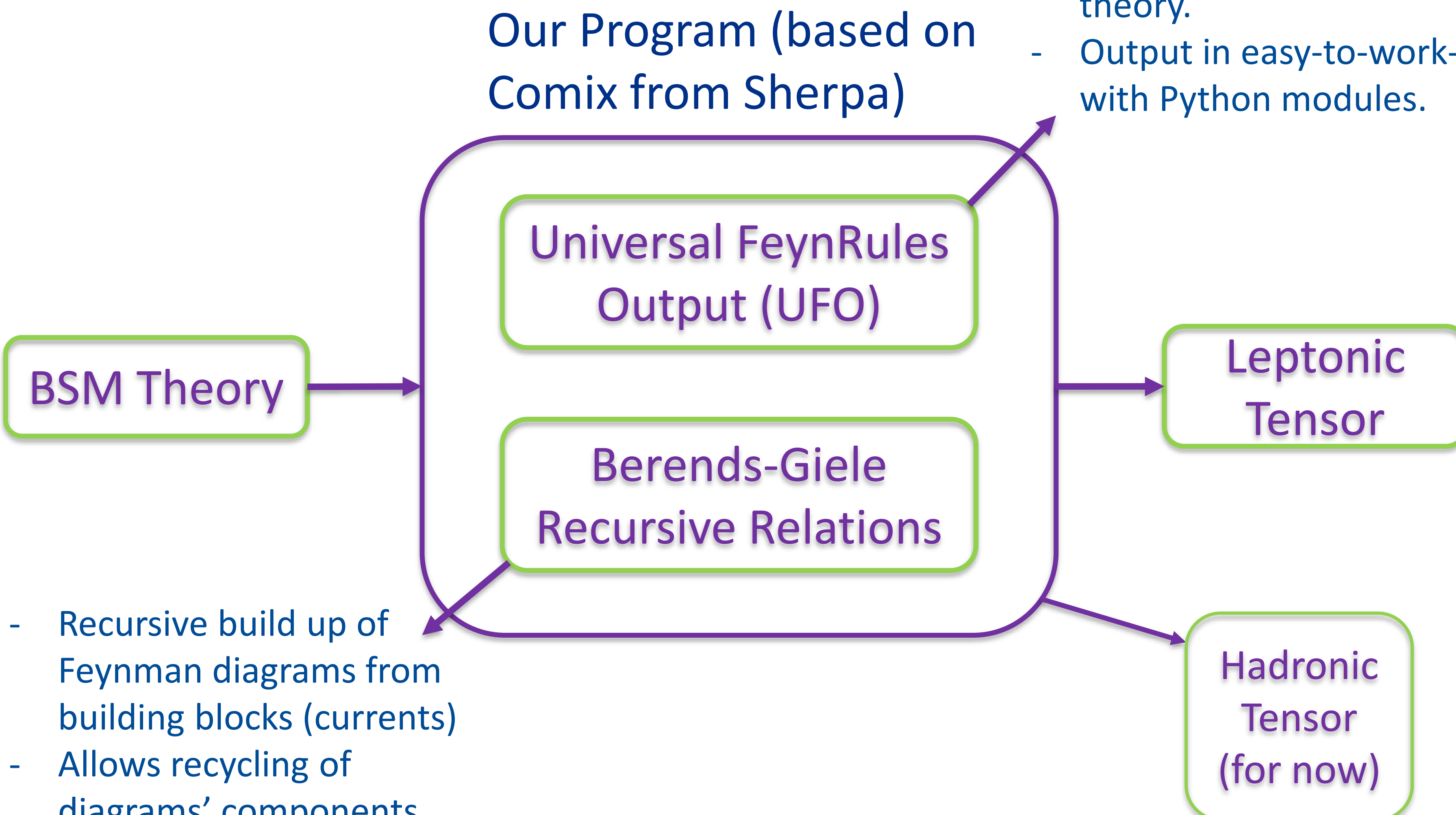


$L^{\mu\nu}_{\mu^-} =$



Methods of our Program

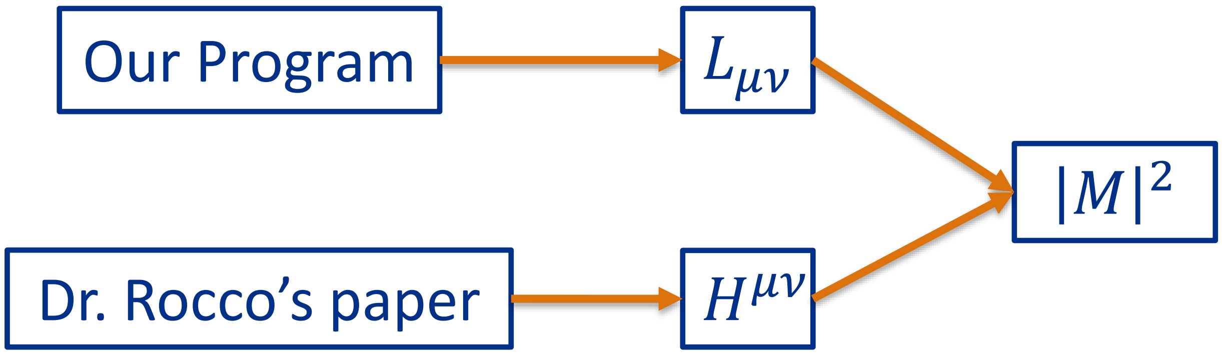
- Contains all necessary information about the theory.
- Output in easy-to-work-with Python modules.



- Recursive build up of Feynman diagrams from building blocks (currents)
- Allows recycling of diagrams' components. Highly efficient.

Validation Methods

- To validate results with literature, we need:
  - $\sigma = \int d\Pi \, |M|^2$
  - $H^{\mu\nu}$: $|M|^2 = L_{\mu\nu} H^{\mu\nu}$
- $H^{\mu\nu}$ comes from Noemi Rocco's *Ab Initio* Calculations of Lepton-Nucleus Scattering (DOI: [10.3389/fphy.2020.00116](https://doi.org/10.3389/fphy.2020.00116))

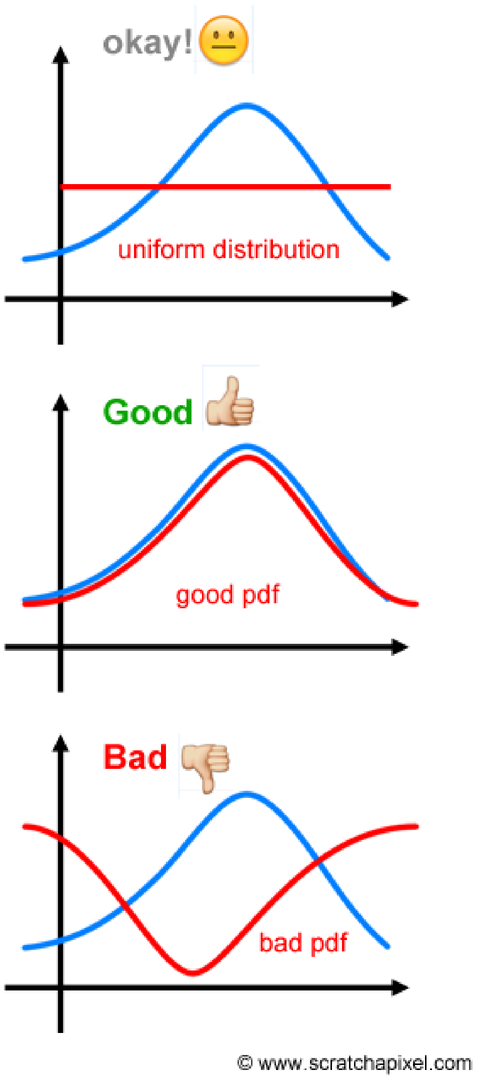


Validation Methods

- σ calculation: too complicated, use numerical methods.
- Multichannel Monte Carlo phase space integrator
  - Random sampling to estimate integral.
  - Relies on different prob. distributions $g_i(x)$ (channels) each with prob. α_i.
  - α_i can be optimized to reduce variance (error).
- Numerical integral:

$$g(x) = \sum_i \alpha_i g_i(x)\,; w(x) = \frac{f(x)}{g(x)}$$

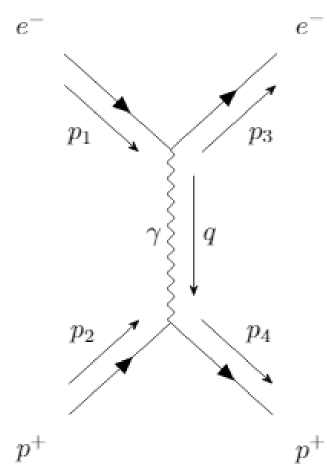
$$\langle w \rangle = \int w(x) g(x) dx = \int f(x) dx$$



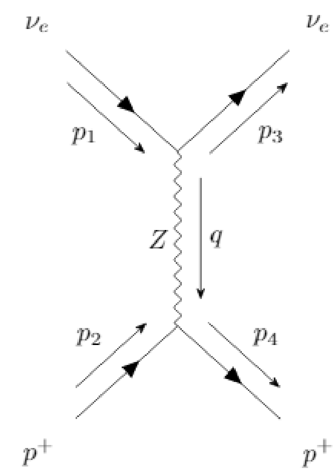
Results

- Validation results of 2 SM processes:
  - $e^- p^+ \rightarrow e^- p^+$
  - $\nu_e p^+ \rightarrow \nu_e p^+$
- Results from previous version of program. Main differences:
  - Nucleus/hadrons treated as point-like particles ($H^{\mu\nu}$).
  - Phase space integration (Rambo)
- Using $L_{\mu\nu}$, calculated $|M|^2$ vs $\cos(\theta)$ for $E_{\text{CM}} = \{20, 60, 100, 140, 180, 200\}$ (GeV), and σ vs E_{CM}.

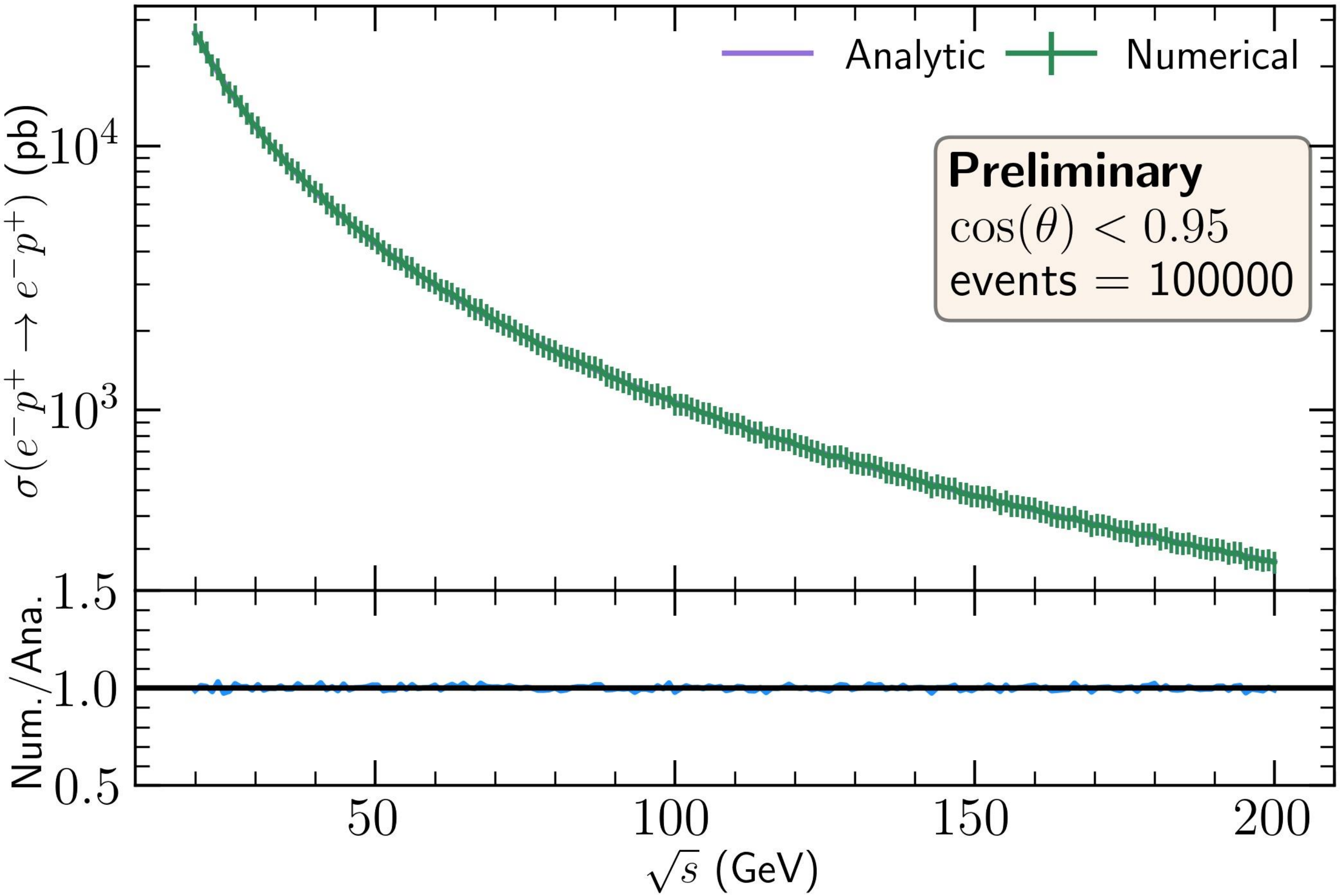
$$e^- p^+ \rightarrow e^- p^+$$



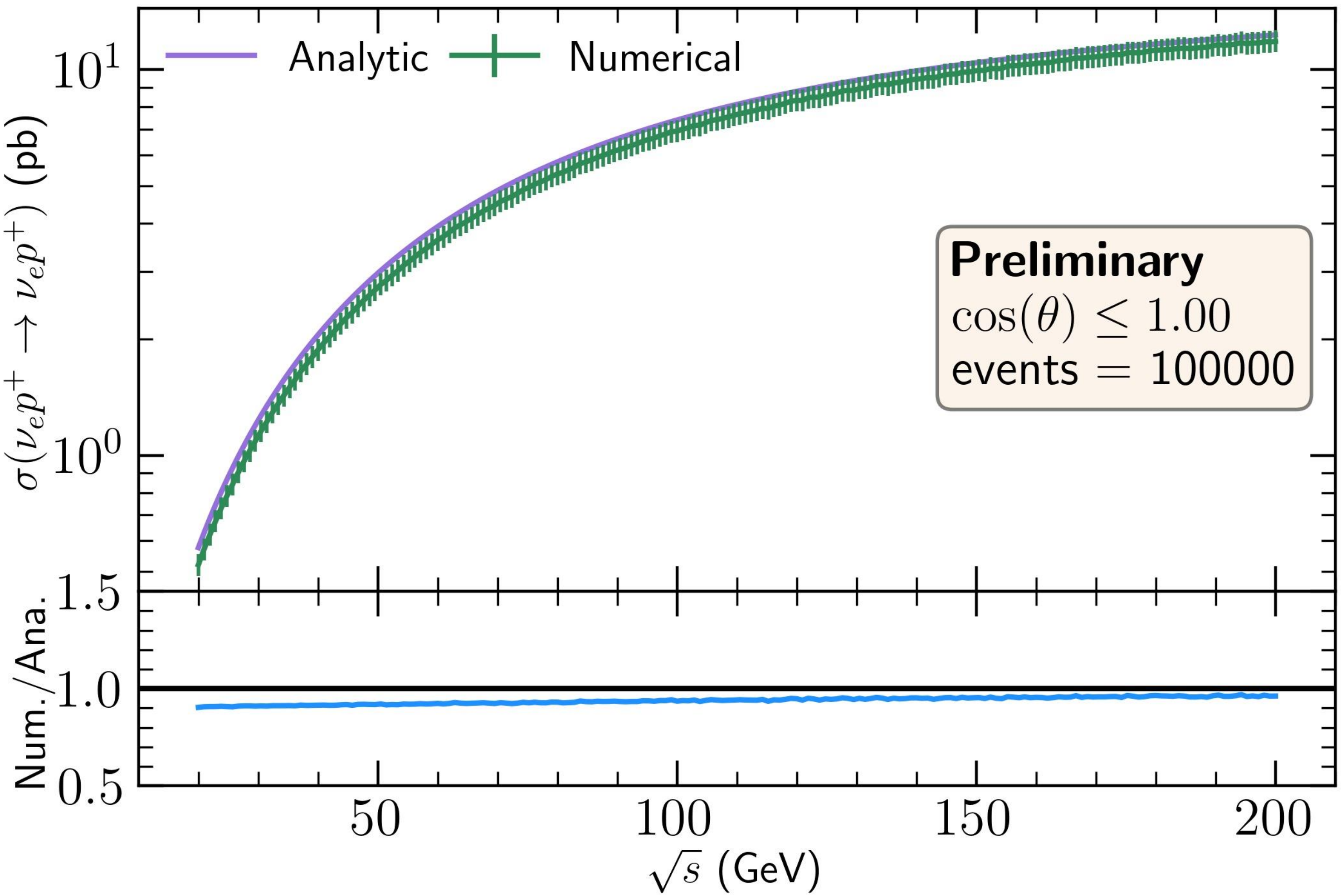
$$\nu_e p^+ \rightarrow \nu_e p^+$$



$e^- p^+ \to e^- p^+$: σ vs E_{CM}



$\nu_e p^+ \rightarrow \nu_e p^+$: σ vs E_{CM}



Conclusion and Future Steps

- Numerical and analytic results in good agreement.
- Promising for more complex SM processes as well as BSM models.
- Future steps:
  - Produce plots from newest version (multichannel, nuclear physics effects)
  - Test neutrino trident processes
  - Validate results from literature
  - Produce distributions to look for in DUNE based on BSM models that explain MiniBooNE excess.

Thank you