Exploring Precision EW Measurement Potential of $\mathrm{e}^+\mathrm{e}^-$ Colliders

Status Report to EF04

Graham W. Wilson

University of Kansas

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See previous report on ILC EW potential, talk at ICHEP2020, and references in LOI for more details.

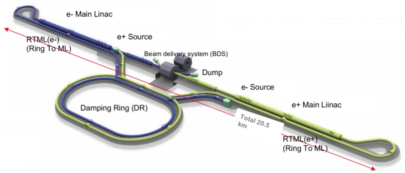
LOI = SNOWMASS21-EF4_EF0-AF3_AF0-IF3_IF5_GrahamWilson-119

ILC

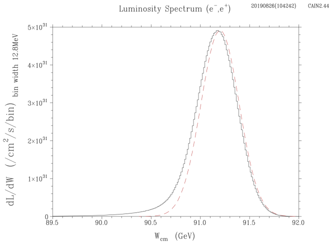
The ILC linear e^+e^- collider has been designed with an emphasis on an **initial-stage Higgs factory** that starts at $\sqrt{s} = 250$ GeV and is **expandable in energy** to run at higher energies for pair production of top quarks and Higgs bosons, and potentially to 1 TeV and more.

The **unique feature** of **longitudinally polarized electron and positron beams** and the **higher energies** open up many new measurement possibilities. These are very complementary to those feasible with e^+e^- circular colliders.

The ILC is designed primarily to explore the 200 – 1000 GeV energy frontier regime. This has been the focus in making the case for the project. It is also capable of running at the **Z** and **WW** threshold.



See Benno List's talk for ILC details



Z running – see Yokoya, Kubo, Okugi

LOI Questions

1. An overarching question is how well can ILC running at lower $\sqrt{s}$, particularly near the Z-pole, perform **statistically** and **systematically** for measurements of PEW observables including those already explored at SLC/LEP?
2. Would this offer significant advantages over only running at energies above ZH threshold?
3. A related question is how such running with ILC compares statistically and systematically with the various circular $\mathrm{e}^+\mathrm{e}^-$ collider proposals?

On the one hand, the circular approach now targets enormous luminosity at low energy, but on the other hand, is therefore enormous and expensive. If realized for $\mathrm{e}^+\mathrm{e}^-$ would likely be on a longer time horizon than ILC.

For both collider types, whether one can exploit the very large statistics and not be dominated by systematics is at the heart of these questions.

Key issue: systematic control for the absolute scale of center-of-mass energy (in collision...) and reconstructed mass at **all** center-of-mass energies

LOI Studies in Progress

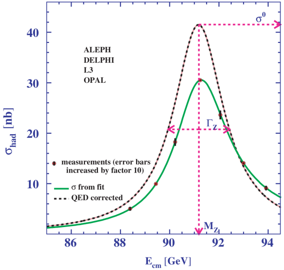
Studies are being undertaken:

1. to understand ILC capabilities for a precision measurement of the Z lineshape observables with a **scan** using polarized beams,
2. to further explore an experimental strategy for $\sqrt{s}$ determination using di-leptons, and
3. to further explore M_{W} capabilities synergistic with a concurrent Higgs program.

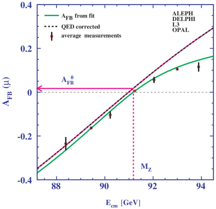
Today's focus: reporting progress on experimental issues associated with **item 2** which are a pre-requisite for getting the most out of a polarized Z scan (item 1).

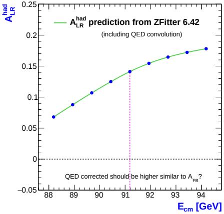
Polarized Beams Z Scan for Z LineShape and Asymmetries

Essentially, perform LEP/SLC-style measurements in all channels but also with $\sqrt{s}$ dependence of the polarized asymmetries, A_{LR} and $A^{f}_{FB,LR}$, in addition to A_{FB}. (Also polarized $\nu\overline{\nu}\gamma$ scan.) Not constrained to LEP-style scan points.



LEP: $\Delta M_{\mathrm{Z}} = 2100$ MeV, $\Delta\Gamma_{\mathrm{Z}} = 2300$ MeV





With 0.1 ab^{-1} polarized scan around M_{Z}, find **statistical** uncertainties of 35 keV on M_{Z}, and 80 keV on Γ_{Z}, from LEP-style fit to $(M_{\mathrm{Z}}, \Gamma_{\mathrm{Z}}, \sigma^0_{\mathrm{had}}, R^0_e, R^0_\mu, R^0_\tau)$ using ZFITTER for QED convolution. Started using model-independent S-matrix approach code.

Exploiting this fully needs in-depth study of $\sqrt{s}$ **calibration systematics**

ILC $\mathcal{L}$ is sufficient for M_{Z}

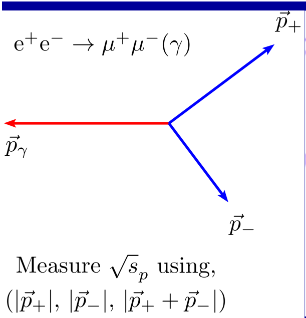
Γ_{Z} systematic uncertainty depends on $\Delta(\sqrt{s}_+ - \sqrt{s}_-)$, so expect $\Delta\Gamma_{\mathrm{Z}} < \Delta M_{\mathrm{Z}}$

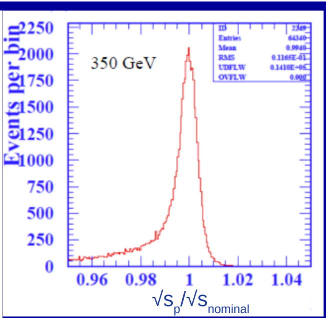
Center-of-Mass Energy Measurement

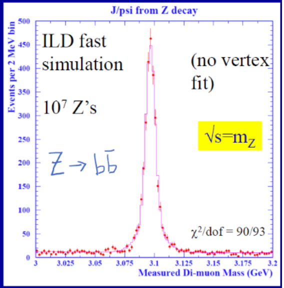
Critical input for $M_{\rm t}$, $M_{\rm W}$, $M_{\rm H}$, $M_{\rm Z}$, $M_{\rm X}$, $\Gamma_{\rm Z}$ measurements

1. Standard precision of $\mathcal{O}(10^{-4})$ in $\sqrt{s}$ for $M_{\rm t}$ straightforward
2. Targeting precision of $\mathcal{O}(10^{-5})$ in $\sqrt{s}$ for $M_{\rm W}$ given likely systematics
3. For $M_{\rm Z}$ - helps to do even better. Now targeting of $\mathcal{O}(10^{-6})$.

Use dilepton **momenta** method, with $\sqrt{s_p} \equiv E_+ + E_- + |\vec{p}_{+-}|$ as $\sqrt{s}$ estimator.
Tie detector p-scale to particle mass scales (J/ψ known to 1.9 ppm).







Measure $<\sqrt{s}>$ and luminosity spectrum with same events. Expect statistical uncertainty of 1.0 ppm on p-scale per 1.2M $J/\psi \to \mu^+\mu^-$ (4×10^9 hadronic Z's).

Introduction to Center-of-Mass Energy Issues

- The proposed $\sqrt{s}_p$ method uses only the momenta of leptons in dilepton events.
- Critical issue for $\sqrt{s}_p$ method: calibrating the **tracker momentum scale**.
- Can use $\mathrm{K}^0_{\mathrm{S}}$, Λ, $J/\psi \to \mu^+\mu^-$ (mass known to 1.9 ppm).

For more details see studies of $\sqrt{s}_p$ from ECFA LC2013, and of momentum-scale from AWLC 2014. Recent $\mathrm{K}^0_{\mathrm{S}}$, Λ studies at LCWS 2021 – much higher precision feasible ... few **ppm** (not limited by parent mass knowledge or J/ψ statistics).

Today,

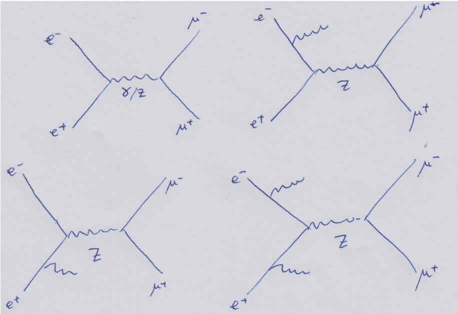
- Look more carefully at the $\sqrt{s}_p$ method prospects with $\mu^+\mu^-$
- Brief overview of the "new" concept in recent tracker momentum scale studies (LCWS2021 talk).
- Include crossing angle, full simulation and reconstruction with ILD, track error matrices, and updated ILC $\sqrt{s} = 250$ GeV beam spectrum
- In progress, treatment of detected ISR/FSR photons and vertex fitting
- Bonus. Physics: M_{Z}. Beam knowledge: **luminosity spectrum**, $\mathrm{dL}/\mathrm{d}\sqrt{s}$.

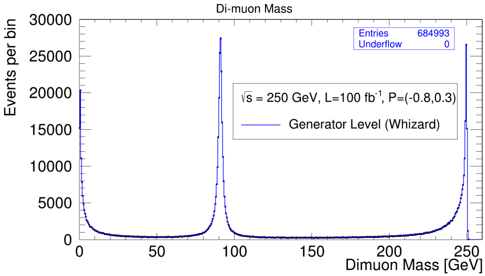
Dimuons

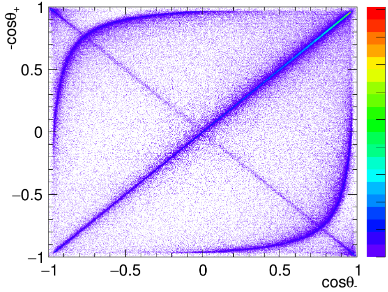
Three main kinematic regimes.

1. **Low** mass, $m_{\mu\mu} < 50$ GeV
2. **Medium** mass, $50 < m_{\mu\mu} < 150$ GeV
3. **High** mass, $m_{\mu\mu} > 150$ GeV

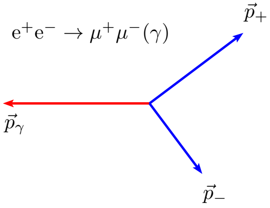
- Back-to-back events in the full energy peak.
- Significant radiative return (ISR) to the Z and to low mass.







$\mathrm{e^+e^-} \to \mu^+\mu^-(\gamma)$



Measure $\sqrt{s}_p$ using,
$(|\vec{p}_+|, |\vec{p}_-|, |\vec{p}_+ + \vec{p}_-|)$

Assuming,

- Equal beam energies, E_b
- The lab is the CM frame, $(\sqrt{s} = 2\,E_\mathrm{b},\ \sum \vec{p}_i = 0)$
- The system recoiling against the dimuon is massless

$$\sqrt{s} = \sqrt{s}_p \equiv E_+ + E_- + |\vec{p}_+ + \vec{p}_-|$$

$$\sqrt{s}_p = \sqrt{p_+^2 + m_\mu^2} + \sqrt{p_-^2 + m_\mu^2} + |\vec{p}_+ + \vec{p}_-|$$

An estimate of $\sqrt{s}$ using only the (precisely measurable) muon momenta

New approach to tracker momentum scale

See LCWS2021 talk for details. Use Armenteros-Podolanski kinematic construction for 2-body decays (AP).

1. Explore AP method using mainly $\mathrm{K^0_S} \to \pi^+\pi^-$, $\Lambda \to \mathrm{p}\pi^-$ (inspired by Rodríguez et al.). **Much higher statistics than J/ψ alone.**
2. If proven realistic, **enables precision Z program** (polarized lineshape scan)
3. Bonus: potential for **large improvement in** parent and child particle **masses**

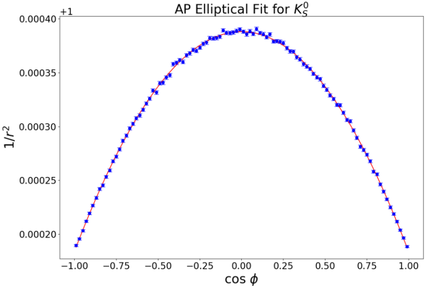
For a "V-decay", $M^0 \to m_1^+ m_2^-$, decompose the child particle lab momenta into components transverse and parallel to the parent momentum. The distribution of (child p_T, $\alpha \equiv \frac{p_L^+ - p_L^-}{p_L^+ + p_L^-}$) is a semi-ellipse with parameters relating the CM decay angle, θ^*, β, and the masses, (M, m_1, m_2), that determine, p^*.

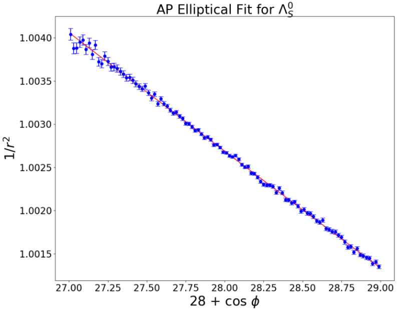
By obtaining sensitivity to both the parent and child masses, and positing improving ourselves the measurements of more ubiquitous parents ($\mathrm{K^0_S}$ and Λ), can obtain high sensitivity to the momentum scale

Proving the feasibility of sub-10 ppm momentum-scale uncertainty needs much work when typical existing experiments are at best at the 100 ppm level

Tracker momentum scale sensitivity estimate

Used sample of 250M hadronic Z's at $\sqrt{s} = 91.2$ GeV. Fit $\mathrm{K^0_S}, \Lambda, \overline{\Lambda}$ in various momentum bins.

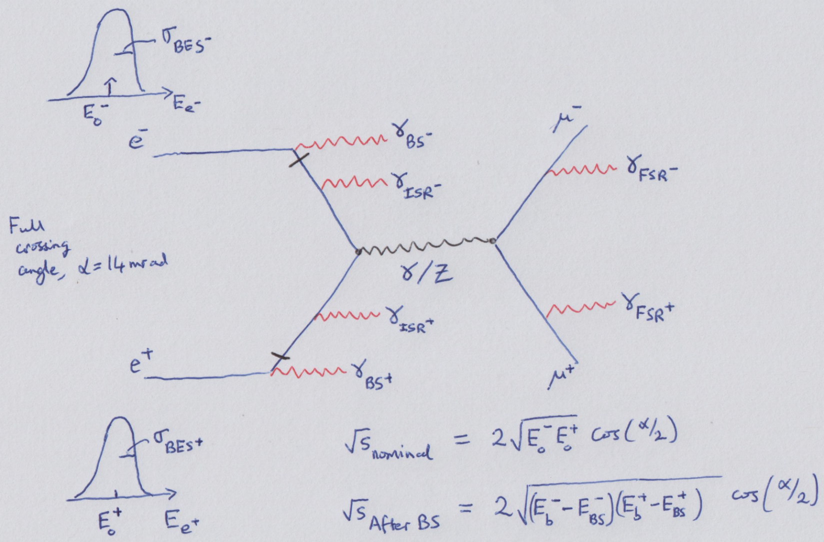




1. $m_{\mathrm{K^0_S}}$: 0.48 ppm
2. m_Λ: 0.072 ppm
3. m_π: 0.46 ppm
4. S_p: 0.57 ppm

- Fit fixes proton mass
- Factors of (54, 75, 3) improvement over PDG for $(\mathrm{K^0_S}, \Lambda/\overline{\Lambda}, \pi^\pm)$
- Momentum-scale to **2.5 ppm stat.** per 10M hadronic Z, ILC Z run has 400 such samples.

Returning to $\sqrt{s}_p$ and Adding More Realism



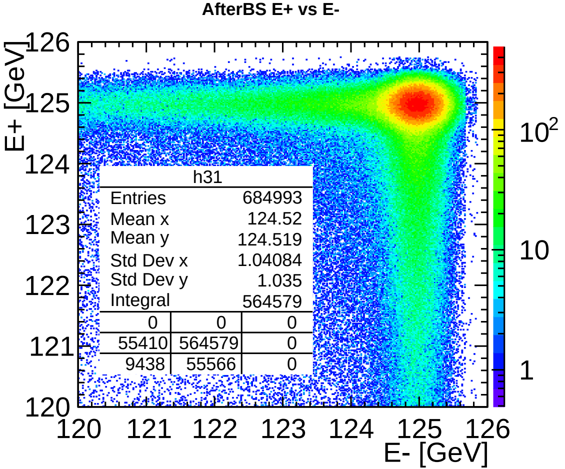
See backup for more detailed explanations

What do we really want to measure?

Ideally, the 2-d distribution of the **absolute beam energies** after beamstrahlung. From this we would know the distribution of both $\sqrt{s}$ and the initial state momentum vector (especially the z component).

Now let's look at the related 1-d distributions $(E_+, E_-, \sqrt{s},\ p_z)$ with empirical fits.

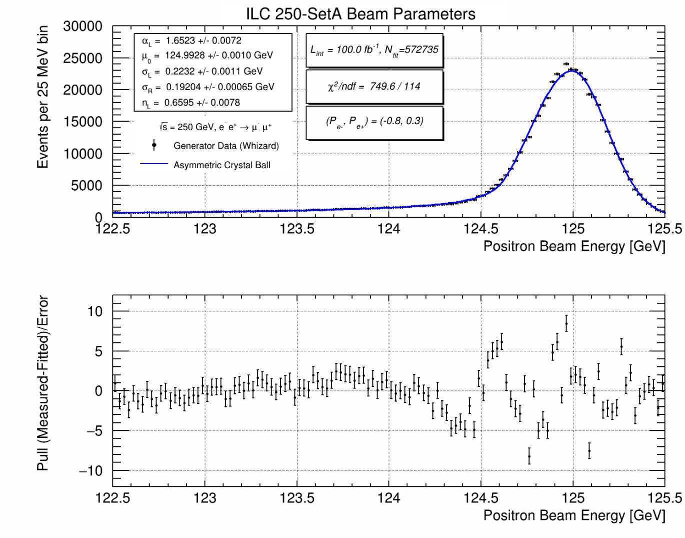
[dL/d$\sqrt{s}$: see work by Frary, Miller, Moenig, Sailer, Poss]



Whizard 250 GeV SetA $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-(\gamma)$ events

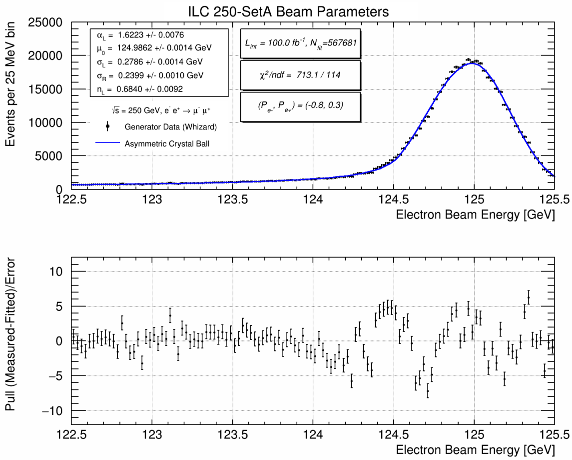
Positron Beam Energy (After Beamstrahlung)

Fits use asymmetric Crystal Ball with 5 parameters (details in backup)



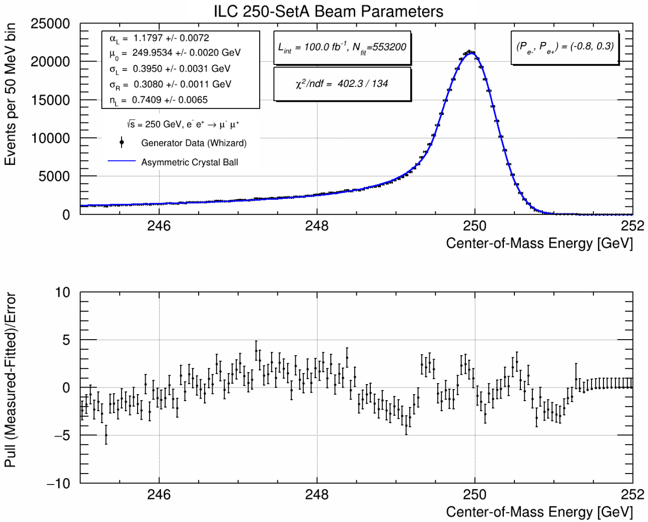
$$\sigma_{\mathrm{R}}/E = 0.1536 \pm 0.0005\% \text{ (cf 0.152\% in TDR)}$$

Electron Beam Energy (After Beamstrahlung)



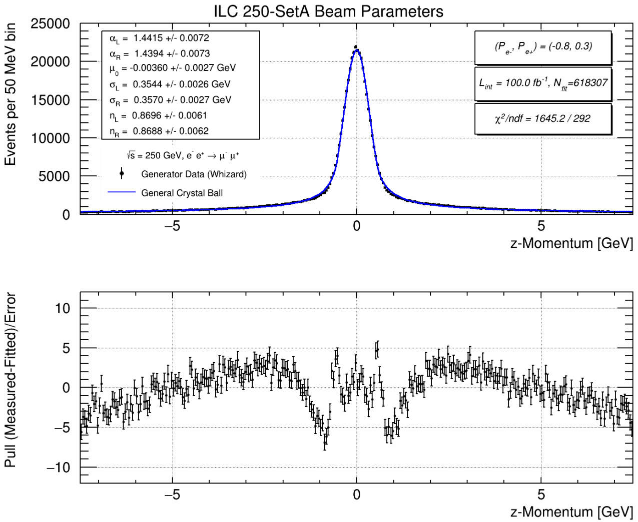
$$\sigma_{\mathrm{R}}/E = 0.1919 \pm 0.0008\% \text{ (cf } 0.190\% \text{ in TDR)}$$

Note an undulator bypass could reduce this spread when one e^- cycle is used purely for e^+ production.



$$\sigma_{\mathrm{R}}/\sqrt{s} = 0.1232 \pm 0.0004\% \text{ (cf } 0.122\% \text{ in TDR (} 0.190\% \oplus 0.152\%)/2)$$

z-Momentum of e^+e^- system (After Beamstrahlung)



$\sigma/\sqrt{s} = 0.1416 \pm 0.0007\%$ (cf 0.122% from beam energy spread alone)

Initial State Kinematics with Crossing Angle

Define the two beam energies (after beamstrahlung) as E_{b}^{-} and E_{b}^{+} for the electron beam and positron beam respectively.
Initial-state energy-momentum 4-vector (neglecting m_{e})

$$E = E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}$$

$$p_x = (E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}) \sin(\alpha/2)$$

$$p_y = 0$$

$$p_z = (E_{\mathrm{b}}^{-} - E_{\mathrm{b}}^{+}) \cos(\alpha/2)$$

The corresponding center-of-mass energy is

$$\sqrt{s} = 2\sqrt{E_{\mathrm{b}}^{-} E_{\mathrm{b}}^{+}} \cos(\alpha/2)$$

Hence if α is known, evaluation of the center-of-mass energy of this collision amounts to measuring the two beam energies. Introducing,

$$E_{\mathrm{ave}} \equiv \frac{E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}}{2} \; , \overline{\Delta E_{\mathrm{b}}} \equiv \frac{E_{\mathrm{b}}^{-} - E_{\mathrm{b}}^{+}}{2}$$

then with this notation,

$$\sqrt{s} = 2\sqrt{E_{\mathrm{ave}}^{2} - (\overline{\Delta E_{\mathrm{b}}})^{2}} \cos(\alpha/2)$$

Final State Kinematics and Equating to Initial State

Let's look at the final state of the $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-(\gamma)$ process. Denote the μ^+ as particle 1, the μ^- as particle 2, and the rest-of-the event (RoE) as system 3. We can write this final-state system 4-vector as

$$(E_1 + E_2 + E_3,\ \vec{p_1} + \vec{p_2} + \vec{p_3})$$

Then applying $(E, \vec{p})$ conservation and assuming $m_3 = 0$ we obtain,

$$(E_1 + E_2 + E_3) = E_1 + E_2 + p_3 = 2\ E_{\mathrm{ave}} \tag{1}$$

$$\vec{p_1} + \vec{p_2} + \vec{p_3} = (2\ E_{\mathrm{ave}} \sin(\alpha/2), 0, 2\ \overline{\Delta E_{\mathrm{b}}} \cos(\alpha/2)) \equiv \vec{p}_{\mathrm{initial}} \tag{2}$$

In general the RoE may not be fully detected and needs to be inferred using $(E, \vec{p})$ conservation. We have 4 equations and 5 unknowns, namely the 3 components of the RoE momentum ($\vec{p_3}$) and E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$.
One approach is to solve for E_{ave} for various assumptions on $\overline{\Delta E_{\mathrm{b}}}$. Specifically we then focus on using the simplifying assumption that $\overline{\Delta E_{\mathrm{b}}} = 0$. Note this is often a poor assumption event-by-event for the p_z conservation component.

The Averaged Beam Energy Quadratic

Using the outlined approach results in a quadratic equation in E_{ave}, ($AE_{\mathrm{ave}}^2 + BE_{\mathrm{ave}} + C = 0$), with coefficients of

$$A = \cos^2(\alpha/2)$$

$$B = -E_{12} + p_{12}^x \sin(\alpha/2)$$

$$C = (M_{12}^2)/4 + p_{12}^z \overline{\Delta E_{\mathrm{b}}} \cos(\alpha/2) - \overline{\Delta E_{\mathrm{b}}}^2 \cos^2(\alpha/2)$$

Based on this, there are three particular cases of interest to solve for E_{ave}.

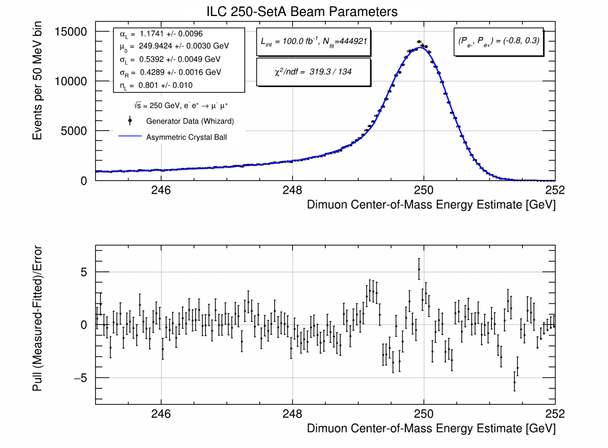
1. Zero crossing angle, $\alpha = 0$, and zero beam energy difference.
2. Crossing angle and zero beam energy difference.
3. Crossing angle and non-zero beam energy difference.

The original formula,

$$\sqrt{s} = E_1 + E_2 + |\vec{p}_{12}|$$

arises trivially in the first case. In the rest of this talk I will use the $\sqrt{s}$ estimate from the largest positive solution of the second case as what I now mean by $\sqrt{s}_p$. Obviously it is also a purely muon momentum dependent quantity.

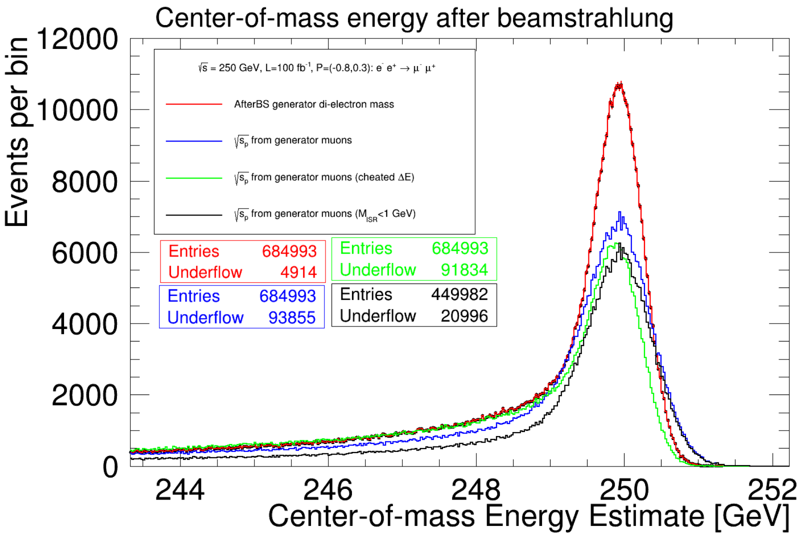
Dimuon Estimate of Center-of-Mass Energy (After BS)



$\sigma_{\mathrm{R}}/\sqrt{s} = 0.1716 \pm 0.0006\%$ (cf 0.1232% with true $\sqrt{s}$)

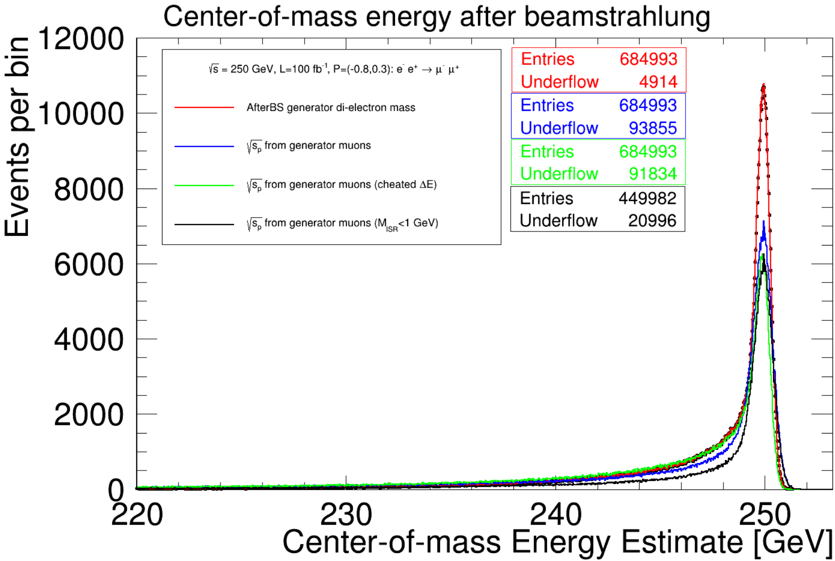
- This is the generator-level $\sqrt{s}_p$ calculated from the 2 muons
- Why so broad? Why fewer events?
- Likely because some events violate the assumptions that $\overline{\Delta E_{\mathrm{b}}} = 0$ and $m_3 = 0$
- The former is no surprise given the p_z distribution
- The latter can be associated with events with 2 or more non-collinear ISR/FSR photons

Comparisons (After BS)

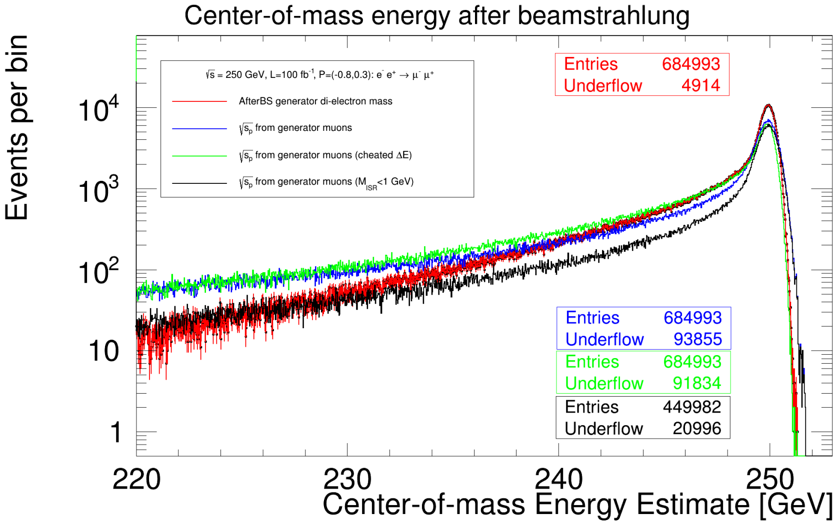


Note: Underflow statistics refer to $<$ 220 GeV. Next 2 slides - same but wider scale

Comparisons I (After BS) Linear



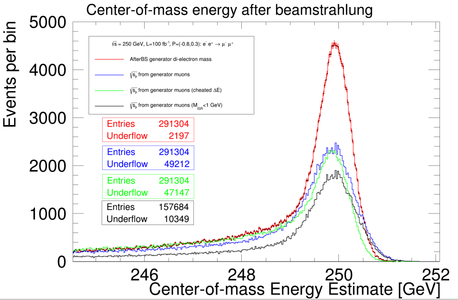
Comparisons II (After BS) Log

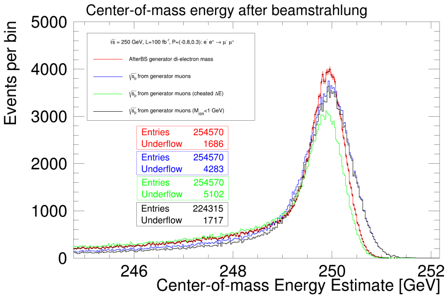


What's Going On?

$50 < m_{\mu\mu}^{\text{gen}} < 150$ GeV

$m_{\mu\mu}^{\text{gen}} > 150$ GeV





- For lower dimuon mass events, only about half are reconstructed close to $\sqrt{s}$
- Most higher dimuon mass events reconstructed close to the original $\sqrt{s}$

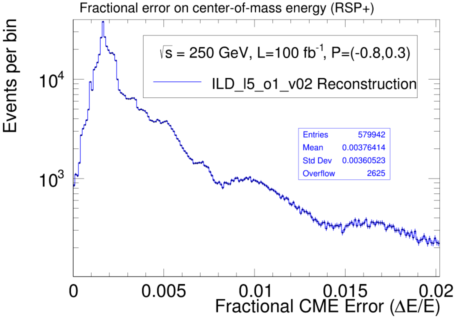
Conclusion

Lower dimuon mass events are more likely to violate the assumptions.

Event Selection Requirements

Currently rather simple.
Use latest full ILD simulation/reconstruction at 250 GeV.

- Require exactly two identified muons
- Opposite sign pair
- Require uncertainty on estimated $\sqrt{s}_p$ of the event of less than 0.8% based on propagating track-based error matrices
- Categorize reconstruction quality as gold (<0.15%), silver ([0.15, 0.30]%), bronze ([0.30, 0.80]%)
- Require the two muons pass a vertex fit with p-value > 1 %



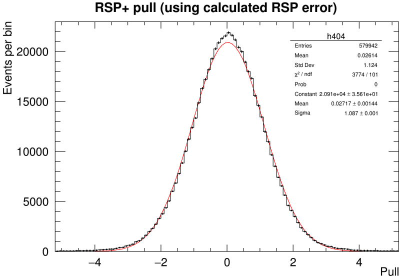
Selection efficiencies for (80%/30%) beam polarizations:

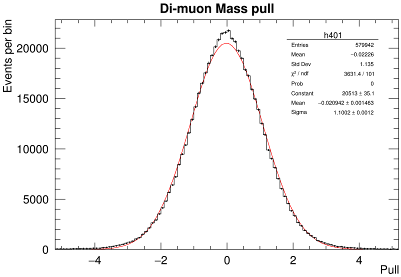
- $\varepsilon_{-+} = 69.77 \pm 0.06$ %
- $\varepsilon_{+-} = 67.35 \pm 0.06$ %
- $\varepsilon_{--} = 69.47 \pm 0.05$ %
- $\varepsilon_{++} = 67.72 \pm 0.06$ %

Backgrounds not yet studied in detail, ($\tau^+\tau^-$ is small:0.15%, of no import for the $\sqrt{s}$ peak region).

Dimuon Pull Distributions

- Pull $\equiv$ (meas - true)/error.
- Track-based estimates of the errors on both the $\sqrt{s}_p$ quantity (left) and the di-muon mass (right) agree well with the modeled uncertainties for reconstructed dimuon events.





- In both cases the fitted rms over this range is about 10% larger than ideal. Central range well described. Suspect tails should be non-Gaussian given the non-Gaussian tails of multiple scattering.
- In practice this is rather encouraging

Vertex Fit: Exploit ILC nanobeams

Given that the track errors are well modeled and the 2 muons should originate from a common vertex consistent with the interaction point, we can perform:

- Vertex Fit: Constrain the two tracks to a common point in 3-d
- Beam-spot Constrained Vertex Fit

The ILC beam-spot size is $(\sigma_x, \sigma_y) = (515, 7.7)$ nm, $\sigma_z = 0.202$ mm

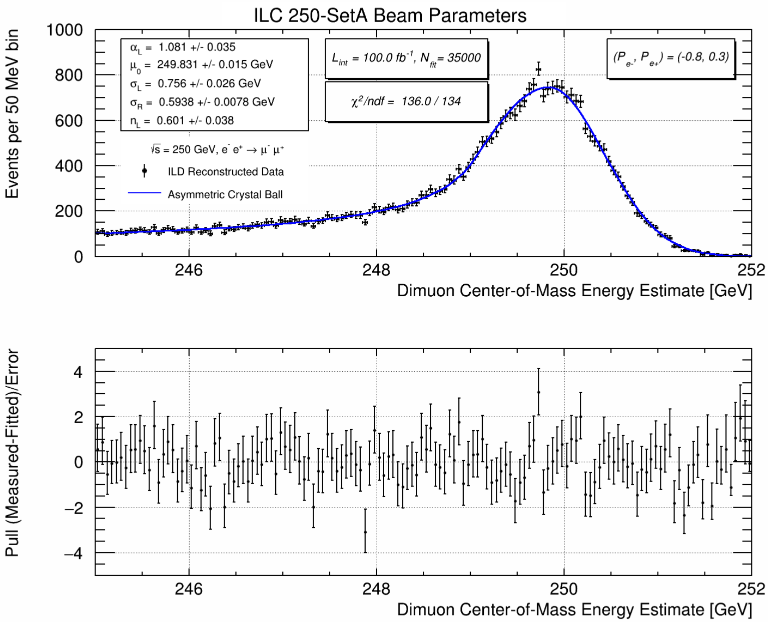
- Vertex fit along same lines as AWLC2014 talk has been re-implemented using the fully simulated data
- Also have explored beam-spot constraints

What good is this?

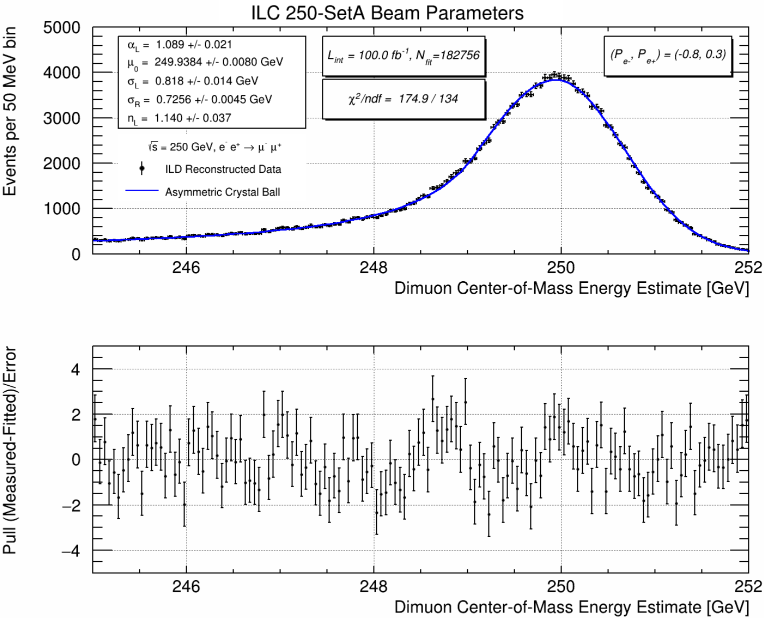
- Residual background rejection (eg. $\tau^+\tau^-$ reduced by factor of 20)
- Additional handle for rejecting or deweighting mis-measured events
- Some modest improvement in precision of di-muon kinematic quantities
- Also useful for $\mathrm{H} \to \mu^+\mu^-$ and for ZH recoil
- Interaction point measurement ($\mathcal{O}(1\mu\mathrm{m})$ resolution per event) could be useful to correlate with (E_-, E_+) for understanding beamstrahlung

Note: simulated data does not currently simulate the transverse beam-spot ellipse

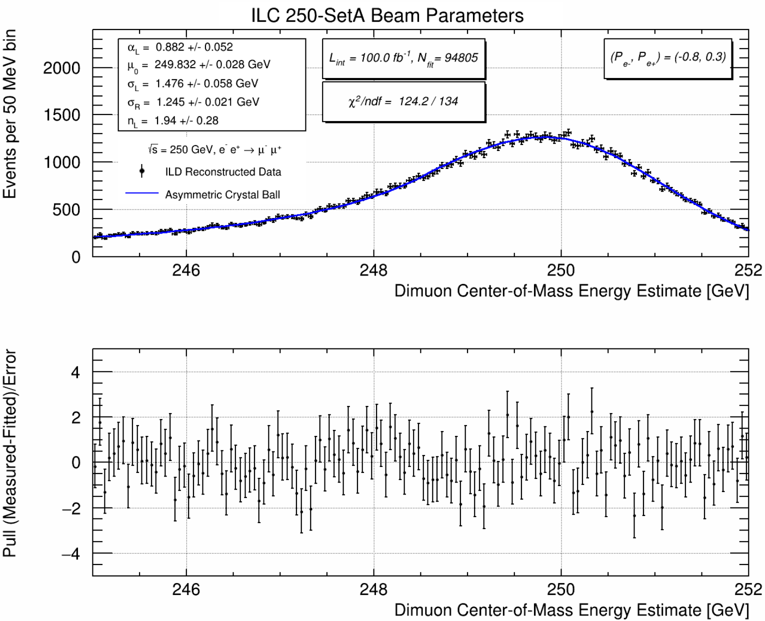
Gold Quality Dimuon PFOs (After BS)



Silver Quality Dimuon PFOs (After BS)



Bronze Quality Dimuon PFOs (After BS)



Strategy for Absolute $\sqrt{s}$ and Estimate of Precision

Prior Estimation Method

- Guesstimate how well the peak position of the Gaussian can be measured using the observed $\sqrt{s}_p$ distributions in bins of fractional error

Current Thinking

- The luminosity spectrum and absolute center-of-mass energy are the same problem or at least very related. How well one can determine the absolute scale depends on knowledge of the shape (input also from Bhabhas).
- Beam energy spread likely to be well constrained by spectrometer data
- Likely need either a convolution fit (CF) or a reweighting fit
- We are currently working on a CF by parametrizing the underlying (E_-, E_+) distribution, and modeling quantities related to $\sqrt{s}$ and p_z after convolving with detector resolution (and ISR, FSR and cross-section effects)

Current Estimation Method

- Follow a similar approach to before, but using estimates of the statistical error on μ_0 for 5-parameter Crystal Ball fits to fully simulated data with the 4 shape parameters fixed to their best fit values. Fits are done in the various resolution categories (example gold, silver, bronze fits in backup slides).
- Next slide has these estimates

$\sqrt{s}$ Sensitivity Estimate at $\sqrt{s} = 250$ GeV

Statistical uncertainties in ppm on $\sqrt{s}$ for $\mu^+\mu^-$ channel

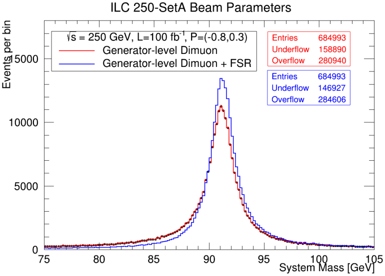
L_{int} [ab^{-1}]	Poln [%]	Gold	Silver	Bronze	G+S+B
0.9	$-80, +30$	6.5	3.1	8.5	2.7
0.9	$+80, -30$	7.7	3.4	9.6	3.0
0.1	$-80, -30$	26	12.1	33	10.4
0.1	$+80, +30$	29	13.0	41	11.4
2.0	–	4.8	2.2	6.2	**1.9**

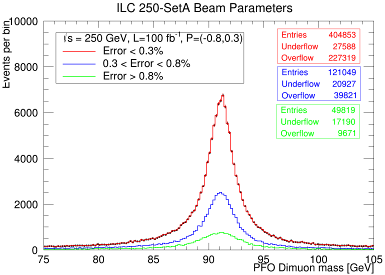
Fractional errors on μ_0 parameter (mode of peak) when fitting with 5-parameter Crystal Ball function with all 4 shape parameters fixed to their best-fit values.

Also the $\mathrm{e}^+\mathrm{e}^-$ channel should be used. The additional benefit of the much larger statistics from more forward Bhabhas is offset by the poorer track momentum resolution at forward angles.

Measuring M_Z using $m_{\mu^+\mu^-}$ with high energy running

Look at $\sqrt{s} = 250$ GeV running with latest beam parameters and full simulation





Adding in FSR photon(s) reduces the peak width to be consistent with Γ_Z. Improves statistical sensitivity on mode by 10–20%.

$m_{\mu^+\mu^-}$ resolution is much less than Γ_Z. Sensitivity estimates from prior study (next slide) with smeared MC will be reasonable.

Main systematics:

1. momentum-scale
2. FSR modeling/treatment
3. Electron p-scale in the e^+e^- channel

Also direct measurement of Γ_Z ?

Measuring M_{Z} from $m_{\mu^+\mu^-}$

Revisited old study of $\sqrt{s}_p$ at $\sqrt{s} = 250$, 350, 500, 1000 GeV. Used smeared MC. Fitted $m_{\mu^+\mu^-} \in [75, 105]$ GeV with sum of two Voigtians. Statistical uncertainties on the peak parameter, M_{Z}, scaled to full ILC program using simulations with TDR beam parameters

Statistical uncertainties for $\mu^+\mu^-$ channel

$\sqrt{s}$ [GeV]	L_{int} [ab^{-1}]	Poln [%]	Sharing [%]	ΔM_{Z} [MeV]
250	2.0	80/30	(45,45,5,5)	1.20
350	0.2	80/30	(67.5,22.5,5,5)	5.99
500	4.0	80/30	(40,40,10,10)	2.55
1000	8.0	80/20	(40,40,10,10)	5.75
All	14.2	–	–	1.05

- Current PDG uncertainty on M_{Z} is 2.1 MeV
- FSR makes effective Breit-Wigner width larger and shifts the peak
- Treatment of FSR and especially inclusion of $\mathrm{e}^+\mathrm{e}^-$ channel should decrease stat. uncertainty to **0.7 MeV**
- Sensitivity dominated by $\sqrt{s} = 250$ GeV running
- Main systematic - tracker p-scale. Target at most 2.5 ppm in this context.

Concluding Remarks on Plans

LOI has 3 main thrusts

1. New study on polarized Z-scan. While anchored in old studies of "Giga-Z" – much broader in scope and ambition. Very much welcome collaboration.
2. Further exploration based on existing studies of center-of-mass energy calibration using di-leptons. Significant progress in this area.
3. Further exploration based on existing studies and LEP2-style W mass measurements using WW production. Much room for additional work and collaboration.

In all cases welcome further collaboration.

- KU graduate student, Justin Anguiano, worked on some of the WW aspects of M_{W} 2011.12451
- Collaborating with others including Jenny List and Michael Peskin.
- KU graduate student, Brendon Madison, now working on aspects of the center-of-mass energy studies.

Summary of Progress

Progress

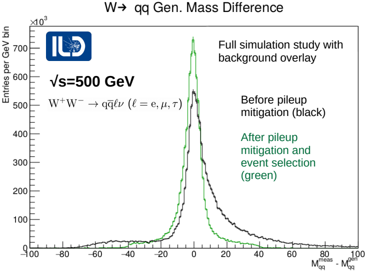
- New high precision method for momentum-scale using especially K^0_S and Λ. Promises 2.5 ppm uncertainty per 10M hadronic Zs.
- More detailed investigation of dimuons for $\sqrt{s}$ and dL/d$\sqrt{s}$ reconstruction
- Measurement of M_Z using dimuon mass for $\sqrt{s} \gg M_\mathrm{Z}$ to 1.0 MeV - dominated by $\sqrt{s} = 250$ GeV data

Conclusions

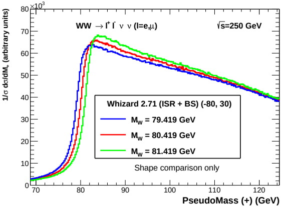
- ILC tracking detectors have the potential to measure beam energy related quantities with precision similar to the intrinsic energy spread using dimuon events (and also wide-angle Bhabha events)
- At $\sqrt{s} = 250$ GeV, dimuon estimate of 2 ppm precision on $\sqrt{s}$. More than sufficient (10 ppm needed) to not be a limiting factor for measurements such as M_W.
- Potential to improve M_Z by a factor of three using 250 GeV di-lepton data
- Applying the same techniques to running at the Z-pole will enable a high precision electroweak measurement program for ILC that takes advantage of absolute center-of-mass energy scale knowledge

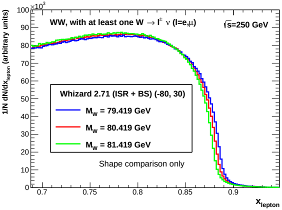
Backup Slides

$M_{\rm W}$, $\Gamma_{\rm W}$ measurements concurrent with Higgs program



- **Hadronic mass study**, J. Anguiano (KU).
- Stat. $\Delta M_{\rm W} = 2.4$ MeV for $1.6\ {\rm ab}^{-1}$ (-80%, +30%).
- Can be improved, but $m_{\rm had}$-only measurement likely limited by JES systematic
- Expect improvements with **constrained fit** and $\sqrt{s} = 250$ GeV data set

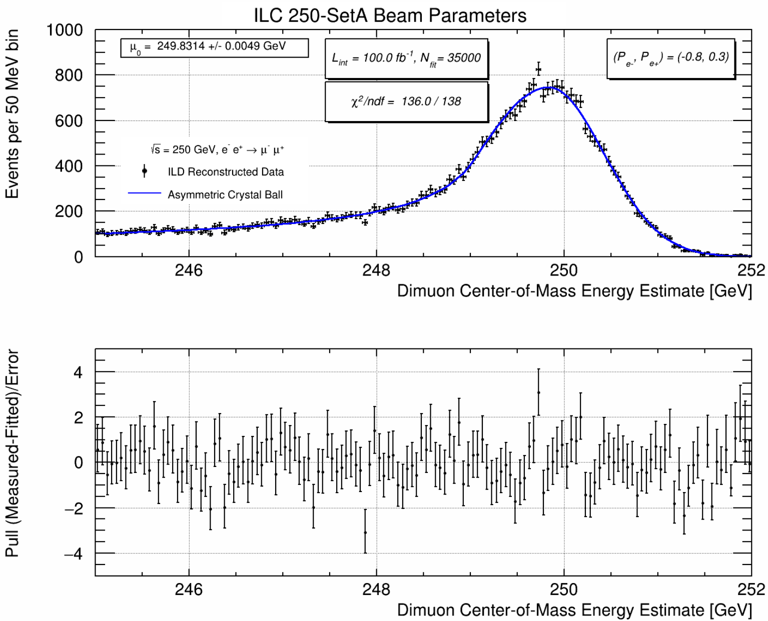




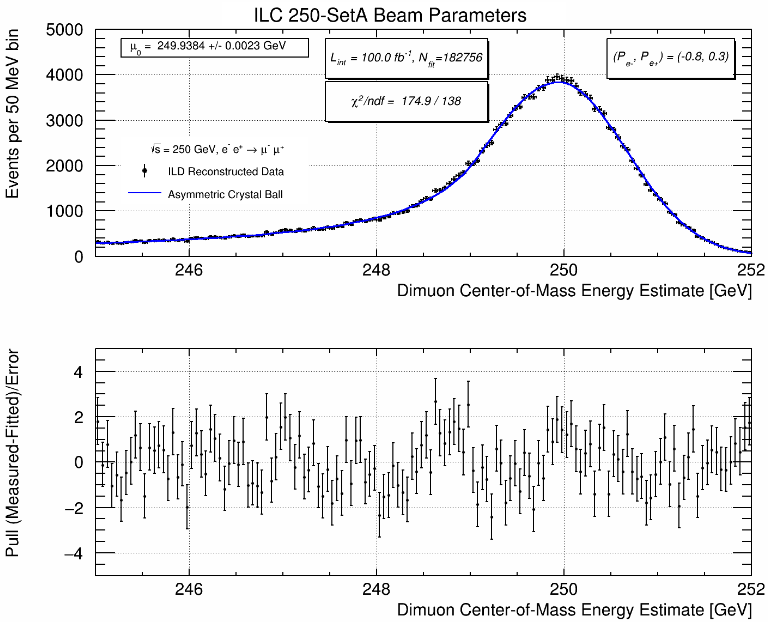
Sensitivity to $M_{\rm W}$ with lepton distributions: **dilepton pseudomasses**, lepton **endpoints**

- Stat. $\Delta M_{\rm W} = 4.4$ MeV for $2\ {\rm ab}^{-1}$ (45,45,5,5) at $\sqrt{s} = 250$ GeV
- **Leptonic observables** (shape-only): M_+, M_-, $x_\ell \equiv E_\ell / E_{\rm b}$. Exptl. systematics small.

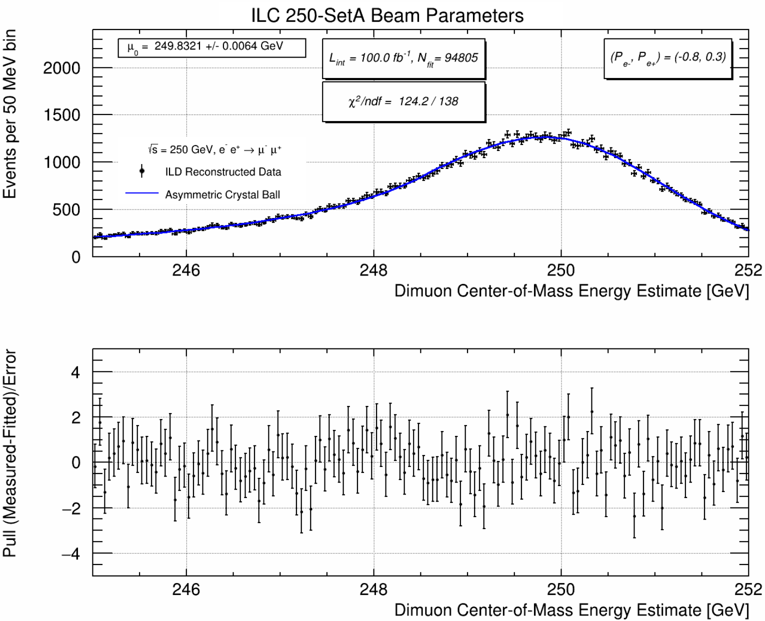
Gold Quality Dimuon PFOs (After BS)



Silver Quality Dimuon PFOs (After BS)



Bronze Quality Dimuon PFOs (After BS)



Beam Effects

The main idea is to use the kinematics of $\mathrm{e^+e^-} \to \mu^+\mu^-(\gamma)$ events and measurements of the final-state particles to measure the distribution of the center-of-mass energy of collisions.

We identify 3 effects needed to make a more realistic model of the collision:

0. Nominal. Each beam is a δ-function centered at a particular beam energy.
1. **Beam energy spread.** Each beam has a Gaussian distribution with rms width, σ_E, centered at a particular beam energy.
2. **Beamstrahlung.** The collective interaction of the two beams leads to radiation of collinear photons from the beams, resulting in the colliding e^+ and e^- having a *beamstrahlung-reduced center-of-mass energy*.
3. **Initial-state-radiation (ISR).** All $\mathrm{e^+e^-}$ physics processes may have ISR, where the invariant mass of the annihilating e^+ and e^- and the resulting particle system is further reduced cf 2 due to the emitted ISR photon(s).

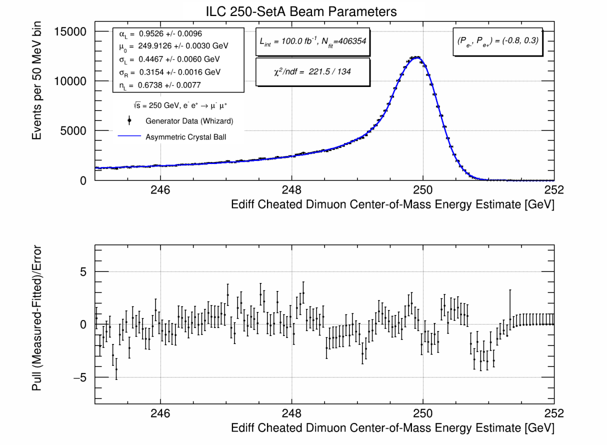
We are primarily concerned with evaluating the **beamstrahlung-reduced center-of-mass energy**. This is *after* beam energy spread and beamstrahlung radiation, but *before* emission of any ISR photons. We should allow for differences in the energy of each beam and for a **beam crossing angle**, α, defined as the horizontal plane angle between the two beam lines. For ILC, α, is 14 mrad.

Aside on Crystal Ball Empirical Fit Functions

- The 1-d distributions generally feature a Gaussian **peak** associated with beam energy spread and a long **tail** with harder beamstrahlung
- These can be fit qualitatively well - although not well enough - with a Crystal Ball function. This piece-wise function has a Gaussian core and a power-law tail with a continuous first-derivative at the transition points.
- The generalized asymmetric double-sided Crystal Ball is

$$f(E; \mu_0, \sigma_L, \alpha_L, n_L, \sigma_R, \alpha_R, n_R)$$

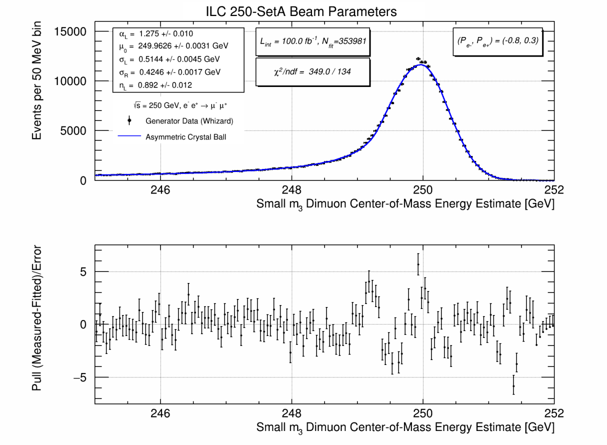
  where μ_0 is the Gaussian peak **mode**, σ_i are the Gaussian **widths** (on L&R), α_i are the Gaussian/power-law **transition points** in units of σ_i (on L&R), and n_i are the power law **exponents** (on L&R)
- With the beam energy related distributions, only a 5-parameter version is applicable with parameters, $\mu_0, \sigma_L, \alpha_L, n_L, \sigma_R$ with the right-hand power-law tail disabled. The classic 1-sided Crystal Ball (4-parameters) $\mu_0, \sigma_L, \alpha_L, n_L$ fits are included for reference in the backup slides.
- See RooCrystalBall for implementation details



$\sigma_R/\sqrt{s} = 0.1259 \pm 0.0007\%$ (cf 0.1232% with true $\sqrt{s}$)

- This is the generator-level $\sqrt{s}_p$ calculated from the 2 muons
- But using the true $\overline{\Delta E_b}$ in the equations
- Why so few events in range?

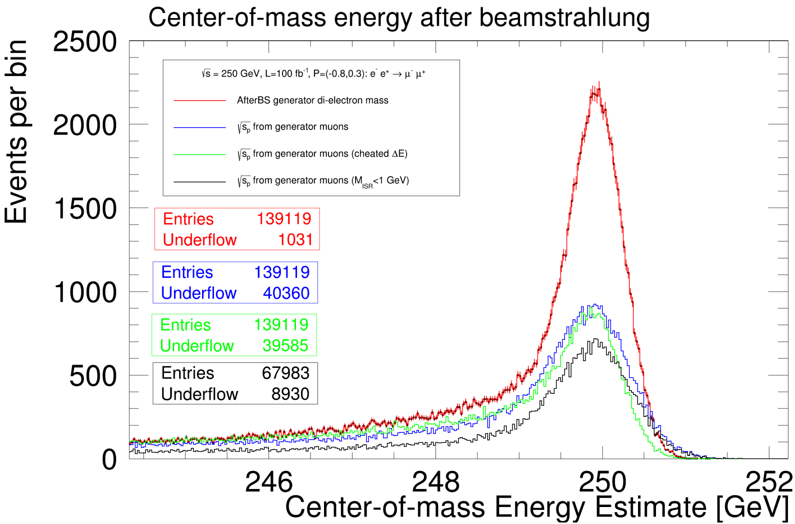
Dimuon Estimate of $\sqrt{s}$ (Low m_3) (After BS)



$\sigma_{\mathrm{R}}/\sqrt{s} = 0.1698 \pm 0.0007\%$ (cf 0.1232% with true $\sqrt{s}$)

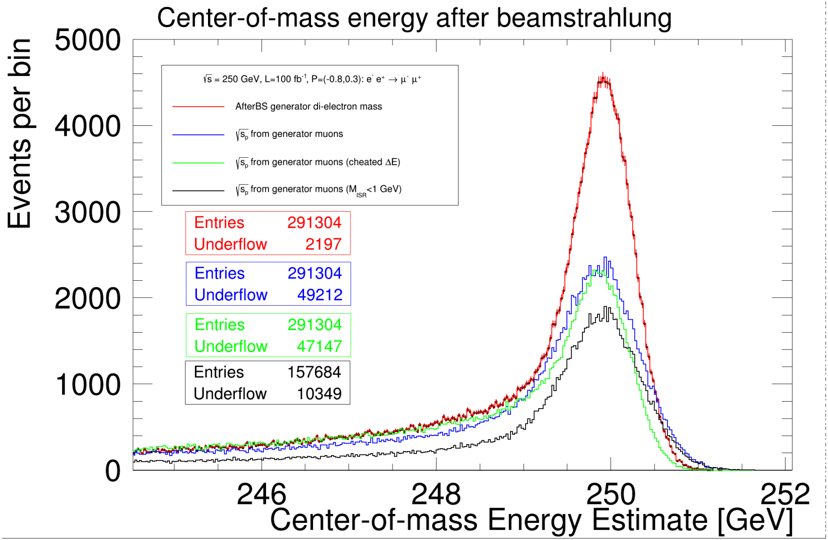
- This is the generator-level $\sqrt{s}_p$ calculated from the 2 muons
- For events with ISR photon system mass < 1 GeV
- Looks like the p_z issue dominates

Comparisons III Low Dimuon Mass (After BS) Zoomed



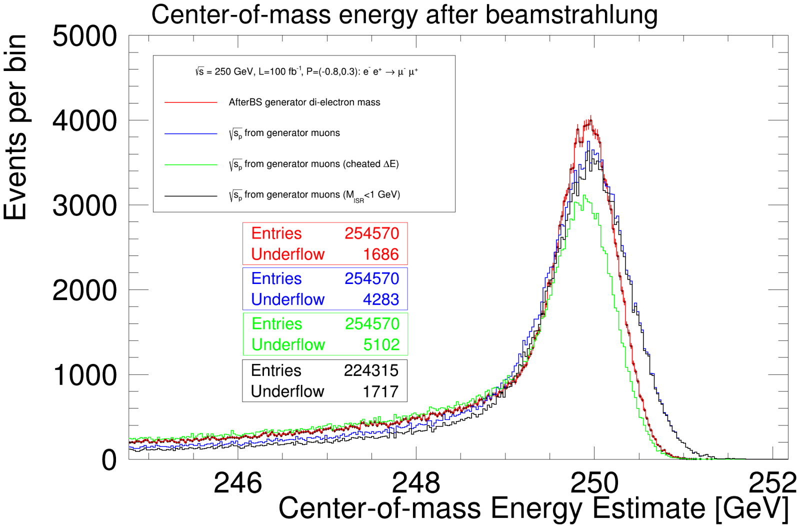
Note: Underflow statistics still refer to < 220 GeV.

Comparisons III Medium Dimuon Mass (After BS) Zoomed



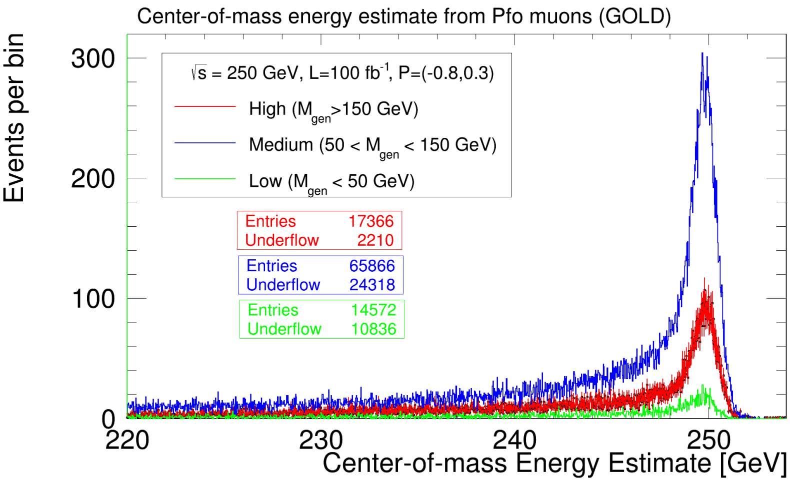
Note: Underflow statistics still refer to < 220 GeV.

Comparisons III High Dimuon Mass (After BS) Zoomed



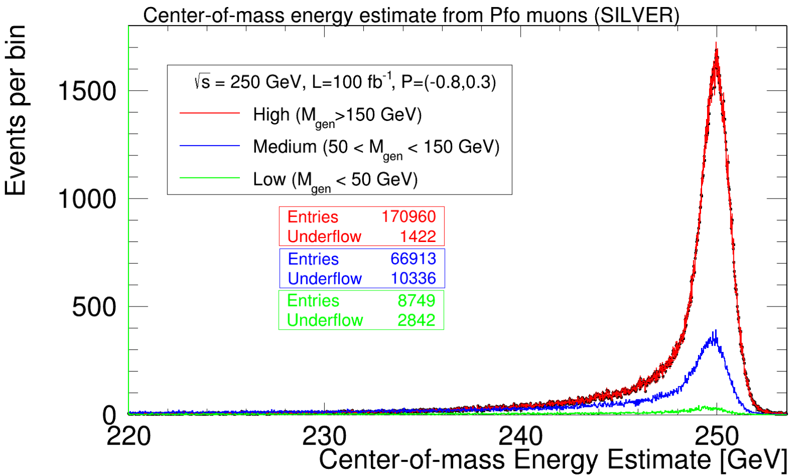
Note: Underflow statistics still refer to < 220 GeV.

Gold Quality Dimuon PFOs (After BS)



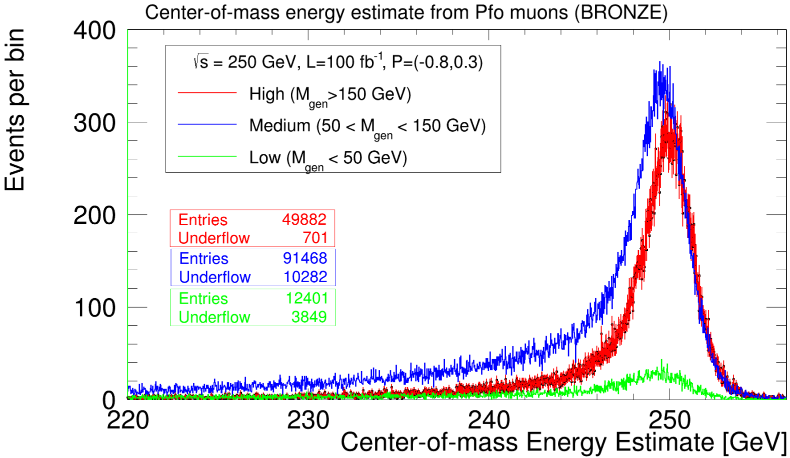
Mostly Z-like

Silver Quality Dimuon PFOs (After BS)



Mostly high mass

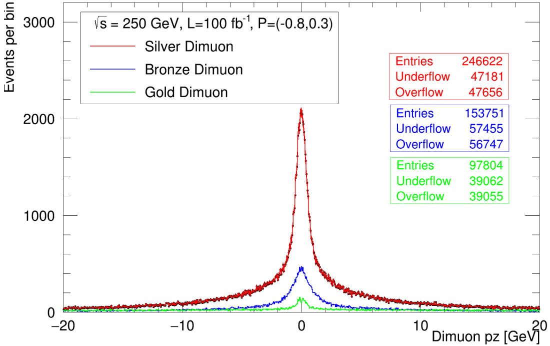
Bronze Quality Dimuon PFOs (After BS)



Mix of high mass and Z-like. Z-like with one forward muon?

Measuring the z-imbalance

Likely can use both p_z and acolinearity (for high mass events).



Will be sensitive to energy asymmetries. The suggestion by Tim Barklow in 2005 (which I now understand) is to measure

$$E_{\mu^+\mu^-} + p_z(\mu^+\mu^-) = (E_+ + E_-) + (E_- - E_+) = 2E_-$$

$$E_{\mu^+\mu^-} - p_z(\mu^+\mu^-) = (E_+ + E_-) - (E_- - E_+) = 2E_+$$

$\sqrt{s}$ Sensitivity Estimate at $\sqrt{s} = 250$ GeV

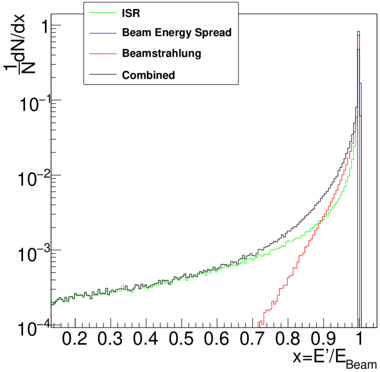
Statistical uncertainties in ppm on $\sqrt{s}$ for $\mu^+\mu^-$ channel

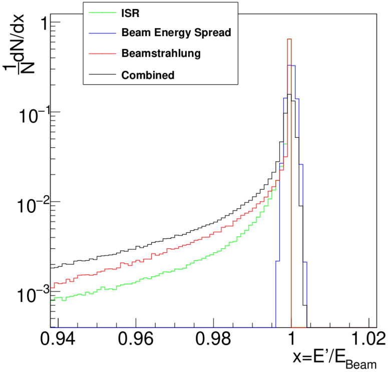
L_{int} [ab^{-1}]	Poln [%]	Gold	Silver	Bronze	G+S+B
0.9	$-80, +30$	11.1	4.8	16	4.3
0.9	$+80, -30$	12.0	5.5	18	4.8
0.1	$-80, -30$	43	19	64	16
0.1	$+80, +30$	46	21	68	18
2.0	–	7.9	3.5	11.7	**3.1**

Fractional errors on μ_0 parameter (mode of peak) when fitting with 4-parameter symmetric Crystal Ball function with all four parameters floating.

This is more conservative and likely too pessimistic. It does degrade from the pure statistical uncertainty of perfectly known shape parameters given the need to determine the shape parameters.

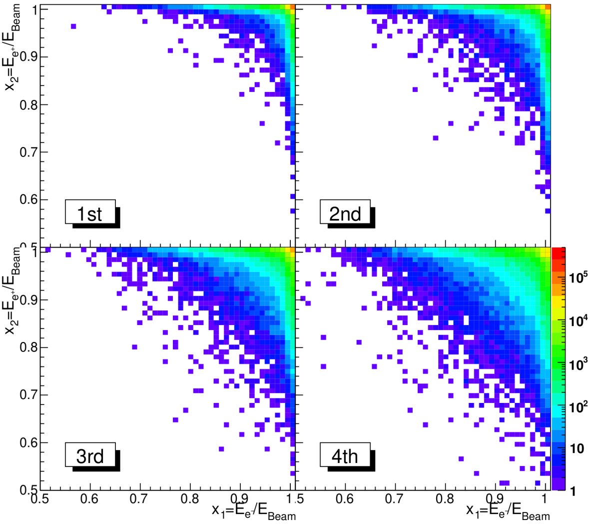
ISR and Beamstrahlung





This is for ILC $\sqrt{s} = 500$ GeV TDR parameters from Andre Sailer's diploma thesis. ISR is the dominant effect in the far tail.

Beamstrahlung

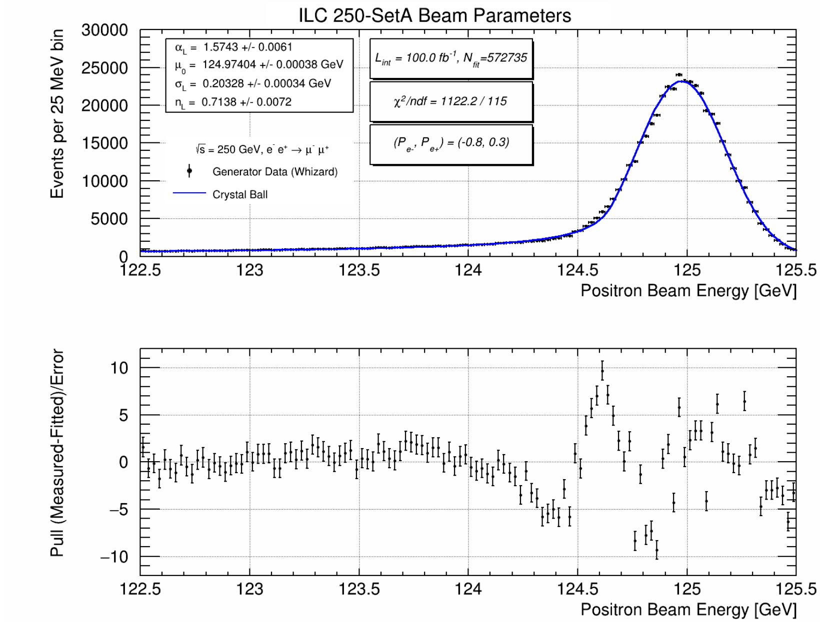


This is for ILC $\sqrt{s} = 500$ GeV TDR parameters from Andre Sailer's diploma thesis. Each plot is a consecutive collision time quartile.

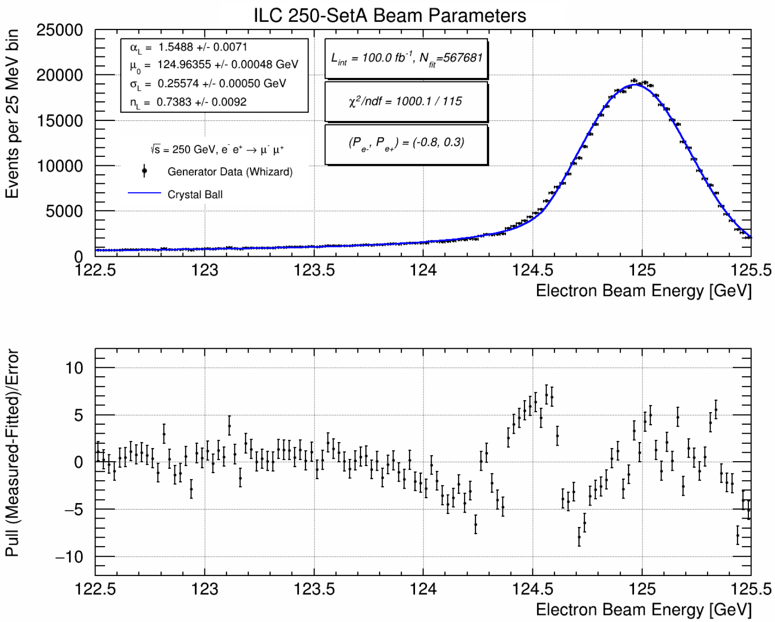
Fit Considerations

- Most of these are 4-parameter Crystal Ball fits. Particularly for those with more sharply resolved features, the χ^2 is substantially worse than the 5-parameter asymmetric fits shown earlier.
- The fits generally need the additional σ_{R} parameter to describe the beam energy spread feature while σ_{L} accommodates the convolution of beam energy spread with soft beamstrahlung.
- On the other hand these 4-parameter fits may better represent the statistical error on the mode parameter when able to better constrain the shape of the distributions such as with external knowledge of the beam energy spread.

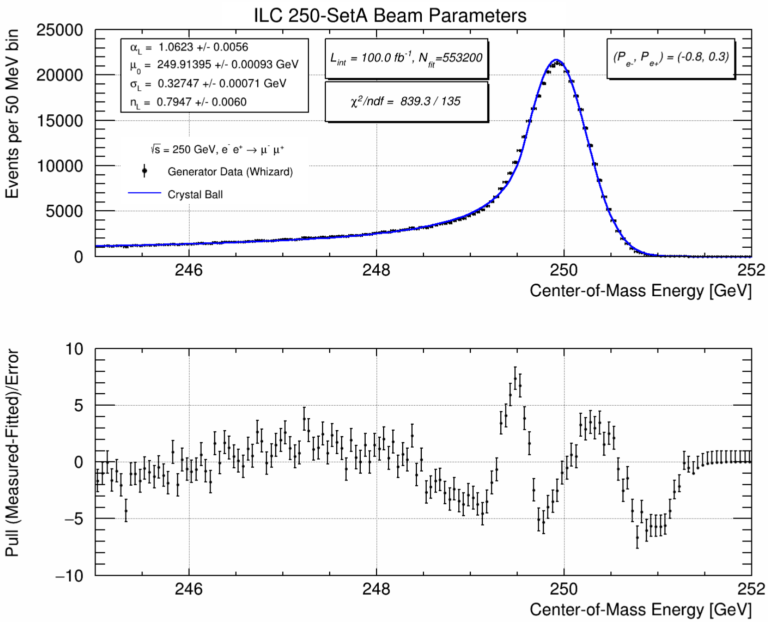
Positron Beam Energy (After Beamstrahlung)



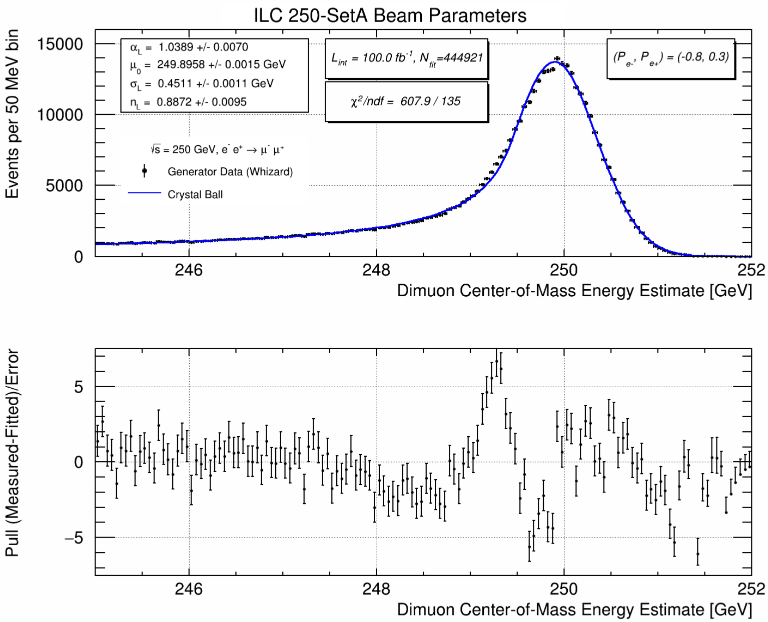
Electron Beam Energy (After Beamstrahlung)



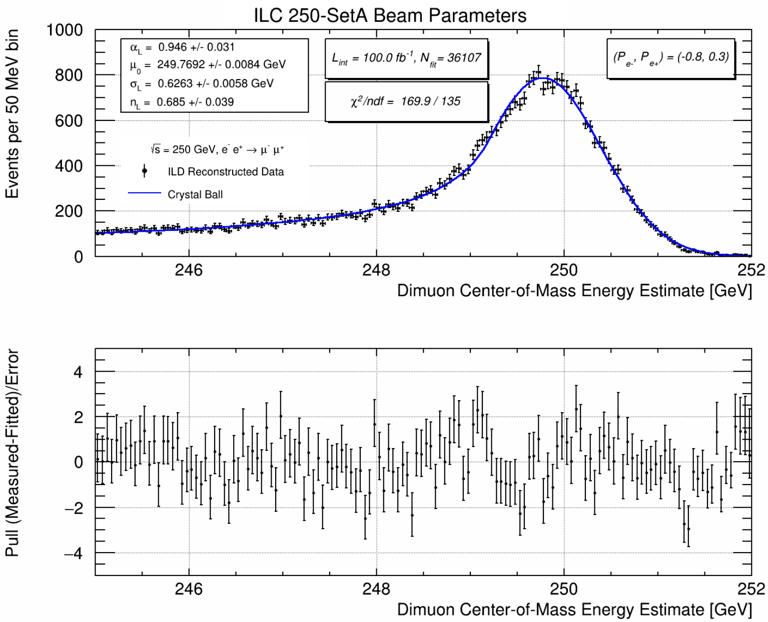
Center-of-Mass Energy (After Beamstrahlung)



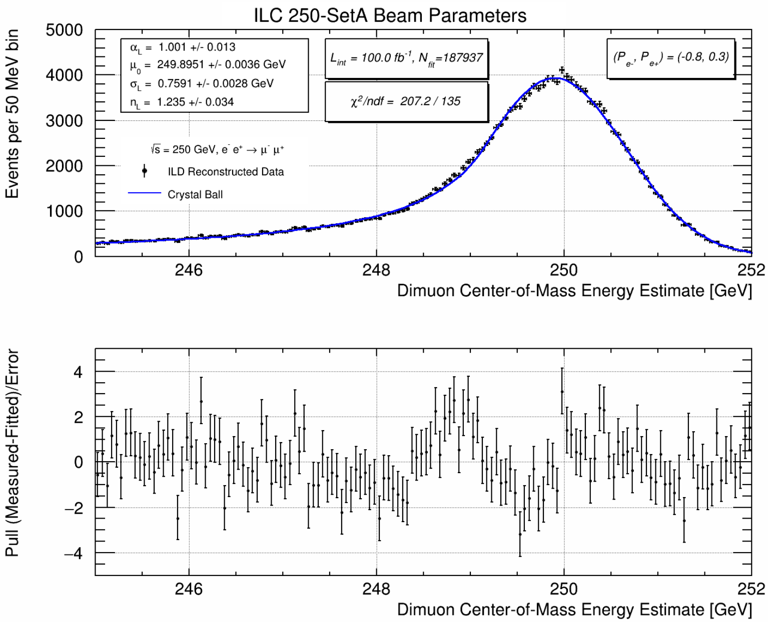
Dimuon Estimate of Center-of-Mass Energy (After BS)



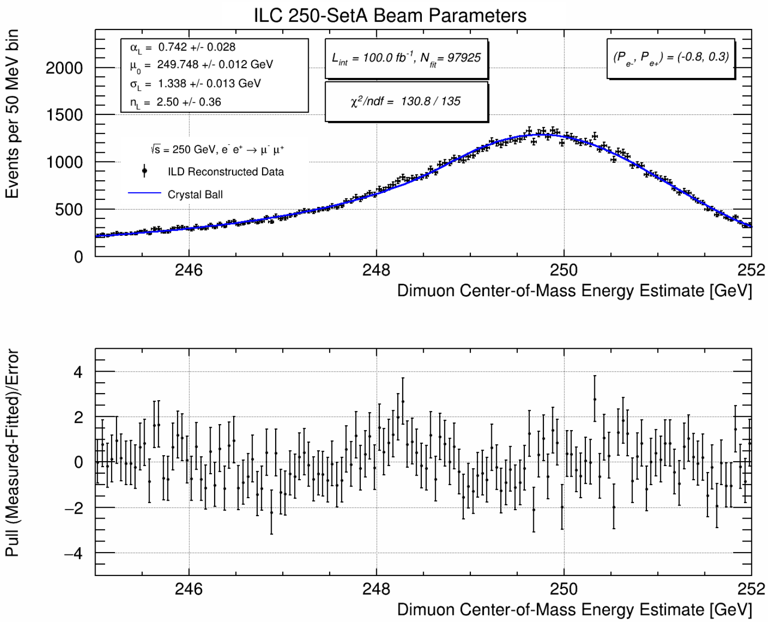
Gold Quality Dimuon PFOs (After BS)

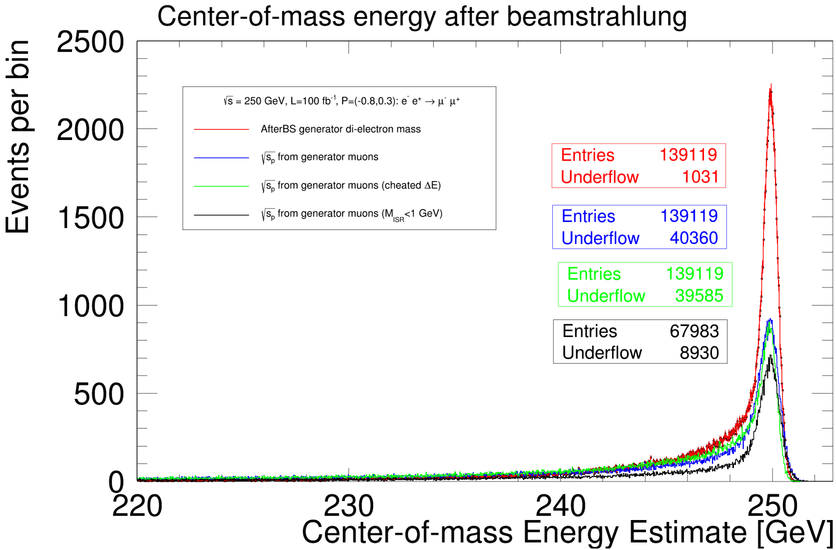


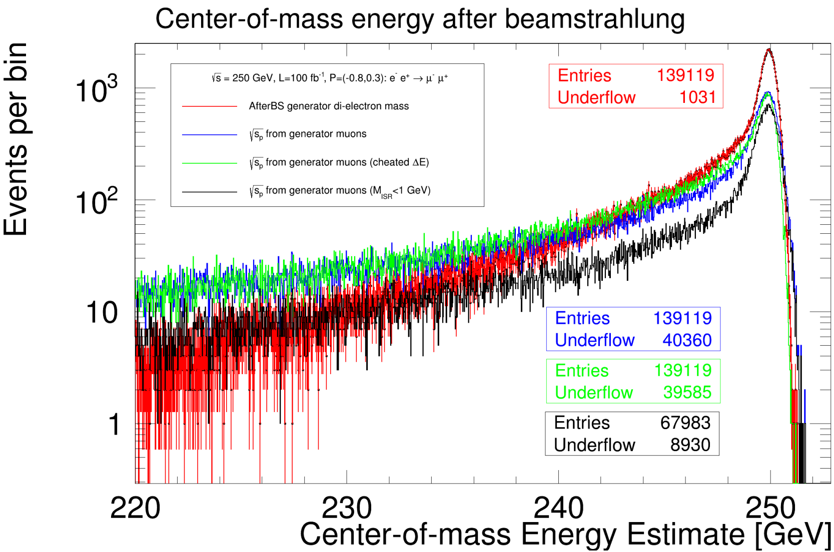
Silver Quality Dimuon PFOs (After BS)

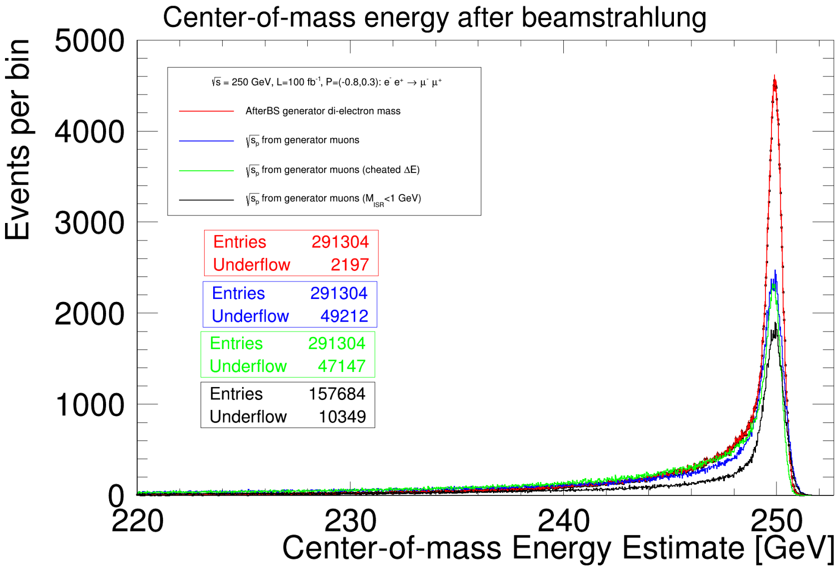


Bronze Quality Dimuon PFOs (After BS)

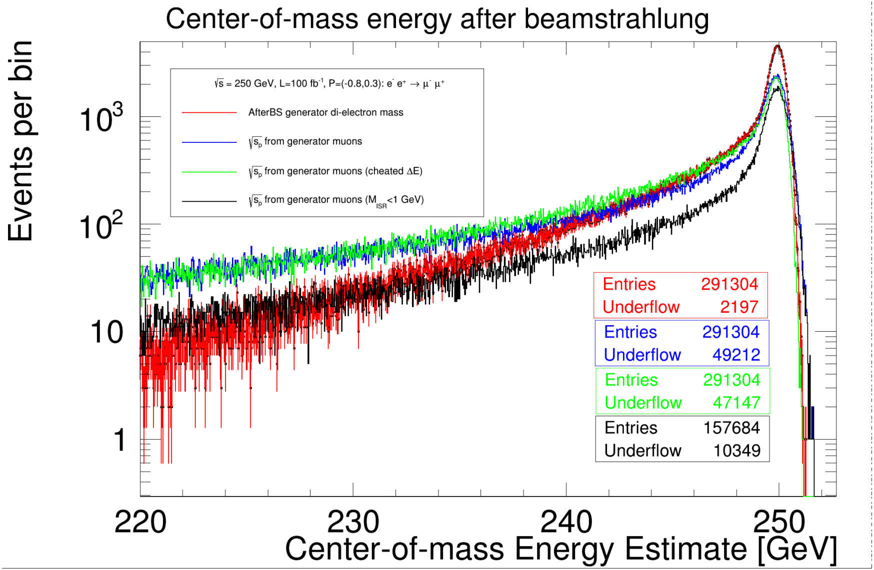


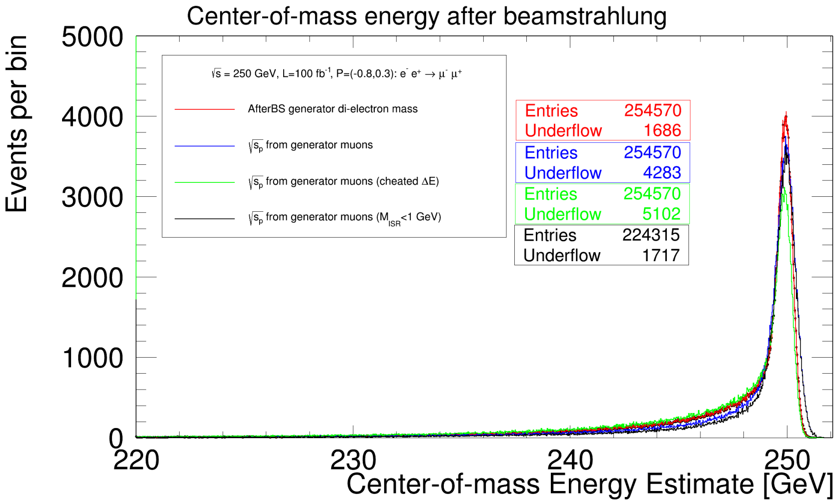

Center-of-mass energy after beamstrahlung
Events per bin
2500
2000
1500
1000
500
0
220
230
240
250
Center-of-mass Energy Estimate [GeV]
√s = 250 GeV, L=100 fb-1, P=(-0.8,0.3): e- e+ → μ- μ+
AfterBS generator di-electron mass
√sp from generator muons
√sp from generator muons (cheated ΔE)
√sp from generator muons (MISR<1 GeV)
Entries 139119
Underflow 1031
Entries 139119
Underflow 40360
Entries 139119
Underflow 39585
Entries 67983
Underflow 8930


Center-of-mass energy after beamstrahlung
√s = 250 GeV, L=100 fb⁻¹, P=(-0.8,0.3): e⁻ e⁺ → μ⁻ μ⁺
AfterBS generator di-electron mass
√s_p from generator muons
√s_p from generator muons (cheated ΔE)
√s_p from generator muons (M_ISR<1 GeV)
Entries 139119
Underflow 1031
Entries 139119
Underflow 40360
Entries 139119
Underflow 39585
Entries 67983
Underflow 8930
Events per bin
10³
10²
10
1
220
230
240
250
Center-of-mass Energy Estimate [GeV]


Center-of-mass energy after beamstrahlung
Events per bin
5000
4000
3000
2000
1000
0
220
230
240
250
Center-of-mass Energy Estimate [GeV]
√s = 250 GeV, L=100 fb⁻¹, P=(-0.8,0.3): e⁻ e⁺ → μ⁻ μ⁺
AfterBS generator di-electron mass
√s_p from generator muons
√s_p from generator muons (cheated ΔE)
√s_p from generator muons (M_ISR<1 GeV)
Entries 291304
Underflow 2197
Entries 291304
Underflow 49212
Entries 291304
Underflow 47147
Entries 157684
Underflow 10349

Comparisons II Medium Dimuon Mass (After BS)




Center-of-mass energy after beamstrahlung
Events per bin
5000
4000
3000
2000
1000
0
220
230
240
250
Center-of-mass Energy Estimate [GeV]
√s = 250 GeV, L=100 fb⁻¹, P=(-0.8,0.3): e⁻ e⁺ → μ⁻ μ⁺
AfterBS generator di-electron mass
√s_p from generator muons
√s_p from generator muons (cheated ΔE)
√s_p from generator muons (M_ISR<1 GeV)
Entries 254570
Underflow 1686
Entries 254570
Underflow 4283
Entries 254570
Underflow 5102
Entries 224315
Underflow 1717

Comparisons II High Dimuon Mass (After BS)