Electroweak parton distributions and fragmentations for high-energy lepton colliders

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In collaboration with **Tao Han** and **Keping Xie**

[T. Han, Y. Ma, K.Xie 2007.14300]

[T. Han, Y. Ma, K.Xie 2103.09844]

[T.Han, Y.Ma, K.Xie, work in progress]

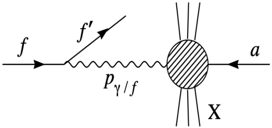
Background: PDF of a lepton

"Equivalent photon approximation (EPA)" [C. F. von Weizsacker, Z. Phys. 88, 612 (1934)]

Treat photon as a parton constituent in the electron [E. J. Williams, Phys. Rev. 45, 729 (1934)]

$$\sigma(\ell^- + a \to \ell^- + X) = \int \mathrm{d}x\, f_{\gamma/\ell}\, \hat{\sigma}(\gamma a \to X)$$

$$f_{\gamma/\ell,\mathrm{EPA}}(x_\gamma, Q^2) = \frac{\alpha}{2\pi} \frac{1+(1-x_\gamma)^2}{x_\gamma} \ln \frac{Q^2}{m_\ell^2}$$



Extra terms:

[Frixione, Mangano, Nason, Ridolfi 2103.09844]
[Budnev, Ginzburg, Meledin, Serbo, Phys. Rept.(1975)]

Applications at muon collider

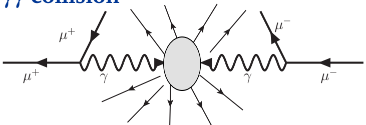
- Production cross sections

$$\sigma(\ell^+\ell^- \to F + X) = \int_{\tau_0}^{1} d\tau \sum_{ij} \frac{d\mathscr{L}_{ij}}{d\tau}\ \hat{\sigma}(ij \to F),\ \tau = \hat{s}/s$$

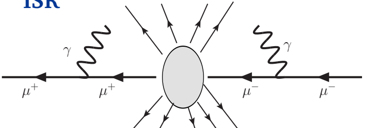
- Partonic luminosities

$$\frac{d\mathscr{L}_{ij}}{d\tau} = \frac{1}{1+\delta_{ij}} \int_{\tau}^{1} \frac{d\xi}{\xi} \left[f_i(\xi, Q^2) f_j\left(\frac{\tau}{\xi}, Q^2\right) + (i \leftrightarrow j)\right]$$

$\gamma\gamma$ collision



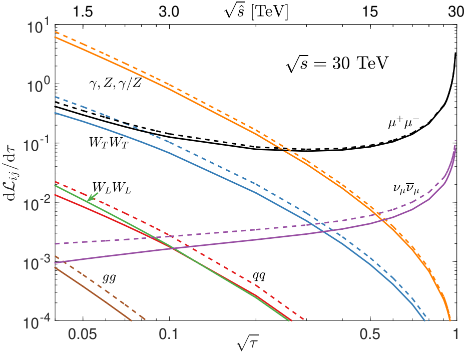
ISR



HE muon collider: An EW version of HE LHC

- All SM particles are partons when the machine energy is high
- We are able to determine the partons with their different polarizations

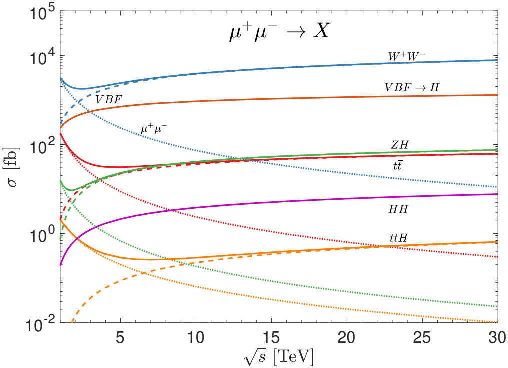
The EW parton luminosities of a 30 TeV muon collider



[T. Han, Y. Ma, K.Xie 2007.14300]

The full picture: Semi-inclusive processes

Just like in hadronic collisions: $\mu^+\mu^- \to$ exclusive particles + remnants



[T. Han, Y. Ma, K.Xie 2007.14300]

Some observations:

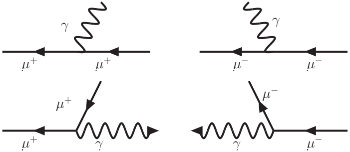
- The annihilations decrease as $1/s$.
- ISR needs to be considered, which can give over 10% enhancement.
- The fusions increase as $\ln^p(s)$, which take over at high energies.
- The large collinear logarithm $\ln\big(s/m_\mu^2\big)$ needs to be resummed, set $Q = \sqrt{\hat{s}}/2$.

Go beyond the EPA at a high-energy muon collider

We have been doing:

- $\ell^+\ell^-$ annihilation

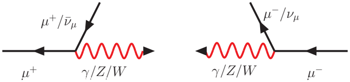
- EPA and ISR



- "Effective W Approx." (EWA)

[G. Kane, W. Repko, and W. Rolnick, PLB 148 (1984) 367]

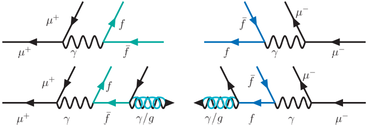
[S. Dawson, NPB 249 (1985) 42]



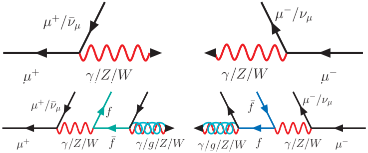
We will add:

[T. Han, Y. Ma, K.Xie 2007.14300, 2103.09844]

- Above $\mu_{\rm QCD}$: QED$\otimes$QCD
  q/g emerge



- Above $\mu_{\rm EW} = M_Z$: EW$\otimes$QCD
  EW partons emerge



In the end, everything is parton, i.e. the full SM PDFs.

The PDFs for a muon collider

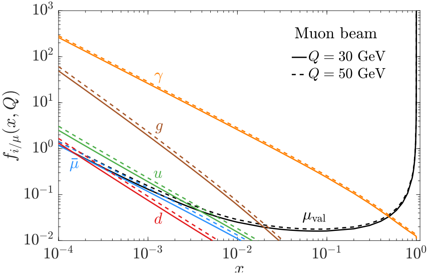
- **QED⊗QCD PDFs**:
  $f_{\mu_{\rm val}}, f_\gamma, f_{\ell_{\rm sea}}, f_q, f_g$
- Scale uncertainty: 20% for $f_{g/\mu}$
- The averaged momentum fractions $\langle x_i \rangle = \int x f_i(x) \mathrm{d}x$

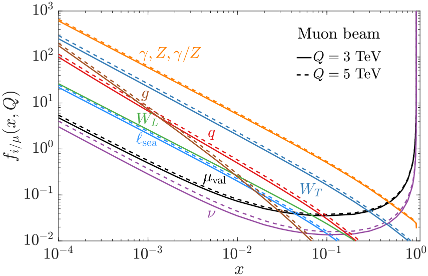
$Q(\mu^\pm)$	$\mu_{\rm val}$	γ	ℓsea	q	g
30 GeV	98.2	1.72	0.019	0.024	0.0043
50 GeV	98.0	1.87	0.023	0.029	0.0051
M_Z	97.9	2.06	0.028	0.035	0.0062

- **EW PDFs: All SM particles**

Q	μ	$\gamma, Z, \gamma Z$	$W^\pm$	ν	ℓsea	q	g
M_Z	97.9	2.06	0	0	0.028	0.035	0.0062
3 TeV	91.5	3.61	1.10	3.59	0.069	0.13	0.019
5 TeV	89.9	3.82	1.24	4.82	0.077	0.16	0.022

- Scale uncertainty: $\sim 20\%$ between $Q = 3$ TeV and $Q = 5$ TeV
- The EW correction is not small: $\sim 100\%$ for $f_{d/\mu}$ due to **relatively large SU(2) gauge coupling**.
- W_L does not evolve



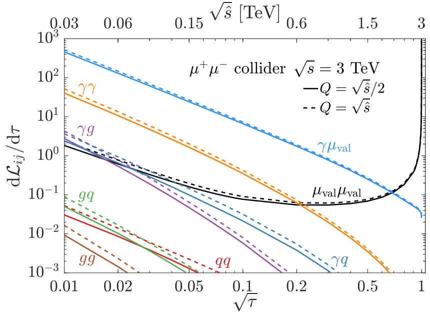


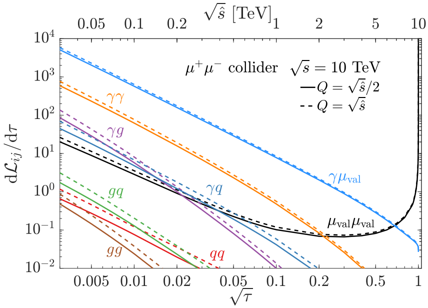
Parton luminosities at a possible muon collider

Consider a 3 TeV and a 10 TeV machine

- Partonic luminosities for

$$\mu^+\mu^-, \gamma\mu, \gamma\gamma, qq, \gamma q, \gamma g, gq, \text{ and } gg$$





- The partonic luminosity of $\gamma g + \gamma q$ is $\sim 20\%$ of the $\gamma\gamma$ one
- The partonic luminosities of qq, gq, and gg are $\sim 0.5\%$ of the $\gamma\gamma$ one
- Given the stronger QCD coupling, **sizable QCD cross sections are expected**.
- Scale uncertainty is $\sim 20\%(\sim 50\%)$ for photon (gluon) initiated processes.

Jet production at a possible lepron collider

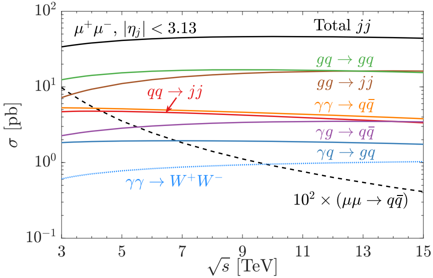
- Low-p_T range: photon induced non-perturbative hadronic production

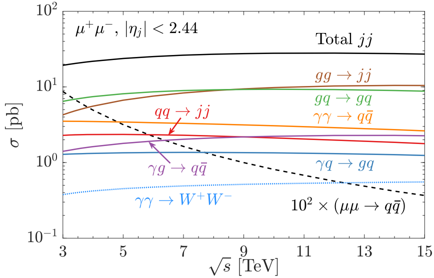
  [Chen, Barklow, and Peskin, hep-ph/9305247; Drees and Godbole, PRL 67, 1189,T. Barklow, etal, LCD-2011-020]

- High-p_T range [$p_T > (4 + \sqrt{s}/3\,\text{TeV})$ GeV]: perturbatively computable

$$\gamma\gamma \to q\bar{q},\ \gamma g \to q\bar{q},\ \gamma q \to gq,$$
$$qq \to qq\ (gg),\ gq \to gq \text{ and } gg \to gg\ (q\bar{q}).$$

- $Q = \sqrt{\hat{s}}/2$, due to large $\alpha_s \ln(Q^2)$, a $30 \sim 40\%$ enhancement if $Q = \sqrt{\hat{s}}$





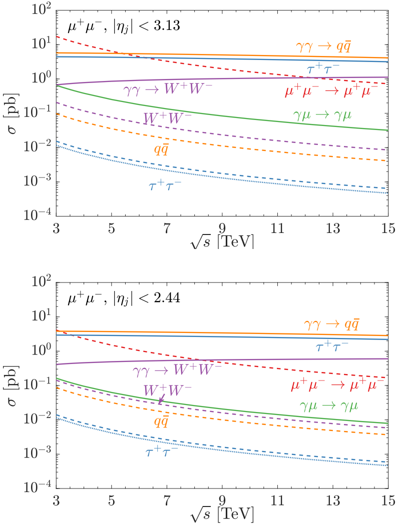
- Including the QCD contribution leads to much larger total cross section.
- gg initiated cross sections are large for its large multiplicity;
- gq initiated cross sections are large for its large luminosity.
- $\gamma\gamma$ initiated cross sections here are smaller than the EPA results.

Refresh the picture of high-energy muon colliders

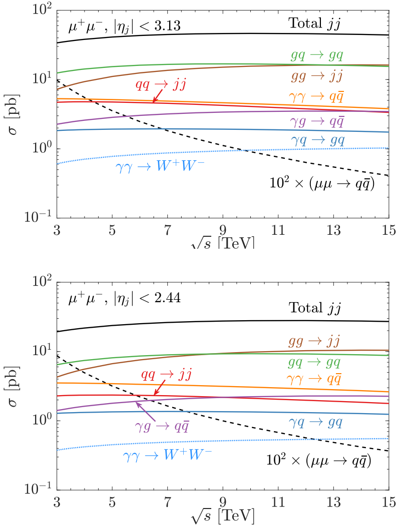
What is the dominant process at a high-energy muon collider?

- **Quark/gluon initiated jet production dominates**

Before:



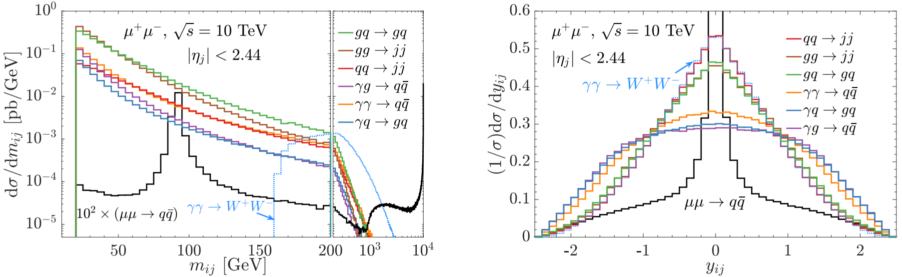
After:



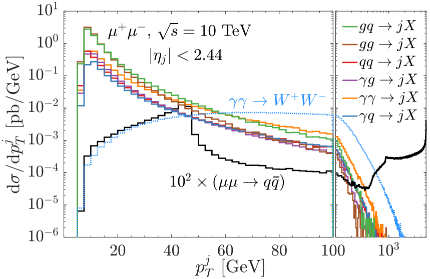
Di-jet distributions at a muon collider

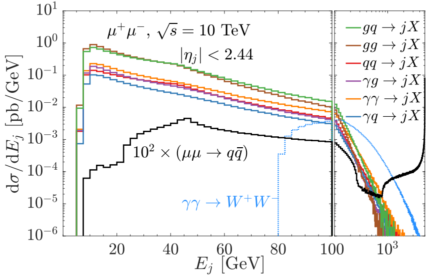
Rather a conservative set up: $\theta = 10^\circ$

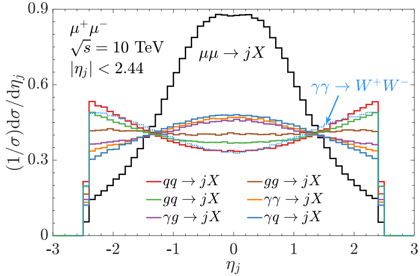
- Some physics:
  Two different mechanisms: $\mu^+\mu^-$ **annihilation** VS **Fusion processes**
  - Annihilation is more than 2 orders of magnitude smaller than fusion process.
  - Annihilation peaks at $m_{ij} \sim \sqrt{s}$;
  - Fusion processes peak near m_{ij} threshold.
  - Annihilation is very central, spread out due to ISR;
  - Fusion processes spread out, especially for γq and γg initiated ones.



Inclusive jet distributions at a muon collider





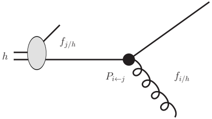


- Jet production dominates over WW production until $p_T > 60$ GeV;
- WW production takes over around energy ~ 200 GeV.
- QCD contributions are mostly forward-backward; $\gamma\gamma$, γq, and γg initiated processes are more isotropic.

Work in progress: EW FSR

- At future super high-energy colliders, collinear splittings also happen to energetic final state particles ⇒ **EW jet factories**
- One treatment is the electroweak fragmentation functions (EW FFs)
- Both parton distributions (f_i) and fragmentations (d_i) are controlled by the DGLAP equations.
- The evolutions (splittings) are in opposite directions.

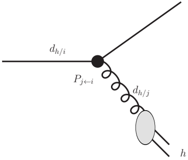
EW PDF



$$\frac{\mathrm{d}f_{i/h}}{\mathrm{d}\log\mu^2} = P_{ij}\otimes f_{j/h}$$

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu^2}\begin{pmatrix} e \\ \bar{e} \\ \gamma \end{pmatrix} = \begin{pmatrix} P_{\ell\ell} & 0 & P_{\ell\gamma} \\ 0 & P_{\ell\ell} & P_{\ell\gamma} \\ P_{\gamma\ell} & P_{\gamma\ell} & P_{\gamma\gamma} \end{pmatrix}\otimes\begin{pmatrix} e \\ \bar{e} \\ \gamma \end{pmatrix}.$$

EW FF



$$\frac{\mathrm{d}d_{h/i}}{\mathrm{d}\log\mu^2} = d_{h/j}\otimes P_{ji}$$

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu^2}\begin{pmatrix} e & \bar{e} & \gamma \end{pmatrix} = \begin{pmatrix} e & \bar{e} & \gamma \end{pmatrix}\otimes\begin{pmatrix} P_{\ell\ell} & 0 & P_{\ell\gamma} \\ 0 & P_{\ell\ell} & P_{\ell\gamma} \\ P_{\gamma\ell} & P_{\gamma\ell} & P_{\gamma\gamma} \end{pmatrix}.$$

Summary and prospects

Physics: EW PDFs and EW FFs

- At very high energies, the collinear splittings dominate.
- ISR: EW bosons should be treated as partons
  - High-energy lepton colliders ⇒ "Vector boson colliders".
  - High-energy hadron colliders ⇒ corrections from the EW sector.
  - Other SM particles enter the picture via DGLAP ⇒ everything is parton.
- FSR: EW FFs (work in progress)
  - The energetic final state particles also have collinear radiations ⇒ "(EW) jet factories".
  - Evolutions are also controlled by DGLAP, in opposite directions to the PDFs.

Machine: A high-energy muon collider is an EW version of HE LHC

- There are many things to work on: SUSY, DM, Higgs, etc.
- Two classes of processes: $\mu^+\mu^-$ annihilation VS fusions

  [T. Han, Y. Ma, K.Xie 2007.14300]
- **The main background of is the jet production**:
  - Low p_T range: non-perturbative $\gamma\gamma$ initiated hadronic production dominates

    [Chen, Barklow, and Peskin, hep-ph/9305247; Drees and Godbole, PRL 67, 1189,T. Barklow, etal, LCD-2011-020]
  - High p_T range, q and g initiated jet production dominates

    [T. Han, Y. Ma, K.Xie 2103.09844]

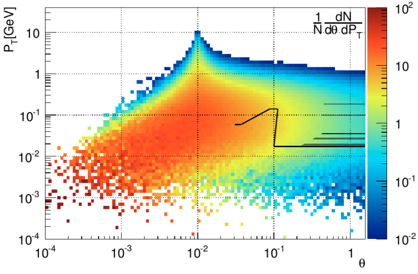
Jet production of possible lepton colliders (low p_T)

- Large photon induced non-perturbative hadronic production

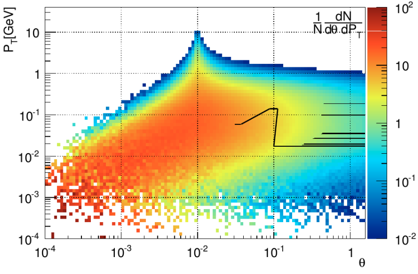
  [Drees and Godbole, PRL 67 1189, hep-ph/9203219]

  [Chen, Barklow, and Peskin, hep-ph/9305247; Godbole, Grau, Mohan, Pancheri, SrivastavaNuovo Cim. C 034S1]

  - $\sigma_{\gamma\gamma}$ may reach micro-barns level at TeV c.m. energies
  - $\sigma_{\ell\ell}$ may reach nano-barns, after folding in the $\gamma\gamma$ luminosity
- The events populate at low p_T regime
  So we can separate from this non-perturbative range via a p_T cut.



(a) Pythia sample



(b) SLAC sample

[T. Barklow, D. Dannheim, M. O. Sahin, and D. Schulte, LCD-2011-020]

The PDF evolution

- The DGLAP equations

$$\frac{\mathrm{d}f_i}{\mathrm{d}\log Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P_{ij}^I \otimes f_j$$

- The initial conditions

$$f_{\ell/\ell}(x, m_\ell^2) = \delta(1-x)$$

- Three regions and two matchings
  - $m_\ell < Q < \mu_{\rm QCD}$: QED
  - $Q = \mu_{\rm QCD} \lesssim 1$ GeV: $f_q \propto P_{q\gamma} \otimes f_\gamma, f_g = 0$
  - $\mu_{\rm QCD} < Q < \mu_{\rm EW}$: QED$\otimes$QCD
  - $Q = \mu_{\rm EW} = M_Z$: $f_\nu = f_t = f_W = f_Z = f_{\gamma Z} = 0$
  - $\mu_{\rm EW} < Q$: EW$\otimes$QCD.

$$\begin{pmatrix} f_B \\ f_{W^3} \\ f_{BW^3} \end{pmatrix} = \begin{pmatrix} c_W^2 & s_W^2 & -2c_W s_W \\ s_W^2 & c_W^2 & 2c_W s_W \\ c_W s_W & -c_W s_W & c_W^2 - s_W^2 \end{pmatrix} \begin{pmatrix} f_\gamma \\ f_Z \\ f_{\gamma Z} \end{pmatrix}$$

- We work in the (B, W) basis. The technical details can be referred to the backup slides.

The QED⊗QCD DGLAP evolution

- The singlets and gauge bosons

$$\frac{\mathrm{d}}{\mathrm{d}\log Q^2}\begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix} = \begin{pmatrix} P_{\ell\ell} & 0 & 0 & 2N_\ell P_{\ell\gamma} & 0 \\ 0 & P_{uu} & 0 & 2N_u P_{u\gamma} & 2N_u P_{ug} \\ 0 & 0 & P_{dd} & 2N_d P_{d\gamma} & 2N_d P_{dg} \\ P_{\gamma\ell} & P_{\gamma u} & P_{\gamma d} & P_{\gamma\gamma} & 0 \\ 0 & P_{gu} & P_{gd} & 0 & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix}$$

- The non-singlets

$$\frac{\mathrm{d}}{\mathrm{d}\log Q^2} f_{NS} = P_{ff} \otimes f_{NS}.$$

- The averaged momentum fractions of the PDFs: $f_{\ell_{\mathrm{val}}}, f_\gamma, f_{\ell_{\mathrm{sea}}}, f_q, f_g$

$$\langle x_i \rangle = \int x f_i(x)\mathrm{d}x,\ \sum_i \langle x_i \rangle = 1$$

$$\frac{\langle x_q \rangle}{\langle x_{\ell\mathrm{sea}} \rangle} \lesssim \frac{N_c\left[\sum_i (e_{u_i}^2 + e_{\bar{u}_i}^2) + \sum_i (e_{d_i}^2 + e_{\bar{d}_i}^2)\right]}{e_{\bar{\ell}_{\mathrm{val}}}^2 + \sum_{i\neq \ell\mathrm{val}} (e_{\ell_i}^2 + e_{\bar{\ell}_i}^2)} = \frac{22/3}{5}$$

The EW isospin (T) and charge-parity (CP) basis

- The leptonic doublet and singlet in the (T,CP) basis

$$f_\ell^{0\pm} = \frac{1}{4}\left[(f_{\nu_L} + f_{\ell_L}) \pm (f_{\bar{\nu}_L} + f_{\bar{e}_L})\right], \quad f_\ell^{1\pm} = \frac{1}{4}\left[(f_{\nu_L} - f_{\ell_L}) \pm (f_{\bar{\nu}_L} - f_{\bar{e}_L})\right].$$

$$f_e^{0\pm} = \frac{1}{2}[f_{e_R} \pm f_{\bar{e}_R}]$$

- Similar for the quark doublet and singlets.
- The bosonic

$$f_B^{0\pm} = f_{B_+} \pm f_{B_-},\ f_{BW}^{1\pm} = f_{BW_+} \pm f_{BW_-},$$

$$f_W^{0\pm} = \frac{1}{3}\left[\left(f_{W_+^+} + f_{W_+^-} + f_{W_+^3}\right) \pm \left(f_{W_-^+} + f_{W_-^-} + f_{W_-^3}\right)\right],$$

$$f_W^{1\pm} = \frac{1}{2}\left[\left(f_{W_+^+} - f_{W_+^-}\right) \mp \left(f_{W_-^+} - f_{W_-^-}\right)\right],$$

$$f_W^{2\pm} = \frac{1}{6}\left[\left(f_{W_+^+} + f_{W_+^-} - 2f_{W_+^3}\right) \pm \left(f_{W_-^+} + f_{W_-^-} - 2f_{W_-^3}\right)\right].$$

The EW PDFs in the singlet/non-singlet basis

Construct the singlets and non-singlets

- Singlets

$$f_L^{0,1\pm} = \sum_i^{N_g} f_\ell^{0,1\pm}, \quad f_E^{0\pm} = \sum_i^{N_g} f_e^{0\pm},$$

- Non-singlets

$$f_{L,NS}^{0,1\pm} = f_{\ell_1}^{0,1\pm} - f_{\ell_2}^{0,1\pm}, \quad f_{E,NS}^{0\pm} = f_{e_1}^{0\pm} - f_{e_2}^{0\pm}$$

- The trivial non-singlets

$$f_{L,23}^{0,1\pm} = f_{E,23}^{0\pm} = 0$$

Reconstruct the PDFs for each flavors

- The leptonic PDFs

$$f_{\ell_1}^{0,1\pm} = \frac{f_L^{0,1\pm} + (N_g - 1) f_{L,NS}^{0,1\pm}}{N_g}, \quad f_{\ell_2}^{0,1\pm} = f_{\ell_3}^{0,1\pm} = \frac{f_L^{0,1\pm} - f_{L,NS}^{0,1\pm}}{N_g},$$

$$f_{e_1}^{0\pm} = \frac{f_E^{0\pm} + (N_g - 1) f_{E,NS}^{0\pm}}{N_g}, \quad f_{e_2}^{0\pm} = f_{e_3}^{0\pm} = \frac{f_E^{0\pm} - f_{E,NS}^{0\pm}}{N_g}.$$

- The quark components can be constructed as singlets/non-singlets, and reconstructed correspondingly as well.

The DGLAP in the singlet and non-singlet basis

$$\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} f_L^{0\pm} \\ f_Q^{0\pm} \\ f_E^{0\pm} \\ f_U^{0\pm} \\ f_D^{0\pm} \\ f_B^{0\pm} \\ f_W^{0\pm} \\ f_g^{0\pm} \end{pmatrix} = \begin{pmatrix} P_{LL}^{0\pm} & 0 & 0 & 0 & 0 & P_{LB}^{0\pm} & P_{LW}^{0\pm} & 0 \\ 0 & P_{QQ}^{0\pm} & 0 & 0 & 0 & P_{QB}^{0\pm} & P_{QW}^{0\pm} & P_{Qg}^{0\pm} \\ 0 & 0 & P_{EE}^{0\pm} & 0 & 0 & P_{EB}^{0\pm} & 0 & 0 \\ 0 & 0 & 0 & P_{UU}^{0\pm} & 0 & P_{UB}^{0\pm} & 0 & P_{Ug}^{0\pm} \\ 0 & 0 & 0 & 0 & P_{DD}^{0\pm} & P_{DB}^{0\pm} & 0 & P_{Dg}^{0\pm} \\ P_{BL}^{0\pm} & P_{BQ}^{0\pm} & P_{BE}^{0\pm} & P_{BU}^{0\pm} & P_{BD}^{0\pm} & P_{BB}^{0\pm} & 0 & 0 \\ P_{WL}^{0\pm} & P_{WQ}^{0\pm} & 0 & 0 & 0 & 0 & P_{WW}^{0\pm} & 0 \\ 0 & P_{gQ}^{0\pm} & 0 & P_{gU}^{0\pm} & P_{gD}^{0\pm} & 0 & 0 & P_{gg}^{0\pm} \end{pmatrix} \otimes \begin{pmatrix} f_L^{0\pm} \\ f_Q^{0\pm} \\ f_E^{0\pm} \\ f_U^{0\pm} \\ f_D^{0\pm} \\ f_B^{0\pm} \\ f_W^{0\pm} \\ f_g^{0\pm} \end{pmatrix}$$

$$\frac{\mathrm{d}}{\mathrm{d}L}\begin{pmatrix} f_L^{1\pm} \\ f_Q^{1\pm} \\ f_W^{1\pm} \\ f_{BW}^{1\pm} \end{pmatrix} = \begin{pmatrix} P_{LL}^{1\pm} & 0 & P_{LW}^{1\pm} & P_{LM}^{1\pm} \\ 0 & P_{QQ}^{1\pm} & P_{QW}^{1\pm} & P_{QM}^{1\pm} \\ P_{WL}^{1\pm} & P_{WQ}^{1\pm} & P_{WW}^{1\pm} & 0 \\ P_{ML}^{1\pm} & P_{MQ}^{1\pm} & 0 & P_{MM}^{1\pm} \end{pmatrix} \otimes \begin{pmatrix} f_L^{1\pm} \\ f_Q^{1\pm} \\ f_W^{1\pm} \\ f_{BW}^{1\pm} \end{pmatrix}$$

$$\frac{\mathrm{d}}{\mathrm{d}L} f_W^{2\pm} = P_{WW}^{2\pm} \otimes f_{WW}^{2\pm}$$

The splitting functions can be constructed in terms of Refs. [Chen et al. 1611.00788, Bauer et al. 1703.08562,1808.08831]